NETHERLANDS GEODETIC COMMISSION

PUBLICATIONS ON GEODESY NEW SERIES

VOLUME 4 NUMBER 4

MEASUREMENT OF THE BASE AND BASE EXTENSION NET "AFSLUITDIJK"

by

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1972

RIJKSCOMMISSIE VOOR GEODESIE, KANAALWEG 4, DELFT, NETHERLANDS

PRINTED BY W. D. MEINEMA B.V., DELFT, NETHERLANDS

ISBN 90 6132 217 0

PREFACE

The task of establishing a new base line in the northwest of The Netherlands was entrusted to the subcommission Triangulation of the Netherlands Geodetic Commission. The measurements themselves were performed under supervision of members of the staff of the Netherlands Triangulation Service (Mr. HAARSMA) and of the Sub-Department of Geodesy of the Delft University of Technology (Messrs. BAKKER, KRIJGER and DE MUNCK). The first stage of their work consisted of a detailed reconnaissance, test measurements, and an examination of the precision attainable in the measurement of the base and its extension to a side of the primary network. The results of these studies were laid down in a report submitted to the subcommission Triangulation. This preliminary report, parts of which are included in the present report, was discussed and approved in full by the Netherlands Geodetic Commission at a meeting held in December, 1964.

The actual measurements were carried out under the responsibility of the authors of this publication. Mr. HAARSMA was in charge of the preparations and angle measurements, Mr. BAKKER of the invar wire measurements, Mr. DE MUNCK of the geodimeter measurements and Mr. KRIJGER of the theoretical studies. In accordance with their duties they made the following contributions to the present report: Mr. HAARSMA: Introduction, sections 1.1, 2.1.1, 2.2, Mr. BAKKER: section 1.2, Mr. DE MUNCK: section 1.3 and Mr. KRIJGER: sections 2.1.2, 2.1.3, 2.1.4, 2.3 and chapter 3.

The Netherlands Geodetic Commission wish to express their sincere thanks to all who contributed to the realization of this project, in particular to the four authors of this publication and to the Deutsches Geodätisches Forschungsinstitut at Munich, Abteilung I, for their assistance with the invar wire measurements.

Delft, April 1972

The President of the subcommission Triangulation

W. BAARDA

CONTENTS

INTRODUCTION

At the Third Symposium on the New Adjustment of the European Triangulation (Munich, 1962) measuring of a new base line in the northwestern part of The Netherlands was recommended to replace the old base near Stroe that was lost as a result of road reconstruction (see Bulletin Géodésique, No. 67, March 1963, p. 69, Resolution No. 3).

After the decision had been taken to carry into effect the above recommendation, the authors of the present report were requested to make a study of the theoretical and practical aspects of this project and to organize and supervise the actual measuring of this new base.

The selection of the site was practically limited to the dam between the provinces Noord-Holland and Friesland, known as the "Afsluitdijk". As endpoints of the base were chosen the towers of the sluices at both ends of the dam, called "Stevinsluizen" and "Lorentz-sluizen" (see Fig. 1). While the base at Stroe (measured in 1913) was situated in the centre of the country, the primary triangulation of The Netherlands is now enclosed by the new base Afsluitdijk, the German bases Meppen (remeasured in 1960) and Bonn (measured in 1892), and the Belgian base Zeebrugge (measured in 1938).

Since both ends of the new base are not incorporated into the primary network, measuring of a base extension network was necessary to extend the base to the length of a normal side of the primary triangulation. The choice of the stations for this extension network was limited to the primary stations Workum, Eierland and Sexbierum. By including the lower order stations Burgwerd and Staveren a network was obtained that could serve two objects at the same time (which had been the intention from the very beginning). These objects were:

a. Checking the scale of the primary triangulation network of The Netherlands and contributing in this way to the new adjustment of the European triangulation.
b. Obtaining accurately determined distances for the calibration of electro-magnetic distance measurement instruments.

These objects can be combined very well since both require measurements of very high accuracy. The extension network thus obtained contains distances passing almost entirely over land or over water and others passing partly over water and partly over land (see Fig. 1). In view of (b) this offers the opportunity of gaining an insight into the behaviour of the various instruments under different circumstances.

The base Afsluitdijk differs from other bases in that the ratio between its length (24 km) and the sides of the extension network is about 2:3. For this reason there was no need to measure the various directions of the extension network a different number of times (which normally is required when extending a comparatively short base to a side of a primary triangulation).

The base itself was measured using invar wires. In addition, geodimeter measurements were carried out from a pillar, erected in the middle of the base and at equal distance of both endpoints. The main purpose of the latter measurements was to obtain information about the accuracy when a certain distance (in this case half the base length) is doubled using a geodimeter whereby the invar wire measurements served as a check.

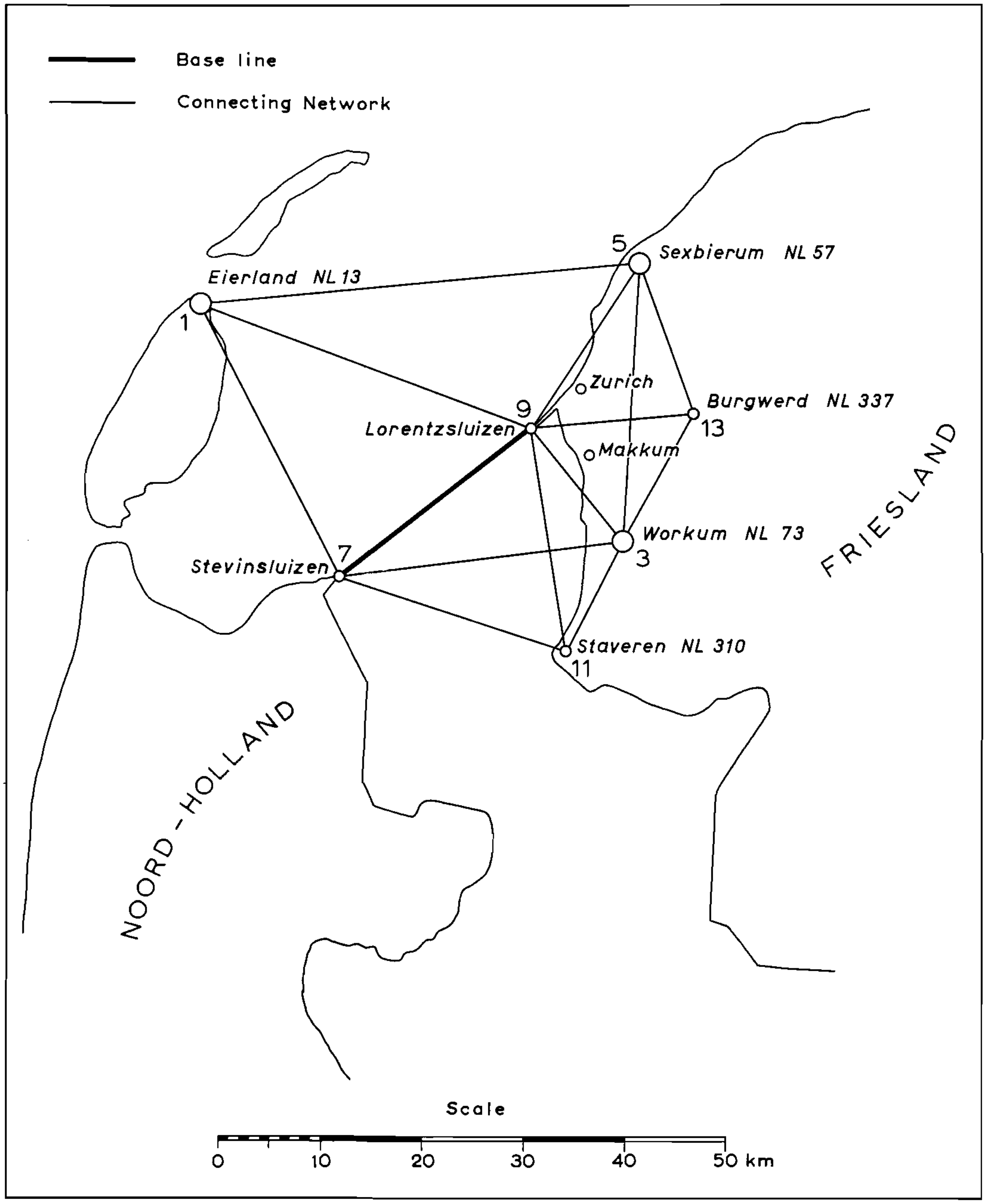


Fig. 1

All measurements mentioned above were carried out in the years 1964–1967 according to the following schedule:

1964	preparations
	test measurements for the base extension network
1964–1966	geodimeter measurements
1965	invar wire measurements
	levelling of the base
	first measurement of the base extension network
1966–1967	remeasurement of the base extension network (considered necessary when the first measurements showed unacceptable misclosures).

The present report consists of three chapters. Chapter 1 gives a description of the base line measurement, subdivided in the sections: preparations (1.1), invar wire measurements (1.2) and geodimeter measurements (1.3). The base extension network is described in detail in chapter 2, in the sections: preparations (2.1), angle measurements (2.2) and analysis of the results (2.3). Finally in chapter 3 some theoretical aspects are considered.

Chapter 1

BASE LINE MEASUREMENT

1.1 Preparations

The base was set out on the flat, 15-m wide grass verge between the road and the top of the dam. This resulted in two slight bends, about 2 km from the Lorentzsluizen (see Fig. 1.1.1). The distance between the two deflection points is about 400 m. The base was divided into 21 sections, numbered I–XXI. Most of the sections had a length of about 1200 m. The end-points of the sections and the two deflection points were marked by two concentric iron pipes, having diameters of 10 cm and 15 cm respectively. The inner pipe was driven into the ground to a depth of 1 m and the outer one to a depth of 50 cm. The top of both pipes was 70 cm above the surface. The inner pipe was filled with concrete and provided with a small cylindrical brass tube for centring theodolites or fixing signals or Jäderin marks (Fig. 1.1.2). The purpose of the outer pipe was to protect the inner one against displacement or damage; when not in use it was closed by a cover.

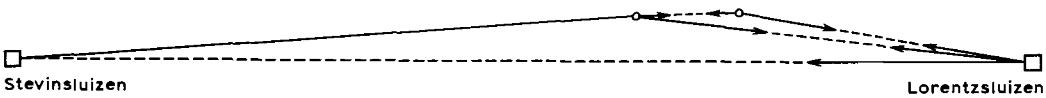


Fig. 1.1.1

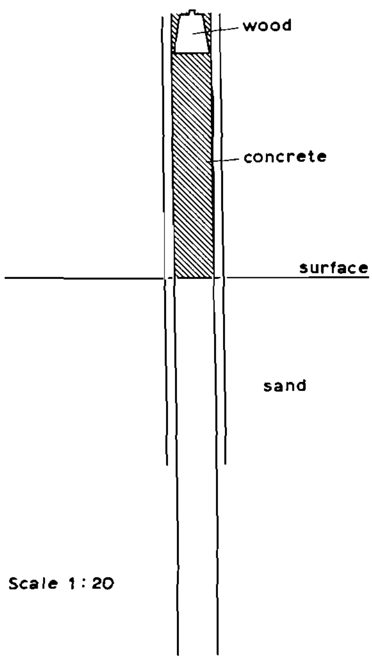


Fig. 1.1.2

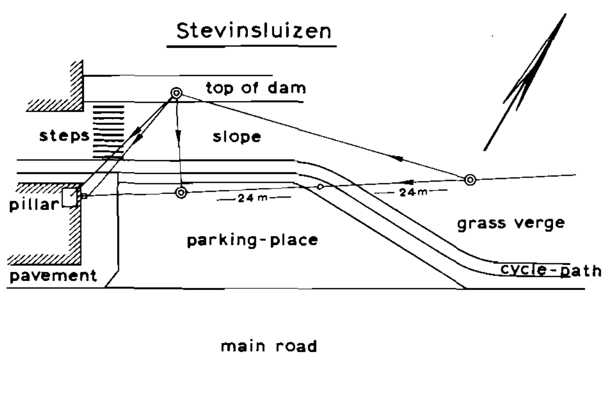


Fig. 1.1.3

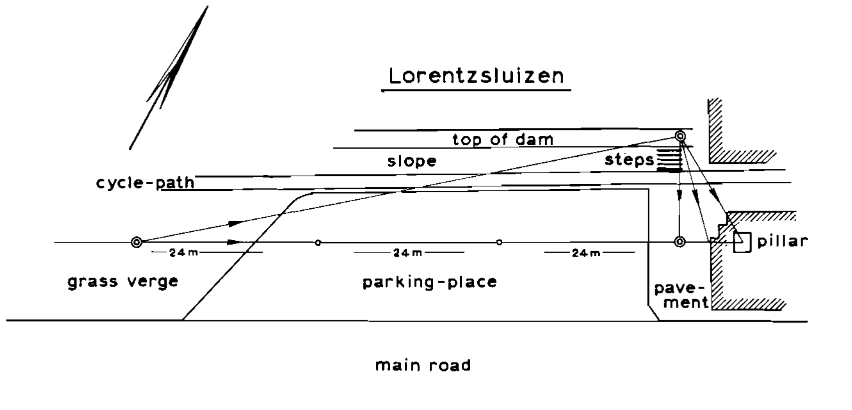


Fig. 1.1.4

The alignment of the section endpoints was done with great care. For the longest distance, Stevinsluizen – first deflection point (Fig. 1.1.1) the following method was applied. Searchlights were placed on both points and a theodolite was set up in the middle, using the km-stones along the road. Measuring the angle between the searchlights and knowing the approximate distances, the exact position of the theodeolite station on the straight line connecting the two points could be determined. All other section endpoints were aligned by theodolite.

Each section was divided into 24-m bays. The endpoints of the bays were marked by 1.5 m wooden stakes, driven into the ground to a depth of 80 cm. Alignment was carried out using a Wild T3 theodolite and a signal at the far end of the section. From one station 12 bays could be aligned at a time, whereafter the theodolite was moved forward. The exact in-line position on the top of the stakes was marked by a nail which was replaced by a Jäderin mark one day before the invar wire measurements took place. In this way a non-alignment correction was avoided.

At the endpoints of the first and last section it was not possible to erect surveying towers of sufficient stability for the base extension network and therefore a small brick pillar was built on the concrete roof of both sluice towers. A brass bolt with a small hole was inserted in the top of each pillar and these two points serve as terminals of the new base line. The connection between the terminal points and the endpoints of the invar wire measurements was carried out by indirect observations (Fig. 1.1.3 and 1.1.4). Two permanent marks were placed at eye-level in the foot of each tower, in line with the base.

1.2 Invar wire measurements

The invar wire measurements, including the standardizations, were carried out in the period August 30–September 28, 1965. Apart from a few days during the first week the measurements were favoured by extremely fine autumn weather. It was calm and slightly hazy and the prevailing wind was southwest, i.e. coinciding with the direction of the base. During the measurements on the Afsluitdijk only two days were lost on account of unfavourable weather.

The following four parties assisted with the invar wire measurements:

- a party from the "Deutsches Geodätisches Forschungsinstitut, Abteilung I, Munich (complemented with three students of geodesy of the Delft University of Technology) with invar wire No. 510 (spare No. 511).
- a party from the Netherlands Triangulation Service with invar wire No. 90 (spare No. 89).
- a party consisting of students of geodesy of the Delft University of Technology with invar wire No. 91 (spare No. 585).
- a party from the Cadastral Survey with invar wire No. 586 (spare No. 587).

The wires were standardized on the standard base line Loenermark. The first days were spent to make aquaintance, to study the theory and to gain the necessary practical experience. In the remaining two days of the first week three standardization programmes were completed with the wires that were to be used and two programmes with the wires kept in reserve. Because of rain a fourth programme had to be cancelled but in the second standardization period (September 27 and 28) the planned four programmes could be realized. The invar wire measurements on the base Afsluitdijk were performed in the period September 6–September 23, 1965. The obtained least square estimations of the total length of the base are summarized on page 24. The first part (1.2.1) of this section deals with the standardization of the wires on the Loenermark. It is subdivided in: the length transfer measurements from the interference base to the invar wire base (1.2.1.1), the determination of the length of the invar wire base (1.2.1.2) and the calibration of the invar wires (1.2.1.3). The second part (1.2.2) consists of a summary of the invar wire measurement on the base Afsluitdijk (1.2.2.1) and some reflections on the precision obtained (1.2.2.2).

It should be emphasized that, unless otherwise stated, values of calibrated lengths of wires and of lengths of sections in this report are all based on the scale of the Väisälä quartz meter system as given in [1]. However, afterwards it became known, that these values are affected by a systematic scale error of $+1.03.10^{-6}$ [5, p. 8, 23].

1.2.1 *Standardization of the wires on the base Loenermark*

1.2.1.1 Length transfer measurements from interference base to invar wire base

The interference base of the Loenermark standard base is not suitable for the standardization of invar wires. The pillars built for the interference measurements become obstacles when wires have to be standardized. Therefore a special calibration distance has been constructed parallel to the line of the interference pillars [1, p. 42]. This so-called invar wire base was established in the summer of 1960 and was first used for the standardization of wires needed for the remeasurement of the triangulation base line near Meppen in Germany.

The two endstations and the centre station of the invar wire base at Loenermark were marked by small pillars of reinforced concrete provided with Jäderin marks. The distance between these marks can be derived from the distances between the underground reference marks by means of length transfer measurements. Since the Jäderin marks might be subject to small displacements, each standardization of wires should be accompanied by a verification of the calibration distance. The length transfer measurements are very simple when the Jäderin mark and the underground reference mark are above each other (as is the case in nearly all standard bases). The transfer programme is then reduced to a simple plumbing up of the underground reference mark. In the Loenermark the latter marks have been placed vertically below the interference pillars in order to facilitate the transfer of the length between the mirrors to these underground reference marks. An inherent drawback however is that it increases the programme for transferring the interference length to the invar wire base.

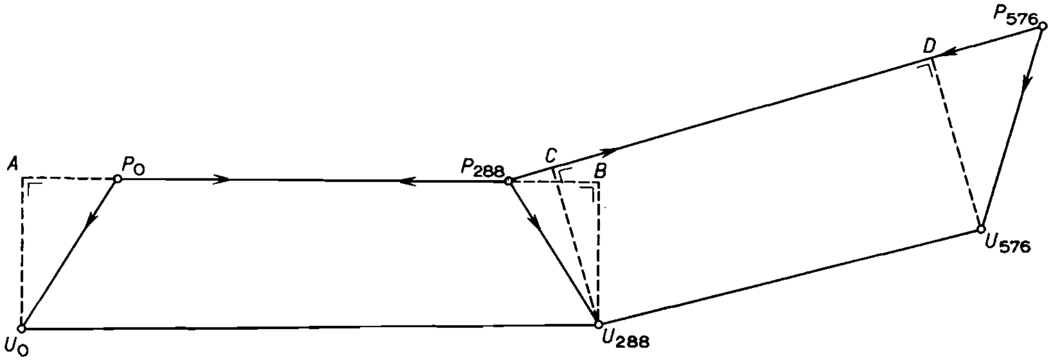


Fig. 1.2.1

The precision to be aimed at is conditioned by the standard deviation of 30 μ – referred to the Väisälä quartz meter system – that was obtained in the measurement of the distance between the underground reference marks. Consequently attempts should be made to maintain this high precision and therefore is should be tried to achieve this same standard deviation in the transfer measurements. As these measurements are needed in the two endstations of the base, the variance of either of them should not exceed 450 μ^2. The situation of the interference - and the invar wire base is sketched in Fig. 1.2.1 in which U_0, U_{288}, U_{576} are the underground reference marks and P_0, P_{288}, P_{576} the Jäderin marks. The following data, obtained in 1960, may be considered as non-stochastic quantities.

distances (m)		angles (gr)	
$U_0 - P_0$	4.4881	$P_0 - P_{288} - P_{576}$	199.9979
$U_{288} - P_{288}$	4.4875	$U_0 - U_{288} - U_{576}$	199.9991
$U_{576} - P_{576}$	4.4921		

Two independent methods of length transfer were applied. Unfortunately part of the equipment got out of order in the first calibration period (prior to the Afsluitdijk base measurement) so that only the results of the second period are available. Experience gained since 1960 had shown that displacements of the Jäderin marks are not very likely within a period of one month. The calibration results, as described in section 1.2.1.3, confirm this supposition. A description of both methods is given below.

First method

A Wild T3 theodolite is approximately centred above the Jäderin mark and the directions indicated in Fig. 1.2.1 are measured. A Wild T2 is set up in a direction perpendicular to the direction of the base line in order to determine the eccentricity of the Wild T3. The

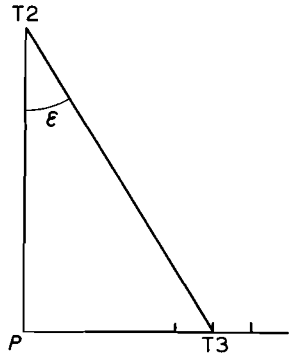


Fig. 1.2.2

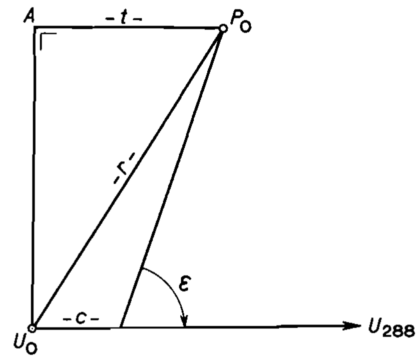


Fig. 1.2.3

distance between both theodolites is taken equal to the distance between the Jäderin mark and the underground mark (Fig. 1.2.2). With the Wild T2 the angle between the reading index on the Jäderin mark and the sighting index on the telescope of the Wild T3 is measured in two diametrical positions of the alidade of the Wild T3. The mean angle ε is the correction which should be applied to the angle measured with the Wild T3.

The underground mark is made visible using a centring bar provided with two spirit levels with a sensitivity of 6 centesimal seconds per scale unit. A plastic conical adapter is attached to its lower end to prevent damage of the underground marks. The top of the bar is fitted with a sharp black needle whereas a piece of blackboard serves as a background. The observational programme of the Wild T3 consisted of 16 sets and halfway the programme both telescope and centring bar were reversed to eliminate possible eccentricity effects.

The results are summarized in table 1.2.1 from which can be derived that the variance

Table 1.2.1

station	date 1965	angle	face left face right	centring bar		ε	corrected angle	mean
				level correction	eccentricity			
P_0	Sept. 28	$P_{288}-P_0-U_0$	100.05282 100.05647	+24 +49	390	−192	100.05309	100.05296
	Sept. 29		100.05286 100.05783	+21 −72	404	−225	100.05284	
P_{576}	Sept. 29	$P_{288}-P_{576}-U_{576}$	300.03909 300.00850	+11 +28	3042*	+ 47	300.02442	300.02445
	Sept. 29		300.04066 300.01014	+ 3 − 3	3058	− 92	300.02448	
P_{288}	Sept. 30	$P_0-P_{288}-U_{288}$	299.98308 299.95002	−63 +180	3063	−333	299.96380	299.96380
		$P_0-P_{288}-P_{576}$	199.99808 199.99782				199.99795	199.99795

* Change of plastic adapter.

in the transfer quantity t obtained with one observational programme amounts to about 250 μ^2 or $\sigma_t = 16$ μ.

Second method

The second method is based on the most simple observational programme that can be thought of, namely the direct measurement of the direction to the Jäderin mark with a theodolite position on the pillar head itself. The determination of the transfer quantity t is then composed of three elementary operations:

a. the location of a point on the pillar with respect to the underground reference mark i.e. the determination of the distance c (see Fig. 1.2.3);
b. the measurement of angle ε (Fig. 1.2.3) at this point subtended by the nearby Jäderin mark and a target set up over the underground reference mark at the other end of the base;
c. the measurement of distance r towards the Jäderin mark (this quantity is also needed when the first method is used).

At first sight the plan seemed not feasible as the demands of precision for operation (a) could not be met with by the plummets available on the market. Consequently a special plummet was developed in the workshop of the Sub-Department of Geodesy of the Delft University of Technology which appears to work very satisfactorily. A brief description of this instrument is given below.

The new plumbing unit (see Fig. 1.2.4) is constructed in such a way that it can be used in combination with the Watts microptic No. 2. This theodolite is detachable from an

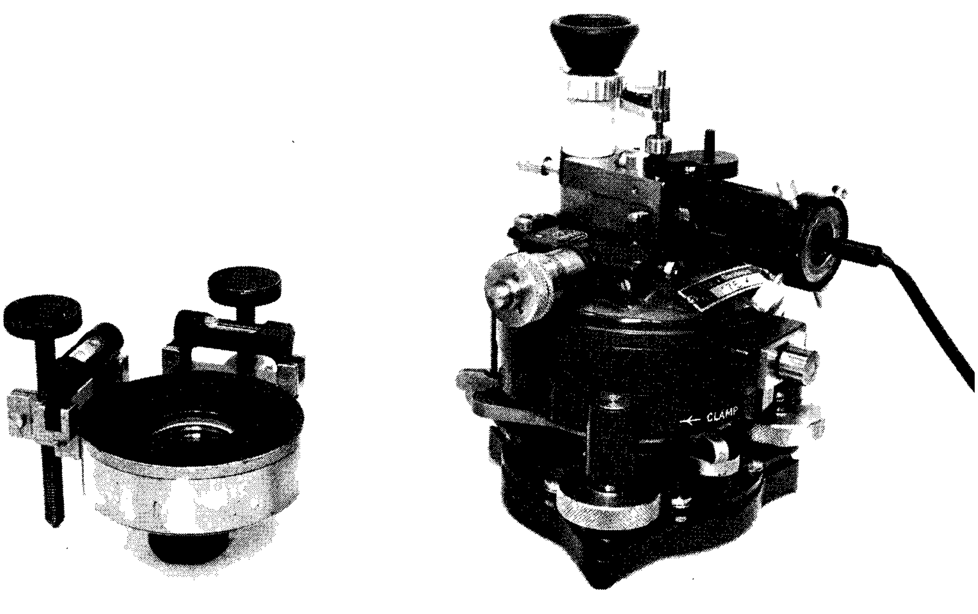


Fig. 1.2.4

independent levelling base so that it can be exchanged with targets, plummets and tape index heads. At the Loenermark the levelling base was roughly centred over the underground reference mark on the pillar head and then cemented to the concrete. The Watts 3-tripod traverse equipment comprises also a special micrometer tape index head. The index mark is mounted in such a way that it can slide relative to the base fitting by means of a micrometer. The zero reading refers to the centre of the levelling base and one revolution of the drum corresponds to a displacement of 1 mm. The reading unit is 0.1 mm and 0.01 mm can easily be estimated. This micrometer head now has been transformed into a special plumbing device with which the distance c between the centre of the levelling base and the underground reference mark in the direction of the interference base is measured.

This new centring method is based on the following principle. Light coming from a point source of light and passing a diffraction screen of concentric rings will produce a diffraction picture in any plane that is parallel to the screen. The interference pattern consists of concentric circular fringes which centre is always situated on the straight line joining the light source and the centre of the diffraction screen. Based on these principles a highly accurate alignment method has been developed by the late Prof. Dr. A. C. S. VAN HEEL (Department of Applied Physics, Delft University of Technology). This method has been modified to a centring method by the introduction of a level surface realised by a mercury bath. The bottom of the mercury container is provided with a plastic base which fits in the hole (diameter 2 mm) made in the underground reference mark. The container is placed on this reference mark; with a pair of cross levels and footscrews it can be roughly levelled. The mercury bath is hermetically sealed and covered by a zone plate and a lens. The zone plate is an optical glass partly covered by a magnesium coating of concentric rings and thus acting as a diffraction screen. The light source and the observing unit are both mounted on the micrometer head. The point source of light is realised by a diaphragm (diameter) 0.5 mm) illuminated by a flashlight bulb (4.5 V; 0.2 Amp.). The light is reflected by an aluminated prism, passes the zone plate, reflects from the mercury surface, passes the zone plate in the inverse direction and produces a diffraction pattern on a reticule that is placed in the focal plane of a magnifying glass. A diaphragm is placed between this glass and the observers eye to intercept all the light not coming from the zone plate. If it were possible to make the point source of light coincident with the centre of the reticule (intersection of crosswires) this common point is centred over the reference mark when it is observed in the centre of the fringes. The micrometer reading should then refer to the distance c. In reality the centre of the reticule does not coincide with the light source and to eliminate this effect the base fitting is reversed in the base. The mean of both micrometer readings will now correspond to the desired distance c. The reversal of the base fitting also eliminates a possible eccentricity of the micrometer zero with respect to the centre of the base. For that same reason the observational programme includes measurements in two opposite positions of the mercury bath. The eccentricity of the zone plate with respect to the reference mark as well as the effect of a possible deviation from plan parallelism of the plate is then also eliminated. A complete programme of observation took 10 to 15 minutes. It was repeated after the measurement of the angle ε had been carried out. The angle measurements were performed with a Watts microptic No. 2 which was provided with a base fitting specific to the Watts 3-tripod traverse equipment. A sensitive striding level was available so that the directions could be duly corrected for a non-vertical position of the vertical axis. To eliminate possible errors in the graduation on the circle, the angle was measured on the (0 gr) and (100 gr) division line in all series. The allied problem of the determination of the combined error in these two division lines could be solved by carrying out the following experiment. The telescope was aimed at a well observable target. The circle was set with its (0 gr) division line on the reading index and the micrometer was read. The telescope was then moved by 100 gr as well as possible. The alidade was clamped, the reading index was brought exactly on the line in question and the micrometer was read. The circle was then reset, the micrometer reading was repeated on the (0 gr) line of the division and thereafter the telescope was moved again through the next 100 gr. This above mentioned procedure was repeated four times, the last angle including again the direction to the target. The sum of the angles obtained in this manner should equal 400 gr. As the measurements include the (0 gr) and (100 gr) lines only, the misclosure equals four times the combined graduation error in these two lines.

The micrometer, covering a range of 2000 centesimal seconds, has also been calibrated by measuring a constant angle of about 200 centesimal seconds on different parts of the scale. The linear scale error was investigated by measuring an angle nearly equal to the range of the scale. The angle ε was observed in 8 sets. The base fitting was reversed in the levelling base halfway the sets in order to eliminate a possible eccentricity of the vertical axis with respect to the centre of the base.

An analysis of the observations revealed that, using the observational programme outlined above, the variance of the transfer quantity t amounts to $\sigma_t^2 = 346\ \mu^2$, thus $\sigma_t \approx 20\ \mu$. For both methods the results of the transfer measurements are summarized in table 1.2.2 (see also Fig. 1.2.1).

Table 1.2.2

transfer quantity t	first method	second method	transfer correction to	first method	second method	mean
AP_0	3733 μ	3678 μ	$P_0 - P_{288}$	−6285 μ	−6310 μ	−6297 μ
BP_{288}	2552 μ	2632 μ	$P_{288} - P_{576}$	+4143 μ	+4119 μ	+4132 μ
CP_{288}	2407 μ	2487 μ	$P_0 - P_{576}$	−2142 μ	−2191 μ	−2165 μ
DP_{576}	1736 μ	1634 μ				

Although the differences between the transfer corrections of both methods could not be explained by their variances, nevertheless a simple weighted mean is taken. The standard deviation of these means is estimated at $\hat{\sigma}_{tc} = 25\ \mu$. From observations carried out in November 1967 and from the calibration results on both sections of the invar wire base (section 1.2.1.3) it appears that this estimation is on the very safe side.

1.2.1.2 Determination of the length of the invar wire base

In 1964 the wooden stakes [1, p. 42], marking the endpoints of the bays of 24 m, were replaced by small concrete pillars. In the top of the pillars small brass tubes were inserted into which the Jäderin marks can be screwed prior to each calibration. The invar wire base was double levelled using a Koni 007 levelling instrument, set up in the middle of each bay. The results are summarized in table 1.2.3.

Table 1.2.3

pillar	height above datum (m)	height difference (m)	pillar	height above datum (m)	height difference (m)
P_0	39.9530	+0.1063	P_{288}	38.6915	−0.3971
2	40.0593	−0.2101	14	38.2944	−0.0597
3	39.8492	−0.1618	15	38.2347	−0.0741
4	39.6874	−0.1761	16	38.1606	+0.1147
5	39.5113	−0.1269	17	38.2753	−0.1437
6	39.3844	−0.1777	18	38.1316	−0.2113
7	39.2067	−0.0078	19	37.9203	−0.3307
8	39.1989	−0.2780	20	37.5896	−0.2204
9	38.9209	+0.0072	21	37.3692	−0.0401
10	38.9281	−0.1368	22	37.3291	+0.1167
11	38.7913	−0.2427	23	37.4458	+0.0370
12	38.5486	+0.1429	24	37.4828	−0.1025
P_{288}	38.6915		P_{576}	37.3803	

The mean level of the first half, second half and the whole base are respectively:

first half $P_0 \; - P_{288}$: +39.2840 m

second half $P_{288} - P_{576}$: +37.8558 m

whole base $P_0 \; - P_{576}$: +38.5699 m

The above values were obtained by averaging the mean levels of the bays in the first half, second half and the whole base.

The mean level of the underground mark U_0 is 38.0312 m and the distance between the underground marks reduced to this level surface are:

$U_0 \; - U_{288}$: 288,051.63 mm*

$U_{288} - U_{576}$: 288,040.63 mm*

$U_0 \; - U_{576}$: 576,092.26 mm*

After applying the correction for the convergence of the plumb lines and the transfer corrections found in section 1.2.1.1, the following values for the distances between the Jäderin marks are obtained:

$P_0 \; - P_{288}$: 288,045.389 mm at the mean level of the first half ($P_0 - P_{288}$)

$P_{288} - P_{576}$: 288,044.754 mm at the mean level of the second half ($P_{288} - P_{576}$)

$P_0 \; - P_{576}$: 576,090.143 mm at the mean level of the whole base ($P_0 - P_{576}$)

* Referring to the values obtained during the 1957-determination of the length of the Loenermark base, see [1, p. 40].

1.2.1.3 Calibration of the invar wires

The results of the calibration measurements of the wires used for measuring the base Afsluitdijk are summarized in the tables 1.2.4, 1.2.5 and 1.2.6. A single calibration of a wire consists of a direct and reverse measurement at the invar wire base of the base Loenermark.

Table 1.2.4. Observed lengths at 15 °C in 0.01 mm

wire	section	Sept. 2	Sept. 3	Sept. 3	Sept. 27	Sept. 27	Sept. 28	Sept. 28
510	0 –288	4853	4880	4873	4877	4848	4818	4853
	288–576	4978	5019	5026	5001	5010	4982	4921
	0 –576	9831	9899	9899	9878	9858	9800	9774
	direct – reverse	+32	−16	+48	−40	−97	+89	−38
90		3611	3593	3615	3612	3584	3612	3616
		3758	3761	3754	3738	3734	3751	3740
		7369	7354	7369	7350	7318	7363	7356
		+48	−21	−42	−29	+22	−8	−44
585		6795		6761	6881	6905	6780	6807
		6925		6973	6899	6931	6977	7022
		13720		13734	13780	13836	13757	13829
		−5		+71	+46	+93	+102	+60
91		4355	4301	4438				
		4507	4490	4429				
		8862	8791	8867				
		+23	+2	−91				
586		7606	6703	6721	6875	6852	6888	6879
		6832	6859	6803	6988	7036	7003	7027
		13538	13562	13524	13863	13888	13891	13906
		−54	+91	−17	+35	−18	+59	+29

Table 1.2.5. Height corrections in μ

wire	section	k_1	k_2	k_3	k_4	Σk
510	0 –288	− 6993.8	+ 2.7	−10.2	+1.0	− 7000
	288–576	− 8962.4	+ 7.9	−10.8	+1.3	− 8964
	0 –576	−15956.2	+10.6	−21.0	+2.3	−15964
90		− 6993.8	+ 2.6	− 7.7	+1.0	− 6998
		− 8962.4	+ 8.2	− 8.3	+1.3	− 8961
		−15956.2	+10.8	−16.0	+2.3	−15959
585		− 6993.3	+ 2.7	−14.9	+1.0	− 7005
		− 8962.4	+ 8.1	−15.8	+1.3	− 8969
		−15956.2	+10.8	−30.7	+2.3	−15974
91		− 6993.8	+ 2.7	−10.2	+1.0	− 7000
		− 8962.4	+ 7.9	−10.8	+1.3	− 8964
		−15956.2	+10.6	−21.0	+2.3	−15964
586		− 6993.8	+ 2.7	−14.4	+1.0	− 7005
		− 8962.4	+ 8.1	−15.2	+1.3	− 8968
		−15956.2	+10.8	−29.6	+2.3	−15973

Table 1.2.6. Calibrated lengths of the wires in μ (+24 m)

wire	section	Sept. 2	Sept. 3	Sept. 3	mean	Sept. 27	Sept. 27	Sept. 28	Sept. 28	mean
510	0 -288	+ 321	+ 299	+ 305	+ 308	+ 301	+ 326	+ 350	+ 321	+ 324
	288–576	+ 329	+ 294	+ 288	+ 303	+ 309	+ 301	+ 325	+ 375	+ 327
	0 -576	+ 325	+ 296	+ 297	+ 306	+ 305	+ 314	+ 337	+ 348	+ 326
90		+1365	+1371	+1353	+1360	+1356	+1379	+1356	+1352	+1361
		+1345	+1342	+1348	+1345	+1361	+1365	+1350	+1360	+1359
		+1351	+1357	+1350	+1353	+1359	+1372	+1353	+1356	+1360
585		−1296		−1268	−1282	−1368	−1388	−1284	−1306	−1336
		−1293		−1334	−1314	−1272	−1299	−1337	−1375	−1320
		−1295		−1301	−1298	−1320	−1343	−1310	−1341	−1328
91		+ 737	+ 782	+ 668	+ 729					
		+ 721	+ 735	+ 786	+ 747					
		+ 729	+ 758	+ 727	+ 738					
586		−1222	−1220	−1235	−1226	−1363	−1344	−1374	−1366	−1362
		−1217	−1239	−1192	−1216	−1347	−1386	−1359	−1379	−1368
		−1219	−1229	−1214	−1221	−1356	−1365	−1366	−1373	−1365

The means and differences obtained on the whole base as well as the means for both sections, reduced to 15 °C, are given in table 1.2.4. The corrections for reducing the observed lengths to the mean level of the sections are given in table 1.2.5. These corrections were computed with the formulae given in [2]; the generally accepted notation was used. The correction k_4 is the Tarczy Hornoch correction, the application of which is still being questioned. A discussion whether or not to apply the correction in this case, is not very sensible as the Tarczy-Hornoch correction is negligible for the Loenermark base.

After applying the corrections given in table 1.2.5 the observed lengths of the sections are obtained at the mean level of these sections. The difference between the sum of these lengths and the known lengths of the invar wire base, as given in section 1.2.1.2, divided by the number of bays, will give the calibrated lengths of the wires. These lengths, reduced to a temperature of 15 °C and an actual straining weight of 10 kg+36 gram (the latter number being the additional weight of the straining wires and swivel hooks) are given in table 1.2.6.

In [1, p. 13] HEISKANEN states "it is recommended to place, in addition to the underground bolts at the endpoints of the interference base, a similar bolt in the middle of it, in order to be able to check for possible relative changes in the two parts of the base".

A further analysis of the differences between the calibrated lengths of the wires, as obtained on both halfs of the invar wire base reveals that the mean difference of all the wires is +4.8 μ in the first and −2.8 μ in the second calibration period.

As these differences can be fully accounted for by the stochastic nature of the observations, the following conclusions can be drawn:

1. no changes have occurred in both parts of the interference base since the establishment of the base and the calibration of 1965*;

* The remeasurement of 1969 has indicated that the length of the first half of the interference base has increased by 0.4 mm and that of the second half by 0.2 mm (see [5]).

2. the transfer quantities (see section 1.2.1.1), which could only be obtained in the second calibration period, are also representative for the first period;
3. if the length of the base Afsluitdijk had been computed with the calibrated lengths as found on the two halfs of the invar wire base, both values would differ only 1 mm.

The mean of the calibration values obtained on the whole base are summarized in table 1.2.6. From this table it appears that the wires No. 510 and No. 90 have not changed their length during the measurement of the Afsluitdijk base. The mean value over both calibration periods has been used for the computations. Wire No. 91 received a kink on September 10, 1965, when it was unrolled from the drum. As the measurement of section XIX (see table 1.2.7) clearly demonstrates a shortening, the wire No. 91 was replaced by wire No. 585 from that time onwards. The five sections, which had already been measured with wire No. 91, were calculated using the calibration value as found in the first calibration period. The remaining sections measured with wire No. 585 have been calculated with its calibration value found in the second calibration period.

Objections might be raised against the procedure of using a combination of two not fully controlled wires. However the actual effect is small as is demonstrated by the fact that the ultimate length of the Afsluitdijk base would be decreased by 3.0 mm if the measurements of the wires No. 91 and No. 585 were left out of consideration. Finally, it appears from table 1.2.6 that the wire No. 586 has shortened considerably. In order to demonstrate that this shortening occurred linearly with time, the lengths of all the sections of the Afsluitdijk base were provisionally calculated with the calibrated length of wire No. 586 as found in the first calibration period.

The difference with the mean length obtained with the remaining wires is plotted against the respective days (Fig. 1.2.5). Where necessary the differences were reduced to the standard length of the section (1200 m or 50 bays). In the same graph the difference between both calibrated lengths, found on the Loenermark base and reduced to 50 bays, is plotted. It can be easily seen that, by an unknown cause, a lengthening of wire No. 586 occurred between September 3 and 6, after which the wire returned by leaps to its original length. Once having attained this state a regular shortening commences. As a consequence of this behaviour the calibration values of No. 586, which have been used for computing the

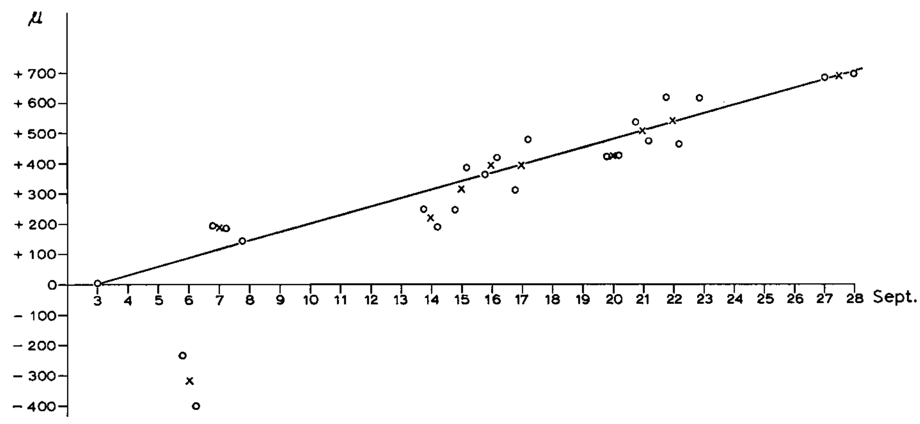


Fig. 1.2.5

measurements of the sections of the Afsluitdijk base, are interpolated linearly between the values in both calibration periods. The measurements of the sections XII and XIII on September 6 with wire No. 586, have not been used. They were replaced by the remeasurements with the wires No. 585 and No. 586 on September 13.

The question whether to use linearly interpolated calibration values or one simple mean for the whole period, is rather academic, as the calculated base length will be the same in both cases.

1.2.2 *Invar wire measurements on the base Afsluitdijk*

1.2.2.1 Summary of the measurements

The invar wire measurements on the base Afsluitdijk were performed between September 6 and September 23, 1965. All sections were measured twice (in direct and reverse direction) by each party. The base, having two deflection points, can be considered as a three-sided polygon. The reduction to a straight line was realized by measuring horizontal angles at the station Lorentzsluizen (see section 2.2.3). A preliminary study (section 2.1.2) had shown that their contribution to the final variance of the base length is of the same order as the length measurement itself provided the standard deviation of these angles does not exceed a value of 3 centesimal seconds. This precision was easily attained. No angle measurements were needed at the other stations of the polygon because the same study had indicated that their contribution to the final precision was very small. The spherical length of the straight base was computed using Legendre's rule. The distance between the endpoints of the invar wire measurements and the terminals of the base on top of the sluice towers was determined by angle measurements from a pillar on top of the dam (see Fig. 1.1.3 and 1.1.4 and section 2.2.4). It may safely be assumed that the combined contribution to the variance of the base length of both the connection measurements and the reduction to a straight line is in any case less than the variance of the length measurement by invar wire.

The levelling of the sections took place in alternation with the invar wire measurements. Normally two sections per day were double levelled, once in each direction. Six Jäderin marks (indicating the endpoints of the bays) were observed from one instrument set-up. An additional control was obtained by including the existing bench marks on the dam. Because of the small differences in height the accuracy required was obtained without any difficulty.

The results of the invar wire measurements, with reference to the 1957 length determination of the base Loenermark, are shown in table 1.2.7. The measurements were first reduced to a temperature of 15°C. This work was done in the field, using tables that were based on the temperature coefficients determined by the Bureau International des Poids et Mesures at Paris. The coefficients of wires Nos. 89, 90, 91 were determined in the beginning of 1965 and those of wires Nos. 585, 586 and 587 in 1960. According to B.I.P.M. the coefficients of the latter wires could still be used in 1965. The temperature coefficient of wire No. 510 was only determined after the completion of the base measurement and therefore a provisional value was used for the first reduction computations. Afterwards the reductions were recomputed using the proper coefficient.

In table 1.2.7 the mean of the two measurements (direct and reverse) and the difference

Table 1.2.7

section	date: 1965, Sept.	number of bays	wire No.	mean length in 0.01 mm at 15°C	difference v in 0.01 mm	calibration correction in in 0.01 mm	correction term k_1 in 0.01 mm	correction term k_2 in 0.01 mm	correction term k_3 in 0.01 mm	mean level of the section in mm	corr. to N.A.P. datum level in 0.01 mm	length at mean level in mm	length at N.A.P. datum level in mm
1	2	3	4	5	6	7	8	9	10	11	12	13	14
I	8	50	510	+ 1991	− 78	+ 1580	−572.1	+0.1	− 0.4	4975	93.3	1200,029.986	
			90	− 3180	+109	+ 6785			+ 0.6			030.336	
			91	+ 6	− 33	+ 3690						031.240	
			586	+ 9880	− 10	− 6260			− 2.1			030.459	
												1200,030.506	1200,029.573
II	14	50	510	17786	− 1	+ 1580	−390.4		− 3.8	4835	90.7	1200,189.718	
			90	12778	− 50	+ 6785			− 2.7			191.699	
			585	26240	− 24	− 6640			− 5.6			192.040	
			586	*25869	+232	− 6415			− 5.5			* 190.581	
	23		586	26349	+160	− 6785			− 5.6			191.680	
												1200,191.284	1200,190.377
III	14	50	510	3330	−167	+ 1580	−339.4		− 0.6	4492	84.2	1200,045.700	
			90	− 1728	+ 43	+ 6785			+ 0.4			047.180	
			585	11672	+ 74	− 6640			− 2.6			046.900	
			586	11306	+ 47	− 6415			− 2.5			045.490	
												1200,046.318	1200,045.476
IV	15	50	510	11629	− 8	+ 1580	−247.0		− 2.5	4768	89.4	1200,129.595	
			90	6522	−103	+ 6785			− 1.4			130.586	
			585	20066	− 21	− 6640			− 4.4			131.746	
			586	19665	+152	− 6465			− 4.4			129.486	
												1200,130.353	1200,129.459
V	7	50	510	14648	− 88	+ 1580	−343.5	+0.1	− 3.1	4810	90.2	1200,158.815	
			90	9627	+ 96	+ 6785			− 2.0			160.666	
			91	12576	− 64	+ 3690			− 2.7			159.199	
			586	22604	− 71	− 6210			− 4.8			160.458	
												1200,159.785	1200,158.883
VI	7	50	510	9296	+120	+ 1580	−308.7		− 2.0	4794	90.0	1200,105.653	
			90	* 4176	+200	+ 6785			− 0.9			* 106.514	
			91	7228	− 80	+ 3690			− 1.6			106.077	
			586	17213	+ 48	− 6210			− 3.7			106.906	
	23		90	4206	+ 63	+ 6785			− 0.9			106.814	
												1200,106.362	1200,105.462
VII	15	50	510	5583	+ 76	+ 1580	−363.3		− 1.2	4811	90.2	1200,067.985	
			90	573	− 6	+ 6785			− 0.1			069.946	
			585	13933	+ 10	− 6640			− 3.0			069.267	
			586	13772	+ 40	− 6465			− 3.0			069.407	
												1200,069.151	1200,068.249
VIII	16	50	510	1643	− 5	+ 1580	−176.9		− 0.3	4817	90.3	1200,030.458	
			90	− 3314	− 77	+ 6785			+ 0.7			032.948	
			585	* 10033	+280	− 6640			− 2.2			* 032.139	
			586	9836	− 63	− 6520			− 2.2			031.369	
	23		585	9880	+116	− 6640			− 2.2			030.609	
												1200,031.346	1200,030.443

Table 1.2.7 *(cont.)*

section	date: 1965, Sept.	number of bays	wire No.	mean length in 0.01 mm at 15°C	difference v in 0.01 mm	calibration correction in in 0.01 mm	correction term k_1 in 0.01 mm	correction term k_2 in 0.01 mm	correction term k_3 in 0.01 mm	mean level of the section in mm	corr. to N.A.P. datum level in 0.01 mm	length at mean level in mm	length at N.A.P. datum level in mm
1	2	3	4	5	6	7	8	9	10	11	12	13	14
IX	16	49	510	3955	− 82	+ 1548	−201.6		− 0.8	4835	88.3	1176,053.006	
			90	− 870	+ 6	+ 6649			+ 0.2			055.776	
			585	12182	+150	− 6507			− 2.7			054.707	
			586	12056	+ 3	− 6390			− 2.7			054.617	
												1176,054.527	1176,053.644
X	17	50	510	5268	+ 51	+ 1580	−290.2		− 1.3	4930	92.4	1200,065.565	
			90	177	− 96	+ 6785						066.718	
			585	13650	+ 61	− 6640			− 3.0			067.168	
			586	13365	+203	− 6575			− 2.9			064.969	
												1200,066.105	1200,065.181
XI	17	50	510	11875	−123	+ 1580	−141.3		− 2.6	4895	91.8	1200,133.111	
			90	6772	+ 12	+ 6785			− 1.5			134.142	
			585	20270	−105	− 6640			− 4.5			134.842	
			586	20140	− 55	− 6575			− 4.4			134.193	
												1200,134.023	1200,133.155
XII	6	51	510	14268	+153	+ 1612	−218.1		− 3.1	5037	96.4	1224,156.588	
			90	* 8998	−205	+ 6921			− 1.8			156.991	
			91	12087	−104	+ 3764			− 2.6			156.303	
			586	* 21872	+ 64	− 6283			− 4.8			153.461	
	13		90	8988	+ 37	+ 6921			− 1.8			156.891	
			586	22309	− 44	− 6492			− 4.9			155.940	
												1224,156.430	1224,155.466
XIII	6	50	510	+ 6797	+ 91	+ 1580	−413.0		− 1.5	5032	94.4	1200,079.625	
			90	1597	−118	+ 6785			− 0.3			** 079.687	
			91	4909	+485	+ 3690			− 1.1			081.849	
			586	* 14167	−134	− 6160			− 3.1			* 075.907	
	13		510	6699	40	+ 1580			− 1.4			** 078.646	
			585	15042	+ 73	− 6640			− 3.3			079.857	
												1200,080.127	1200,079.183
XIV	21	50	510	16704	+ 7	+ 1580	−352.7		− 3.7	4862	91.1	1200,179.276	
			90	11656	− 84	+ 6785			− 2.6			180.857	
			585	24952	−127	− 6640			− 5.5			179.538	
			586	24989	− 37	− 6680			− 5.5			179.508	
												1200,179.795	1200,178.884
XV	21	50	510	14408	− 8	+ 1580	−148.8		− 3.1	4937	92.6	1200,158.361	
			90	9300	+ 11	+ 6785			− 2.0			159.342	
			585	22716	+103	− 6640			− 5.0			159.222	
			586	22637	+131	− 6680			− 5.0			158.032	
												1200,158.739	1200,157.813
XVI	22	50	510	20869	+ 27	+ 1580	−326.5		− 4.6	5013	94.0	1200,221.179	
			90	15772	− 93	+ 6785			− 3.5			222.270	
			585	29202	+117	− 6640			− 5.3			222.302	
			586	29108	− 52	− 6730			− 5.3			220.462	
												1200,221.554	1200,220.64

Table 1.2.7 *(cont.)*

section	date: 1965, Sept.	number of bays	wire No.	mean length in 0.01 mm at 15°C	difference v in 0.01 mm	calibration correction in in 0.01 mm	correction term k_1 in 0.01 mm	correction term k_2 in 0.01 mm	correction term k_3 in 0.01 mm	mean level of the section in mm	corr. to N.A.P. datum level in 0.01 mm	length at mean level in mm	length at N.A.P. datum level in mm
1	2	3	4	5	6	7	8	9	10	11	12	13	14
XVII	22	57	510	22415	+ 19	+ 1801	−415.2		− 4.9	4831	103.3	1368,237.957	
			90	16712	− 32	+ 7735			− 3.7			240.281	
			585	32012	+168	− 7570			− 7.0			240.198	
			586	32040	+109	− 7672			− 7.0			239.458	
												1368,239.474	1368,238.41
XVIII	20	57	510	13236	− 52	+ 1801	−472.8		− 2.9	4769	102.0	1368,145.613	
			90	7453	+ 96	+ 7735			− 1.6			147.136	
			585	22652	+ 22	− 7570			− 5.0			146.042	
			586	22541	+197	− 7553			− 5.0			145.102	
												1368,145.973	1368,144.953
XIX	13	15	510	6145	− 7	+ 474	−304.7		− 1.4	4843	27.3	360,063.120	
			90	4668	+ 36	+ 2036			− 1.0			063.983	
			585	8718	+ 72	− 1992			− 1.9			064.194	
			586	8605	+ 45	− 1920			− 1.9			063.784	
												360,063.773	360,063.500
XX	13	38	510	4395	− 47	+ 1201	−390.8		− 0.9	4847	69.1	912,052.043	
			90	556	− 72	+ 5157			− 0.1			053.221	
			585	10768	+ 50	− 5046			− 2.4			053.288	
			586	10392	− 29	− 4837			− 2.3			051.619	
												912,052.542	912,051.851
XXI	20	38	510	24590	−147	+ 1201	−896.4	+0.2	− 5.3	4981	71.0	912,248.895	
			90	20709	− 26	+ 5157			− 4.6			249.652	
			585	30960	−129	− 5046			− 6.8			250.110	
			586	30822	+ 81	− 5035			− 6.8			248.840	
												912,249.374	912,248.664
sum of sections		1005	510	230782		+31758	−7313.4	+0.4	−50.0	4861	1832.0	24,122,551.764	
			90	128974		+136379			−28.1			2,580.131	$\sigma_l^2 = 44 mm^2$
			91/585	346679		−81607			−76.9			2,576.841	$\sigma_l = 6.7$ mm
			586	394671		−131109			−85.1			2,561.637	
												24,122,567.593	24,122,549.273
XIX	10	15	510	* 6313	− 1	+ 474	−304.7		− 1.4			*360,064.809	
			90	* 4664	− 9	+ 2036			− 1.0			* 063.943	
			91	* 5893	− 56	+ 1107			− 1.3			* 066.940	
			586	* 8597	+ 51	− 1893			− 1.9			* 063.974	
												360,064.242	
XXI	23	35	510	22408	− 9	+ 1106	−346.1		− 4.9	4918	64.6	840,231.630	
			90	18888	+ 32	+ 4750			− 4.1			232.878	
			585	28314	+137	− 4648			− 6.4			233.135	
			586	28311	+ 3	− 4750			− 6.4			232.085	
												840,232.432	840,231.786
XXI	20	3	510	2182	− 48	+ 95	−550.3		− 0.4	5707	6.4	72,017.283	
			90	1864	− 33	+ 407			− 0.4			017.203	
			585	2702	− 10	− 398			− 0.6			017.531	
			586	2686	+ 20	− 398			− 0.6			017.371	
												72,017.347	72,017.28

* rejected ** mean value taken

between them are given in 0.01 mm in columns 5 and 6 respectively. From section 1.2.2.2 it follows that a tolerance for the difference v can be given by

$$|v| < 2\sqrt{2bl}$$

with $b = 0.0300\ \text{mm}^2\text{hm}^{-1}$ and $l = 12$ hm, one finds:

$$|v| < 1.7\ \text{mm}$$

When this value was exceeded, the section was remeasured on another day. The measurements not meeting the tolerance were rejected and are indicated with an asterisk.

In column 7 the calibration corrections are given. These corrections are based on the calibrated length of the wires found at the Loenermark standard base, given in table 1.2.6.

Table 1.2.8. Length in m at N.A.P. datum level

		1957(f)	1957(t)	1969
Stevinsluis	– junction X/XI	11,995.169	11,995.181	11,995.194
junction X/XI	– junction XVIII/XIX	9,961.308	9,961.319	9.961.329
junction XVIII/XIX	– junction XIX/XX	360.063	360.064	360.064
junction XIX/XX	– Lorentzsluis	1,839.207	1,839.209	1,839.211
junction XVIII/XIX	– Lorentzsluis	2,192.083	2,192.085	2,192.087
Stevinsluis	**– Lorentzsluis**	**23,970.066**	**23,970.091**	**23,970.116**
Stevinsluis	– geodimeter pillar	11,994.331	11,994.343	11,994.356
geodimeter pillar	– Lorentzsluis	11,994.323	11,994.335	11,994.348

The first four values in table 1.2.8 are directly derived from the invar wire measurements, as given in table 1.2.7, and using the results of the connection measurements between the endpoints of the invar wire measurements and terminals of the base, being:

Stevinsluizen – section I: 18,292.18 mm
section XXI – Lorentzsluizen: 14,907.02 mm $\}$ see section 2.2.4, p. 75

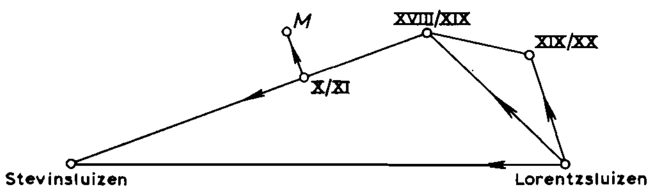


Fig. 1.2.6

The last four lengths in table 1.2.8 were determined by using the following ancillary measurements (see Fig. 1.2.6).

Connection of geodimeterpillar M to base (see section 2.2.5, p. 75):

angle Stevinsluizen – X/XI-M: 97.1711 gr
distance X/XI-M: 19,208.08 mm

Angle measurement at station Lorentzsluizen:

Stevinsluizen: 0.00000 gr
junction XVIII/XIX: 24.69260 gr
junction XIX/XX: 26.96244 gr

The corrections to the mean level of the section are tabulated in the columns 8, 9 and 10. They are computed according to the formulae given in [2]. Because of the small differences in height the k_2-term is less than 1 μ for almost all sections whereas the k_3-term never exceeds 7 μ. The Tarczy Hornoch correction is negligible. The mean height of the section with reference to N.A.P. datum level and the correction to this datum are given in the columns 11 and 12. For the whole base the latter correction amounts to only 18.32 mm. From this it is obvious that an accurate value for the radius of curvature of the level surface is not necessary. This radius has been taken at 6378 km. In column 13 the length of the section is given at its own mean level, whereas in column 14 the length of the section is given at the N.A.P. datum level. The gravity difference between the base Afsluitdijk and the standard base Loenermark is about 85 mgal. This results in a gravity correction of only 0.59 mm for the whole base length. Because of its smallness, this correction was not applied to the lengths of the individual sections as given in the columns 13 and 14. In the ultimate results for the whole base and its two halves this gravity correction is duly taken into account. Finally the lengths of the main distances at N.A.P. datum level are given in table 1.2.8.

The values in the first column (1957f) are based on the results of the determination of the length of the Loenermark base in 1957, using the erroneous scale of the Väisälä quartz meter system, (see [1, p. 40]). Those of the second column (1957t) refer to the same determination using the correct scale (see [5, p. 8, 23]). The values mentioned in the third column (1969) are based on the remeasurement of the Loenermark base in 1969 [5].

1.2.2.2 Reflections on the precision

The precision that was to be expected in the Afsluitdijk base was estimated beforehand by analysing the remeasurement of the German base Meppen and its associating calibration measurements on the standard bases Munich and Loenermark in 1961.

Let two lengths l_i and l_k be observed and referred to the same standard as to which the length of the wire has been related. Let this standard be the invar wire base of one of the standard bases as determined according to the Väisälä-method. To be more concrete let us focus the attention to the Loenermark reference system.

The variance and covariance of the two lengths l_i and l_k evaluated in this reference system can be represented by (see [3]):

$$\sigma_{l_i}^2 = a l_i^2 + b l_i$$

$$\sigma_{l_i l_k} = a l_i l_k$$

When n wires are used and the length is observed m times with each wire, the variance of the mean length $\bar{l}$ is given by:

$$\sigma_{\bar{l}_i}^2 = (a l_i^2 + b l_i m^{-1}) n^{-1}$$

For a difference v between a direct and reverse measurement follows:

$$\sigma_v^2 = 2 b l_i$$

and an estimate of σ_v^2 is obtained from n differences v by:

$$\hat{\sigma}_v^2 = \frac{[vv]}{n} \quad \text{and thus} \quad \hat{b} = \frac{\hat{\sigma}_v^2}{2l_i}$$

From respectively 56 and 68 differences found on the standard bases Munich and Loenermark the following b-values were obtained:

$$\hat{b} = 0.0097 \text{ mm}^2\text{hm}^{-1} \text{ and } \hat{b} = 0.0115 \text{ mm}^2\text{hm}^{-1}$$

From differences obtained on the various sections of the Meppen base the following results were found:

sections	0–51	51–139	139–219	219–293
vv	11.645	15.095	11.627	9.342
n	20	11	10	10
$\hat{\sigma}_v^2$	0.582	1.372	1.163	0.934
$2l$	24.48	42.24	38.40	35.52
$\hat{b}$	0.0237	0.325	0.0303	0.0263

From the above analysis two different b values may be deduced namely:

$b = 0.0150 \text{ mm}^2\text{hm}^{-1}$ for standard bases
$b = 0.0300 \text{ mm}^2\text{hm}^{-1}$ for triangulation bases

The smaller b-value for standard bases is quite understandable as the selection of the site is such that optimum measuring conditions are warranted (flat terrain, shelter from sun and wind). The sections of the Afsluitdijk base were to be measured twice. A tolerance for the differences v can now be given by $|v| < 2\sigma_v < 2\sqrt{bl}$. It should be noted that the above b-values are based on five scale readings for each bay and on the availability of reliable temperature coefficients. They are characteristic for the European way of base measurement, introduced by the Deutsche Geodätische Kommission.

As to the parameter a it must be ascertained that should the measurements of triangulation- and standard base take place under exactly the same conditions and should the length of the wire only change linearly with time, the only contribution to this parameter would come from the variance with which the calibrated length is obtained on a standard base. An increase of the number of calibrations before and after the base measurement would then logically lead to a decrease of the variance of the ultimate base length. From experience however it is known that a strictly lineair change of the length of the wire is not very likely to occur, especially when the wire is continually used for a long period as is the case for the measurement of a triangulation base. A reliable estimation of the precision is then only obtained when more wires are applied and an intercomparison is made between the ultimate base length found with the various wires. The variability of the length of the wire sets a limit to the observational programme on a standard base. From experience it is known that four calibrations before and after the base measurement, each calibration consisting of a direct and reverse measurement, will suffice. The base near Meppen was measured direct and reverse with 10 wires which had been calibrated on the standard bases Munich and Loenermark. The variance of the base length obtained with one wire was:

$$\sigma_l^2 = al^2 + \frac{bl}{2}, \quad \text{and an estimation of } \sigma_l^2 \text{ is given by: } \hat{\sigma}_l^2 = \frac{[vv]}{9}$$

in which v are the differences between the lengths obtained with each wire and their overall mean.

The following estimations, based on respectively the calibrated wire lengths as found on the standard bases Munich and Loenermark, were obtained:

$$\hat{\sigma}_l^2 = 15.65 \text{ mm}^2 \qquad \text{and} \qquad \hat{\sigma}_l^2 = 13.06 \text{ mm}^2$$

A better estimation is the mean of both values, i.e. $\hat{\sigma}_l^2 = 14.34$ mm^2. With $l = 70.3$ hm and $b = 0.0300$ mm^{2}hm^{-1} one finds: $a = 0.0027$ mm^{2}hm^{-2}, or rounded off:

$$\boxed{a = 0.0030 \text{ mm}^2\text{hm}^{-2}}$$

For the base Afsluitdijk 4 wires were to be used. With $l = 240$ hm, $n = 4$, $m = 2$ the estimated standard deviation becomes: $\hat{\sigma}_l = 6.7$ mm. By coincidence exactly the same estimation is found from the actual observations (see table 1.2.7, page 23).

It should be born in mind that the above mentioned value of a is based on:

1. four calibrations on the standard base, immediately before and after the base measurement;
2. a measuring procedure and equipment as is used for the base Afsluitdijk;
3. a non-stochastic length of the invar wire base of the standard base Loenermark.

The variance of the latter base, with respect to the standard represented by the Väisälä quartz meters, should be taken into account if the base Afsluitdijk is determined in this Väisälä quartz meter system.

An estimation of the variance of the length of the invar wire base with respect to the interference base of the standard base Loenermark is:

$$\sigma_{tc}^2 = 625\,\mu^2 \; (\hat{\sigma}_{tc} = 25\mu, \text{ see page 15})$$

An estimation of the variance of the length of the interference base of the Loenermark base with respect to the system of the Väisälä quartz meters is [1, p. 40]:

$$\hat{\sigma}^2_{\text{int}} = 900\,\mu^2$$

thus:

$$\hat{\sigma}^2_{\text{inv}} = 1525\,\mu^2 \qquad \text{or} \qquad \hat{\sigma}_{\text{inv}} = 38\,\mu$$

Consequently the standard deviation of the standard meter, which is incorporated in the invar wire base, with respect to the system of Väisälä quartz meters, can be estimated at:

$$\hat{\sigma} = 0.066\ \mu$$

This latter standard deviation can be taken into account by adding a term of the form $a_{ad}l^2$ in the general variance formula (p. 25), with:

$$a_{ad} = 0.00005\ \mathrm{mm^2hm^{-2}}$$

The above standard deviations are typical for a model, that is based on the same weather- and other conditions on both the standard- and the triangulation base. When these conditions are not identical the actual precision of the base length will be smaller. An analysis of some of the systematic effects, which might have occurred, will now be considered.

Temperature

Each party recorded the temperature using a well calibrated mercury thermometer, not shielded from the sun. Because of radiation effects it was not very likely that mercury bulb and invar wire would have the same temperature. Therefore a special temperature station was established in between both sections that were to be measured each day.

At this station every five minutes the temperature was recorded using a thermometer after HONKASALO [4] (unfortunately out of order after a few days), an electrical resistance thermometer of own make, an Assmann thermometer provided with a fan, and a mercury thermometer as used by the parties. The means of these recordings, taken over the period of the measurement of the section and the means recorded by the parties were compared with each other. The differences between the recordings of the electrical resistance thermometer and the mercury thermometer are apparently not correlating with the intensity of neither wind nor sun. Finally the recordings of each thermometer are averaged over the whole observational period and it appears that the difference between the overall means of the four thermometers of the parties and the electrical resistance thermometer is less than 0.1 °C. This is equivalent to a difference of 1.3 mm in the length of the base. Consequently it is justified to assume that the obtained length of the base is not affected by systematic temperature effects.

Wind

Although force and direction of wind were recorded at the temperature station, their effects on the base length were not taken into consideration and no corrections were computed. This procedure is normal practice. The recording of the wind force had only as result that the length measurements were stopped when the wind force exceeded a velocity of 7 m/sec. There is strong evidence to presume that neither direction nor force of wind has affected the final results. Firstly as a rule the wind force was slightly larger in the early morning (during the first measurement) than later in the morning when the second measurement took place. The differences between both measurements however do not show any preference for sign. Secondly on September 10, when the wind was too strong, section XIX was nevertheless measured, as it concerned a demonstration. The section was remeasured on September 13 in calm weather and the difference between the two measurements amounts to 0.4 mm only (see table 1.2.7).

Friction

The pulley friction, amounting to a few grammes, was checked every day before measurements started. The eccentricity of and the swinging in the bearings was measured daily. They never exceeded 0.015 mm and 0.15 mm, respectively; therefore corrections were not necessary.

Finally a review is given of the various components contributing to the ultimate estimation of the variance of the length of the Afsluitdijk base in the Väisälä quartz meter system.

invar wire measurements	44 mm²	(see table 1.2.7, p. 23)
reduction to a straight line	46 mm²	(see 2.1.2, p. 39)
transfer from Loenermark standard to Väisälä quartz meter standard	3 mm²	(see [1, p. 40])
various minor effects (rough estimate)	7 mm²	(see above)
Total variance	100 mm²	

The estimated standard deviation of the base length L thus amounts to $\sigma_L = 10$ mm.

1.3 Geodimeter measurements

1.3.1 *Description of method*

In order to check the invar wire measurements but more in particular to investigate the possibility of doubling half the base length, the distances from a point M in the middle of the base to the endpoints *Ss* (Stevinsluis) en *Ls* (Lorentzsluis) were compared using a geodimeter (NASM-4D). The geodimeter was placed on the brick pillar at M and the distances *MSs* and *MLs* were measured immediately after each other on one frequency. This procedure was repeated as many times per night as possible during a number of nights. In this manner one can expect that most instrumental errors are eliminated and that the meteorological errors are substantially reduced. Of course the centring of geodimeter and of both reflectors remains very important, as well as the eccentricity constants of the reflectors. Similar ideas are described by SCHÖLDSTROM [6] and by MCLEAN [8].

Although not necessary for this purpose, all distances were reduced in the normal way

Table 1.3.1

station	height above N.A.P. datum
M	+ 8.9 m
Ss	+14.3 m
Ls	+14.5 m

for meteorological effects, slope, height, curvature, frequency deviations, wavelength, additional constant of the geodimeter, etc. The barometric pressures were obtained by adjusting the data issued by K.N.M.I. (Royal Netherlands Meteorological Institute) [7]. The temperatures were obtained in the same way and also from own measurements during the observations. The differences ($MSs-MLs$) were also calculated without applying a temperature correction (i.e. identical temperatures for both sections were taken). Both distances have about the same slope, see table 1.3.1.

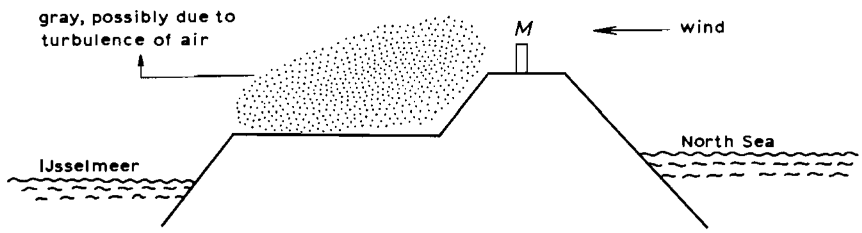


Fig. 1.3.1

However the terrain conditions are quite different: the light path MSs follows the dam lengthwise, for the greater part just behind the top of the dam, while the path MLs, because of the two bends near Lorentzsluizen, crosses the water of "IJsselmeer" (the former Zuyderzee). At most nights it was not possible to measure the distance MSs because of the turbulence of the air behind the top of the dam (see Fig. 1.3.1). In spite of the different terrain conditions the differences between the two geodimeter distances agree very well with the invar wire measurements as will be shown in section 1.3.2.

1.3.2 *Results*

Table 1.3.2 shows the results of all the geodimeter measurements mentioned above. Columns 4, 5 and 6 give the differences ($MSs-MLs$) in millimeters in which the measurements are respectively reduced in accordance with the temperatures supplied by K.N.M.I., own temperature measurements, and without applying a temperature correction. For the first night columns 5 and 6 are identical because the temperature was only measured near the geodimeter. In the columns 7 and 8 only the last three digits of the sums ($MSs+MLs$) in mm are indicated for the various temperatures mentioned above. Comparison of the deviation in the differences with the corresponding deviation in the sums enables one to find the gain in accuracy of the differences over normal geodimeter measurements. In the columns 9–13 the mean values per night are given. In the coloms 4–13 of table 1.3.2 means were taken and the standard deviations were calculated with:

$\sigma_1^2 = \Sigma vv/(N-1)$ for the standard deviation per measurement, in which N is the number of measurements (columns 4–8);

$\sigma_{1m}^2 = \Sigma vv/(N-1)\ (N)$ for the standard deviation of the mean for each of the columns 4–8;

$\sigma_2^2 = \Sigma vv/(n-1)$ for the standard deviation per night, in which n is the number of nights (columns 9–13); and

$\sigma_{2m}^2 = \Sigma vv/(n-1)(n)$ for the standard deviation of the mean of each of the columns 9–13.

In the case the measurements in one night are nearly uncorrelated, the averaging of the columns 4–8 will be most real, as will be the standard deviations σ_1 and σ_{1m}. This appears to be the case with the differences ($MSs-MLs$). If the measurements are highly correlated per night, the columns 9–13 will be more real. This is the case for the sums ($MSs+MLs$).

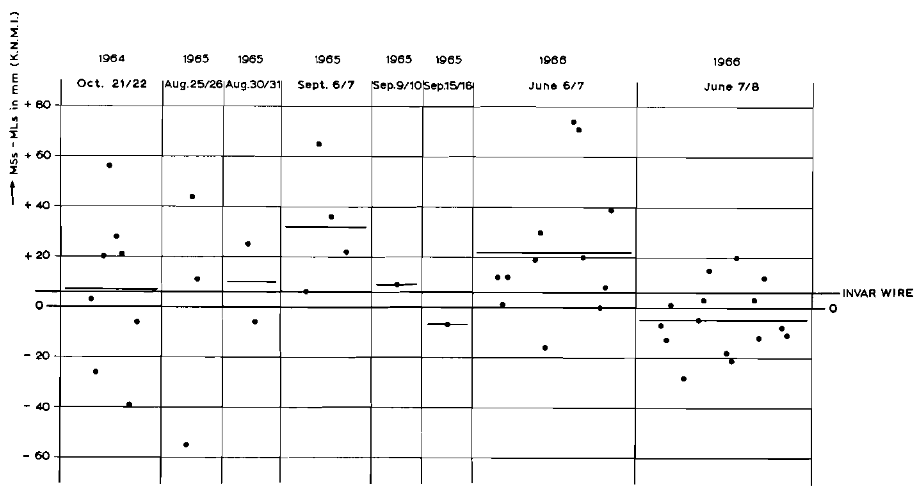


Fig. 1.3.2

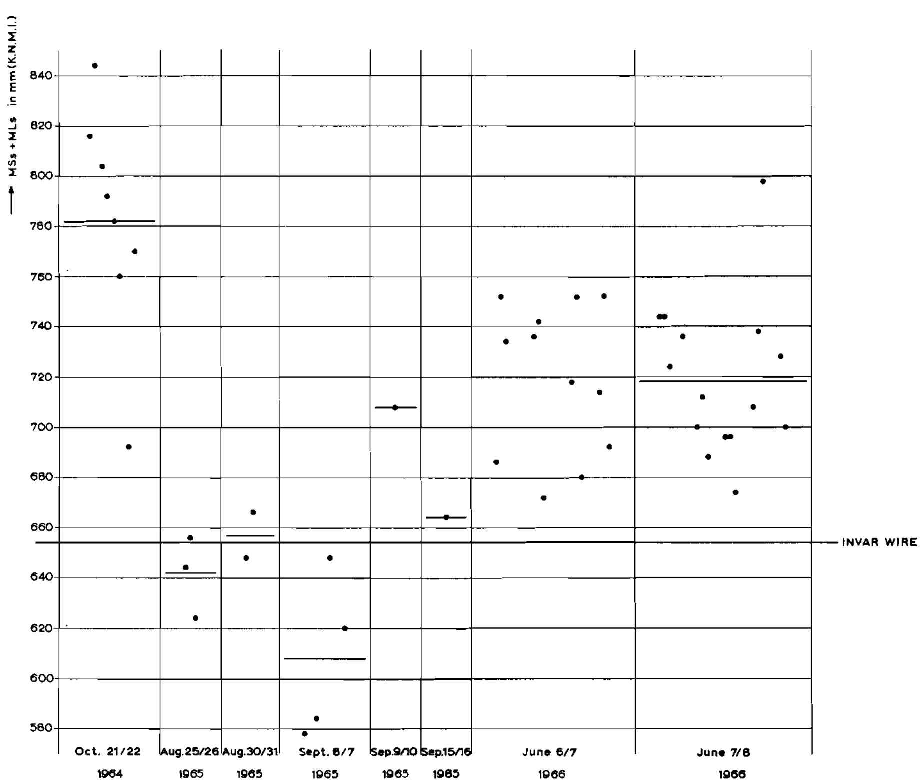


Fig. 1.3.3

Table 1.3.2

date	time M.E.T.	frequency	measurements in mm					mean per night in mm				
			MSs−MLs			*MSs+MLs*		*MSs−MLs*			*MSs+MLs*	
			K.N.M.I. temp.	own temp.	no temp.	K.N.M.I. temp.	own temp.	K.N.M.I. temp.	own temp.	no temp.	K.N.M.I. temp.	own temp.
1	2	3	4	5	6	7	8	9	10	11	12	13
1964 Oct. 22	02.10	1	+ 3	+ 3	+ 3	816	820					
	02.25	1	−26	−26	−26	844	850					
	02.40	1	+20	+18	+18	804	810					
	02.55	1	+56	+55	+55	792	800	+ 7	+ 6	+ 7	782	790
	03.10	1	+28	+27	+27	782	792					
	03.25	1	+21	+20	+20	760	770					
	03.45	2	−39	−41	−41	692	700					
	04.00	3	− 6	− 8	− 8	770	778					
1965 Aug. 25	23.15	1	−55	−70	−61	644	666					
	23.15	2	+43	+28	+37	656	674	0	−15	− 6	642	660
	23.15	3	+11	− 4	+ 5	624	638					
1965 Aug. 31	00.45	1	+25	+21	+20	648	664					
	01.00	2	− 6	−10	−11	666	682	+10	+ 6	+ 5	657	672
1965 Sept. 6	22.15	1	+ 6	−12	+ 5	578	620					
	22.45	2	+65	+46	+65	584	628	+32	+13	+31	608	646
	23.15	3	+36	+19	+34	648	676					
	24.50	1	+22	− 2	+20	620	648					
1965 Sept. 9	24.00	2	+ 9	− 6	+ 5	708	718	+ 9	− 6	+ 5	708	718
1965 Sept. 15	22.50	1	− 7	−19	− 8	664	680	− 7	−19	− 8	664	680

1966	23.00	1	+12	+11	+12	686	762					
June 6	23.00	2	+ 1	0	+ 1	752	796					
	23.00	3	+12	+11	+12	734	780					
1966	00.10	1	+19	+17	+17	736	780					
June 7	00.10	2	+30	+28	+28	742	782					
	00.10	3	−16	−18	−18	672	716	+22	+20	+22	720	764
	01.15	1	+74	+71	+74	718	764					
	01.15	2	+71	+68	+71	752	798					
	01.15	3	+20	+17	+20	680	724					
	01.55	1	0	− 5	0	714	760					
	01.55	2	+ 8	+ 3	8	752	796					
	01.55	3	+39	+34	39	692	736					
1966	21.45	1	− 7	+ 1	− 7	744	764					
June 7	21.45	2	−13	− 5	−13	744	764					
	21.45	3	+ 1	+ 9	+ 1	724	744					
	22.15	1	−28	−20	−28	736	752					
	22.40	1	− 5	+ 2	− 5	700	718					
	22.40	2	+ 3	+10	+ 3	712	730					
	22.40	3	+15	+22	+15	688	706					
	23.20	1	−18	−11	−18	696	716	− 5	+ 3	− 5	718	734
	23.20	2	−21	−14	−21	696	718					
	23.20	3	+20	+27	+20	674	696					
	24.00	1	+ 3	+10	+ 3	708	734					
	24.00	2	−12	− 5	−12	738	764					
	24.00	3	+12	+19	+12	798	724					
	24.40	2	− 8	− 1	− 8	728	756					
	24.40	3	−11	− 4	−11	700	728					
		mean	+ 8.8	+ 6.8	+ 8.0	710	734	+ 8.5	+ 1.0	+ 6.3	687	708
		σ	27	27	27	56	54	13	12	14	53	52
		σ_m	4	4	4	8	8	5	4	5	22	20

A visual picture of the measurements of $(MSs-MLs)$ and $(MSs+MLs)$ is given in the Figs. 1.3.2 and 1.3.3. Only the results calculated from the temperatures supplied by K.N.M.I. are plotted. The results of the invar wire measurements are also indicated in the diagrams,

Table 1.3.3 gives a recapitulation of the above mentioned results. The invar wire values are taken from section 1.2, the geodimeter values for $(MSs-MLs)$ are from table 1.3.2, columns 4, 5, 6, the values for $(MSs+MLs)$ from the columns 12 and 13.

Table 1.3.3

	$MSs-MLs$	$MSs+MLs$
	(table 1.2.8, p. 24; table 1.3.2, col. 4, 5, 6)	(table 1.2.8, p. 24; table 1.3.2, col. 12, 13)
invar wire (1957t)	+8 mm, $\sigma=1$ mm	23,988,678 mm, $\sigma=10$ mm
invar wire (1969)	+8 mm, $\sigma=1$ mm	23,988,704 mm, $\sigma=10$ mm
geod. KNMI temp.	+9 mm, $\sigma=4$ mm	23,988,687 mm, $\sigma=22$ mm
geod. own temp.	+7 mm, $\sigma=4$ mm	23,988,708 mm, $\sigma=20$ mm
geod. no temp.	+8 mm, $\sigma=4$ mm	

1.3.3 *Conclusions*

- The difference between two almost equal distances may be measured electro-optically with a very high accuracy and precision. Particularly the systematic errors appear to be very small compared with normal electro-optical distance measurement.
- Doubling of an invar wire base of 12 km is possible, almost without any loss of accuracy.
- The method might be useful for deformation measurements, for example for barrages.
- There are no differences in errors between the invar wire measurement of both halves of the base of more than a few mm.

Chapter 2

BASE EXTENSION NET

2.1 Preparations

In planning the base extension network attention was paid to the following subjects:

a. The method to be applied for angle measurement.
b. The additional direction measurements required for reducing the distance measured by invar wire in a justified way to a straight line (the base has two bends, see Fig. 1.1.1).
c. The layout of the network. Consideration was given to: the stations to be included, the precision attainable in determining the lengths of the relevant sides, the most suitable side of the triangle Eierland-Sexbierum-Workum of the primary network to which the base should be extended.
d. The possibility to signalize observation errors in the chosen network (Fig. 1) using statistical tests and investigate the effect of such errors on the quantities to be determined.

In order to find an answer to the problems mentioned above, test measurements and theoretical studies were carried out before the actual measurements were started.

2.1.1 *Test measurements*

In the periods June 23–July 30 and August 17–September 11, 1964, horizontal angles were measured at the station Lorentzsluizen on the days when this was possible. The instrument used was a Wild T3 theodolite (No. 58204) and at the other stations of the proposed base extension network (Eierland, Stevinsluizen, Staveren, Workum, Burgwerd and Sexbierum) 50-watt, 6-volt Bosch-Eisemann searchlights with automatic switches were posted. Because of poor visibility, the searchlight at Eierland was later replaced by a more powerful Francis searchlight with a high-pressure mercury-vapour bulb.

As the directions to be measured at Lorentzsluizen pass over land and over water, it was not very likely that they could all be measured in one observation period. Therefore the method with reference mark was chosen instead of the method of Schreiber that is normally used for primary work in The Netherlands. In order to have a check on the observations, two church towers at a distance of about 4 km from the station Lorentzsluizen were selected as referance marks. Reference point No. 1 was the rod on the church tower of Makkum; as check point No. 2 served the church tower of Zurich (Fig. 1).

All the angles were observed in the same position of the horizontal circle of the theodolite to evade the influence of possible errors in the graduation. The observations were made in the afternoon between 4 p.m. and sunset and after August 28 also in the morning between 7 a.m. and 9 a.m. In one observation period eight sets of measures were taken of each angle followed by eight sets of measures of the angle between the two reference marks at the end of the observation period (provided the atmospheric conditions had not deteriorated in the mean time). A set of measures is the mean of the readings face left and face right of the theodolite.

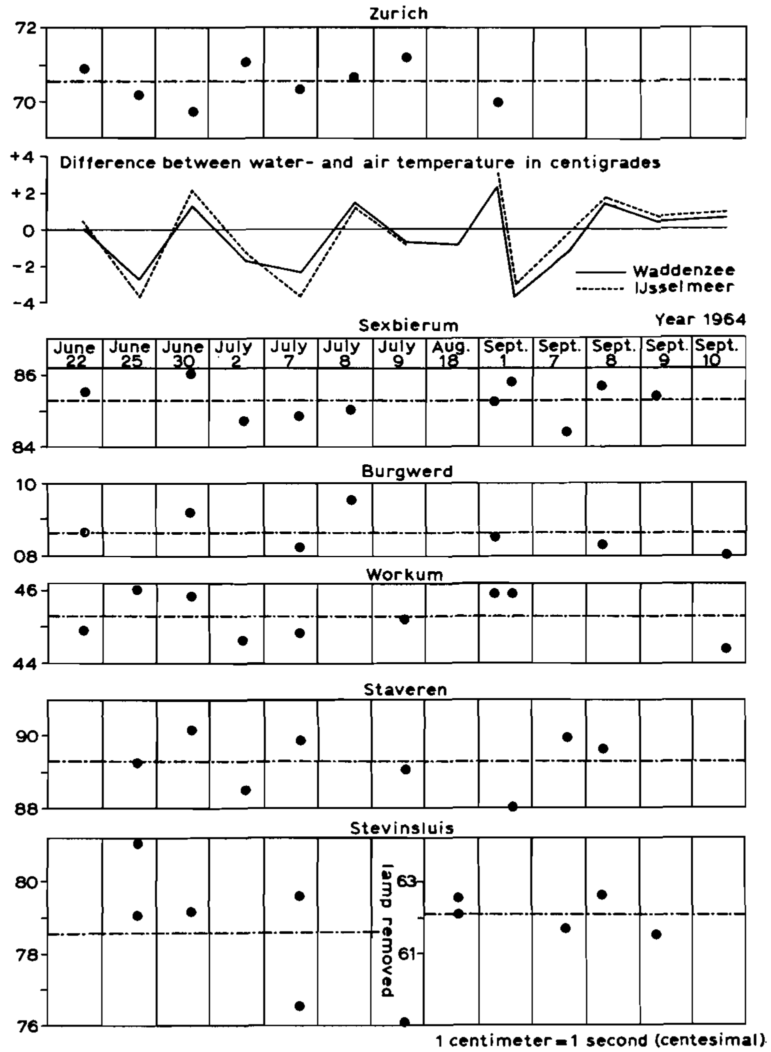


Fig. 2.1.1

The results of the test measurements are shown in Fig. 2.1.1. The black spots are the means of the observed angles per period while the dashed lines indicate the overall means. The scale on the left gives the (centesimal) seconds of the means. Only four sets of measures were possible to the station Burgwerd on June 22, denoted by half a black spot. The station Eierland is not included in this figure because the prevailing atmospheric conditions permitted only one complete set of measures to be taken in the test periods. The searchlight on the station Stevinsluizen was inadvertently removed between July 9 and August 18 and not replaced in its original position, shown by different results in the two test periods. From the test measurements the following conclusions can be drawn:

- Lateral refraction appears to be small; only the observations to the station Stevinsluizen before the removal of the searchlight showed some signs of it (concluded from the *F*-test with a statistical certainty of 95%).

- No reliable observations are possible when the difference between water- and air temperature is more than 4°C. Consequently spring and autumn are the best time of the year for the angle measurements.
- Because of the direction of the sun with respect to the various stations, observations have to be made both in the morning and in the afternoon.
- The heat shimmer during the rest of the day restricts the measurements to just after sunrise and just before sunset.

2.1.2 *Reduction of the distance measured by invar wire to a straight line*

Data

The base Afsluitdijk has two slights bends, about 2 km from the Lorentzsluizen, see Fig. 1.1.1. The situation (not to scale) is given in Fig. 2.1.2. With invar wires are measured

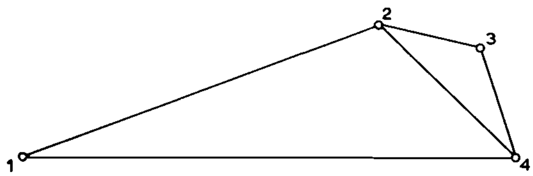


Fig. 2.1.2

the distances $\underline{s}_{12}$, $\underline{s}_{23}$, $\underline{s}_{34}$; to be computed is the distance $\underline{s}_{14}$. The following geometrical data define the problem:

$$\left.\begin{array}{llr} s_{12} \approx 21.9 \text{ km} & \alpha_{214} \approx \alpha_{243} \approx & 2.5 \text{ gr} \\ s_{23} \approx 0.4 \text{ km} & \alpha_{421} \approx & 172.5 \text{ gr} \\ s_{34} \approx 1.9 \text{ km} & \alpha_{324} \approx & 12.5 \text{ gr} \\ s_{14} \approx 24.0 \text{ km} & \alpha_{432} \approx & 185.0 \text{ gr} \\ & \alpha_{142} \approx & 25.0 \text{ gr} \end{array}\right\} \quad \ldots \ldots \ldots \quad (2.1.2.1)$$

Of importance is furthermore the precision with which the invar wire measurements can be performed. Assuming the use of 4 invar wires and a careful execution of the measurements, according section 1.2.2.1 the following matrix of variances and covariances can be taken (directions are denoted by $\underline{r}$):

variance	$\underline{\ln s_{ij}}$	$\underline{\ln s_{kl}}$	$\underline{r}_{ij}$	$\underline{r}_{kl}$
$\underline{\ln s_{ij}}$	$\left(0.075+\frac{0.038}{(s_{ij})_{km}}\right)\cdot 10^{-12}$	$0.075\cdot 10^{-12}$	0	0
$\underline{\ln s_{kl}}$	$0.075\cdot 10^{-12}$	$\left(0.075+\frac{0.038}{(s_{kl})_{km}}\right).10^{-12}$	0	0
$\underline{r}_{ij}$	0	0	σ_r^2	0
$\underline{r}_{kl}$	0	0	0	σ_r^2

(2.1.2.2)

Since the required precision for the directions is to be determined from the problem posed, σ_r is for the time being indefinite.

From (2.1.2.2) follows with (2.1.2.1):

var. $\times 10^{12}$	$\ln \underline{s}_{12}$	$\ln \underline{s}_{23}$	$\ln \underline{s}_{34}$
$\ln \underline{s}_{12}$	0.077	0.075	0.075
$\ln \underline{s}_{23}$	0.075	0.170	0.075
$\ln \underline{s}_{34}$	0.075	0.075	0.095

(2.1.2.3)

The problem

The questions that arise are: (1) What directions are to be measured and (2) The precision to be aimed at in measuring these directions. When answering these questions attention should be paid to the following matters:

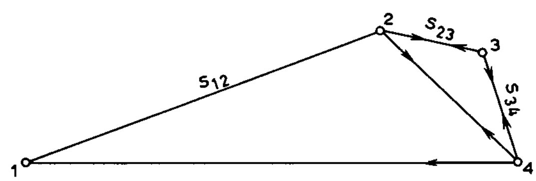


Fig. 2.1.3

Firstly, measuring of $\underline{r}_{12}$ requires in point 2 (Fig. 2.1.2) a signal of about 8m above ground level because of the earth's curvature and the height of the observation station in point 1 (also about 8 m). For the same reasons raising the instrument at point 2 to a height of 8 m would be necessary for measuring the direction $\underline{r}_{21}$. Especially constructing the raised instrument set-up would be rather expensive.

Secondly, the directions $\underline{r}_{12}$ and $\underline{r}_{21}$ pass exactly over the body of the dam which certainly will have an adverse effect on the quality of the results.

It was therefore obvious to search for another solution of the problem, see Fig. 2.1.3. The directions to be measured are indicated by arrows. To express $\underline{s}_{14}$ linearly in $\underline{s}_{12}$, $\underline{s}_{23}$, $\underline{s}_{34}$, $\underline{r}_{23}$, $\underline{r}_{24}$, $\underline{r}_{41}$, $\underline{r}_{42}$ and $\underline{r}_{43}$ the formulas (2.3.3.1)–(2.3.3.3) can be used. Substitution of (2.1.2.1) in this formulas gives:

$$\underline{\Delta \ln s_{24}} = 0.171\,\underline{\Delta \ln s_{23}} + 0.825\,\underline{\Delta \ln s_{34}} - 0.033(\underline{\Delta r_{24}} - \underline{\Delta r_{23}}) - 0.033(\underline{\Delta r_{43}} - \underline{\Delta r_{42}}) \tag{2.1.2.4}$$

$$\underline{\Delta \alpha_{214}} = 0.095(\underline{\Delta r_{42}} - \underline{\Delta r_{41}}) - 0.039\,\underline{\Delta \ln s_{12}} + 0.039\,\underline{\Delta \ln s_{24}} \quad \ldots\ldots \tag{2.1.2.5}$$

$$\underline{\Delta \ln s_{14}} = 0.912\,\underline{\Delta \ln s_{12}} + 0.089\,\underline{\Delta \ln s_{24}} - 0.036\,\underline{\Delta \alpha_{214}} - 0.036(\underline{\Delta r_{42}} - \underline{\Delta r_{41}}) \tag{2.1.2.6}$$

to which can be added:

$$\underline{\Delta\alpha}_{421} = -\underline{\Delta\alpha}_{214} - \underline{\Delta r}_{42} + \underline{\Delta r}_{41} \quad \ldots \quad (2.1.2.7)$$

Substitution of (2.1.2.4) in (2.1.2.5) and (2.1.2.6) and subsequently substitution of (2.1.2.5) in (2.1.2.6) and (2.1.2.7) gives:

$$\boxed{\begin{aligned}\underline{\Delta\alpha}_{214} = &-0.039\,\underline{\Delta\ln s}_{12} + 0.007\,\underline{\Delta\ln s}_{23} + 0.032\,\underline{\Delta\ln s}_{34} - \\ &-0.001(\underline{\Delta r}_{24} - \underline{\Delta r}_{23}) - 0.095\,\underline{\Delta r}_{41} + 0.096\,\underline{\Delta r}_{42} \\ &-0.001\,\underline{\Delta r}_{43}\end{aligned}} \quad \ldots \quad (2.1.2.8)$$

$$\boxed{\begin{aligned}\underline{\Delta\ln s}_{14} = &\;0.913\,\underline{\Delta\ln s}_{12} + 0.015\,\underline{\Delta\ln s}_{23} + 0.072\,\underline{\Delta\ln s}_{34} - \\ &-0.003(\underline{\Delta r}_{24} - \underline{\Delta r}_{23}) + 0.039\,\underline{\Delta r}_{41} - 0.037\,\underline{\Delta r}_{42} \\ &-0.003\,\underline{\Delta r}_{43}\end{aligned}} \quad \ldots \quad (2.1.2.9)$$

$$\boxed{\begin{aligned}\underline{\Delta\alpha}_{421} = &+0.039\,\underline{\Delta\ln s}_{12} - 0.007\,\underline{\Delta\ln s}_{23} - 0.032\,\underline{\Delta\ln s}_{34} + \\ &+0.001(\underline{\Delta r}_{24} - \underline{\Delta r}_{23}) + 1.095\,\underline{\Delta r}_{41} - 1.096\,\underline{\Delta r}_{42} \\ &+0.001\,\underline{\Delta r}_{43}\end{aligned}} \quad \ldots \quad (2.1.2.10)$$

From (2.1.2.8)–(2.1.2.10) follows with (2.1.2.2) and (2.1.2.3):

$$\boxed{\sigma^2_{\alpha_{214}} = 0.29 \times 10^{-16} + 0.018\sigma_r^2} \quad \ldots \quad (2.1.2.11)$$

$$\boxed{\sigma^2_{\alpha_{421}} = 0.29 \times 10^{-16} + 2.400\sigma_r^2} \quad \ldots \quad (2.1.2.12)$$

$$\boxed{\sigma^2_{\ln s_{14}} = 0.077 \times 10^{-12} + 0.0029\sigma_r^2} \quad \ldots \quad (2.1.2.13)$$

If σ^2 is expressed in centesimal seconds, then from (2.1.2.13) it follows:

$$\boxed{(\sigma_{s_{14}})^2_{\mathrm{mm}} = 179 + 4.12(\sigma_r)^2_{\mathrm{cent.sec}}} \quad \ldots \quad (2.1.2.14)$$

Conclusions

1. Omitting the measurement of $\underline{r}_{21}$ is justified. From (2.1.2.14) it follows that the influence of the direction measurement on $\underline{s}_{14}$ is small compared with that of the distance measurement. Furthermore (2.1.2.12) shows that the indirect determination of $\underline{\alpha}_{421}$ with (2.1.2.7) can take place with almost the same precision as the direct determination using $\underline{r}_{24}$ and $\underline{r}_{21}$. Only for check-purposes measuring of $\underline{r}_{21}$ would have sense.
2. For the same reasons the measurements of $\underline{r}_{12}$ can be omitted. The indirect determination of $\underline{\alpha}_{214}$ with (2.1.2.5) can take place with greater precision than the direct determination, as is apparent from (2.1.2.11).

3. If the measurement of $\underline{r}_{21}$ and $\underline{r}_{12}$ is omitted, a check on $\underline{r}_{41}$ can then be obtained by including one or more directions of the base extension network in the series of direction measurement in point 4.
4. Table 2.1.1 and Fig. 2.1.4 give the relation between σ_r and $\sigma_{s_{14}}$, according to (2.1.2.14). Starting from an admissible increase of 40% in $\sigma_{s_{14}}$ in consequence of the direction measurement, which means equal contributions of the variances of length and direction measurements, it follows from (2.1.2.14) that the measurements in the points 2, 3 and 4 should be performed with a precision of:

$$\sigma_r \leqslant 3 \quad \text{centesimal seconds} \tag{2.1.2.15}$$

Table 2.1.1

(σ_r) cent.sec	$(\sigma_{s_{14}})$ mm
1	7
2	8
3	9
4	11
5	12
6	14
8	17
10	21

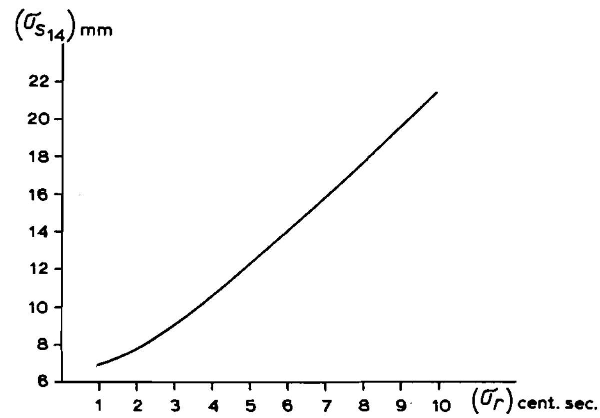


Fig. 2.1.4

5. From (2.1.2.11) it is apparent that the measurement of the directions $\underline{r}_{41}$ and $\underline{r}_{42}$ should be carried out with the greatest care. Measurement of the other directions has very little influence on the final result.

2.1.3 *Layout and precision of the base extension network*

The problem

Fig. 2.1.5 shows the base Afsluitdijk and the base extension network. The points 7 and 9 are the endpoints of the base; the points 1 (Eierland), 3 (Workum) and 5 (Sexbierum) belong to the primary network. Point 13 is needed to obtain some accurately determined distances that can be used for the calibration of electro-magnetic distance measurement instruments. Point 11 serves to check and strengthen the transfer of the length of the base to a side of the primary network. All measurable directions are shown in Fig. 2.1.5. Subject to examination are only the following two questions:

1. To which side of the triangle Eierland-Workum-Sexbierum should the length of the base be transferred and the precision to be aimed at in this transfer.
2. Should point 11 be included in the measurements? The inclusion of point 11 will certainly give a higher precision and reliability of the network chosen. But is this extension justified in view of the costs it involves?

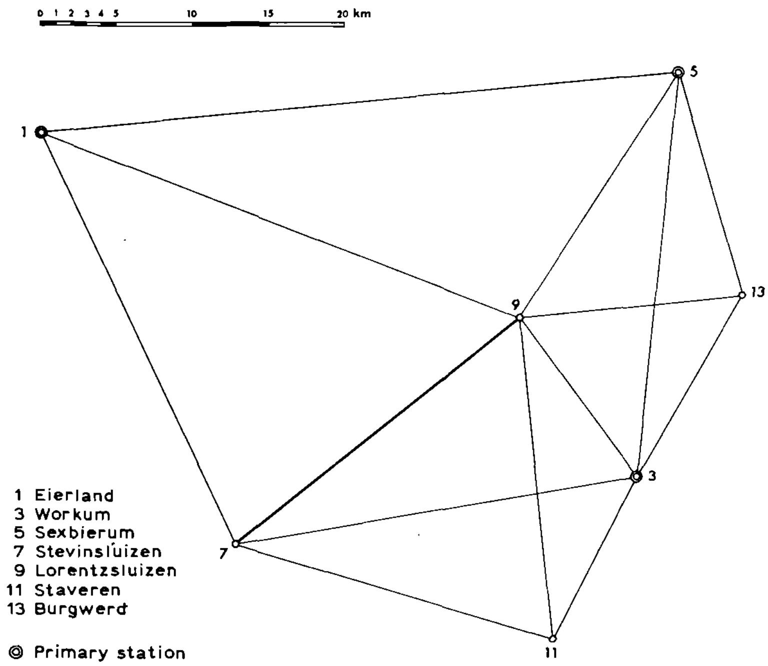


Fig. 2.1.5

The objective of the base extension network is the determination of a number of distances $\underline{s}_{i,j}$. The measurement of the base gives as result the distance $\underline{s}_{7,9}$. The measurement of the extension network results in a number of derived quantities $\underline{v}_{7,9;i,j}$, defined as follows:

$$\underline{v}_{7,9;i,j} = \frac{\underline{s}_{ij}}{\underline{s}_{97}} \qquad (2.1.3.1)$$

Each of the desired distances can be determined using the equation:

$$\underline{s}_{ij} = \underline{s}_{97}\underline{v}_{7,9;i,j} \qquad (2.1.3.2)$$

The analysis of the precision of the base extension network consists then of computing and analysing the variances of a number of distance-ratios to be derived from the network.

The model of the adjustment

In every adjustment a probability model and a condition model is chosen. The probability model is, as far as directions are concerned, based on earlier primary measurements and on the test measurements (see section 2.1.1). For the directions the following equation applies:

$$\sigma_r^2 = 1\,(\text{centesimal second})^2 = \frac{1}{636620^2} = 2.47 \times 10^{-12} \qquad (2.1.3.3)$$

Introducing as factor variance:

$$\sigma^2 = 2.47 \times 10^{-12} \quad (2.1.3.4)$$

then we obtain for the matrix of weight coefficients of the (non-correlative) directions the unit matrix:

$$(g^{ij}) = (\delta^{ij}) \quad (2.1.3.5)$$

(see also [9], chapter 2).

Furthermore it is presumed, that there is no correlation between base- and direction measurements.

For solving the problem of the condition model the method of condition equations is used. In total, we have 28 directions (see Fig. 2.1.5). This means that 11 independent condition equations can be formed, i.e. $(28-3\times 7+4)$. The way in which this is done is given in section 3.3. The coefficients of the condition equations are shown in table 2.3.3 (see folding page at the end). The matrix of the coefficients (u_i^ϱ) is formed by the elements of the rows 1–8, 11, 14 and 17 of the table 2.3.3.

For the investigation of the precision of the distance, derived from the network with (2.1.3.2), it is necessary to express the quantities $v_{7,9;i,j}$ (see (2.1.3.1)) linearly in the observations of the directions of the network. The manner in which this is done is also given in section 3.3. The matrix of the coefficients (Λ_i^R) is given in table 2.1.2. (*Table 2.1.2 is included in folding page at the and of this book*).

Thus all matrices, necessary for the reconnaissance computations can be obtained without observations. The only requirements are the position and the configuration of the network, and the precision of the observations.

Since (g^{ij}), (u_i^ϱ) and (Λ_i^R) are given, the computation of the matrix of weight coefficients of the adjusted observations can be done with the formulas (3.2.3), (3.2.4), (3.2.5) and (3.2.9).

Precision of the adjusted directions and length-ratios

The results of the computations are given in the tables 2.1.3 and 2.1.4. Moreover both these tables are incorporated into Fig. 2.1.6.

Table 2.1.3. Diagonal weight-coefficients adjusted directions

direction	A	B	direction	A	B	direction	A	B
$r_{1,5}$	0.635	0.655	$r_{5,13}$	0.642	0.643	$r_{13,5}$	0.708	0.710
$r_{1,7}$	0.728	0.742	$r_{7,1}$	0.729	0.750	$r_{13,9}$	0.608	0.617
$r_{1,9}$	0.491	0.519	$r_{7,3}$	0.489	0.685	$r_{3,11}$	0.679	–
$r_{3,5}$	0.398	0.417	$r_{7,9}$	0.550	0.588	$r_{7,11}$	0.682	–
$r_{3,7}$	0.505	0.708	$r_{9,1}$	0.683	0.700	$r_{9,11}$	0.584	–
$r_{3,9}$	0.548	0.576	$r_{9,3}$	0.604	0.676	$r_{11,3}$	0.674	–
$r_{3,13}$	0.615	0.624	$r_{9,5}$	0.662	0.669	$r_{11,7}$	0.682	–
$r_{5,1}$	0.675	0.690	$r_{9,7}$	0.639	0.722	$r_{11,9}$	0.513	–
$r_{5,3}$	0.385	0.390	$r_{9,13}$	0.701	0.706			
$r_{5,9}$	0.493	0.510	$r_{13,3}$	0.696	0.703			

A: adjusted with point 11
B: adjusted without point 11

Table 2.1.4. Diagonal weight-coefficients of the ln of the desired length-ratios

derived quantity	A	B	derived quantity	A	B
$\ln v_{7,9,3}$	2.047	3.331	$\ln v_{7,9,1}$	1.098	1.168
$\ln v_{7,9,13}$	2.830	4.048	$\ln v_{9,7;1,5}$	0.989	1.179
$\ln v_{7,9,5}$	3.242	4.163	$\ln v_{9,7,1}$	1.880	2.066
$\ln v_{9,7;3,5}$	2.639	3.545	$\ln v_{9,7;1,3}$	0.900	1.145
$\ln v_{9,7;3,13}$	3.523	4.464	$\ln v_{7,9,11}$	0.920	
$\ln v_{9,7;5,13}$	4.161	5.061	$\ln v_{9,7,11}$	0.633	
$\ln v_{9,7,3}$	0.202	0.274	$\ln v_{9,7;3,11}$	2.856	

A: Adjusted with point 11
B: Adjusted without point 11

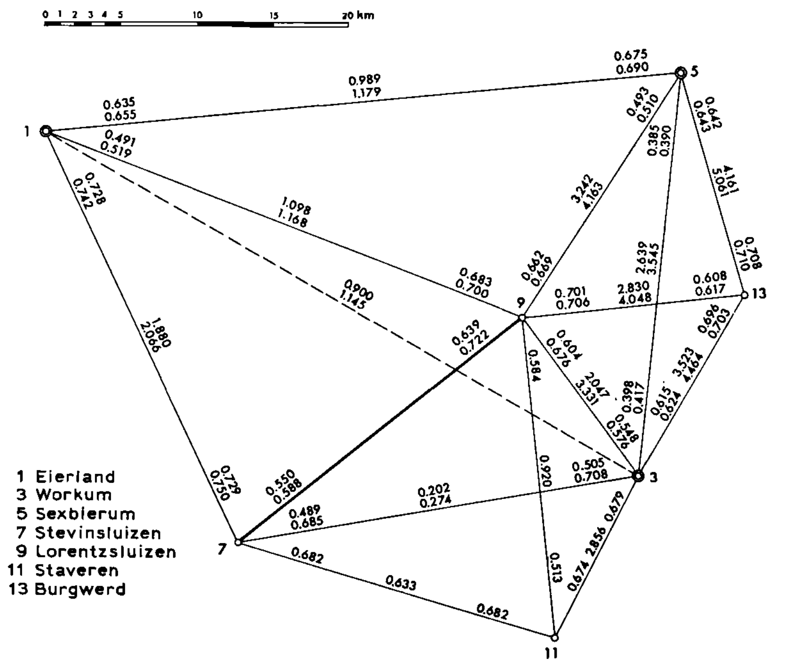


Fig. 2.1.6. The diagonal weight-coefficients of the adjusted directions and of a number of ln v's.

In this figure the diagonal weight-coefficients (proportional to the variance) of each ln v are shown in the middle of the relevant sides. For example the diagonal weight-coefficient of ln $v_{1,7;3,13}$ is given in the middle of the side (3-13). Always two weight-coefficients are mentioned which have the following meaning. The upper one is based on an adjustment including the directions to and from point 11; the lower one is based on an adjustment without these directions.

Precision of the distances

Using the computed weight coefficients of table 2.1.4 the standard deviations of the distances to which they refer can be computed. From (2.1.3.2) follows:

$$\underline{\ln s_{ij}} = \underline{\ln s_{97}} + \underline{\ln v_{7,9;i,j}} \quad \ldots \quad (2.1.3.6)$$

then follows:

$$\sigma^2_{\ln s_{ij}} = \sigma^2_{\ln s_{97}} + \sigma^2_{\ln v_{7,9;i,j}} \quad \ldots \quad (2.1.3.7)$$

(2.1.2.13) gives, applying conclusion 4 of section 2.1.2:

$$\sigma^2_{\ln s_{97}} = 0.16 \times 10^{-12} \quad \ldots \quad (2.1.3.8)$$

Using (2.1.3.4), $\sigma^2_{\ln v_{7,9;i,j}}$ can be derived from table 2.1.4. Then (2.1.3.7) with (2.1.3.8) gives the desired standard deviations. These standard deviations are shown in table 2.1.5 and Fig. 2.1.7.

Table 2.1.5. Standard deviations of the derived quantities $\ln s_{ij}$ and s_{ij}

s_{ij}	$\sigma_{\ln s_{ij}} \cdot 10^6$	$(s_{ij})_{km}$	$(\sigma_{s_{ij}})_{mm}$	s_{ij}	$\sigma_{\ln s_{ij}} \cdot 10^6$	$(s_{ij})_{km}$	$(\sigma_{s_{ij}})_{mm}$
$s_{7,9}$	0.40	24.0	10	$s_{1,9}$	1.69	33.9	57
$s_{3,9}$	2.28	13.0	30	$s_{1,5}$	1.61	42.2	68
$s_{9,13}$	2.67	14.7	39	$s_{1,7}$	2.19	30.1	66
$s_{5,9}$	2.86	19.3	55	$s_{1,3}$	**1.54**	**45.4**	**70**
$s_{3,5}$	2.58	26.9	70	$s_{9,11}$	1.56	21.4	33
$s_{3,13}$	2.98	13.8	41	$s_{7,11}$	1.31	21.9	29
$s_{5,13}$	3.23	15.3	49	$s_{3,11}$	2.68	12.1	32
$s_{3,7}$	0.81	6.8	22				

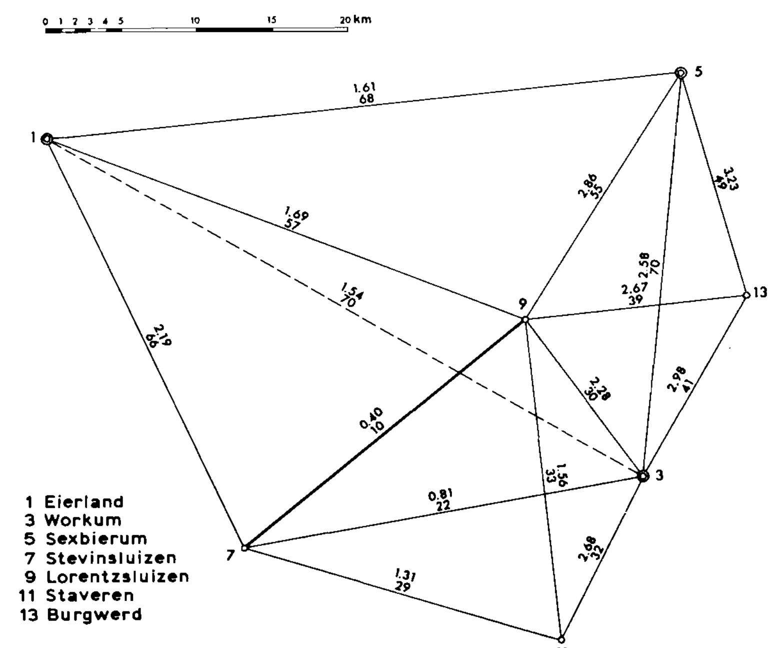


Fig. 2.1.7. The standard deviations $\ln s_{ij}$ and s_{ij}.

The standard deviation of $\ln s_{ij}$ (the upper number in the figure) is always multiplied by 10^6. The standard deviation of s_{ij} (the lower number in the figure) is expressed in mm.

Conclusions

1. From table 2.1.3 and Fig. 2.1.6 it follows that leaving out the directions to and from point 11 has little influence on the majority of the adjusted directions. Only the directions $\underline{r}_{3,7}$ and $\underline{r}_{7,3}$ are exceptions to this rule.
2. From table 2.1.4 and Fig. 2.1.6 it follows that omitting the directions to and from point 11 has considerable consequences for several diagonal weight coefficients of distance-ratios (increases up to 65%). Therefore measuring the directions to and from point 11 is a necessity.
3. Table 2.1.5 shows that the standard deviation of ln s for the distances to be derived from the network, varies between 0.40 and 3.23×10^{-6}; the standard deviation of s varies between 10 and 70 mm.
4. A closer investigation of table 2.1.5 shows, that the lengths of the sides of quadrilateral (3-9-7-11) can be determined with a higher precision that those of quadrilateral (5-9-3-13). This can be of importance for the choice of the distances to be used for the calibration of electromagnetic distance measurement instruments. For the sake of clearness the results for both quadrilaterals are shown in table 2.1.6.

Table 2.1.6

quadrilateral (3-9-7-11)		quadrilateral (5-9-3-13)	
s_{ij}	$\sigma_{\ln s_{ij}} \times 10^6$	s_{ij}	$\sigma_{\ln s_{ij}} \times 10^6$
$s_{3,9}$	2.28	$s_{5,9}$	2.86
$s_{3,7}$	0.81	$s_{3,5}$	2.58
$s_{3,11}$	2.68	$s_{5,13}$	3.23
$s_{7,9}$	0.40	$s_{3,9}$	2.28
$s_{9,11}$	1.56	$s_{9,13}$	2.67
$s_{7,11}$	1.31	$s_{3,13}$	2.98

5. A comparison of the results of $s_{1,3}$, $s_{1,5}$ and $s_{3,5}$ indicates that $\underline{\ln s_{1,3}}$ has the smallest standard deviation. Consequently it was proposed that the length of the base should be extended to the side (1–3) i.e. the side Eierland-Workum of the primary network.

It should be remarked that $s_{1,3}$ does not show the smallest standard deviation compared with those of $s_{1,5}$ and $s_{3,5}$. However within the framework of the New Adjustment of the European Triangulation $\sigma_{\ln s}$ is of greater importance than σ_s.

2.1.4 *Reliability of the base extension network*

What possibilities contains the network to check the measurements on possible errors and what are the consequences of undetected errors on the quantities ultimately desired? To answer these questions, the methods given in [9] and [11], will be used. Checking on errors takes place applying a F-test.

Assuming that the probability model, given in (2.1.3.3)–(2.1.3.5) is correct, testing takes place by using:

$$\frac{\hat{\sigma}^2}{\sigma^2} \gtrless F_{1-\alpha;b,\infty} \qquad (2.1.4.1)$$

with:

$\hat{\sigma}^2$ computed according to (3.2.11)
σ^2 assumed according to (2.1.3.4)
α level of significance, for which is chosen: 0.05
b number of condition equations, in this case: 11

One concludes to the existence of one or more errors when $\hat{\sigma}^2/\sigma^2 > F_{0.95;11,\infty}$. For β, the power of the test, the value 0.80 is chosen. That means, the size of the errors is investigated that leads to rejecting 4 out of the 5 cases. Moreover it is necessary to make some further suppositions regarding possible errors. Since no further information is available about the errors, we suffice with the simple supposition that in one observation x^i an error $\widetilde{\nabla x^i}$ is present. This alternative hypothesis can be made successively for each of the m observations.

According to [11] (3.12) the alternative hypothesis H_{a_p} can be formulated as:

$$\left(\frac{\widetilde{\nabla_{p,0} x^i}}{\sigma}\right) = (c_p^i) \cdot |\nabla_{p,0}| \qquad (2.1.4.2)$$

with i, p: $1, 2, \ldots, m$

For the vector (c_p^i) we have:

$$c_p^i = 1;\ \text{for } i = p$$
$$c_p^i = 0;\ \text{for } i \neq p$$

Again according to [11] $\nabla_{p,0}$ can be obtained from:

$$|\nabla_{p,0}| = \sqrt{\frac{\lambda_0}{N_p}} \qquad (2.1.4.3)$$

with:

$$\lambda_0 = \lambda(\alpha, \beta_0, b, \infty) = \lambda(0.05, 0.80, 11, \infty) = 16.9 \qquad (2.1.4.4)$$

and:

$$N_p = (c_p^j)^* (u_j^\tau)^* (\bar{g}_{\tau\varrho}) (u_i^\varrho) (c_p^i) \qquad (2.1.4.5)$$

In this way for every observation x^i is computed a value for $\nabla_{p,0}$ and thus for $\widetilde{\nabla_{p,0} x^i}$.

As in the extension network only directions occur as observations, the computed quantities $\widetilde{\nabla x^i}$ concern only directions.

Application of the formulas (2.1.4.2)–(2.1.4.5) on the extension network gives the result, as shown in table 2.1.7. The results show a homogeneous picture. All ∇-values satisfy: 5 cent. sec. $< \nabla r <$ 8 cent. sec.

Therefore the extension network has nearly the same "detection-capacity" for errors occuring in each of the investigated directions.

Hence errors of 5 to 8 centesimal seconds in the measurement of the directions can be detected with a probability of 80% if $\hat{\sigma}^2/\sigma^2$ is tested with a significance level of 5%.

Table 2.1.7

$\alpha=0.05 \quad \beta_0=0.80 \quad \sigma=1.57\times10^{-6}$

direction	$\nabla_{p,0}\breve{x}^i/\sigma$	direction	$\nabla_{p,0}\breve{x}^i/\sigma$	direction	$\nabla_{p,0}\breve{x}^i/\sigma$
$r_{1,5}$	6.8	$r_{5,13}$	6.9	$r_{11,3}$	7.2
$r_{1,7}$	7.9	$r_{7,1}$	7.9	$r_{11,7}$	7.3
$r_{1,9}$	5.8	$r_{7,3}$	5.8	$r_{11,9}$	5.9
$r_{3,5}$	5.3	$r_{7,9}$	6.1	$r_{13,3}$	7.5
$r_{3,7}$	5.8	$r_{7,11}$	7.3	$r_{13,5}$	7.6
$r_{3,9}$	6.1	$r_{9,1}$	7.3	$r_{13,9}$	6.6
$r_{3,11}$	7.3	$r_{9,3}$	6.5		
$r_{3,13}$	6.6	$r_{9,5}$	7.1		
$r_{5,1}$	7.2	$r_{9,7}$	6.8		
$r_{5,3}$	5.2	$r_{9,11}$	6.4		
$r_{5,9}$	5.8	$r_{9,13}$	7.5		

This table gives the ∇-quantities dimensionless (in radians). Since $\sigma=1/636620$, the number in the second column can be considered as expressed in centesimal seconds.

Table 2.1.8

Concerns H_{a_i}	$(G_i^R)^*$		
	$\ln v_{9,7;3,5}$	$\ln v_{9,7;1,5}$	$\ln v_{9,7;1,3}$
$r_{1,5}$	−0.44	+0.16	+0.16
$r_{1,7}$	−0.10	−0.45	−0.48
$r_{1,9}$	+0.53	+0.29	+0.32
$r_{3,5}$	−0.19	+0.08	−0.02
$r_{3,7}$	+0.19	−0.04	+0.06
$r_{3,9}$	−0.13	−0.17	−0.13
$r_{3,11}$	+0.08	0	+0.05
$r_{3,13}$	+0.06	+0.14	+0.04
$r_{5,1}$	−0.46	−0.18	+0.06
$r_{5,3}$	+0.33	+0.10	0
$r_{5,9}$	−0.17	−0.06	−0.12
$r_{5,13}$	+0.30	+0.14	+0.05
$r_{7,1}$	−0.32	−0.48	−0.37
$r_{7,3}$	+0.51	+0.31	+0.28
$r_{7,9}$	−0.44	0	−0.08
$r_{7,11}$	+0.25	+0.17	+0.17
$r_{9,1}$	+0.45	+0.12	+0.33
$r_{9,3}$	+0.33	+0.03	0
$r_{9,5}$	−0.23	+0.12	−0.01
$r_{9,7}$	−0.29	−0.15	−0.17
$r_{9,11}$	−0.26	−0.14	−0.18
$r_{9,13}$	0	+0.01	+0.02
$r_{11,3}$	+0.31	+0.17	+0.15
$r_{11,7}$	+0.17	0	+0.05
$r_{11,9}$	−0.48	−0.17	−0.20
$r_{13,3}$	+0.26	+0.12	+0.06
$r_{13,5}$	−0.01	+0.09	+0.04
$r_{13,9}$	−0.25	−0.21	−0.10

Example: An error in $r_{5,3}$ has almost no influence on $\ln v_{9,7;1,3}$, as opposed to an error in $r_{1,7}$. An error in the latter direction is almost for 50% propagated onto $\ln v_{9,7;1,3}$.

However more interesting are the consequences of certain undetected errors. For the measurements are not performed to determine the directions itselves, but to allow computation of the quantity $\ln v_{9,7;1,3}$ and $\ln s_{1,3}$ (see section 2.1.3, conclusion 5). These consequences can, according [9] (3.2.4), be described by the matrix $(G_i^R) = (G^{Rj})(\bar{g}_{ji})$. For multiplication of (G_i^R) with a vector $(\nabla_{p,0}\tilde{x}^i)$ shows the influence of the error, described by $(\nabla_{p,0}\tilde{x}^i)$, on the derived quantity $\underline{X}^r$.

The matrix (G_i^R) is given in table 2.1.8 for the three most important derived quantities $\ln v_{9,7;}\ldots$ for every alternative hypothesis H_{a_p}.

In Fig. 2.1.8 the numbers concerning $\ln v_{9,7;1,3}$ are arranged once more in a more convenient form. The figure shows that $\ln v_{9,7;1,3}$ is mostly influenced by errors in the directions $r_{1,7}$, $r_{1,9}$, $r_{7,1}$, $r_{9,1}$ and $r_{7,3}$.

Multiplication of the matrix (G_i^R) in table 2.1.8 with the ∇-values of the observations, given in table 2.1.7, gives the influences of these, with a probability of 80% detectable errors, on the three derived observation quantities in $\ln v_{9,7;3,5}$, $\ln v_{9,7;1,5}$ and $\ln v_{9,7;1,3}$. The result is given in table 2.1.9, expressed in centesimal seconds.
As to $\ln v_{9,7;1,3}$, the largest values are due to the influence of the 5 alternative hypotheses referring to the directions $r_{1,7}$, $r_{1,9}$, $r_{7,1}$, $r_{9,1}$ and $r_{7,3}$. These values are not alarming. The greatest value is 3.8 centesimal seconds, corresponding to a value of 6.0×10^{-6}. Therefore it can be concluded that the extension network shows an acceptable reliability in all respects.

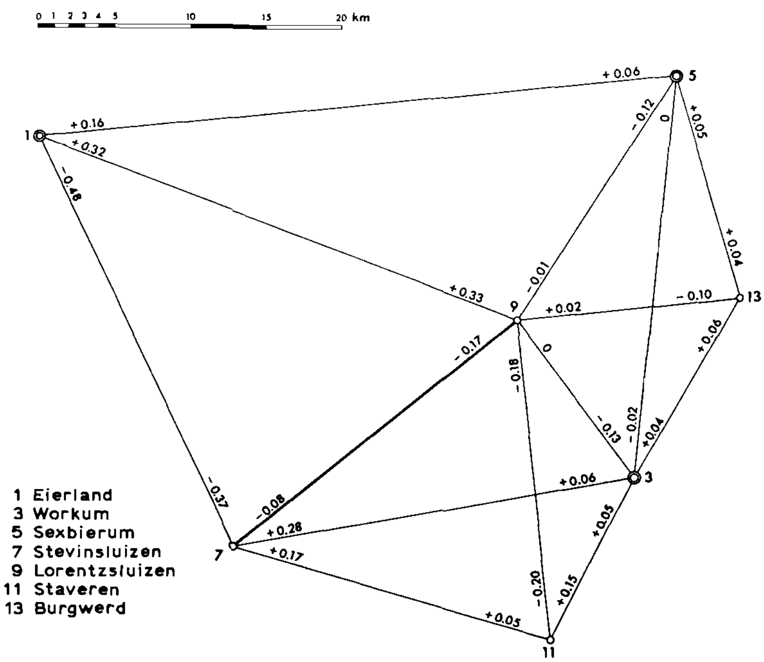


Fig. 2.1.8. $\nabla \ln v_{9,7;1,3}$ is given as a function of ∇r_{ij}.

The influence of a certain ∇r_{ij} on $\ln v_{9,7;1,3}$ is found by multiplying ∇r_{ij} with the corresponding number indicated in the figure.

Table 2.1.9

Concerns H_{a_i}	$(G_i^R)\lvert\nabla_{p,0}^{\sim}x^i\rvert = \frac{\nabla \ln v_{9,7;\ldots}}{\sigma}$ with $\sigma = 1.57\ 10^{-6}$ $\lvert\nabla_{p,0}^{\sim}x^i\rvert$ from table 2.1.7		
	$\ln v_{9,7;3,5}$	$\ln v_{9,7;1,5}$	$\ln v_{9,7;1,3}$
$r_{1,5}$	−3.0	+1.1	+1.1
$r_{1,7}$	−0.8	−3.6	−3.8
$r_{1,9}$	+3.1	+1.7	+1.9
$r_{3,5}$	−1.0	+0.4	−0.1
$r_{3,7}$	+1.1	−0.2	+0.3
$r_{3,9}$	−0.8	−1.0	−0.8
$r_{3,11}$	+0.6	0	+0.4
$r_{3,13}$	+0.4	+0.9	+0.3
$r_{5,1}$	−3.3	−1.3	+0.4
$r_{5,3}$	+1.7	+0.5	0
$r_{5,9}$	−1.0	−0.3	−0.7
$r_{5,13}$	+2.1	+1.0	+0.3
$r_{7,1}$	−2.5	−3.8	−2.9
$r_{7,3}$	+3.0	+1.8	+1.6
$r_{7,9}$	−2.7	0	−0.5
$r_{7,11}$	+1.8	+1.2	+1.2
$r_{9,1}$	+3.3	+0.9	+2.4
$r_{9,3}$	+2.1	+0.2	0
$r_{9,5}$	−1.6	+0.9	−0.1
$r_{9,7}$	−2.0	−1.0	−1.2
$r_{9,11}$	−1.7	−0.9	−1.2
$r_{9,13}$	0	+0.1	+0.2
$r_{11,3}$	+2.2	+1.2	+1.1
$r_{11,7}$	+1.2	0	+0.4
$r_{11,9}$	−2.8	−1.0	−1.2
$r_{13,3}$	+2.0	+0.9	+0.5
$r_{13,5}$	−0.1	+0.7	+0.3
$r_{13,9}$	−1.7	−1.4	−0.6

2.2 Angle measurements

The instruments used for angle measurements were Wild T3 theodolites No. 58204 and No. 74314. Before starting the observations the horizontal circles of both instruments were carefully checked for possible systematic errors in the graduation. Applying the method of HEUVELINK it was found that the circle of instrument No. 74314 had a pronounced periodic error. The characteristics of this error were determined and taken into account in the computation of the results. No appreciable error was discovered in the horizontal circle of instrument No. 58204.

In this project angle measurements were required for the following purposes:

1. Base extension network.
2. Reduction to centre of the stations.
3. Determination of the two deflection points of the base (see Fig. 1.1.1).
4. Connecting the endpoints of the invar wire measurements to the terminals of the base on the towers of Stevinsluizen and Lorentzsluizen (see Figs. 1.1.3 and 1.1.4).

5. Connecting the geodimeter pillar to the base line.
6. Checking the alignment of the section endpoints of the invar wire measurements.

In section 2.2.1 the angle measurements for the base extension network are described in detail, including all observations made. The other angle measurements are briefly mentioned in sections 2.2.2–2.2.6 (without giving the observations); descriptions of stations are included in section 2.2.2.

2.2.1 *Base extension network*

In 1964 some preparations were made at the various stations of the base extension network, as building of observation pillars, observer shelters, etc. All angles were measured for the first time in 1965 but when the results obtained did not satisfy the requirements, the measurements were repeated in 1966 and 1967. All results are analyzed in detail in section 2.3.

The test measurements carried out in 1964 had indicated that the method with reference mark was the most suitable one for this particular network (see section 2.1.1). Consequently this method was also applied for the definite measurements in 1965. At each stations two reference marks were selected at distances of 3–6 km. The angles to be determined between reference point No. 1 and the other stations were measured in 24 positions of the horizontal circle, distributed over at least 4 observation periods. The observations were made moving the telescope clockwise; each observation of an angle was immediately followed by reading its explement (i.e. 400 gr − the angle) in the same position of the horizontal circle. In this way the effect of possible drag of the circle was eliminated. The same method was applied for measuring the angle between the two reference marks; 12 sets of measures were taken (atmospheric conditions permitting) at the end of each observation period in the same position of the horizontal circle. All angle measurements were performed just after sunrise and just before sunset and results of the same angle obtained in either period were combined. The test measurements of 1964 had indicated that there was no objection to this procedure. A weather report was made every hour during the time a station was occupied.

Figs. 2.2.1–2.2.7 give the results obtained in 1965. The rows indicate from top to bottom: the angle between the two reference marks, the angle between reference mark No. 1 and the various other stations, the temperature, cloudiness, and force and direction of the wind. In the columns of the dates the mean of the observations per period are shown by black spots (morning observation on the left line, afternoon observations on the right line). Half a black spot means that only half the measuring programme could be completed. The scale left of the dates gives the (centesimal) seconds and the dashed line the total mean of the angle concerned. The direction of the wind is shown by arrows pointing towards the centres of the circles in the last two rows. The length of the arrow indicates the wind force to which the scale 0–4 (left) refers. The amount of shading is a measure for the cloud cover during the observations.

The observations of 1965 are given in detail in the tables 2.2.1–2.2.7. However an analysis of the results revealed some unacceptable misclores (see section 2.3). Although the mean of the observations per period did not point that way, the influence of lateral refraction was suspected. Since the cause could not be attributed to any particular station or angle, it was decided to remeasure the whole base extension network. The remeasurement was carried out in 1966 and 1967; the results are given in the tables 2.2.8–2.2.14. The method with reference mark was used again at the stations Lorentzsluizen, Workum and Staveren but favourable climatic conditions allowed the method of SCHREIBER (requiring less pointings and less circle readings) to be used at the stations Eierland, Stevinsluizen, Burgwerd and Sexbierum. As the positions of the searchlights were not exactly the same as in 1965, the results of both measurement are not directly comparable. In table 2.2.15 all the angles measured are reduced to centre and put into the same system.

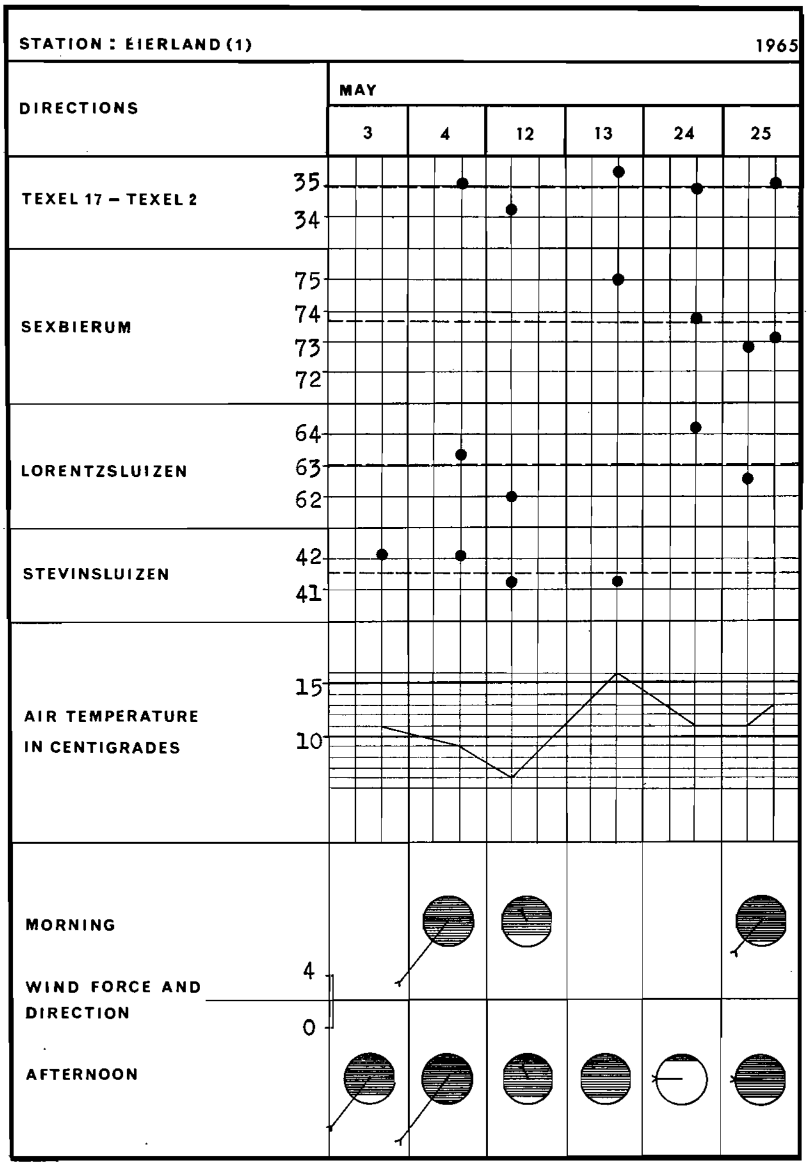
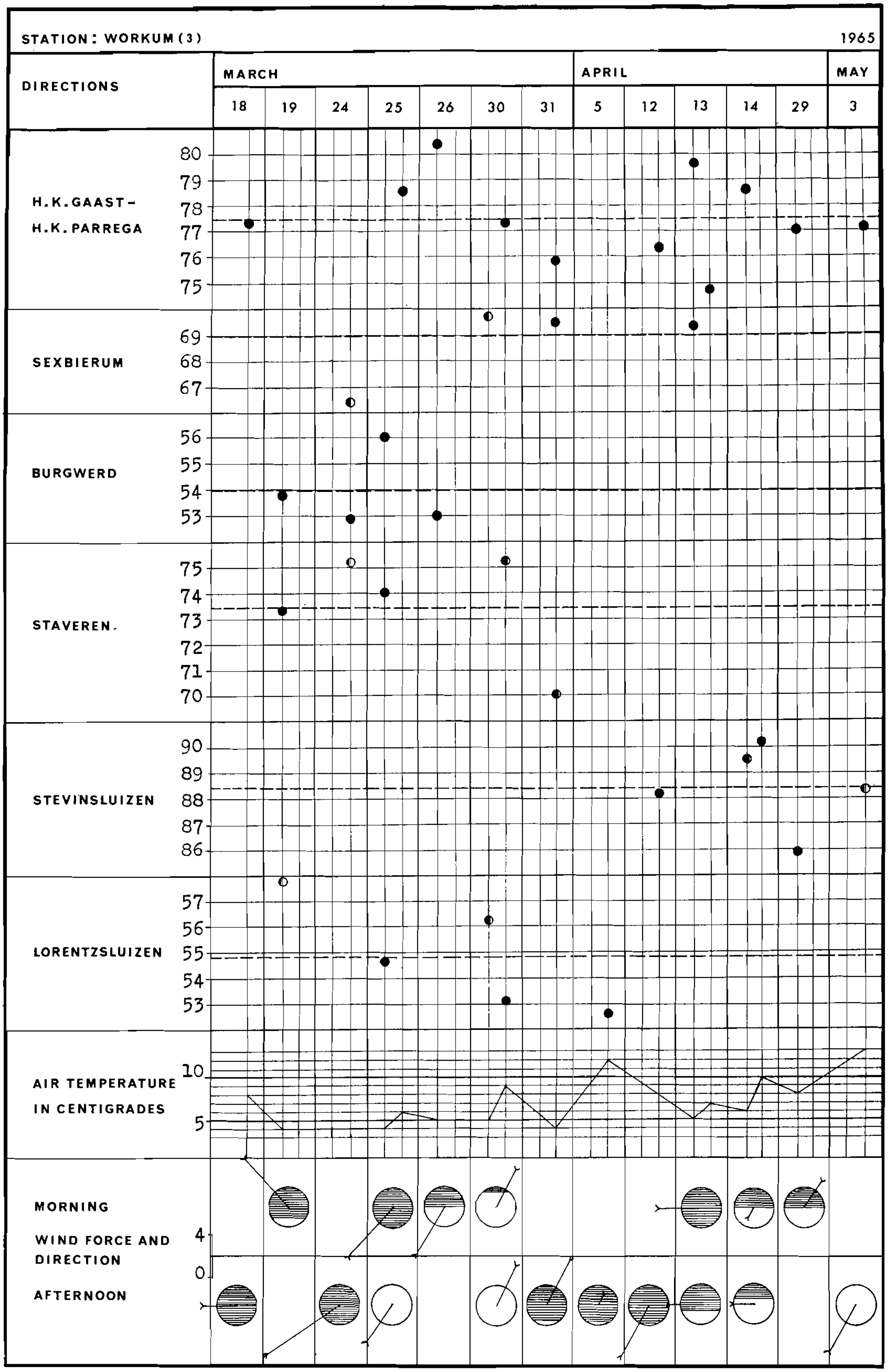


Fig. 2.2.1



STATION: WORKUM (3)
1965
DIRECTIONS
MARCH
APRIL
MAY
18
19
24
25
26
30
31
5
12
13
14
29
3
H.K.GAAST –
H.K.PARREGA
80
79
78
77
76
75
SEXBIERUM
69
68
67
BURGWERD
56
55
54
53
STAVEREN.
75
74
73
72
71
70
STEVINSLUIZEN
90
89
88
87
86
LORENTZSLUIZEN
57
56
55
54
53
AIR TEMPERATURE
IN CENTIGRADES
10
5
MORNING
WIND FORCE AND
DIRECTION
AFTERNOON
4
0

Fig. 2.2.2

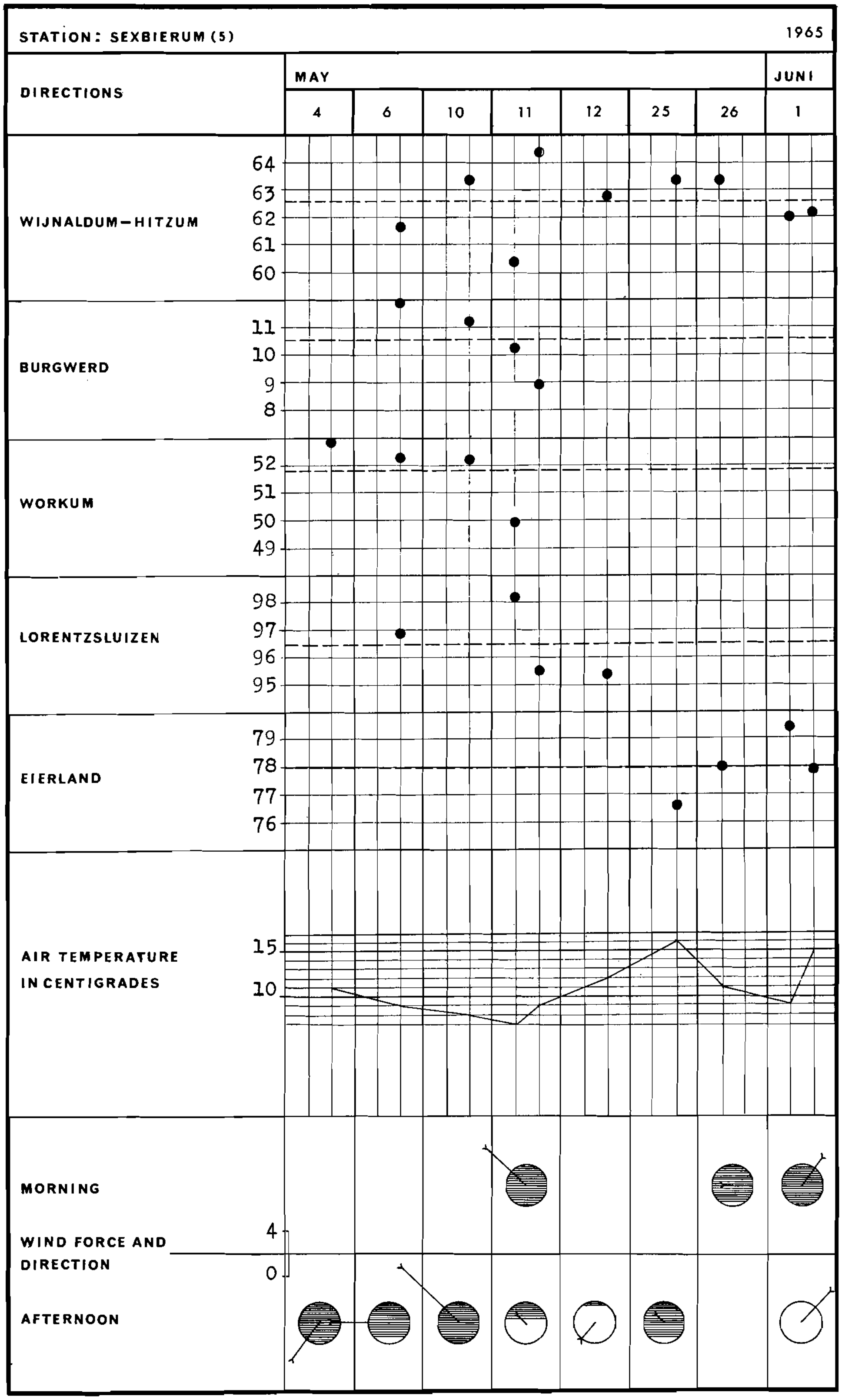

STATION: SEXBIERUM (5)
1965
DIRECTIONS
MAY
JUNI
4
6
10
11
12
25
26
1
WIJNALDUM-HITZUM
64
63
62
61
60
BURGWERD
11
10
9
8
WORKUM
52
51
50
49
LORENTZSLUIZEN
98
97
96
95
EIERLAND
79
78
77
76
AIR TEMPERATURE IN CENTIGRADES
15
10
MORNING
WIND FORCE AND DIRECTION
4
0
AFTERNOON

Fig. 2.2.3

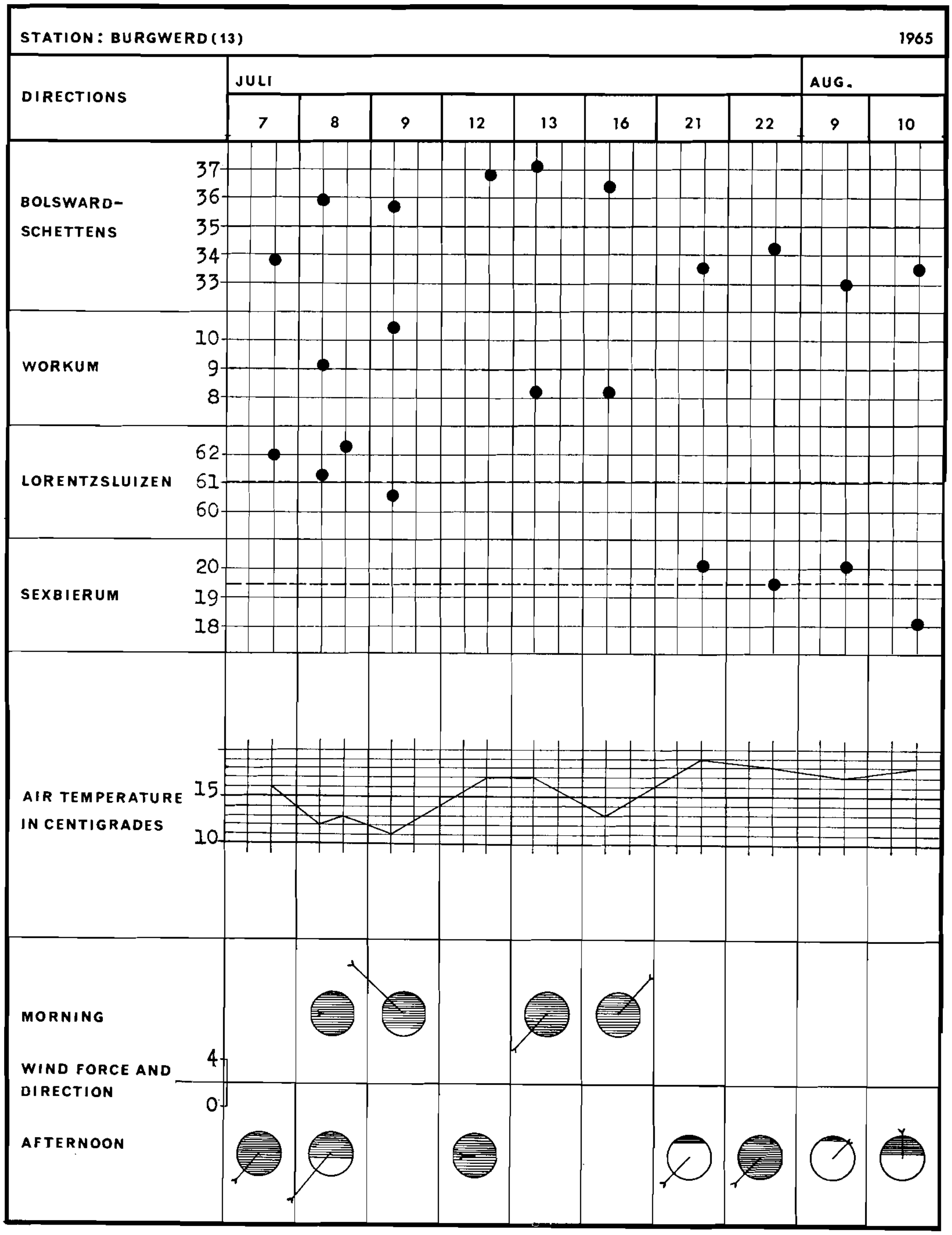

STATION: BURGWERD (13)
1965
DIRECTIONS
JULI
AUG.
7
8
9
12
13
16
21
22
9
10
BOLSWARD-SCHETTENS
37
36
35
34
33
WORKUM
10
9
8
LORENTZSLUIZEN
62
61
60
SEXBIERUM
20
19
18
AIR TEMPERATURE IN CENTIGRADES
15
10
MORNING
WIND FORCE AND DIRECTION
4
0
AFTERNOON

Fig. 2.2.4

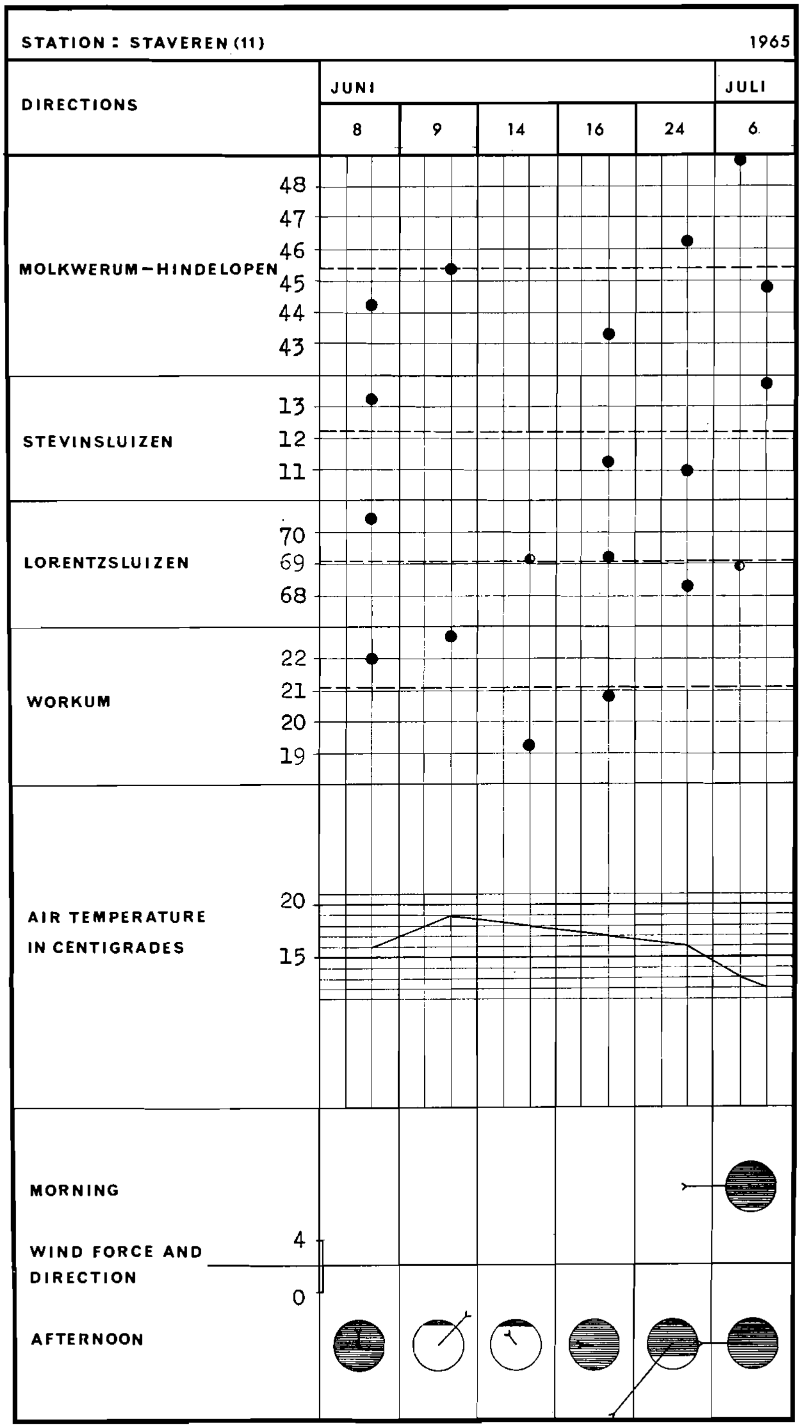

STATION : STAVEREN (11)
1965
DIRECTIONS
JUNI
JULI
8
9
14
16
24
6
MOLKWERUM–HINDELOPEN
48
47
46
45
44
43
STEVINSLUIZEN
13
12
11
LORENTZSLUIZEN
70
69
68
WORKUM
22
21
20
19
AIR TEMPERATURE
IN CENTIGRADES
20
15
MORNING
WIND FORCE AND
DIRECTION
4
0
AFTERNOON

Fig. 2.2.5

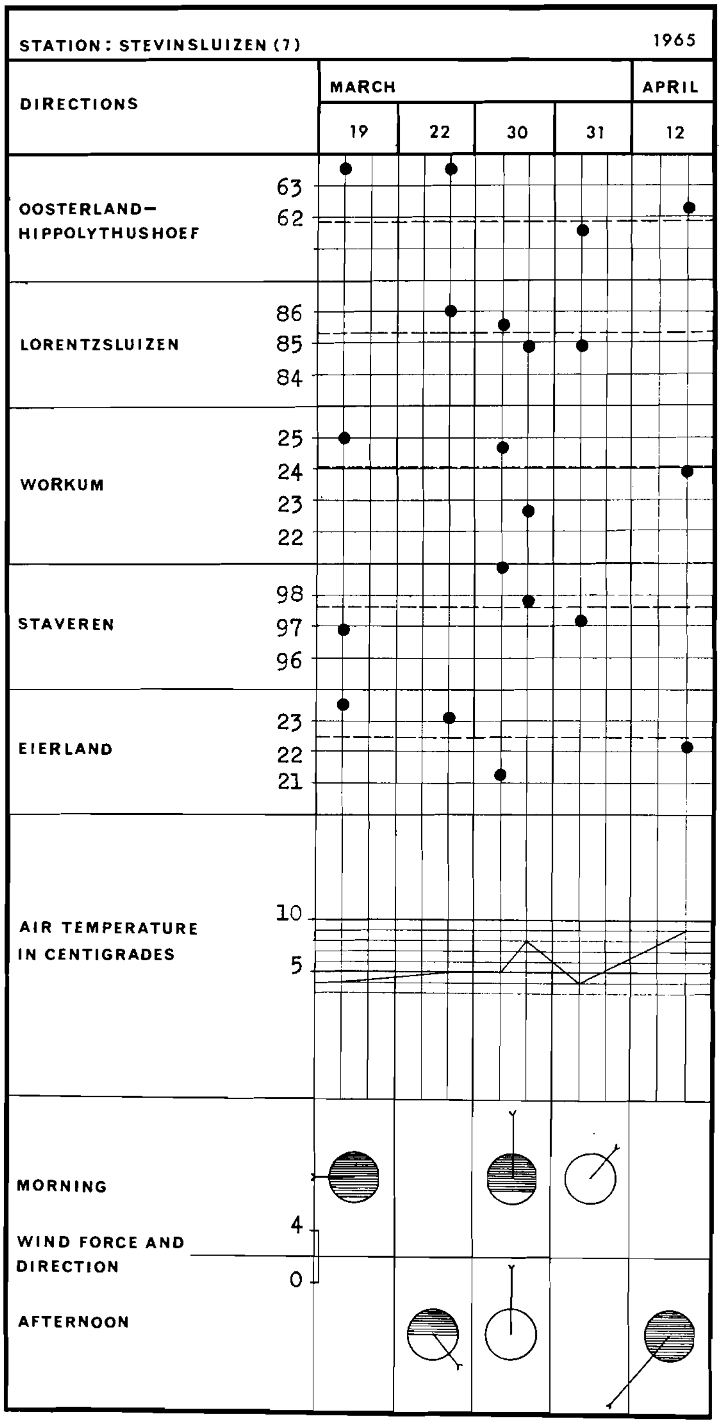
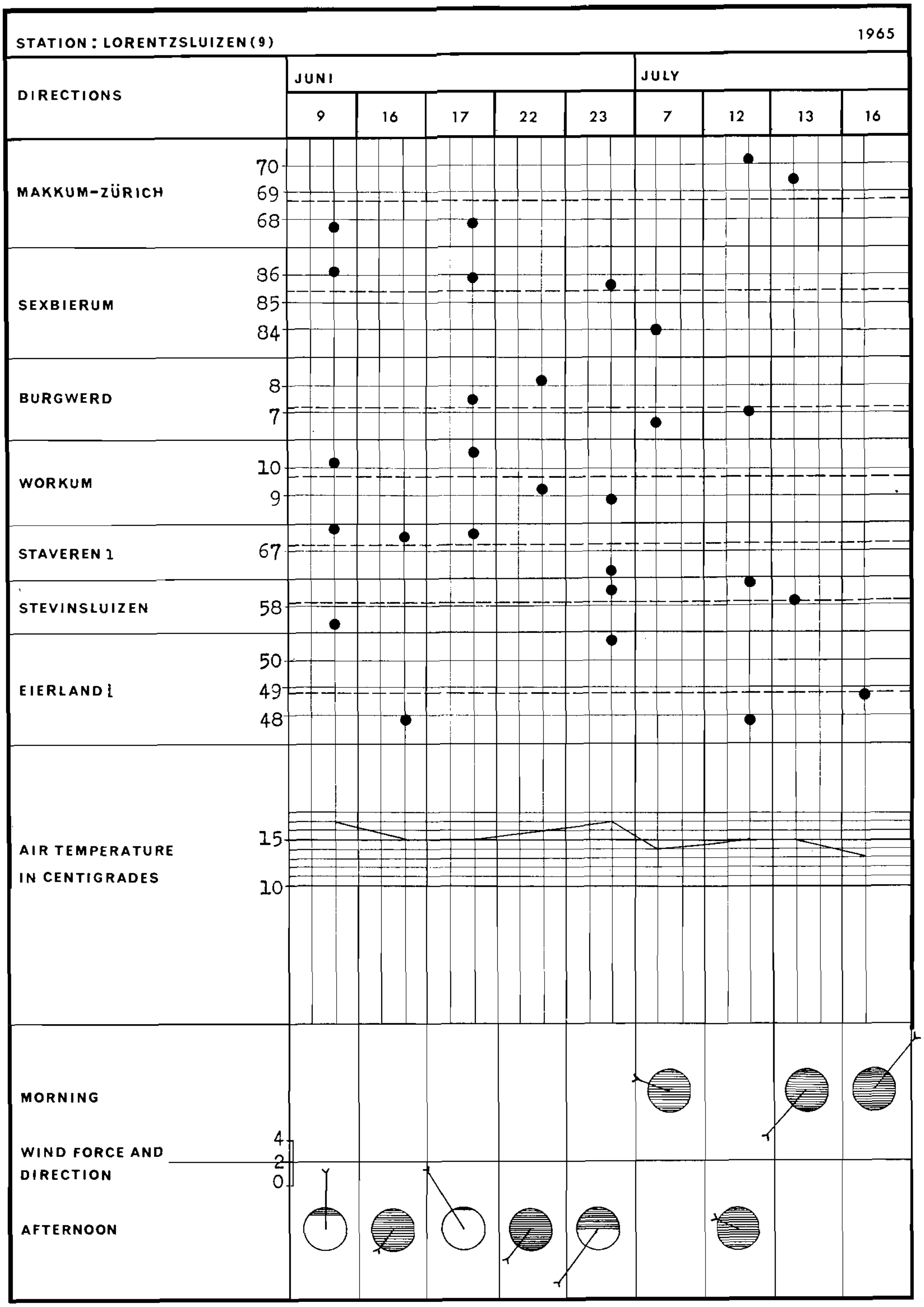

STATION : STEVINSLUIZEN (7)
1965
DIRECTIONS
MARCH
APRIL
19
22
30
31
12
OOSTERLAND–
HIPPOLYTHUSHOEF
63
62
LORENTZSLUIZEN
86
85
84
WORKUM
25
24
23
22
STAVEREN
98
97
96
EIERLAND
23
22
21
AIR TEMPERATURE
IN CENTIGRADES
10
5
MORNING
WIND FORCE AND
DIRECTION
4
0
AFTERNOON

Fig. 2.2.6



STATION : LORENTZSLUIZEN (9)
1965
DIRECTIONS
JUNI
JULY
9
16
17
22
23
7
12
13
16
MAKKUM-ZÜRICH
70
69
68
SEXBIERUM
86
85
84
BURGWERD
8
7
WORKUM
10
9
STAVEREN 1
67
STEVINSLUIZEN
58
EIERLAND 1
50
49
48
AIR TEMPERATURE
IN CENTIGRADES
15
10
MORNING
WIND FORCE AND
DIRECTION
4
2
0
AFTERNOON

Fig. 2.2.7

Table 2.2.1

Station: Eierland (1)

directions	mean values
0: Ref. point No. I	0
1: Sexbierum	312.8773.67
3: Lorentzsluizen	342.5263.09
2: Stevinsluizen	390.8741.65

Observer: H. A. VERHOEF
Instrument: Wild T3 (No. 58204)
Year: 1965

angles	0–1				0–3				0–2			
	May				May				May			
date	13	24	25	25	4	13	24	25	3	4	12	13
time	18.30	18.30	08.10	18.50	17.30	05.45	18.30	07.55	18.45	17.10	05.30	19.00
seconds of the observations	73.7	73.1	71.2	72.6	63.3	62.5	65.3	62.9	45.7	42.5	40.8	44.1
	74.5	73.5	73.8	72.0	63.6	62.7	64.9	62.4	42.3	43.1	41.9	41.3
	76.6	75.3	71.6	72.1	63.3	61.1	62.9	62.0	41.2	41.8	41.4	41.3
	76.0	74.3	75.8	74.8	62.8	60.6	63.3	63.0	41.0	39.9	43.2	40.3
	73.7	74.6	72.4	73.1	63.6	63.3	64.9	62.1	39.1	42.7	40.8	40.1
	75.5	71.7	72.2	73.9	64.0	61.9	64.2	63.5	43.4	42.7	39.3	39.8
means	75.0	73.8	72.8	73.1	63.4	62.0	64.2	62.6	42.1	42.1	41.2	41.2

Table 2.2.2

Station: Workum (3)

directions	mean values
0: Ref. point No. I	0
7: Staveren	263.0373.33
5: Stevinsluizen	322.1288.32
6: Lorentzsluizen	392.4454.23
4: Sexbierum	39.5569.12
8: Burgwerd	66.5553.93

Observer: P. Dijkstra
Instrument: Wild T3 (No. 74314)
Year: 1965

angles	0–7					0–5					0–6				
	March					April				May	March				April
date	19	24	25	30	31	12	14	14	29	3	19	25	30	30	5
time	08.00	18.00	09.15	18.30	18.30	18.30	06.20	18.00	07.35	17.45	09.00	08.30	08.30	18.00	18.00
seconds of the observations	68.0 77.2 72.0 72.5 74.8 75.1	72.7 77.7	73.6 69.2 75.8 72.2 75.6 77.4	74.4 77.0 78.2 75.0 71.5	76.2 68.1 67.9 67.9 70.0	87.3 86.5 87.7 89.1 89.5 88.8	88.5 86.5 93.0 89.9	87.6 89.0 93.1 90.6 91.6 89.0	83.8 90.3 83.6 85.8 84.9 86.9	88.8 87.8	57.9 57.5	52.0 55.3 56.0 52.6 55.7 55.9	55.7 56.3 56.5 56.5	52.8 51.8 56.1 52.3 56.4 49.0	55.1 53.4 53.3 49.1 50.8 53.6
means	73.5	75.2	74.0	75.2	70.0	88.2	89.5	90.2	85.9	88.3	57.7	54.6	56.2	53.1	52.6

angles	0–4					0–8			
	March			April		March			
date	24	30	31	13	13	19	24	25	26
time	17.00	08.15	18.00	07.10	18.20	08.30	17.15	08.50	08.15
seconds of the observations	67.9 66.2 65.1	67.9 69.4 71.7	68.5 72.2 69.8 67.4 69.5 69.5	64.4 69.2 70.3 71.9 68.3 71.6	66.8 70.1 69.8 69.7 74.1 67.7	48.1 54.9 52.3 54.4 56.5 56.4	52.5 53.6 51.7 51.9 53.2 54.5	56.4 59.2 55.0 54.2 55.7 55.6	54.7 51.1 53.5 49.6 55.3 54.1
means	66.4	69.7	69.5	69.3	69.7	53.8	52.9	56.0	53.0

Table 2.2.3

Station: Sexbierum (5)

directions	mean values	
0: Ref. point No. I	0	Observer: P. Dijkstra
12: Burgwerd	349.0810.59	Instrument: Wild T3 (No. 74314)
10: Workum	373.2551.81	Year: 1965
11: Lorentzsluizen	3.1296.49	
9: Eierland	60.6377.98	

angles	0–12				0–10				0–11			
	May				May				May			
date	6	10	11	11	4	6	10	11	6	11	11	12
time	18.15	16.15	05.45	19.20	18.40	18.30	16.00	06.30	19.20	06.10	19.50	18.45
seconds of the observations	12.8	10.1	08.9	06.7	52.7	57.1	50.2	49.5	97.5	98.5	95.4	97.9
	15.5	13.9	10.4	04.7	54.3	52.0	55.5	49.1	96.0	97.6	95.2	96.5
	10.9	13.2	07.7	14.8	50.3	51.1	48.5	48.1	00.6	97.3	94.1	95.5
	08.4	12.5	09.4	07.5	54.5	50.8	56.1	53.2	98.2	98.7	93.1	96.7
	13.9	10.4	14.9	08.1	50.7	53.4	53.0	50.3	93.2	97.8	99.1	90.4
	10.1	07.4	10.1	11.8	54.1	49.4	50.5	49.0	95.1	99.5	96.3	95.6
means	11.9	11.2	10.2	08.9	52.8	52.3	52.3	49.9	96.8	98.2	95.5	95.4

angles	0–9			
	May		June	
date	25	26	1	1
time	19.15	05.00	05.00	19.30
seconds of the observations	78.5	77.2	79.2	76.7
	79.1	79.0	80.1	77.9
	75.0	76.1	80.5	76.0
	71.0	77.4	74.9	76.8
	78.1	78.4	80.7	79.0
	78.1	80.1	81.6	80.1
means	76.6	78.0	79.5	77.8

Table 2.2.4

Station: Stevinsluizen (7)

directions	mean values
0: Ref. point No. I	0
13: Eierland	89.0422.52
15: Lorentzsluizen	174.1185.35
14: Workum	206.2524.08
16: Staveren	235.5697.67

Observer: H. A. VERHOEF
Instrument: Wild T3 (No. 58204)
Year: 1965

angles	0–13				0–15				0–14			
	March			April	March				March			April
date	19	22	30	12	22	30	30	31	19	30	30	12
time	07.00	16.50	08.30	15.25	16.35	08.15	18.15	07.10	07.20	08.45	17.35	15.40
seconds of the observations	23.1	24.3	21.4	22.7	85.8	85.9	84.2	85.4	27.9	24.1	21.8	23.3
	24.5	23.2	21.6	22.4	86.1	85.4	86.3	83.9	22.3	22.0	23.0	23.5
	23.1	25.3	21.8	23.1	85.7	87.2	85.1	85.3	25.2	26.6	21.3	24.8
	23.5	22.6	21.5	22.2	85.7	83.8	86.1	83.9	24.9	25.6	23.2	23.4
	24.4	22.5	20.1	21.7	85.8	86.1	82.6	85.3	22.8	25.0	23.5	25.0
	22.4	20.8	21.6	20.7	86.7	85.3	85.3	85.6	26.0	24.9	23.5	23.4
means	23.5	23.1	21.3	22.1	86.0	85.6	84.9	84.9	25.0	24.7	22.7	23.9

angles	0–16			
	March			
date	19	30	30	31
time	07.40	09.00	17.15	06.35
seconds of the observations	96.8	99.2	99.9	99.0
	98.2	00.0	95.6	97.0
	98.1	99.0	98.9	96.2
	96.6	96.9	98.9	95.6
	95.7	99.0	96.6	96.4
	95.8	99.1	97.0	98.5
means	96.9	98.9	97.8	97.1

Table 2.2.5

Station: Lorentzsluizen (9)

directions	mean values
0: Ref. point No. I	0
19: Sexbierum	317.4085.40
22: Burgwerd	374.4307.30
18: Workum	40.4009.70
21: Staveren	74.3567.30
20: Stevinsluizen	137.9558.20
17: Eierland	204.5448.80

Observer: H. A. VERHOEF
Instrument: Wild T3 (No. 58204)
Year: 1965

angles	0–19				0–22				0–18			
	June			July	June		July		June			
date	9	17	23	7	17	22	7	12	9	17	22	23
time	19.10	19.55	17.15	06.45	19.35	16.55	07.35	19.00	18.50	19.25	16.15	17.00
seconds of the observations	86.5	86.6	84.6	83.9	06.6	07.8	06.1	07.7	11.0	10.2	09.4	08.9
	88.4	85.4	85.0	83.8	09.8	07.4	06.3	07.4	10.1	11.2	08.5	07.8
	86.6	87.5	85.4	85.0	07.9	10.2	07.8	06.0	10.1	10.7	08.9	08.1
	82.5	85.5	85.5	83.4	06.0	06.3	07.4	06.7	09.7	10.0	11.1	08.9
	85.9	85.4	86.2	84.3	07.3	07.7	06.1	08.4	09.6	10.4	08.2	09.6
	86.7	84.9	87.1	83.5	07.1	09.4	06.1	05.8	10.3	10.6	09.3	10.2
means	86.1	85.9	85.6	84.0	07.4	08.1	06.6	07.0	10.1	10.5	09.2	08.9

angles	0–21				0–20				0–17			
	June				June		July		June		July	
date	9	16	17	23	9	23	12	13	16	23	12	16
time	19.55	19.25	20.10	16.40	19.30	16.25	18.35	05.00	19.45	16.10	19.15	05.40
seconds of the observations	68.5	67.5	67.9	67.6	55.3	58.7	60.4	57.3	47.7	50.4	48.5	48.7
	66.8	66.5	67.7	66.2	57.0	60.3	59.2	57.9	47.7	51.1	48.3	48.9
	68.6	67.5	68.0	66.8	58.4	59.4	59.0	58.5	47.5	51.1	47.6	48.4
	66.5	67.2	69.5	65.6	57.9	57.5	56.5	58.6	48.7	49.6	48.1	48.7
	67.6	68.3	65.8	66.7	56.3	57.3	59.3	58.3	46.7	50.6	47.7	48.8
	68.5	68.1	66.7	65.0	58.7	58.2	58.8	58.0	48.9	51.5	47.2	48.7
means	67.8	67.5	67.6	66.3	57.3	58.6	58.9	58.1	47.9	50.7	47.9	48.7

Table 2.2.6

Station: Staveren (11)

directions	mean values
0: Ref. point No. I	0
24: Stevinsluizen	260.5512.33
25: Lorentzsluizen	335.4969.20
23: Workum	372.1621.22

Observer: P. Dijkstra
Instrument: Wild T3 (No. 74314)
Year: 1965

angles	0–24				0–25					0–23			
	June			July	June				July	June			
date	8	16	24	6	8	14	16	24	6	8	9	14	16
time	19.20	17.30	17.30	18.00	19.00	19.45	18.30	18.25	05.50	18.45	19.20	19.55	17.45
seconds of the observations	12.1	12.8	04.5	13.8	68.9	67.5	68.6	68.0	68.1	24.5	27.1	19.1	22.2
	11.4	13.4	11.2	13.8	72.0	70.5	71.0	65.8	68.5	20.6	22.7	19.4	20.9
	13.9	10.9	14.5	15.8	70.5	69.3	70.9	69.2	70.0	18.3	21.9	18.9	18.5
	11.8	10.5	10.1	11.5	67.6		66.9	70.7		21.7	23.9	22.9	21.4
	13.9	09.6	13.2	13.9	73.8		68.0	69.6		22.6	22.5	18.5	21.2
	16.2	10.7	12.3	14.2	69.5		69.2	66.6		24.4	18.3	16.7	21.0
means	13.2	11.3	11.0	13.8	70.4	69.1	69.1	68.3	68.9	22.0	22.7	19.2	20.9

Table 2.2.7

Station: Burgwerd (13)

directions	mean values
0: Ref. point No. I	0
26: Workum	0.1909.02
28: Lorentzsluizen	60.1061.57
27: Sexbierum	149.0219.48

Observer: P. Dijkstra
Instrument: Wild T3 (No. 74314)
Year: 1965

angles	0–26				0–28				0–27			
	July				July				July		August	
date	8	9	13	16	7	8	8	9	21	22	9	10
time	05.30	06.50	05.35	05.10	19.00	06.30	19.20	07.10	19.00	17.30	18.50	18.40
seconds of the observations	10.5	11.1	06.7	07.6	63.1	59.7	62.9	62.5	18.7	15.7	18.5	17.9
	06.7	11.7	10.4	08.1	63.6	60.6	65.1	65.5	20.1	20.8	20.8	19.2
	10.8	11.9	05.9	07.9	58.7	61.5	60.8	59.2	21.9	23.2	19.2	17.9
	10.1	09.4	11.2	08.6	61.9	61.9	62.4	57.6	20.0	20.6	21.2	16.5
	06.9	08.7	08.8	08.7	61.1	61.7	62.8	59.5	19.6	18.1	21.8	17.3
	09.5	10.3	06.4	08.6	63.8	62.5	59.9	59.4	20.5	19.4	19.0	19.6
means	09.1	10.5	08.2	08.2	62.0	61.3	62.3	60.6	20.1	19.6	20.1	18.1

Table 2.2.8

Station: Lorentzsluizen (9)

directions	mean values
0: Ref. point No. I	0
19: Sexbierum	317.4144.62
22: Burgwerd	374.4399.32
18: Workum	40.4248.69
21: Staveren	74.3599.79
20: Stevinsluizen	137.9678.29
17: Eierland	204.5444.77

Observers: H. A. Verhoef (1966)
D. van Loon (1967)
Instrument: Wild T3 (No. 58204)
Years: 1966/1967

angles	0–19				0–22				0–18			
	1966 Aug.				1966 Sept.				1966 Sept.			
date	22	29	30	31	5	6	6	8	5	6	6	7
time	16.15	19.20	05.20	05.20	17.45		17.45	07.20	17.25		17.00	06.00
seconds of the observations	47.0	40.6	45.0	45.9	99.2	99.1	98.3	97.9	46.8	47.9	48.5	49.2
	46.6	43.2	41.8	47.6	03.4	00.6	98.5	96.7	47.7	50.9	46.9	48.5
	44.9	42.8	46.6	49.6	96.3	99.6	98.5	99.3	45.6	51.2	51.5	50.8
	43.5	41.6	47.7	49.8	99.0	00.0	99.9	99.6	45.2	53.2	48.3	47.6
	42.2	40.6	42.0	46.6	01.2	98.4	02.2	99.7	48.9	48.9	48.6	46.8
	46.0	39.0	42.1	48.4	01.2	00.1	96.7	98.4	52.0	46.2	50.0	47.3
means	45.0	41.3	44.2	48.0	00.0	99.6	99.0	98.6	47.7	49.7	49.0	48.4

angles	0–21				0–20				0–17			
	1966 Sept.				'66 Sept.	'67 Aug.	1967 Sept.		1967 Aug.		1967 Sept.	
date	6	7	8	14	14	22	4	5	22	31	4	5
time	17.25	06.35	06.15	14.50		15.15	17.00	14.45	19.45	07.55	19.15	05.45
seconds of the observations	99.6	00.6	97.4	98.5	79.9	78.8	78.0	78.9	40.0	40.8	48.0	43.6
	02.0	01.7	00.4	02.7	80.1	75.0	77.6	76.3	42.9	41.6	47.8	47.3
	02.5	98.3	98.4	99.2	78.7	79.8	77.0	80.0	42.4	43.0	48.2	48.6
	99.2	98.1	99.9	98.7	80.4	78.2	75.8	79.8	40.5	43.4	46.6	47.3
	00.1	01.3	99.0	00.5	76.8	75.0	74.8	82.0	42.4	43.5	48.9	45.8
	00.6	98.0	99.2	99.1	81.2	76.6	76.7	71.6	44.2	44.7	47.0	46.0
means	00.7	99.7	99.0	99.8	79.5	77.2	76.7	79.8	42.1	42.8	47.8	46.4

Table 2.2.9

Station: Staveren (11)

directions	mean values
0: Ref. point No. I	0
24: Stevinsluizen	260.5628.14
25: Lorentzsluizen	335.4913.92
23: Workum	372.1611.35

Observers: H. A. VERHOEF
P. DIJKSTRA (Sept. 22: 16.40–17.15)
Instruments: Wild T3 (Nos. 58204 and 74314)
Year: 1966

angles	0–24				0–25				0–23			
	Sept.				Sept.				Sept.			
date	19	20	22	22	19	20	22	22	20	21	22	22
time	17.55	17.55	06.30	17.15	17.45	17.30	06.45	16.40	17.15	16.50	07.00	17.00
seconds of the observations	32.1	30.1	31.2	19.8	16.5	14.7	15.3	15.9	15.0	10.8	11.6	09.4
	33.1	30.5	30.1	19.5	14.4	16.2	13.7	15.8	12.5	12.5	11.9	08.8
	33.1	29.8	30.9	22.1	13.8	12.7	13.4	14.2	12.2	12.3	12.8	09.0
	30.5	30.8	28.0	21.8	16.2	12.5	13.2	09.4	11.5	10.2	12.5	08.8
	30.0	29.0	29.6	21.8	13.7	13.5	12.5	12.4	11.9	11.9	11.5	11.0
	30.1	29.8	30.6	21.1	16.2	13.6	13.8	10.4	12.3	11.3	12.7	08.0
means	31.5	30.0	30.1	21.0	15.1	13.9	13.6	13.0	12.6	11.5	12.2	09.2

Table 2.2.10

Station: Sexbierum (5)

directions	adjusted values	date/angle	12–10	12–11	12–9	10–11	10–9	11–9
12: Burgwerd	0.0000.00	Aug. 15	22.2	07.3	31.2	82.9	02.4	19.2
10: Workum	24.1825.40		26.8	09.8	30.5	81.4	03.9	16.3
11: Lorentzsluizen	54.0609.50	Aug. 16	26.9	09.0		86.2	03.8	18.4
9: Eierland	111.5528.68		27.2	10.0		85.3	03.5	17.3
			25.7	11.1	27.6	83.8		
Observer: H. A. VERHOEF			25.1	07.5	29.4	82.4		
Instrument: Wild T3 (No. 58.204)		Aug. 17			27.3		04.4	18.9
Year: 1967					26.9		05.7	20.0
		mean	25.6	09.1	28.8	83.7	04.0	18.4

Table 2.2.11

Station: Workum (3)

directions	mean values
0: Ref. point No. I	0
7: Staveren	263.0419.20
5: Stevinsluizen	322.1395.85
6: Lorentzsluizen	392.4468.74
4: Sexbierum	39.5583.95
8: Burgwerd	66.5554.65

Observers: P. Dijkstra, D. van Loon, H. A. Verhoef (1967)
Instruments: Wild T3 (Nos. 74314 and 58204)
Years: 1966/1967

angles	0–7				0–5				0–6			
	1966 Sept.				1966 Sept.				1966 Sept.			
date	6	7	7	8	14	19	20	22	5	6	6	7
time	17.30	08.15	18.20	07.30	16.30	18.15	18.30	08.15	17.35	06.35	17.00	07.40
seconds of the observations	20.2	19.1	18.5	17.1	93.6	92.2	95.6	01.6	69.4	69.0	67.1	67.3
	21.1	17.0	17.8	18.4	95.2	94.5	96.7	99.6	69.2	73.0	64.2	70.3
	20.5	17.0	20.6	20.0	96.7	90.9	94.5	99.3	68.0	68.6	67.2	70.4
	20.9	19.8	17.0	14.3	95.4	89.8	93.5	97.8	69.2	65.6	70.0	70.2
	20.1	19.6	22.9	15.9	97.3	95.8	91.1	01.0	67.4	68.8	71.6	69.6
	19.0	22.3	22.6	19.1	96.8	94.7	95.4	01.5	68.4	65.6	67.6	72.0
means	20.3	19.1	19.9	17.5	95.8	93.0	94.5	00.1	68.6	68.4	67.9	70.0

angles	0–4				0–8			
	1967 Sept.				1966 Sept.			
date	4	5	5	6	6	6	7	7
time	16.45	06.45	15.45	06.45	06.50	17.15	07.55	18.00
seconds of the observations	81.4	84.1	84.8	83.0	53.0	55.0	55.0	56.7
	84.2	83.8	84.7	83.8	57.0	56.2	52.4	52.8
	82.7	84.5	84.6	83.5	58.0	54.9	53.9	53.5
	83.8	84.6	84.1	85.1	56.7	55.7	55.0	56.0
	83.6	84.0	84.7	83.2	54.6	53.7	51.9	53.8
	84.6	83.1	84.4	84.6	51.4	54.4	53.2	56.7
means	83.4	84.0	84.5	83.9	55.1	55.0	53.6	54.9

Table 2.2.12

Station: Eierland (1)

directions	adjusted values
1: Sexbierum	0.0000.00
3: Lorentzsluizen	29.6391.58
2: Stevinsluizen	77.9914.26

Observer: D. van Loon
Instrument: Wild T3 (No. 74314)
Year: 1967

date/angle	1–3	1–2	3–2
June 26	94.0	16.2	19.6
	90.8	14.3	22.6
July 13	93.7	17.1	25.2
	92.2	17.7	24.7
Aug. 7	92.6		22.5
	91.8		22.4
Aug. 9	89.7	11.6	22.6
	90.0	12.8	23.9
Aug. 10		10.6	
		11.6	
mean	91.9	14.0	22.9

Table 2.2.13

Station: Stevinsluizen (7)

directions	adjusted values
13: Eierland	0.0000.00
15: Lorentzsluizen	85.0710.40
14: Workum	117.2211.84
16: Staveren	146.5351.74

Observer: D. van Loon
Instrument: Wild T3 (No. 74314)
Year: 1967

date/angle	13–15	13–14	13–16	15–14	15–16	14–16
April 20	12.0	13.9	55.0	98.8	39.0	36.6
	10.8	15.4	54.4	99.3	43.8	40.7
April 26				02.4	43.6	41.3
				00.6	39.3	41.2
Aug. 16	12.6	13.0	51.2			37.7
	08.4	09.9	48.8			38.5
		11.9	48.4	99.6	46.8	
		12.4	48.4	99.0	43.2	
Aug. 17	07.4					
	10.0					
mean	10.2	12.8	51.0	00.0	42.6	39.3

Table 2.2.14

Station: Burgwerd (13)

directions	adjusted values
26: Workum	0.0000.00
28: Lorentzsluizen	59.9065.76
27: Sexbierum	148.8200.03

Observer: H. A. Verhoef
Instrument: Wild T3 (No. 58204)
Year: 1967

date/angles	26–28	26–27	28–27
Aug. 22	65.3	98.9	32.6
	66.6	00.9	34.0
Aug. 23			34.5
			35.6
Aug. 29	66.6	01.7	32.5
	64.9	00.4	32.0
Aug. 30	65.2	99.1	
	65.1	01.2	
	65.4	00.6	35.2
	64.6	99.8	35.4
mean	65.5	00.3	34.0

Table 2.2.15

direction		measured 1965	corr. for center cent.sec.	measured 1966–'67	corr. for center cent.sec.	centred directions		diff. 1966–1965	w_{1965}	$w_{1966-67}$
No.	from/to					1965	1966/'67			
1	1–5	312.8773.7	+ 55.3	0.0000.0	+ 11.2	0.0000.0	0.0	0.0	−2.2	+0.5
2	1–7	390.8741.7	− 24.1	77.9914.3	− 12.8	77.9888.6	9890.3	+1.7	+0.8	+1.0
3	1–9	342.5263.1	− 13.3	29.6391.6	+ 43.3	29.6420.8	6423.7	+2.9	+1.4	−1.1
4	3–5	39.5569.1	− 47.3	39.5584.0	− 61.9	39.5521.8	5522.1	+0.3	−0.1	−1.8
5	3–7	322.1288.3	+ 89.8	322.1395.9	− 23.7	322.1378.1	1372.2	−5.9	−3.5	+0.7
6	3–9	392.4454.2	−242.7	392.4468.7	−257.5	392.4211.5	4211.2	−0.3	+3.9	+1.2
7	3–11	263.0373.3	+191.3	263.0419.2	+143.5	263.0564.6	0562.7	−1.9	−0.4	−1.2
8	3–13	66.5553.9	−110.5	66.5554.6	−110.5	66.5443.4	5444.1	+0.7	+0.1	+1.3
9	5–1	60.6378.0	− 28.9	111.5528.7	+ 11.2	111.5529.5	5530.9	+1.4	+0.5	−1.9
10	5–3	373.2551.8	+ 23.4	24.1825.4	− 61.9	24.1755.6	1754.5	−1.1	−0.6	+0.7
11	5–9	3.1296.5	+142.3	54.0609.5	+ 18.2	54.0619.2	0618.7	−0.5	+0.9	+1.8
12	5–13	349.0810.6	+ 9.0	0.0000.0	+ 9.0	0.0000.0	0.0	0.0	−0.8	−1.3
13	7–1	89.0422.5	− 71.2	0.0000.0	− 12.8	0.0000.0	0.0	0.0	−1.8	−1.7
14	7–3	206.2524.1	+ 31.0	117.2211.8	− 23.7	117.2203.8	2200.9	−2.9	−1.5	−0.5
15	7–9	174.1185.4	−109.8	85.0710.4	0.0	85.0724.3	0723.2	−1.1	+1.5	+1.7
16	7–11	235.5697.7	+ 52.3	146.5351.7	+ 33.0	146.5398.7	5397.5	−1.2	+1.8	0.0
17	9–1	204.5448.8	+ 43.3	204.5444.8	+ 43.3	204.5492.1	5488.1	−4.0	+0.4	+2.9
18	9–3	40.4009.7	− 14.9	40.4248.7	−257.5	40.3994.8	3991.2	−3.6	−1.6	−1.1
19	9–5	317.4085.4	+ 78.7	317.4144.6	+ 18.1	317.4164.1	4162.7	−1.4	+0.3	−0.9
20	9–7	137.9558.2	+121.0	137.9678.3	0.0	137.9679.2	9678.3	−0.9	+1.8	−1.6
21	9–11	74.3567.3	+ 27.8	74.3599.8	− 8.6	74.3595.1	3591.2	−3.9	−0.8	+0.9
22	9–13	374.4307.3	+ 2.9	374.4399.3	− 88.6	374.4310.2	4310.7	+0.5	+0.2	−0.1
23	11–3	372.1621.2	+ 31.8	372.1511.4	+143.5	372.1653.0	1654.9	+1.9	+2.5	+0.6
24	11–7	260.5512.3	+147.9	260.5628.1	+ 33.0	260.5660.2	5661.1	+0.9	+0.5	−0.7
25	11–9	335.4969.2	− 63.8	335.4913.9	− 8.6	335.4905.4	4905.3	−0.1	−2.4	+0.1
26	13–3	0.1909.0	− 24.1	0.0000.0	−110.5	0.0000.0	0.0	0.0	−0.6	−1.5
27	13–9	149.0219.5	− 11.2	148.8200.0	+ 9.0	148.8323.4	8319.5	−3.9	+0.4	+1.4
28	13–9	60.1061.6	− 88.6	59.9065.8	− 88.6	59.9088.1	9087.7	−0.4	+0.2	+0.1

2.2.2 *Reduction to centre*

The stations of the base extension network are: church towers (Sexbierum, Burgwerd, Workum, Staveren), sluice towers (Stevinsluizen, Lorentzsluizen) and a lighthouse (Eierland). As centres of the stations were chosen the weather-vanes of the church towers (also used as centres in the original determination of these stations), the terminals of the new base line on the sluice towers and the lightning-conductor of the lighthouse. Brass bolts in the walls served as permanent marks for the centres and brick pillars were erected on all buildings for setting up the theodolite and the searchlights. Consequently all angles measured had to be reduced to centre. This was achieved by carrying out small local triangulations at the various stations. The base lines of these triangulations were measured using 30-m steel tapes, checked by the Sub-Department of Geodesy of the Delft University of Technology. The measured distances were reduced to mean sea level and corrected for sagging of the tapes and map projection. The angles were measured with the same T3 theodolites as used for the other angle measurements. Short descriptions and sketches of the stations are given below.

EIERLAND (Fig. 2.2.8)

The reconnaissance of this primary station was carried out in 1895; the angle measurements and the reduction to centre date from 1901. In 1946 a new permanent mark (brass bolt, V5) was placed in one of the walls of the lightkeeper's house. Its position was determined with respect to permanent mark V1 of 1901. In 1950 the latter mark was lost as a result of adding a brickjacket to the outer wall of the lighthouse. Permanent mark V6 was put into the new wall at about the same place of the now hidden mark V1. By reconstructing the measurements of permanent mark V5 of 1946, the positions of V6 with respect to V1 was checked.

Since the station is a lighthouse, it can only be used in the day-time for angle measurements.

SEXBIERUM (Fig. 2.2.9)

The tower of the Reformed church is a primary station, the details of which are: reconnaissance 1886 and 1896, station building 1897, angle measurements 1902. An instrument stand was constructed on the southwest pillar of the railing surrounding the gallery of the spire. The railing was reconstructed in such a way that the observation pillar became completely free from it (see Fig. 2.2.9).

WORKUM (Fig. 2.2.10)

This primary station is the tower of the Reformed church. The details are as follows: reconnaissance 1886 and 1895, angle measurements 1901, reduction to centre 1900. A brass bolt, V7, was placed in the concrete roof surrounding the lantern of the tower. A brick pillar for setting up the theodolite was erected just over this point. As it was not allowed to have a permanent observation pillar on this tower, it was removed as soon as the angle observations for the base extension net were completed.

EIERLAND NL 13

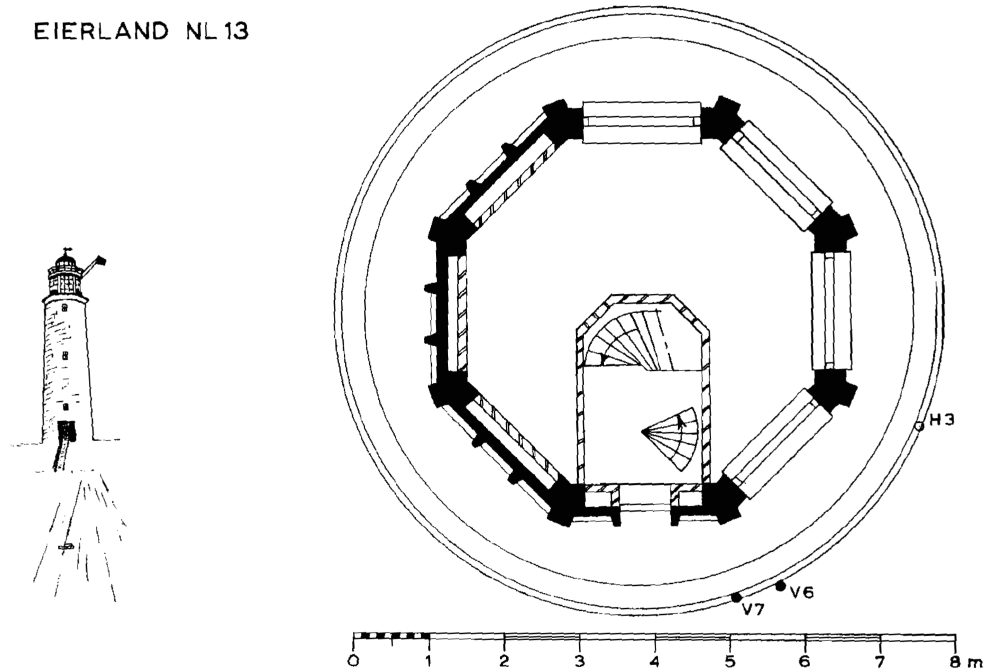


Fig. 2.2.8

SEXBIERUM NL 57

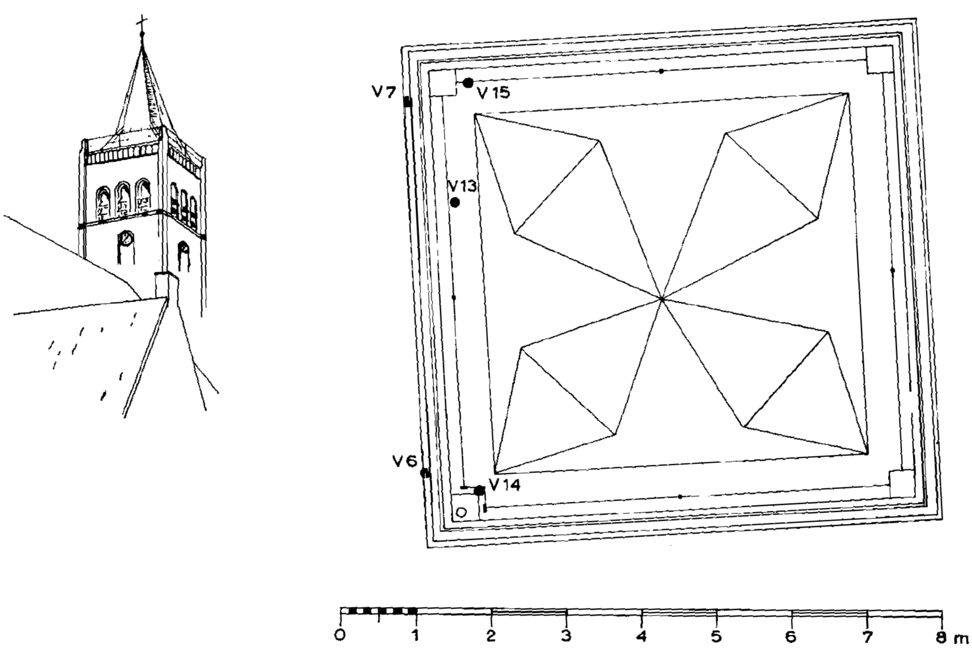


Fig. 2.2.9

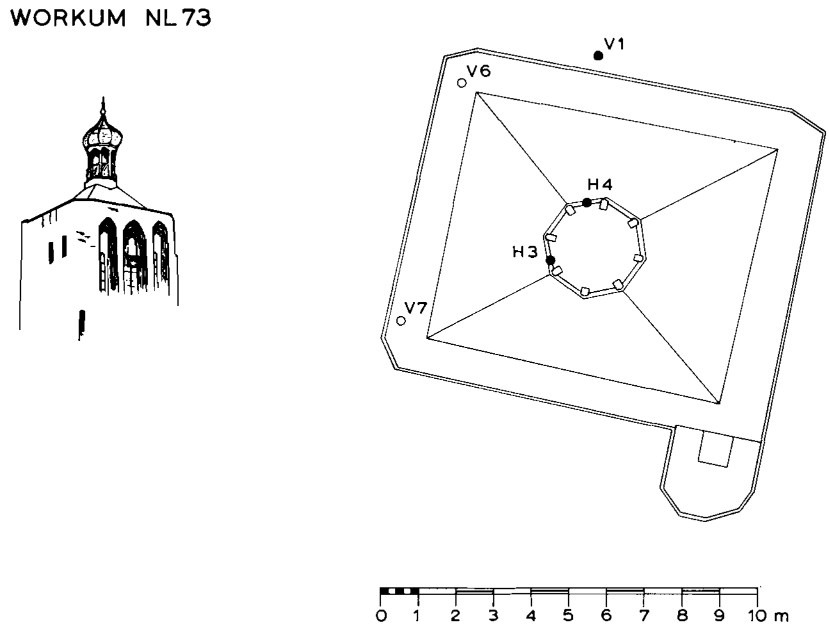


Fig. 2.2.10

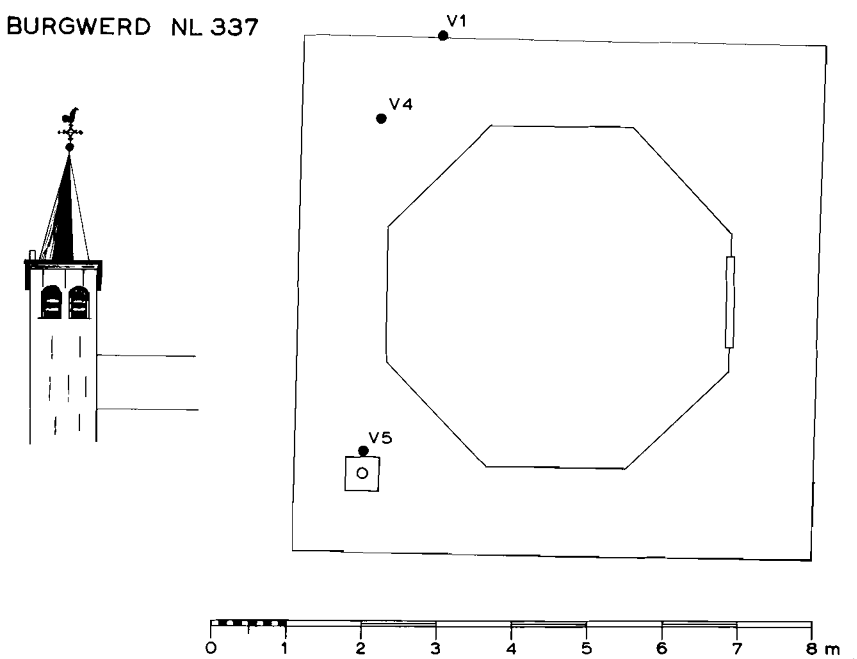


Fig. 2.2.11

BURGWERD (Fig. 2.2.11)

This station (Reformed church) is a first order intermediate point. Originally it was planned as a primary point but in 1901–1902 its position was determined by intersection from three other primary points.

For the base extension network a new permanent pillar was built on the southwest corner of concrete floor of the gallery surrounding the spire.

STAVEREN (Fig. 2.2.12)

The tower of the Reformed church is a lower order point of the triangulation of The Netherlands. An observation pillar was built on the concrete edge just outside the railing of the cupola. The observer had to stand inside the cupola where the church-bells hindered him in his movements. Since a yearly rent has to be paid for the pillar, it will be removed as soon as no further measurements are intended.

STEVINSLUIZEN (Fig. 2.2.13)

The observation pillar was erected on the northeast tower of the sluices. To protect instrument and observer against bad weather, a portable wooden shed was constructed with shutters at eye-level.

LORENTZSLUIZEN (Fig. 2.2.14)

The instrument pillar is situated on the southwest tower of the sluices. For the observations at this station the portable wooden shed was used again.

STAVEREN NL 310

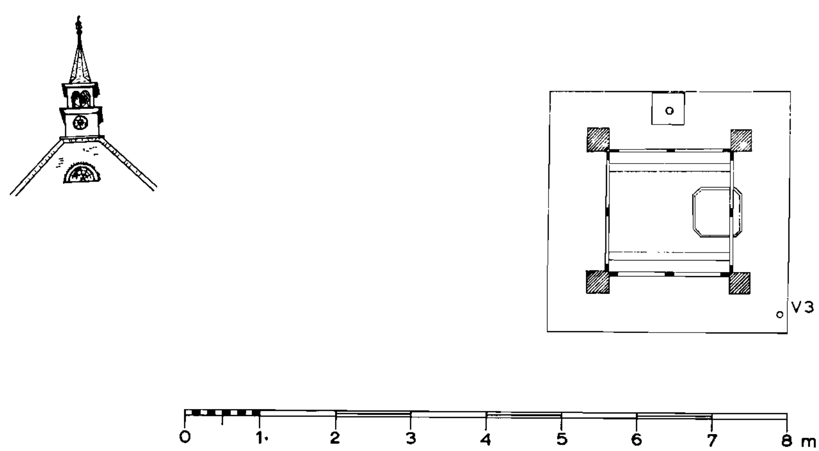


Fig. 2.1.12

STEVINSLUIZEN

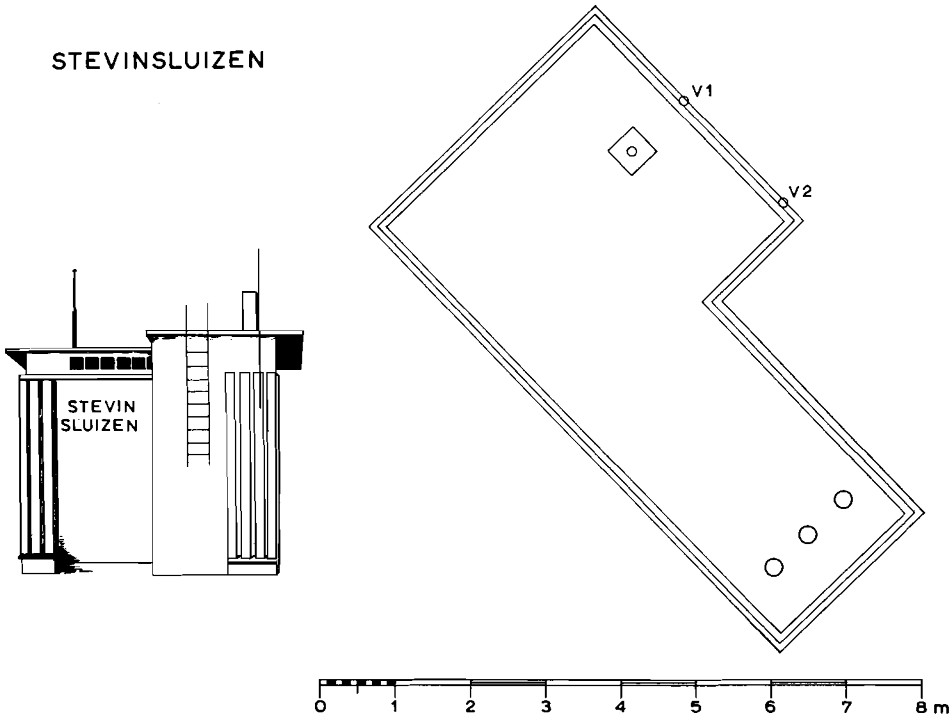


Fig. 2.2.13

LORENTZSLUIZEN NL 311

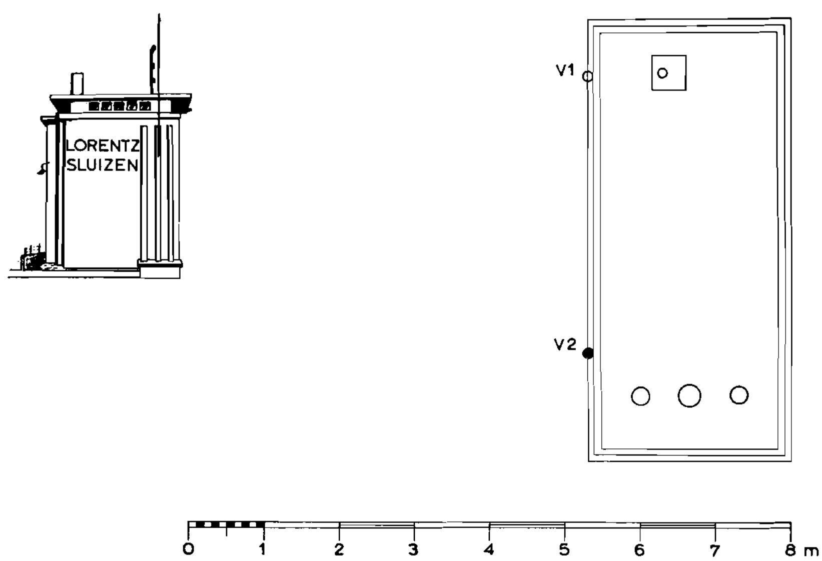


Fig. 2.2.14

2.2.3 *Determination of the deflection points of the base*

The deflection points of the base (see Fig. 1.1.1) were determined by angle measurement at the station Lorentzsluizen. This was done on September 29, October 11, 12 and 13, 1965, a few months after the other angle measurement at this station were completed. Only the angles between the two deflection points and reference point No. 1 of Lorentzsluizen were observed, each angle 24 times. Determining the difficult direction to the other terminal of the base (Stevinsluizen) was in this way avoided, provided the position of reference point No. 1 of Lorentzsluizen had not changed since the angle measurements at this station for the base extension network were completed. To check this, the angle between the two reference points of Lorentzsluizen was remeasured on October 11, 12 and 13, 1965. The results were in good agreement with those obtained previously from which it was concluded that no change had occurred in the mean time (see section 2.3.1).

2.2.4 *Connection between endpoints invar wire measurements and terminals of the base*

The endpoints of the invar wire measurements at ground level had to be connected to the terminals of the base (brass bolts) in the pillars on top of the sluice towers. The way in which this was done is shown by Figs. 2.2.15 and 2.2.16. The iron pipes *S* (Stevinsluizen and *L* (Lorentzsluizen) served as instrument pillar. Both are situated on top of the dam, see Figs. 1.1.3 and 1.1.4. The instrument used was a Wild T3 theodolite and the angles were measured five times, distributed over five days.

2.2.5 *Connection between the geodimeter pillar and the base*

The geodimeter pillar on top of the dam and at equal distance of the endpoints (see section 1.3) was situated near the junction of sections X and XI (see Fig. 2.2.17). The theodolite

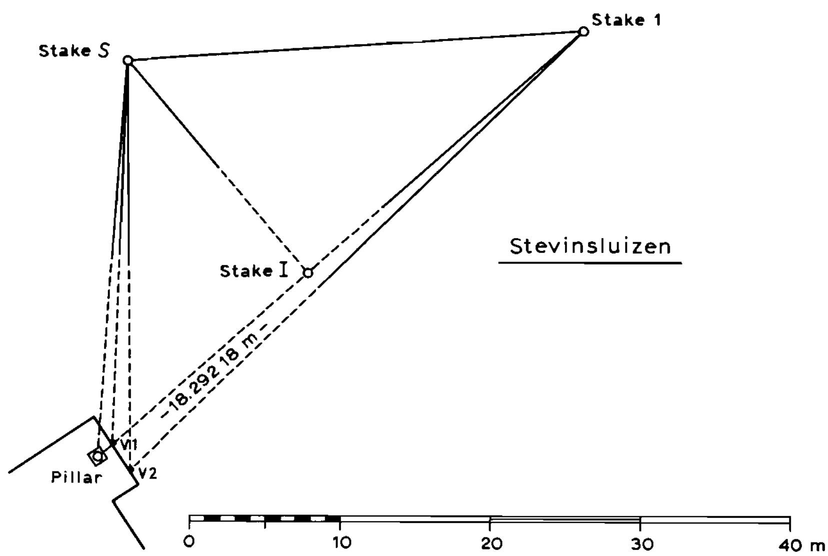


Fig. 2.2.15

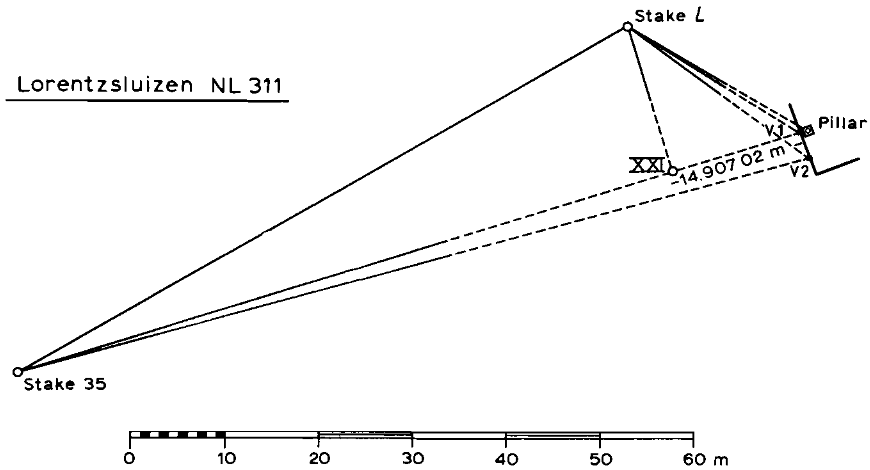


Fig. 2.2.16

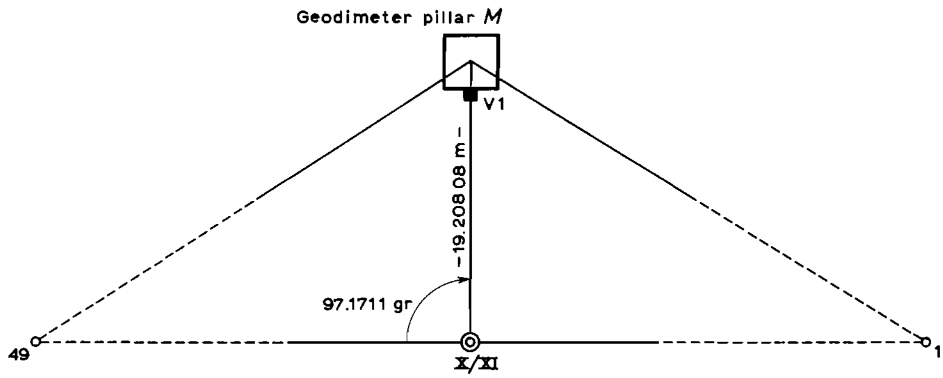


Fig. 2.2.17

was set up on the geodimeter pillar and on the junction pillar. At both points the three directions as indicated in Fig. 2.2.17 were determined. The distance geodimeter pillar – junction pillar was calculated from the two triangles and the mean of the two values thus obtained was taken.

2.2.6 *Alignment of the section endpoints of the base*

The alignment of the three parts of the base was checked by setting up a theodolite at each section endpoint and measuring the angle between the preceding section and the next one. The results indicated that it was not necessary to apply a correction for non-alignment.

2.3 Analysis of the results

As stated in chapter 1 (table 1.2.8), a value of 23,970.091 m can be taken for the distance Stevinsluizen-Lorentzsluizen. By means of the extension network, shown in Fig. 2.3.1, the length of side Eierland-Workum, (1–3), can be derived. In accordance with conclusion 5, section 2.1.3 (page 45), it was decided to use this side of the Netherlands primary network for the new adjustment of the European triangulation.

In section 2.2 are given the angle measurements on which the computations are to be based (tables 2.2.1–2.2.14). Before using the observations it might be useful to make a statistical analysis of the observations itself. This analysis can be distinguished in three parts, referring to resp. the station adjustment, the adjustment of the network and the comparison with previous primary measurements. In this order the analysis is given, after

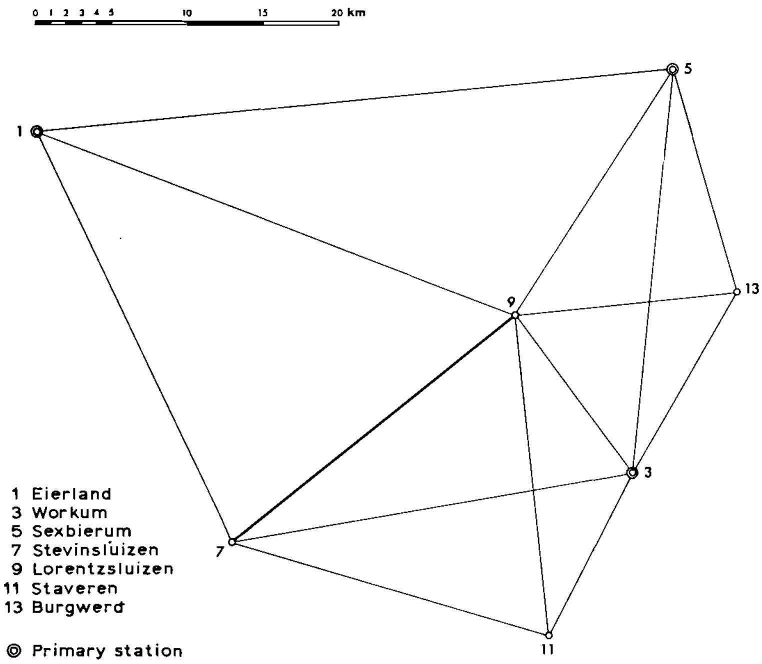


Fig. 2.3.1

dealing first with the angle measurement needed to reduce the invar wire measurement to a straight line connecting the terminals of the base.

2.3.1 *Analysis of the angle measurements needed for the determination of the deflection points*

As proved in section 2.1.2, only at the station Lorentzsluizen angle measurements are required to determine the two deflection points of the base.

Using the same reference directions for both the determination of the deflection points and the measurements of the extension network, measuring of the difficult direction to the station Stevinsluizen is evaded. This method is only admissable when it can be ascertained that the same reference direction applies to both measurements. A check was obtained by repeated measurement of the angle α between the two reference directions at the station Lorentzsluizen. Table 2.3.1 shows the means per day for this angle in centesimal seconds. Taking the mean for measurements e and d separately, an estimate can be computed for the variance factor using the equation:

Table 2.3.1

e: extension network		d: deflection points	
date	α_e	date	α_d
June 9	67.7	Oct. 11	69.0
June 17	67.8	Oct. 12	68.0
July 7	70.1	Oct. 13	67.4
July 13	69.4		
mean $\bar{\alpha}_e$	68.7	mean $\bar{\alpha}_d$	68.1

$$\hat{\sigma}_{\mathrm{I}}^2 = \frac{\Sigma(\alpha_e-\bar{\alpha}_e)^2+\Sigma(\alpha_d-\bar{\alpha}_d)^2}{4-1+3-1} = 1.11$$

and taking the mean of $\bar{\alpha}_e$ and $\bar{\alpha}_d$, a second estimate can be obtained according:

$$\hat{\sigma}_{\mathrm{II}}^2 = \frac{4(\bar{\alpha}_e-\bar{\alpha})^2+3(\bar{\alpha}_d-\bar{\alpha})}{1} = 0.63$$

with:

$$\bar{\alpha} = \frac{4\bar{\alpha}_e+3\bar{\alpha}_d}{7} = 68.4$$

A possible change of the angle can be checked using [9, page 16, (2.3.5)] by comparison of $\hat{\sigma}_{\mathrm{II}}^2/\hat{\sigma}_{\mathrm{I}}^2(=0.57)$ with: $F_{0.95;1,5}=6.61$ (like in the following argumentation, for the significance level is always used the value $\alpha=0.05$). This leads to the conclusion that the angle between the reference directions of the station Lorentzsluizen has not changed in the mean time.

2.3.2 *Check on the occurence of lateral refraction by means of station adjustment*

As mentioned in section 2.2.1 (page 50) the measurements of the various angles at all stations were distributed over at least four observation periods in order to eliminate as much as possible the influence of lateral refraction. For it is not to be expected that refraction will manifest itself in every period in the same way. Of interest is now to check afterwards whether symptoms of refraction can be detected in the measurements. This is possible in the following way.

Suppose a direction r is measured on d different days, numbered $i=1,2,\ldots,d$. Suppose the number of observations per day i equals n_i, numbered $j=1,2,\ldots,n_i$. Then every observation can be denoted by r_{ij}, in which: $i=$ number of the day; $j=$ number of the observation on day i.

Using all the observations it is possible to compute means per day:

$$\bar{r}_i = \frac{\sum_{j=1}^{n_i} r_{ij}}{n_i}$$

At the same time one can obtain a total mean:

$$\bar{r} = \frac{\sum_{i=1}^{d} \bar{r}_i}{d}$$

Supposing $v_{ij} = r_{ij} - \bar{r}_i$ and $\bar{v}_i = r_i - r$ and choosing the variance factor σ^2 in such a way, that the weight coefficient for a single direction equals 1, then it is possible to compute for every direction:

$$\hat{\sigma}_{\mathrm{I}}^2 = \frac{\sum_{i=1}^{d} \sum_{j=1}^{n_i} v_{ij}^2}{\sum_{i=1}^{d} (n_i - 1)} \quad \text{and} \quad \hat{\sigma}_{\mathrm{II}}^2 = \frac{\sum_{i=1}^{d} n_i \bar{v}_i^2}{d-1}$$

A check on the occurrence of errors, whereby in the first place is thought of those caused by lateral refraction, is possible by comparison of:

$$\frac{\hat{\sigma}_{\mathrm{II}}^2}{\hat{\sigma}_{\mathrm{I}}^2} \quad \text{with: } F_{1-\alpha; d-1, \sum_{i=1}^{d} (n_i - 1)}$$

Table 2.3.2. Testing station adjustment

1	2	3	4	5	6
1–5	1.80	5.74	3.2	3.1	4.9
1–7	0.62	5.52	8.9	3.1	4.9
1–9	2.67	1.64	0.6	3.1	4.9
3–5	4.95	6.58	1.3	2.9	4.5
3–7	4.26	15.51	3.6	2.9	4.5
3–9	4.19	16.02	3.8	2.9	4.5
3–11	9.73	20.67	2.1	2.9	4.5
3–13	4.79	12.46	2.6	3.1	4.9
5–1	4.96	8.50	1.7	3.1	4.9
5–3	5.85	10.22	1.7	3.1	4.9
5–9	4.62	10.38	2.2	3.1	4.9
5–13	8.13	10.20	1.2	3.1	4.9
7–1	1.05	5.92	5.6	3.1	4.9
7–3	1.84	6.34	3.4	3.1	4.9
7–9	1.00	1.80	1.8	3.1	4.9
7–11	1.71	4.90	2.9	3.1	4.9
9–1	0.35	10.48	30.0	3.1	4.9
9–3	0.57	3.38	5.9	3.1	4.9
9–5	1.48	5.48	3.7	3.1	4.9
9–7	1.20	2.94	2.4	3.1	4.9
9–11	0.93	2.76	3.0	3.1	4.9
9–13	1.34	2.46	1.8	3.1	4.9
11–3	4.87	13.96	2.9	3.1	4.9
11–7	4.92	11.50	2.3	3.1	4.9
11–9	3.28	3.47	1.1	2.9	4.5
13–3	2.54	7.08	2.8	3.1	4.9
13–5	2.72	5.38	2.0	3.1	4.9
13–9	4.07	3.46	0.9	3.1	4.9

Explanation to columns:

1: direction

2: $\hat{\sigma}_{\mathrm{I}}^2$

3: $\hat{\sigma}_{\mathrm{II}}^2$

4: $\dfrac{\hat{\sigma}_{\mathrm{II}}^2}{\hat{\sigma}_{\mathrm{I}}^2}$

5: $F_{0.95; d-1, \Sigma i(n_i - 1)}$

6: $F_{0.99; d-1, \Sigma i(n_i - 1)}$

The data, needed for the computation, can be taken from the tables 2.2.1–2.2.7. The results are shown in table 2.3.2.

From table 2.3.2 it follows that in general $\hat{\sigma}_{\mathrm{II}}^2$ is larger than $\hat{\sigma}_{\mathrm{I}}^2$ (in 26 out of the 28 cases). In 10 of the 28 cases $\hat{\sigma}_{\mathrm{II}}^2$ is significant larger than $\hat{\sigma}_{\mathrm{I}}^2$ with a level of significance of 0.05 and in 4 of the 28 cases $\hat{\sigma}_{\mathrm{II}}^2$ is significant larger than $\hat{\sigma}_{\mathrm{I}}^2$ with a level of significance of 0.01.

So the difference is striking, and the results give a good justification of the method to divide the measurement of each angle over several days. Frequently differences occur between the observations carried out on several days, while measurements carried out on one day do not show any sign of it. The most obvious explanation is: the influence of refraction is about constant in one observation period, but varies from period to period. Consequently the method applied improves the reliability of the final result.

2.3.3 *Adjustment of the extension network*

Starting point for the analysis are the observations, adjusted per station, as given in table 2.2.15. Since it leads to the most simple procedure, the method of condition equations is used. For the computations 31 condition equations are formed, according the formulas (3.3.5)–(3.3.35). Table 2.3.3 gives the condition equations in numerical form. (*Table* 2.3.3 *is included in folding page at the end of this book*).

Only 11 of the 31 given condition equations mentioned are independent. For this 11 are chosen the equations 1–8, 11, 14 and 17, marked with an asterisk in table 2.3.3 (see also (3.3.36)).

The observations can be analysed according to the testing method given in [9], based on the adjustment of one or more condition equations. For the choice of the condition equations (forming the matrix of coefficients of the condition equations) are three possibilities:

1. All condition equations together.
2. Each of the 31 condition equations separately.
3. Transformation of condition equations in accordance with the method of data-snooping, given in [11], chapter 4.

All these three possibilities are applied. The results are given in resp. the tables 2.3.4, 2.3.3 and 2.2.15. Naturally all tables are based on the same observations; therefore the results are not independent. All computations are based on one assumption for the matrix of variances and covariances of the observations. The covariances σ^{ij} of the directions are taken zero. For the variances σ^{ii} is taken: (1 centesimal second)^2. This assumption is based on data of a large part of the primary triangulation of the Netherlands.

From the testing of the observations of 1965 (see table 2.3.4) it is evident that the results are unsatisfactory. $\hat{\sigma}^2/\sigma^2$ differs significantly from 1, even with a level of significance of

Table 2.3.4. Testing of the entire network

	$\hat{\sigma}^2/\sigma^2$
1965	2.61
1966–'67	1.55

$F_{0.95;11,\infty} = 1.79$

$F_{0.99;11,\infty} = 2.43$

only 0.01. The separate condition equations of table 2.3.3 were further analysed with the object to localize the errors.

Taking into account that $F_{0.95;1,\infty} = 3.84$ and $F_{0.995;1,\infty} = 7.9$, the following conclusions can be drawn:

1. The conditions 3, 8, 14, 15, 17, 18, 19, 21, 26 and 27 show, with $\alpha = 0.05$, a significant deviation. The conditions 14, 17, 18 and 21 give the same picture with a level of significance of 0.005.
2. All conditions with significant deviation are situated in quadrilateral 9-7-11-3, with the exception of condition 17: $W_{(9),3,5,1,7}$. This gives some localization of the errors.
3. It is striking, that the triangles 3-9-7 and 3-9-11 (resp. condition 3 and 8) show large positive misclosures and the triangles 5-9-3 and 13-9-3 (resp. condition 4 and 6) rather large negative misclosures. This warrants the supposition, that an error has been made in measuring the directions $r_{3,9}$ and $r_{9,3}$. However also the conditions 17 and 18 (in which $r_{9,3}$ is not included) and the condition 19 (lacking $r_{3,9}$) show important deviations, so that it is most likely that the error can not be attributed to one particular observation. It was tried to relate the measurements to the circumstances under which they were made. These circumstances are partly summed up in the Figs. 2.2.1.–2.2.7. A clear relation could not be found. *On account of the results obtained and in view of the importance of the network it was decided to repeat the measurements completely.* This was done in the years 1966 and 1967.

The third method of analysis ("data-snooping", see [11], chapter 4) was not developed yet when the decision about remeasurements was taken. However, it was applied afterwards. The w-quantities concerned are given in column 10 of table 2.2.15. The w-quantities are linear functions of the observations and have a $\sqrt{F_{1,\infty}}$-distribution. Each w-quantity is related to one of the directions and has, compared with other linear functions of the observations, a maximal power for an error in the direction to which it refers.

In view of $\sqrt{F_{0.95;1,\infty}} = 1.96$ and $\sqrt{F_{0.999;1,\infty}} = 3.29$ it can be concluded from column 10 of table 2.2.15 that the directions $r_{1,5}$, $r_{3,7}$, $r_{3,9}$, $r_{11,3}$ and $r_{11,9}$ are suspicious, in particular the directions $r_{3,7}$ and $r_{3,9}$. Also from this analysis it follows, that possible errors are to be looked for especially in the quadrilateral 9-7-11-3 but there is no clear indication for an error in one particular direction. This analysis supports the decision foı a complete remeasurement of the extension network.

Naturally also the measurements of 1966–'67 were analyzed. From the testing of $\hat{\sigma}^2/\sigma^2$, obtained from the whole network, it follows (see table 2.3.4), that the remeasurement gives more satisfying results, although the value obtained is still rather high. This conclusion is confirmed by the analysis of the w-quantities of the observations of 1966–'67 (column 11 of table 2.2.15). Only one observation, namely $r_{9,1}$, gives reason to mistrust. Comparison of the columns 10 and 11 shows that there is no clear relation between the corresponding w-quantities of 1965 and 1966–'67.

Therefore, and in view of the fact, that the quality of the measurements presents no convincing differences, it was decided to base the computation of the side of the primary network Eierland-Workum on the mean of the direction measurements of 1965 and 1966–'67.

Evidently the measurements contain some irregularities which can not completely be traced by remeasurement. However they do not repeat themselves in a next measuring season.

2.3.4 *Comparison with previous measurements of the primary network*

Fig. 2.3.1, shows that not two but three stations of the primary network are included in the base extension network, namely the stations Eierland, Workum and Sexbierum. Consequently not only the length of the side resulting from the base extension network can be compared with the value of the same side in the primary network, but also the form of the triangle, formed by the three stations mentioned, can be compared. Since the form of a triangle is determined by one Π-quantity, one can suffice by comparing one angle and one distance-ratio. The angle and the distance-ratio in the station Eierland are taken for this purpose.

Table 2.3.5. Comparison with primary network

	1965	1966–'67	mean of the measurement	from primary network
α_{513}	39.40609	39.40600	39.40605	39.40609
$\ln v_{513}$	0.0714360	0.0714356	0.0714358	0.0714329
$s_{1,3}$	45373.946	45373.860	45373.903	45373.636
$s_{1,5}$	42245.677	42245.616	42245.647	42245.521
$s_{3,5}$	26852.784	26852.676	26852.730	26852.627

Remark: The comparison is made in the plane in which the primary network was originally computed. Therefore corrections had to be applied to the measurements of the base extension network.

Table 2.3.5 shows that there are considerable differences. In this respect not too much value should be attached to the comparison of the lengths of the sides: in the primary network these lengths are obtained by transfer of scale via a number of triangles. More sense has the comparison of $\alpha_{5,1,3}$ and $\ln v_{5,1,3}$. Testing of the differences can be carried out by introducing as null hypothesis:

$$\tilde{\alpha}_{3,1,5} = \tilde{\bar{\alpha}}_{3,1,5}$$
$$\widetilde{\ln v}_{3,1,5} = \widetilde{\ln \bar{v}}_{3,1,5}$$

with $\tilde{\alpha}_{3,1,5}$ en $\widetilde{\ln v}_{3,1,5}$ taken from the extension network, and $\tilde{\bar{\alpha}}_{3,1,5}$ and $\widetilde{\ln \bar{v}}_{3,1,5}$ taken from the primary network. Taking into account the weight coefficients of the four quantities concerned an estimate for σ^2 can be computed. This leads to, see table 2.3.6:

Table 2.3.6

	1965	1966–'67	mean of both measurements	$F_{0.95;2,\infty}$
$\hat{\sigma}^2/\sigma^2$	2.7	2.3	2.6	3.0

Obviously there are some differences between the results of 1965–'67 and the earlier measurements but these differences are not significant. Moreover these results do not indicate salient differences between the measurements of 1965 and 1966–'67. This gives again support to the correctness of the course followed, i.e. to combine the observations of both periods.

2.3.5 *Final result*

On account of the analysis carried out it can be concluded, that for the length of the side of the primary network Eierland-Workum can be taken the value:

45,373.903 m,

with a standard deviation of 70 mm, according to table 2.1.5. For the reliability of the length, see section 2.1.4.

Chapter 3

SOME THEORETICAL ASPECTS

The methods, applied to preparation and analysis of the base extension network, are almost entirely based on the theories of BAARDA, as published in [9]–[13]. In this chapter a summary will be given of that part of the theories that was applied to the problems of the base extension network. It consists of two parts: Functional relations between observation quantities in the plane (section 3.1), and: Adjustment (section 3.2). The formulas of section 3.1 are applied to the base extension network in section 3.3.

3.1 Functional relations in the plane

In accordance with [10] we write for a number of points i, j, k, etc. in a rectangular coordinate system:

Coordinates of point P_i: x_i, y_i . . . . . . (3.1.1)

Combined to a complex number: $z_i = y_i + \mathrm{i}x_i$ . . . . . . (3.1.2)

Differences of coordinates: $y_{ij} = y_j - y_i$ . . . . . . (3.1.3)

$x_{ij} = x_j - x_i$

and complex: $z_{ij} = y_{ij} + \mathrm{i}x_{ij}$ . . . . . . (3.1.4)

Distance, respectively bearings: s_{ij}, A_{ij} . . . . . . (3.1.5)

Combined to a complex number: $\Lambda_{ij} = \ln z_{ij} = \ln s_{ij} + \mathrm{i}A_{ij}$ . . . . . . (3.1.6)

and analogously: $\Lambda_{ij}^{(i)} = \ln z_{ij}^{(i)} = \ln d_{ij} + \mathrm{i}r_{ij}$ . . . . . . (3.1.7)

with: r_{ij} = direction between the points i and j
d_{ij} = distance-measure between i and j

$$\left.\begin{aligned} \ln v_{jik}(\text{distance-ratio}) &= \ln s_{ik} - \ln s_{ij} = \ln d_{ik} - \ln d_{ij} \\ \alpha_{jik} &= A_{ik} - A_{ij} = r_{ik} - r_{ij} \end{aligned}\right\} \quad \ldots\ldots \quad (3.1.8)$$

Again combined to a complex number:

$$\Pi_{jik} = \ln v_{jik} + \mathrm{i}\alpha_{jik} = \Lambda_{ik} - \Lambda_{ij} \quad \ldots\ldots \quad (3.1.9)$$

From (3.1.6) and (3.1.9) follows:

$$\boxed{z_{ik} = z_{ij} e^{\Pi_{jik}}} \qquad (3.1.10)$$

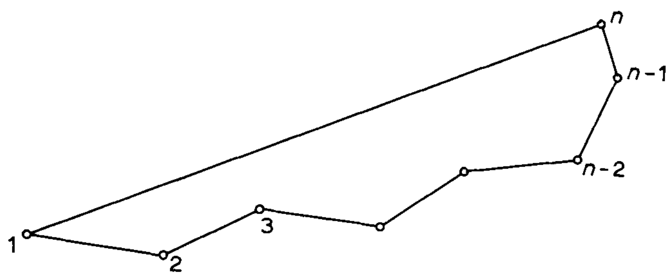


Fig. 3.1.1

Repeated application of (3.1.10) in Fig. 3.1.1 gives:

$$\left.\begin{aligned}
z_{n1} &= z_{n1} \\
z_{12} &= -z_{n1} e^{\Pi_{n12}} \\
z_{23} &= -z_{12} e^{\Pi_{123}} = (-1)^2 z_{n1} e^{\Pi_{n12}+\Pi_{123}} \\
&\dots \\
z_{n-1,n} &= -z_{n-2,n-1} e^{\Pi_{n-2,n-1,n}} = (-1)^{n-1} z_{n1} e^{\Pi_{n12}+\Pi_{123}+\dots+\Pi_{n-2,n-1,n}}
\end{aligned}\right\} \qquad (3.1.11)$$

$$z_{n,1} = -z_{n-1,n} e^{\Pi_{n-1,n,1}} = (-1)^n z_{n1} e^{\Pi_{n12}+\Pi_{123}+\dots+\Pi_{n-1,n,1}} \qquad (3.1.12)$$

Adding of all relations in (3.1.11) gives the network- or coordinate relation $N_{(n),1,2\cdots n-1}$

$$\boxed{\begin{array}{r} z_{n,n} = 0 = z_{n1}(1 - e^{\Pi_{n12}}(1 - e^{\Pi_{123}}(1 - \dots(1 - e^{\Pi_{n-2,n-1,n}})\dots))) \\ \boxed{N_{(n),1,2\dots n-1}} \end{array}} \qquad (3.1.13)$$

From (3.1.12) follows the polygon relation: $V_{1,2\dots n}$

$$\left.\begin{array}{l} e^{\Pi_{n,1,2}+\Pi_{1,2,3}+\dots+\Pi_{n-1,n,1}} = (-1)^n \\ \text{or:} \quad \boxed{\begin{array}{r} \Pi_{n,1,2}+\Pi_{1,2,3}+\dots+\Pi_{n-1,n,1} = n\ln(-1) \\ \boxed{V_{1,2\dots n}} \end{array}} \end{array}\right\} \qquad (3.1.14)$$

(3.1.14) can be split up in real and imaginary part:

$$\underset{\mathrm{Re}\{V_{1,2\cdots n}\}}{\ln v_{n,1,2}+\ln v_{1,2,3}+\dots+\ln v_{n-1,n,1} = 0} \qquad (3.1.14a)$$

$$\underset{\mathrm{Im}\{V_{1,2\cdots n}\} \quad k = \text{integer}}{\alpha_{n,1,2}+\alpha_{1,2,3}+\dots\alpha_{n-1,n,1} = n\pi \pm k2\pi} \qquad (3.1.14b)$$

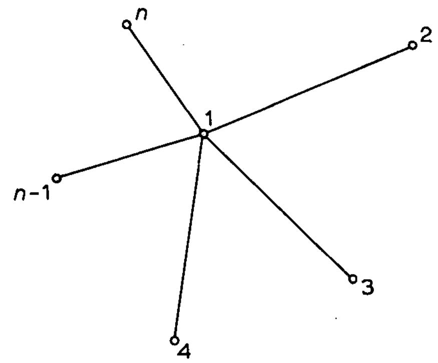


Fig. 3.1.2

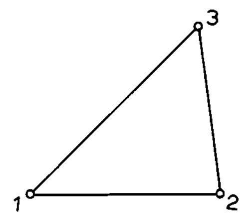


Fig. 3.1.3

Repeated application of (3.1.10) in Fig. 3.1.2 gives:

$$\left.\begin{aligned} z_{12} &= z_{12} \\ z_{13} &= z_{12}e^{\Pi_{213}} \\ z_{14} &= z_{13}e^{\Pi_{314}} = z_{12}e^{\Pi_{213}+\Pi_{314}} \\ z_{1n} &= z_{1,n-1}e^{\Pi_{n-1,1,n}} = z_{12}e^{\Pi_{213}+\Pi_{314}+\ldots+\Pi_{n-1,1,n}} \\ z_{12} &= z_{1,n}e^{\Pi_{n,1,2}} = z_{12}e^{\Pi_{213}+\Pi_{314}+\ldots+\Pi_{n,1,2}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3.1.15)$$

or the central relation $W_{(1),2,\ldots,n}$

$$\boxed{\begin{array}{l} \Pi_{213}+\Pi_{314}+\ldots+\Pi_{n-1,1,n}+\Pi_{n,1,2} = 0 = \mathrm{i}\,k2\pi \\ \hfill W_{(1),2,\ldots n} \quad k = \text{integer} \end{array}} \quad \ldots\ldots\ldots\ldots \quad (3.1.16)$$

Splitting up (3.1.16) in a real and a imaginary part gives:

$$\ln v_{213}+\ln v_{314}+\ldots+\ln v_{n-1,1,n}+\ln v_{n12} = 0 \quad \ldots\ldots\ldots\ldots \quad (3.1.16a)$$
$$\mathrm{Re}\{W_{(1),2,3,\ldots n}\}$$

$$\alpha_{213}+\alpha_{314}+\ldots+\alpha_{n-1,1,n}+\alpha_{n,1,2} = 2\pi \quad \ldots\ldots\ldots\ldots \quad (3.1.16b)$$
$$\mathrm{Im}\{W_{(1),2,3\ldots n}\}$$

Application in a triangle (Fig. 3.1.3)

Write Π_1, Π_2, Π_3 for resp. Π_{312}, Π_{123} and Π_{231}. Then (3.1.14) can be written as:

$$\Pi_1+\Pi_2+\Pi_3 = 3\ln(-1) \qquad V_{1,2,3} \quad \ldots\ldots\ldots\ldots \quad (3.1.17)$$

or:

$$\ln v_1+\ln v_2+\ln v_3 = 0 \qquad \mathrm{Re}\{V_{1,2,3}\} \quad \ldots\ldots\ldots\ldots \quad (3.1.17a)$$

$$\alpha_1+\alpha_3+\alpha_3=\pi \qquad \mathrm{Im}\{V_{1,2,3}\} \quad \ldots \ldots (3.1.17b)$$

Differentiated:

$$\Delta\Pi_1+\Delta\Pi_2+\Delta\Pi_3=0 \quad \ldots \ldots (3.1.18)$$

or:

$$\begin{cases}\Delta\ln v_1+\Delta\ln v_2+\Delta\ln v_3=0 & \ldots \ldots (3.1.18a)\\ \Delta\alpha_1+\Delta\alpha_2+\Delta\alpha_3=0 & \ldots \ldots (3.1.18b)\end{cases}$$

For (3.1.13) can be written:

$$1-e^{\Pi_1}+e^{\Pi_1+\Pi_2}=0$$

Or:

$$e^{-\Pi_1}+e^{\Pi_2}=1 \quad \ldots \ldots (3.1.19)$$

With (3.1.9):

$$\frac{1}{v_1}e^{-i\alpha_1}+v_2e^{i\alpha_2}=1$$

Splitting up in a real and a imaginary part:

$$\frac{1}{v_1}\cos\alpha_1+v_2\cos\alpha_2=1 \quad \ldots \ldots (3.1.20)$$

$$-\frac{1}{v_1}\sin\alpha_1+v_2\sin\alpha_2=0 \quad \ldots \ldots (3.1.21)$$

(3.1.21) with (3.1.17a) gives:

$$\frac{1}{v_1v_2}=\boxed{v_3=\frac{\sin\alpha_2}{\sin\alpha_1}} \quad \ldots \ldots (3.1.22)$$

With some reduction (3.1.20), (3.1.21) and (3.1.17b) give:

$$\boxed{\alpha_3=\arccos\tfrac{1}{2}\left(\frac{v_1}{v_2}-\frac{1}{v_1v_2}-v_1v_2\right)} \quad \ldots \ldots (3.1.23)$$

Linearization of (3.1.19) gives:

$$-e^{-\Pi_1}\Delta\Pi_1+e^{\Pi_2}\Delta\Pi_2=0 \quad \ldots \ldots (3.1.24)$$

Or:

$$-\frac{1}{v_1}(\cos\alpha_1 - \mathrm{i}\sin\alpha_1)\Delta\Pi_1 + v_2(\cos\alpha_2 + \mathrm{i}\sin\alpha_2)\Delta\Pi_2 = 0 \qquad (3.1.25)$$

With (3.1.22):

$$-\frac{\sin\alpha_2\sin\alpha_1}{\sin\alpha_3}(\cot\alpha_1 - \mathrm{i})\Delta\Pi_1 + \frac{\sin\alpha_1\sin\alpha_2}{\sin\alpha_3}(\cot\alpha_2 + \mathrm{i})\Delta\Pi_2 = 0 \qquad (3.1.26)$$

Or:

$$-(\cot\alpha_1 - \mathrm{i})(\Delta\ln v_1 + \mathrm{i}\,\Delta\alpha_1) + (\cot\alpha_2 + \mathrm{i})(\Delta\ln v_2 + \mathrm{i}\,\alpha_2) = 0 \qquad (3.1.27)$$

Splitting up in a real and a imaginary part gives:

$$-\cot\alpha_1\,\Delta\ln v_1 - \Delta\alpha_1 + \cot\alpha_2\,\Delta\ln v_2 - \Delta\alpha_2 = 0 \qquad (3.1.28a)$$

$$\Delta\ln v_1 - \cot\alpha_1\Delta\alpha_1 + \Delta\ln v_2 + \cot\alpha_2\Delta\alpha_2 = 0 \qquad (3.1.28b)$$

With (18) is then:

$$\boxed{\begin{aligned} \Delta\alpha_3 &= \cot\alpha_1\Delta\ln v_1 - \cot\alpha_2\Delta\ln v_2 \\ \Delta\ln v_3 &= -\cot\alpha_1\Delta\alpha_1 + \cot\alpha_2\Delta\alpha_2 \end{aligned}} \qquad \begin{aligned} &(3.1.29a) \\ &(3.1.29b) \end{aligned}$$

Compare (3.1.29a) with (3.1.23) and (3.1.29b) with (3.1.22). In this way $\Delta\alpha_3$ is expressed in $\Delta\ln v_2$ and $\Delta\ln v_1$, and $\Delta\ln v_3$ in $\Delta\alpha_1$ and $\Delta\alpha_2$. Other relations can be formed, e.g. $\Delta\ln v_1$ can be expressed in $\Delta\ln v_2$ and $\Delta\alpha_2$ by multiplying (3.1.28a) with $-\cot\alpha_1$ and adding to (3.1.28b):

$$(1+\cot^2\alpha_1)\Delta\ln v_1 + (1-\cot\alpha_1\cot\alpha_2)\Delta\ln v_2 + (\cot\alpha_1 + \cot\alpha_2)\Delta\alpha_2 = 0$$

or:

$$\Delta\ln v_1 = -\frac{1-\cot\alpha_1\cot\alpha_2}{1+\cot^2\alpha_1}\Delta\ln v_2 - \frac{\cot\alpha_1+\cot\alpha_2}{1+\cot^2\alpha_1}\Delta\alpha_2 \qquad (3.1.30)$$

(3.1.20) can be written with (3.1.8) as:

$$\frac{s_{13}}{s_{12}}\cos\alpha_1 + \frac{s_{23}}{s_{12}}\cos\alpha_2 = 1$$

or:

$$s_{12} = s_{13}\cos\alpha_1 + s_{23}\cos\alpha_2$$

and (3.1.21) as:

$$\ln\sin\alpha_1 = \ln\sin\alpha_2 - \ln s_{13} + \ln s_{23}$$

Linearization gives:

$$s_{12}\Delta \ln s_{12} = s_{13}\cos\alpha_1\Delta \ln s_{13} + s_{23}\cos\alpha_2\Delta \ln s_{23} - s_{13}\sin\alpha_1\Delta\alpha_1 - s_{23}\sin\alpha_2\Delta\alpha_2 \quad \text{(3.1.31a)}$$

$$\cot\alpha_1\Delta\alpha_1 = \cot\alpha_2\Delta\alpha_2 - \Delta \ln s_{13} + \Delta \ln s_{23} \quad \text{(3.1.31b)}$$

Application in a quadrilateral (Fig. 3.1.4)

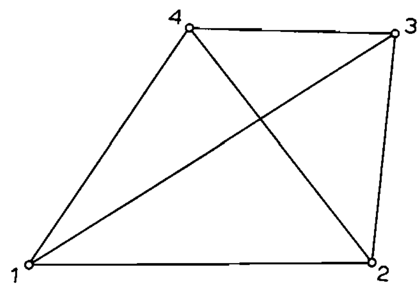


Fig. 3.1.4

In a quadrilateral (3.1.14) can be applied in several ways:

$$\ln v_{412} + \ln v_{123} + \ln v_{234} + \ln v_{341} = 0 \quad \text{(3.1.32a)}$$

$$\alpha_{412} + \alpha_{123} + \alpha_{234} + \alpha_{341} = 0 \quad \text{(3.1.32b)}$$

$V_{1,2,3,4}$

Or:

$$\ln v_{312} + \ln v_{124} - \ln v_{342} - \ln v_{134} = 0 \quad \text{(3.1.33a)}$$

$$\alpha_{312} + \alpha_{124} - \alpha_{342} - \alpha_{134} = 0 \quad \text{(3.1.33b)}$$

$V_{1,2,4,3}$

Or:

$$\ln v_{241} + \ln v_{413} - \ln v_{231} - \ln v_{423} = 0 \quad \text{(3.1.34a)}$$

$$\alpha_{241} + \alpha_{413} - \alpha_{231} - \alpha_{423} = 0 \quad \text{(3.1.34b)}$$

$V_{1,3,2,4}$

The formulas (3.1.32a), (3.1.33a) and (3.1.34a) can, with 3.1.22, be used to derive other relations between the angles of a quadrilateral:

$$\ln\sin\alpha_{241} - \ln\sin\alpha_{124} + \ln\sin\alpha_{312} - \ln\sin\alpha_{231} + \ln\sin\alpha_{423} - \ln\sin\alpha_{342} + \ln\sin\alpha_{134} - \ln\sin\alpha_{413} = 0 \quad \text{(3.1.35a)}$$

$\mathrm{Re}\{\bar{V}_{1,2,3,4}\}$

$$\begin{aligned}&\ln\sin\alpha_{231}-\ln\sin\alpha_{123}+\ln\sin\alpha_{412}-\ln\sin\alpha_{241}+\\&+\ln\sin\alpha_{423}-\ln\sin\alpha_{234}+\ln\sin\alpha_{341}-\ln\sin\alpha_{413}=0\end{aligned} \quad \mathrm{Re}\{\overline{V}_{1,2,4,3}\} \quad \ldots\ldots \quad (3.1.35b)$$

$$\begin{aligned}&\ln\sin\alpha_{124}-\ln\sin\alpha_{412}+\ln\sin\alpha_{341}-\ln\sin\alpha_{134}+\\&+\ln\sin\alpha_{312}-\ln\sin\alpha_{123}+\ln\sin\alpha_{234}-\ln\sin\alpha_{342}=0\end{aligned} \quad \mathrm{Re}\{\overline{V}_{1,3,2,4}\} \quad \ldots\ldots \quad (3.1.35c)$$

The notation $\overline{V}$ for indicating the type of relation is used to indicate that the ln v-quantities are expressed in α-quantities with (3.1.22) and the α-quantities in ln v-quantities with (3.1.23).

Differentiating, using (3.1.29b), gives:

$$\begin{aligned}&+\cot\alpha_{241}\Delta\alpha_{241}-\cot\alpha_{124}\Delta\alpha_{124}+\cot\alpha_{312}\Delta\alpha_{312}-\cot\alpha_{231}\Delta\alpha_{231}\\&+\cot\alpha_{423}\Delta\alpha_{423}-\cot\alpha_{342}\Delta\alpha_{342}+\cot\alpha_{134}\Delta\alpha_{134}-\cot\alpha_{413}\Delta\alpha_{413}=0\end{aligned} \qquad (3.1.36a)$$

and analogous, formulas (3.1.36b) and (3.1.36c) using (3.1.33a) and (3.1.34a).

Also (3.1.16) can be applied in the quadrilateral, e.g.:

$$\ln v_{413}+\ln v_{312}+\ln v_{214}=0$$

Or:

$$\ln v_{413}+\ln v_{312}-\ln v_{412}=0 \quad \mathrm{Re}\{W_{(1),4,3,2}\} \quad \ldots\ldots \quad (3.1.37)$$

Or with (3.1.22):

$$\ln\sin\alpha_{341}-\ln\sin\alpha_{134}+\ln\sin\alpha_{231}-\ln\sin\alpha_{123}+\ln\sin\alpha_{124}-\ln\sin\alpha_{241}=0 \qquad (3.1.38)$$

and differentiated:

$$\begin{aligned}&\cot\alpha_{341}\Delta\alpha_{341}-\cot\alpha_{134}\Delta\alpha_{134}+\cot\alpha_{231}\Delta\alpha_{231}-\\&-\cot\alpha_{123}\Delta\alpha_{123}+\cot\alpha_{124}\Delta\alpha_{124}-\cot\alpha_{241}\Delta\alpha_{241}=0\end{aligned} \quad \mathrm{Re}\{\overline{W}_{(1),4,3,2}\} \quad \ldots\ldots \quad (3.1.39)$$

Application in a centre-network (Fig. 3.1.5)

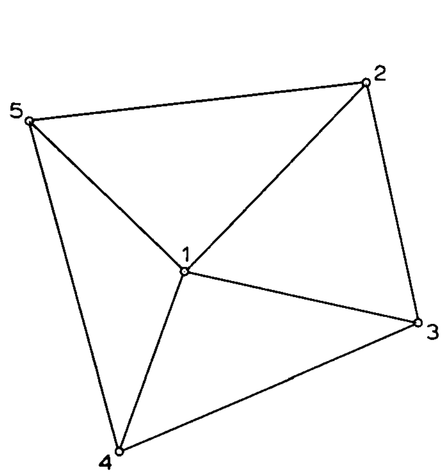


Fig. 3.1.5

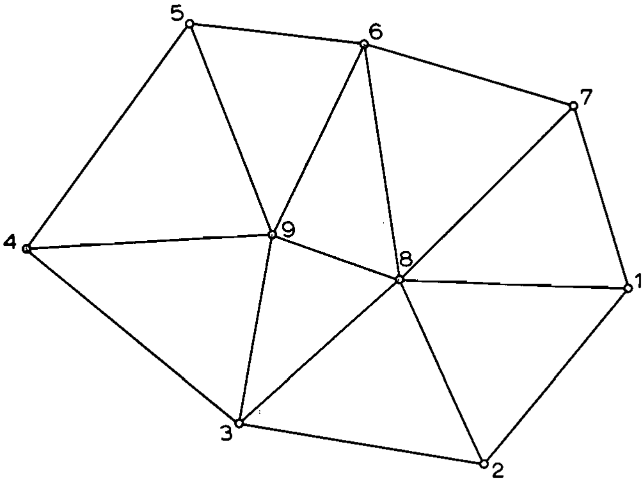


Fig. 3.1.6

Application of (3.1.16a) in the centre-network of Fig. 3.1.5 gives:

$$\boxed{\ln v_{213}+\ln v_{314}+\ln v_{415}+\ln v_{512}=0 \qquad \boxed{\mathrm{Re}\{W_{(1),2,3,4,5}\}}}$$

and again using (3.1.22):

$$\ln\sin\alpha_{321}-\ln\sin\alpha_{132}+\ln\sin\alpha_{431}-\ln\sin\alpha_{143}+$$
$$+\ln\sin\alpha_{541}-\ln\sin\alpha_{154}+\ln\sin\alpha_{251}-\ln\sin\alpha_{125}=0 \quad \text{(3.1.40)}$$

Differentiated:

$$\boxed{\begin{aligned}&\cot\alpha_{321}\Delta\alpha_{321}-\cot\alpha_{132}\Delta\alpha_{132}+\cot\alpha_{431}\Delta\alpha_{431}-\cot\alpha_{143}\Delta\alpha_{143}+\\&+\cot\alpha_{541}\Delta\alpha_{541}-\cot\alpha_{154}\Delta\alpha_{154}+\cot\alpha_{251}\Delta\alpha_{251}-\cot\alpha_{125}\Delta\alpha_{125}=0\\&\qquad\qquad\boxed{\mathrm{Re}\{\overline{W}_{(1),2,3,4,5}\}}\end{aligned}} \quad \text{(3.1.41)}$$

Application in a triangulation network (Fig. 3.1.6)

Define a distance-ratio in a more general form than in (3.1.8) as:

$$v_{i,j;k,l}=\frac{s_{kl}}{s_{ij}}$$

Assuming that i,j and k,l refer to sides of the network, one can express $v_{i,j;k,l}$ in the angles of the network, via a chain, e.g.

$$v_{1,8;5,6} = \frac{s_{56}}{s_{18}} = \frac{s_{56}}{s_{69}} = \frac{s_{69}}{s_{68}} = \frac{s_{68}}{s_{78}} = \frac{s_{78}}{s_{18}} = v_{965}v_{869}v_{786}v_{187} \quad \ldots\ldots \quad (3.1.42)$$

After linearization, with (3.1.29b):

$$\ln v_{1,8;5,6} = \cot\alpha_{659}\,\Delta\alpha_{659} - \cot\alpha_{956}\,\Delta\alpha_{956} + \cot\alpha_{698}\,\Delta\alpha_{698} - \cot\alpha_{986}\,\Delta\alpha_{986}$$
$$-\cot\alpha_{876}\,\Delta\alpha_{876} + \cot\alpha_{678}\,\Delta\alpha_{678} - \cot\alpha_{817}\,\Delta\alpha_{817} + \cot\alpha_{178}\,\Delta\alpha_{178} \qquad (3.1.43)$$

3.2 Adjustment

In accordance with [9] and [10] we indicate the adjustment, using the method of condition equations, as follows:

(x^i) outcome of practical measurement $i,j: 1,2 \ldots m$
$(\underline{x}^i)$ vector of stochastic variables
$(\tilde{x}^i)$ vector of means of $\underline{x}^i$
$(\underline{X}^i)$ vector of estimators of $(\tilde{x}^i)$

$\sigma_{\underline{x}^i\underline{x}^j} = \sigma^2 \cdot \overline{(\underline{x}^i),(\underline{x}^j)^*} = \sigma^2(g^{ij})$ σ^2 (g^{ij})	covariance matrix variance factor matrix of weight coefficients	(3.2.1)
$(u_i^\varrho)(\tilde{x}^i)-(u_0^\varrho) = 0$ $(u_i^\varrho)(\tilde{x}^i - a_0^i) = 0$	„laws of nature„ (model relations from which follow condition equations) $\varrho, \tau: 1,2 \ldots b$ (b = number of conditions)	
null hypothesis H_0		

Adjustment

$(\underline{y}^\varrho) = (u_i^\varrho)(\underline{x}^i - a_0^i)$ vector of misclosure variates . . . . (3.2.2)

$(g^{i\tau}) = (g^{ij})(u_j^\tau)^*$ . . . . (3.2.3)

$(g^{\varrho\tau}) = (u_i^\varrho)(g^{ij})(u_j^\tau)^*$ matrix of coefficients of normal equations . . . (3.2.4)

$(\bar{g}_{\tau\varrho}) = (g^{\varrho\tau})^{-1}$ . . . . (3.2.5)

$(\underline{\varepsilon}^i) = (g^{i\tau})(\bar{g}_{\tau\varrho})(-\underline{y}^\varrho)$ ("corrections") . . . . (3.2.6)

$(\underline{X}^i) = (\underline{x}^i) + (\underline{\varepsilon}^i)$ . . . . (3.2.7)

$(\underline{X}^r - a_0^r) = (A_i^r)(\underline{X}^i - a_0^i)$ derived variates . . . . (3.2.8)

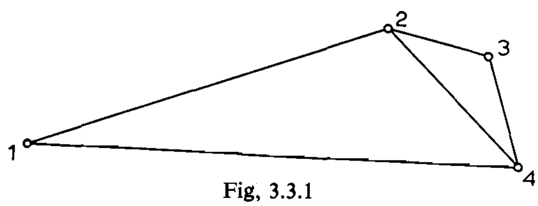
$$\left.\begin{aligned}\overline{(X^i),(X^j)^*} &= (G^{ij}) = (g^{ij}) - (g^{i\tau})(\bar{g}_{\tau\varrho})(g^{\varrho j})\\ \overline{(X^r),(X^j)^*} &= (G^{rj}) = (\Lambda^r_i)(G^{ij})\\ \overline{(X^r),(X^s)^*} &= (G^{rs}) = (\Lambda^r_i)(G^{ij})(\Lambda^s_j)^*\end{aligned}\right\}\begin{array}{l}\text{weight coefficients}\\ \text{of variates and}\\ \text{derived variates}\end{array} \quad \ldots\ldots \quad (3.2.9)$$

$$\underline{E} = (\underline{y}^\tau)^*(\bar{g}_{\tau\varrho})(\underline{y}^\varrho) \qquad \text{shifting variate} \ldots\ldots \quad (3.2.10)$$

$$\hat{\sigma}^2 = \frac{\underline{E}}{b} \qquad \text{estimator for } \sigma^2 \ldots\ldots \quad (3.2.11)$$

3.3 Application to the base extension network

Reduction of the base (having two deflection points) to a straight line



Fig, 3.3.1

The problem is, in the situation given in Fig. 3.3.1, to express $\underline{s}_{14}$ linearly in $\underline{s}_{12}$, $\underline{s}_{23}$, $\underline{s}_{34}$, $\underline{r}_{23}$, $\underline{r}_{24}$, $\underline{r}_{41}$, $\underline{r}_{42}$ and $\underline{r}_{43}$.

In triangle 2-3-4 is then with (3.1.31a):

$$s_{24}\underline{\Delta \ln s_{24}} = s_{23}\cos\alpha_{324}\underline{\Delta \ln s_{23}} + s_{34}\cos\alpha_{243}\underline{\Delta \ln s_{34}} - $$
$$- s_{23}\sin\alpha_{324}(\underline{\Delta r_{24}} - \underline{\Delta r_{23}}) - s_{34}\sin\alpha_{243}(\underline{\Delta r_{43}} - \underline{\Delta r_{42}}) \quad \ldots \quad (3.3.1)$$

(3.1.31b) gives:

$$\cot\alpha_{214}\underline{\Delta\alpha_{214}} = \cot\alpha_{142}(\underline{\Delta r_{42}} - \underline{\Delta r_{41}}) - \underline{\Delta \ln s_{12}} + \underline{\Delta \ln s_{24}} \quad \ldots\ldots \quad (3.3.2)$$

Again with (3.1.31a):

$$s_{14}\underline{\Delta \ln s_{14}} = s_{42}\cos\alpha_{214}\underline{\Delta \ln s_{12}} + s_{24}\cos\alpha_{142}\underline{\Delta \ln s_{24}} - $$
$$- s_{12}\sin\underline{\Delta\alpha_{214}}\,\underline{\Delta\alpha_{214}} - s_{24}\sin\alpha_{142}(\underline{\Delta r_{42}} - \underline{\Delta r_{41}}) \quad \ldots\ldots \quad (3.3.3)$$

Substitution of (3.3.1) in (3.3.2) and of (3.3.1) and (3.3.2) in (3.3.3) gives the expression desired.

Forming of the condition model

Some of the formulas of section 3.1 can be used to derive condition equations in Fig. 3.3.2. If in relation (3.1.17b) observation variates are linked to the angles α_1, α_2 and α_3, we have as condition equation between the means of the variates:

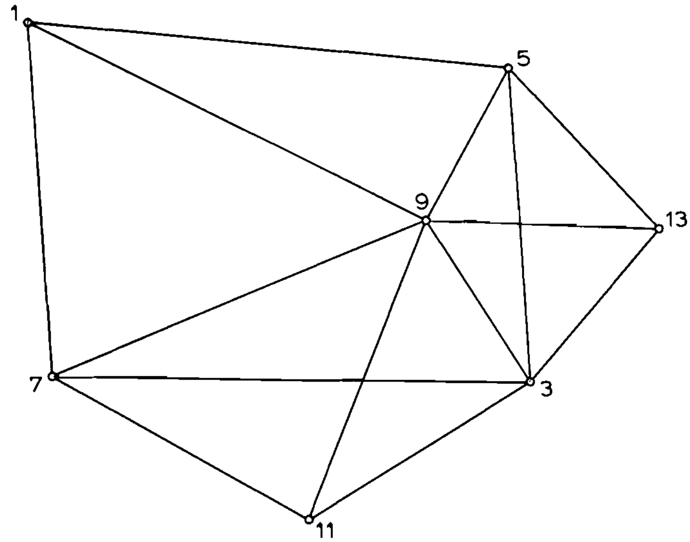


Fig. 3.3.2

$$\boxed{\tilde{\alpha}_1+\tilde{\alpha}_2+\tilde{\alpha}_3 = \Pi}\ \boxed{\mathrm{Im}\{V_{1,2,3}\}} \quad \ldots \quad (3.3.4)$$

In Fig. 3.3.2 this gives the following 10 condition equations:

$\mathrm{Im}\{V_{7,9,1}\}$ . . . . (3.3.5) $\mathrm{Im}\{V_{1,9,5}\}$ . . . . (3.3.6)

$\mathrm{Im}\{V_{9,7,3}\}$ . . . . (3.3.7) $\mathrm{Im}\{V_{9,3,5}\}$ . . . . (3.3.8)

$\mathrm{Im}\{V_{5,9,13}\}$ . . . (3.3.9) $\mathrm{Im}\{V_{13,9,3}\}$ . . . (3.3.10)

$\mathrm{Im}\{V_{9,7,11}\}$ . . . (3.3.11) $\mathrm{Im}\{V_{3,9,11}\}$ . . . (3.3.12)

$\mathrm{Im}\{V_{5,3,13}\}$ . . . (3.3.13) $\mathrm{Im}\{V_{3,7,11}\}$ . . . (3.3.14)

In the same way applying (3.1.35a), (3.1.35b) and (3.1.35c) gives:

$\mathrm{Re}\{\bar{V}_{5,9,3,13}\}$ . . . (3.3.15) $\mathrm{Re}\{\bar{V}_{5,3,9,13}\}$ . . . (3.3.16)

$\mathrm{Re}\{\bar{V}_{5,9,13,3}\}$ . . . (3.3.17) $\mathrm{Re}\{\bar{V}_{9,7,11,3}\}$ . . . (3.3.18)

$\mathrm{Re}\{\bar{V}_{9,11,7,3}\}$ . . . (3.3.19) $\mathrm{Re}\{\bar{V}_{9,7,3,11}\}$ . . . (3.3.20)

e.g. (3.3.19):

$$\ln\sin\tilde{\alpha}_{11,3,7}-\ln\sin\tilde{\alpha}_{7,11,3}+\ln\sin\tilde{\alpha}_{9,7,11}-\ln\sin\tilde{\alpha}_{11,9,7}+$$
$$+\ln\sin\tilde{\alpha}_{9,11,3}-\ln\sin\tilde{\alpha}_{11,3,9}+\ln\sin\tilde{\alpha}_{3,9,7}-\ln\sin\tilde{\alpha}_{9,7,3} = 0$$

Applying (3.1.40) gives one condition equation:

$$\mathrm{Re}\{W_{(9),3,5,1,7}\} \quad (3.3.21)$$

(3.1.38) gives 8 condition equations:

$$\mathrm{Re}\{\overline{W}_{(9),3,11,7}\} \quad (3.3.22) \qquad \mathrm{Re}\{\overline{W}_{(3),11,7,9}\} \quad (3.3.23)$$

$$\mathrm{Re}\{\overline{W}_{(11),7,9,3}\} \quad (3.3.24) \qquad \mathrm{Re}\{\overline{W}_{(7),9,3,11}\} \quad (3.3.25)$$

$$\mathrm{Re}\{\overline{W}_{(5),13,3,9}\} \quad (3.3.26) \qquad \mathrm{Re}\{\overline{W}_{(13),3,9,5}\} \quad (3.3.27)$$

$$\mathrm{Re}\{\overline{W}_{(3),9,5,13}\} \quad (3.3.28) \qquad \mathrm{Re}\{\overline{W}_{(9),5,13,3}\} \quad (3.3.29)$$

Finally applying (3.1.32a), (3.1.33a) and (3.1.34a) gives the 6 condition equations:

$$\mathrm{Im}\{V_{9,7,11,3}\} \quad (3.3.30) \qquad \mathrm{Im}\{V_{9,11,7,3}\} \quad (3.3.31)$$

$$\mathrm{Im}\{V_{9,7,3,11}\} \quad (3.3.32) \qquad \mathrm{Im}\{V_{5,9,3,13}\} \quad (3.3.33)$$

$$\mathrm{Im}\{V_{5,9,13,3}\} \quad (3.3.34) \qquad \mathrm{Im}\{V_{5,3,9,13}\} \quad (3.3.35)$$

It is obvious that these 31 condition equations are mutually dependent, e.g.: (3.3.7)+(3.3.14) = (3.3.30). However it is possible to choose 11 independent condition equations. The equations (3.3.5) – (3.3.12), (3.3.15), (3.3.18) and (3.3.21) were selected for this purpose:

After linearization this gives (see also (3.1.36a) and (3.1.41)):

$$\tilde{r}_{1,7}-\tilde{r}_{1,9}-\tilde{r}_{7,1}+\tilde{r}_{7,9}+\tilde{r}_{9,1}-\tilde{r}_{9,7}=\pi$$

$$-\tilde{r}_{1,5}+\tilde{r}_{1,9}+\tilde{r}_{5,1}-\tilde{r}_{5,9}-\tilde{r}_{9,1}+\tilde{r}_{9,5}=\pi$$

$$-\tilde{r}_{3,7}+\tilde{r}_{3,9}+\tilde{r}_{7,3}-\tilde{r}_{7,9}-\tilde{r}_{9,3}+\tilde{r}_{9,7}=\pi$$

$$\tilde{r}_{3,5}-\tilde{r}_{3,9}-\tilde{r}_{5,3}+\tilde{r}_{5,9}+\tilde{r}_{9,3}-\tilde{r}_{9,5}=\pi$$

$$\tilde{r}_{5,9}-\tilde{r}_{5,13}+\tilde{r}_{9,13}-\tilde{r}_{9,5}+\tilde{r}_{13,5}-\tilde{r}_{13,9}=\pi$$

$$-\tilde{r}_{3,9}+\tilde{r}_{3,13}+\tilde{r}_{9,3}-\tilde{r}_{9,13}-\tilde{r}_{13,3}+\tilde{r}_{13,9}=\pi$$

$$-\tilde{r}_{7,9}+\tilde{r}_{7,11}+\tilde{r}_{9,7}-\tilde{r}_{9,11}-\tilde{r}_{11,7}+\tilde{r}_{11,9}=\pi$$

$$\tilde{r}_{3,9}-\tilde{r}_{3,11}-\tilde{r}_{9,3}+\tilde{r}_{9,11}+\tilde{r}_{11,9}-\tilde{r}_{11,3}=\pi$$

$$\begin{aligned}
&(\cot\alpha_{9,3,5}+\cot\alpha_{5,3,13})\,\Delta r_{3,5}+(-\cot\alpha_{9,3,5})\,\Delta r_{3,9}+(-\cot\alpha_{5,3,13})\,\Delta r_{3,13}+\\
&+(\cot\alpha_{13,5,3}+\cot\alpha_{3,5,9})\,\Delta r_{5,3}+(-\cot\alpha_{3,5,9})\,\Delta r_{5,9}+(-\cot\alpha_{13,5,3})\,\Delta r_{5,13}+\\
&+(-\cot\alpha_{13,9,3})\,\Delta r_{9,3}+(-\cot\alpha_{5,9,13})\,\Delta r_{9,5}+(\cot\alpha_{13,9,3}+\cot\alpha_{5,9,13})\,\Delta r_{9,13}+\\
&+(-\cot\alpha_{3,13,9})\,\Delta r_{13,3}+(-\cot\alpha_{9,13,5})\,\Delta r_{13,5}+(\cot\alpha_{3,13,9}+\cot\alpha_{9,13,5})\,\Delta r_{13,9}=0
\end{aligned}$$

$$(\cot\alpha_{11,3,7}+\cot\alpha_{7,3,9})\Delta r_{3,7}+(-\cot\alpha_{7,3,9})\Delta r_{3,9}+(-\cot\alpha_{11,3,7})\Delta r_{3,11}+$$
$$+(\cot\alpha_{3,7,11}+\cot\alpha_{9,7,3})\Delta r_{7,3}+(-\cot\alpha_{9,7,3})\Delta r_{7,9}+(-\cot\alpha_{3,7,11})\Delta r_{7,11}+$$
$$+(-\cot\alpha_{3,9,11})\Delta r_{9,3}+(-\cot\alpha_{11,9,7})\Delta r_{9,7}+(\cot\alpha_{3,9,11}+\cot\alpha_{11,9,7})\Delta r_{9,11}+$$
$$+(-\cot\alpha_{9,11,3})\Delta r_{11,3}+(-\cot\alpha_{7,11,9})\Delta r_{11,7}+(\cot\alpha_{7,11,9}+\cot_{9,11,3})\Delta r_{11,9}=0$$

$$(\cot\alpha_{5,1,9}\,\Delta r_{1,5}+\cot\alpha_{9,1,7}\,\Delta r_{1,7}+(-\cot\alpha_{5,1,9}-\cot\alpha_{9,1,7})\Delta r_{1,9}+$$
$$+\cot\alpha_{9,3,5}\,\Delta r_{3,5}+\cot\alpha_{7,3,9}\,\Delta r_{3,7}+(-\cot\alpha_{9,3,5}-\cot\alpha_{7,3,9})\Delta r_{3,9}+$$
$$+\cot\alpha_{9,5,1}\,\Delta r_{5,1}+\cot\alpha_{3,5,9}\,\Delta r_{5,3}+(-\cot\alpha_{9,5,1}-\cot\alpha_{3,5,9})\Delta r_{5,9}+$$
$$+\cot\alpha_{1,7,9}\,\Delta r_{7,1}+\cot\alpha_{9,7,3}\,\Delta r_{7,3}+(-\cot\alpha_{1,7,9}-\cot\alpha_{9,7,3})\Delta r_{7,9}=0 \tag{3.3.36}$$

Computing distance ratios

The problem is to express in Fig. 3.3.2 the derived variates

$$\underline{v}_{9,7;1,3},\ \underline{v}_{7,9;1,5},\ \underline{v}_{9,7,1},\ \underline{v}_{7,9;3,5},\ \underline{v}_{9,7,3},\ \underline{v}_{7,9,3},\ \underline{v}_{7,9;3,11},\ \underline{v}_{7,9;3,13},\ \underline{v}_{7,9,5},$$
$$\underline{v}_{7,9;5,13},\ \underline{v}_{9,7,11},\ \underline{v}_{7,9,11}\ \underline{v}_{7,9,13}$$

in the observed variates (the directions).

With the exception of $\underline{v}_{9,7;1,3}$, all distance ratio variates are determined by the same method. According (3.1.42) the distance ratio is written as the product of one or more distance ratios, each of which can be expressed with the method of (3.1.43) in two measured angles.

Table 3.3.1

distance ratio	triangles
$v_{7,9,3}$	(7-9-3)
$v_{7,9,13}$	(7-9-3) and (3-9-13)
$v_{7,9,5}$	(7-9-3) and (3-5-9)
$v_{7,9;3,5}$	(7-9-3) and (3-5-9)
$v_{7,9;3,13}$	(7-9-3) and (3-9-13)
$v_{7,9;5,13}$	(7-9-3), (3-5-9) and (9-5-13)
$v_{9,7,3}$	(7-9-3)
$v_{7,9,1}$	(1-7-9)
$v_{7,9;1,5}$	(1-7-9) and (1-5-9)
$v_{9,7,1}$	(1-7-9)
$v_{7,9,11}$	(7-9-11)
$v_{9,7,11}$	(7-9-11)
$v_{7,9;3,11}$	(7-9-3) and (3-9-11)

In table 3.3.1 are given the triangles, by means of which each of the distance ratios can be expressed in angles.

For $\underline{\Delta \ln v}_{9,7;1,3}$ holds good:

$$\underline{\Delta \ln v}_{9,7;1,3} = \underline{\Delta \ln v}_{9,7,1} + \underline{\Delta \ln v}_{7,1,3} \quad \ldots \quad (3.3.37)$$

With (3.1.30):

$$\underline{\Delta \ln v}_{7,1,3} = -\underline{\Delta \ln v}_{3,1,7} = \frac{1-\cot\alpha_{3,1,7}\cot\alpha_{1,7,3}}{1+\cot^2\alpha_{3,1,7}}\underline{\Delta \ln v}_{1,7,3} +$$

$$+\frac{\cot\alpha_{3,1,7}+\cot\alpha_{1,7,3}}{1+\cot^2\alpha_{3,1,7}}\underline{\Delta\alpha}_{1,7,3} \quad \ldots \quad (3.3.38)$$

$\underline{\Delta \ln v}_{1,7,3}$ is obtained as follows:

$$\underline{\Delta \ln v}_{1,7,3} = \underline{\Delta \ln v}_{1,7,9} + \underline{\Delta \ln v}_{9,7,3} = \cot\alpha_{9,1,7}\underline{\Delta\alpha}_{9,1,7} - \cot\alpha_{7,9,1}\underline{\Delta\alpha}_{7,9,1}$$

$$+\cot\alpha_{3,9,7}\underline{\Delta\alpha}_{3,9,7} - \cot\alpha_{7,3,9}\underline{\Delta\alpha}_{7,3,9} \quad \ldots \quad (3.3.39)$$

with:

$$\underline{\Delta \ln v}_{9,7,1} = \cot\alpha_{7,9,1}\underline{\Delta\alpha}_{7,9,1} - \cot\alpha_{9,1,7}\underline{\Delta\alpha}_{9,1,7} \quad \ldots \quad (3.3.40)$$

Substitution of (3.3.39) in (3.3.38), of (3.3.38) and (3.3.40) in (3.3.37) and expressing angles in directions gives the desired result.

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Table 2.1.2 (A_i^R)

R	i / derived quantity	1	2	3	4	5	6	7	8	9	10	11	12	13
		obser												
		$r_{1,5}$	$r_{1,7}$	$r_{1,9}$	$r_{3,5}$	$r_{3,7}$	$r_{3,9}$	$r_{3,11}$	$r_{3,13}$	$r_{5,1}$	$r_{5,3}$	$r_{5,9}$	$r_{5,13}$	$r_{7,1}$
1	$\ln v_{7,9,3}$					+0.503	−0.503							
2	$\ln v_{7,9,13}$					+0.503	−0.935		+0.432					
3	$\ln v_{7,9,5}$				+1.098	+0.503	−1.600				+1.970	−1.970		
4	$\ln v_{9,7;3,5}$					+0.503	−0.503				+1.970	−1.970		
5	$\ln v_{9,7;3,13}$					+0.503	−0.503							
6	$\ln v_{9,7;5,13}$				+1.098	+0.503	−1.600				+1.970	−1.970		
7	$\ln v_{9,7,3}$					+0.503	−0.503							
8	$\ln v_{7,9,1}$		−1.051	+1.051										−0.23
9	$\ln v_{9,7;1,5}$		−1.051	+1.051						−0.789		+0.789		−0.23
10	$\ln v_{9,7,1}$		−1.051	+1.051										
11	$\ln v_{9,7;1,3}$		−0.573	+0.573		+0.229	−0.229							−0.37
12	$\ln v_{7,9,11}$													
13	$\ln v_{9,7,11}$													
14	$\ln v_{9,7;3,11}$					+0.503	−0.503							

Note: All zero elements are omitted

Table 2.3.3

No.	number / condition	1	2	3	4	5	6	7	8	9	10	11	12	13
		c												
		$r_{1,5}$	$r_{1,7}$	$r_{1,9}$	$r_{3,5}$	$r_{3,7}$	$r_{3,9}$	$r_{3,11}$	$r_{3,13}$	$r_{5,1}$	$r_{5,3}$	$r_{5,9}$	$r_{5,13}$	$r_{7,1}$
1*	$V_{7,9,1}$		+1	−1										−1
2*	$V_{1,9,5}$	−1		+1						+1		−1		
3*	$V_{9,7,3}$					−1	+1							
4*	$V_{9,3,5}$				+1		−1				−1	+1		
5*	$V_{5,9,13}$											+1		−1
6*	$V_{13,9,3}$						−1		+1					
7*	$V_{9,7,11}$													
8*	$V_{3,9,11}$						+1	−1						
9	$V_{5,3,13}$				−1				+1		+1		−1	
10	$V_{3,7,11}$					+1		−1						
11*	$\bar{V}_{5,9,3,13}$				−3.310		+1.095		+2.216		−4.477	+1.971	+2.506	
12	$\bar{V}_{5,3,9,13}$				+2.216		−0.430		−1.785		−1.971	+1.091	+0.880	
13	$\bar{V}_{5,9,13,3}$				+1.095		−0.664		−0.430		−2.506	+0.880	+1.626	
14*	$\bar{V}_{9,7,11,3}$					−1.253	+0.504	+0.749						
15	$\bar{V}_{9,11,7,3}$					−0.749	−0.497	+1.246						
16	$\bar{V}_{9,7,3,11}$					+0.504	−1.001	+0.497						
17*	$\bar{W}_{(9),3,5,1,7}$	−1.990	−1.053	+3.044	−1.095	−0.504	+1.598			−0.789	−1.971	+2.760		−0.23
18	$\bar{W}_{(9),3,11,7}$					−0.504	+1.001	−0.497						
19	$\bar{W}_{(3),11,7,9}$													
20	$\bar{W}_{(11),7,9,3}$					−0.749	−0.497	+1.246						
21	$\bar{W}_{(7),9,3,11}$					−1.253	+0.504	+0.749						
22	$\bar{W}_{(5),13,3,9}$				−3.310		+1.095		+2.216					
23	$\bar{W}_{(13),3,9,5}$				−2.216		+0.430		+1.785		−2.506	+0.880	+1.626	
24	$\bar{W}_{(3),9,5,13}$										−4.477	+1.971	+2.506	
25	$\bar{W}_{(9),5,13,3}$				−1.095		+0.664		+0.430		−1.971	+1.091	+0.880	
26	$V_{9,7,11,3}$						+1	−1						
27	$V_{9,11,7,3}$					−1	+1							
28	$V_{9,7,3,11}$					+1		−1						
29	$V_{5,9,3,13}$						−1		+1			+1	−1	
30	$V_{5,9,13,3}$				+1		0		−1		−1	+1		
31	$V_{5,3,9,13}$				−1		+1				+1		−1	

Note: All zero elements in the table are omitted

observation															
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
$r_{5,9}$	$r_{5,13}$	$r_{7,1}$	$r_{7,3}$	$r_{7,9}$	$r_{7,11}$	$r_{9,1}$	$r_{9,3}$	$r_{9,5}$	$r_{9,7}$	$r_{9,11}$	$r_{9,13}$	$r_{11,3}$	$r_{11,7}$	$r_{11,9}$	$r_{13,3}$
			+1.809	−1.809											
			+1.809	−1.809											+0.727
−1.970			+1.809	−1.809											
−1.970			+1.809	−1.809			−0.379	+0.379							
			+1.809	−1.809			+0.592				−0.592				+0.727
−1.970			+1.809	−1.809				−0.799			+0.799				
							−0.039		+0.039						
		−0.239		+0.239											
+0.789		−0.239		+0.239		+0.206		−0.206							
						+0.581			−0.581						
		−0.378	+0.378			+0.317	−0.018		−0.299						
				−0.692	+0.692								+0.416	−0.416	
									+0.643	−0.643			+0.416	−0.416	
			+1.809	+1.809			−1.649			+1.699		−1.543		−1.543	

observation															
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
$r_{5,9}$	$r_{5,13}$	$r_{7,1}$	$r_{7,3}$	$r_{7,9}$	$r_{7,11}$	$r_{9,1}$	$r_{9,3}$	$r_{9,5}$	$r_{9,7}$	$r_{9,11}$	$r_{9,13}$	$r_{11,3}$	$r_{11,7}$	$r_{11,9}$	$r_{13,3}$
		−1		+1		+1			−1						
−1						−1		+1							
			+1	−1			−1		+1						
+1							+1	−1							
+1		−1						−1			+1				
							+1				−1				−1
				−1	+1				+1	−1			−1	+1	
							−1			+1		+1		−1	
	−1														−1
			−1		+1							+1	−1		
+1.971	+2.506						+0.592	+0.801			−1.393				+0.729
+1.091	+0.880						+0.378	−1.178			+0.801				+1.693
+0.880	+1.626						+0.970	−0.378			−0.592				+0.964
			−3.825	+1.809	+2.016		+1.693		+0.643	−2.337		+1.539	+0.416	−1.955	
			+1.809	−1.117	−0.692		+0.038		+0.605	−0.643		−1.724	+0.184	+1.539	
			−2.016	+0.692	+1.324		+1.655		+0.038	−1.693		+0.184	−0.600	+0.416	
+2.760		−0.239	−1.809	+2.048											
			−1.809	+1.117	+0.692							+1.539	+0.416	−1.955	
			−3.825	+1.809	+2.016		+1.655		+0.038	−1.693		+1.724	−0.184	−1.539	
			−2.016	+0.692	+1.324		+1.693		+0.643	−2.337					
							+0.038		+0.605	−0.643		−0.184	+0.600	−0.416	
							−0.378	+1.178							−0.964
+0.880	+1.626						+0.592	+0.801							
+1.971	+2.506						+0.970	−0.378			−0.592				+1.693
+1.091	+0.880														+0.729
				−1	+1		−1		+1			+1	−1		
			+1		−1		−1			+1			+1	−1	
			−1	+1					−1	+1		+1		−1	
+1	−1						+1	−1							−1
+1								−1			+1				+1
	−1						−1				+1				

18	19	20	21	22	23	24	25	26	27	28
$r_{9,3}$	$r_{9,5}$	$r_{9,7}$	$r_{9,11}$	$r_{9,13}$	$r_{11,3}$	$r_{11,7}$	$r_{11,9}$	$r_{13,3}$	$r_{13,5}$	$r_{13,9}$
								+0.727		−0.727
−0.379	+0.379									
+0.592				−0.592				+0.727		−0.727
	−0.799			+0.799					−0.177	+0.177
−0.039		+0.039								
	−0.206									
		−0.581								
−0.018		−0.299								
						+0.416	−0.416			
		+0.643	−0.643			+0.416	−0.416			
−1.649			+1.699		−1.543		−1.543			

18	19	20	21	22	23	24	25	26	27	28	$(-y^{\varrho})$ cent.sec.	$(\sigma_{-y\varrho})$ cent.sec.	δ^2/σ^2
$r_{9,3}$	$r_{9,5}$	$r_{9,7}$	$r_{9,11}$	$r_{9,13}$	$r_{11,3}$	$r_{11,7}$	$r_{11,9}$	$r_{13,3}$	$r_{13,5}$	$r_{13,9}$			
		−1									+ 0.5	2.4	0.04
	+1										+ 1.9	2.4	0.62
−1		+1									+ 5.1	2.4	4.34
+1	−1										− 2.8	2.4	1.33
	−1			+1					+1	−1	+ 1.1	2.4	0.22
+1				−1				−1		+1	− 3.2	2.4	1.73
		+1	−1			−1	+1				− 0.4	2.4	0.02
−1			+1		+1		−1				+ 6.5	2.4	6.95
								−1	+1		+ 0.7	2.4	0.09
					+1	−1					+ 1.0	2.4	0.16
+0.592	+0.801			−1.393				+0.729	+0.176	−0.905	+ 4.4	7.2	0.37
+0.378	−1.178			+0.801				+1.693	−0.964	−0.729	− 1.7	4.7	0.13
+0.970	−0.378			−0.592				+0.964	−1.140	+0.176	− 2.6	3.9	0.46
+1.693		+0.643	−2.337		+1.539	+0.416	−1.955				+18.0	6.3	8.22
+0.038		+0.605	−0.643		−1.724	+0.184	+1.539				− 7.6	3.7	4.23
+1.655		+0.038	−1.693		+0.184	−0.600	+0.416				− 1.1	3.6	0.09
											+17.4	6.1	8.00
					+1.539	+0.416	−1.955				+13.4	3.6	13.87
+1.655		+0.038	−1.693		+1.724	−0.184	−1.539				+12.2	5.7	4.52
+1.693		+0.643	−2.337								+ 4.7	4.2	1.24
+0.038		+0.605	−0.643		−0.184	+0.600	−0.416				+ 5.8	1.9	9.01
−0.378	+1.178							−0.964	+1.140	−0.176	+ 4.3	4.6	0.87
+0.592	+0.801										+ 1.7	4.6	0.14
+0.970	−0.378			−0.592				+1.693	−0.964	−0.729	+ 0.0	6.0	0
								+0.729	+0.176	−0.905	+ 2.7	3.0	0.79
−1		+1			+1	−1					+ 6.1	2.8	4.64
−1			+1			+1	−1				+ 5.5	2.8	3.74
		−1	+1		+1		−1				+ 1.4	2.8	0.23
+1	−1							−1	+1		− 2.1	2.8	0.54
	−1			+1				+1		−1	+ 0.4	2.8	0.02
−1				+1					+1	−1	+ 4.0	2.8	1.96