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OBSERVATIONS  
IN THE NETHERLANDS, 1947-1973

by

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## LIST OF SYMBOLS

$\varphi, \varphi_g$	latitude (astronomic or geodetic)	
$\lambda, \lambda_g$	longitude (positive west of Greenwich)	
$A, A_g$	azimuth of the reference mark	
$z$	zenith distance	}
$a$	azimuth	
$q$	parallactic angle	
$t$	local hour angle	
$t^G$	Greenwich hour angle	
$\alpha$	apparent right ascension	
$\delta$	apparent declination	} of the star
$\psi$	horizontal angle R.M.-star	
$M$	Mean value of the level reading: $M = \frac{1}{2}(l+r)$	
$M_0$	reference position of the bubble (approximately corresponding with a vertical alidade axis)	
$p$	level value per division	
$UT$	Universal Time	
$LMT$	Local Mean Time	
$GAST$	Greenwich Apparent Sidereal Time	
$\hat{\sigma}$	estimate of standard deviation	
$\beta$	width of the contact strips of the self-recording micrometer	
$\tau$	lost motion of the self-recording micrometer	
$F_k$	distances of the contacts with respect to their centre point	
$N$	number of contacts ( $k = 1, 2 \dots N$ )	
$s$	number of stars ( $i = 1, 2 \dots s$ )	
$n$	number of series	

## GEODETIC-ASTRONOMICAL OBSERVATIONS IN THE NETHERLANDS, 1947–1973

### 1 Introduction

At the end of the 19th century astronomical observations were carried out in The Netherlands with the object of establishing a reference ellipsoid for the national triangulation. For this purpose latitude and azimuth were determined in 13 first order points, regularly distributed over the network [1]. The methods applied for these determinations were meridian altitudes of stars (latitude) and Polaris observations (azimuth). Starting from the astronomical data obtained in each of these points and using the geodetic data of the network, 13 values for latitude and azimuth were computed for Amersfoort, the central station of the net. The mean of these 13 values was assumed to be the geodetic latitude of Amersfoort and the azimuth Amersfoort–Utrecht [3].

The longitude of Amersfoort, although less important for the ellipsoid, was derived from the longitude of Leiden and from the longitude difference Amersfoort–Leiden, computed from the geodetic data.

The data of Amersfoort obtained in this way are:

$$\begin{aligned} \text{latitude } \varphi &= 52^{\circ}09'22''.178 \\ \text{longitude } \lambda &= - 5^{\circ}23'15''.500 \\ \text{azimuth Amersfoort–Utrecht } A &= 248^{\circ}35'19''.891 \end{aligned}$$

Laplace stations for azimuth control were not used in the adjustment of the network. In a small country like The Netherlands this is admissible. The need for Laplace stations came much later, namely after the second world war, when it was decided to readjust the European network as a whole. Three twin Laplace stations were planned in The Netherlands for this purpose, i.e. Leeuwarden–Ameland, Zierikzee–Goedereede and Ubachsberg–Tongeren. The measurements were carried out using the following methods:

1. Longitude determination by meridian transit of stars. Leeuwarden (1947), Ameland (1947), Zierikzee (1949) and Goedereede (1950); see section 3.
2. Simultaneous determination of latitude, longitude and azimuth using the Black method. Ubachsberg (1968), Tongeren (1968); see section 4.
3. Azimuth by Polaris. Goedereede (1969), Zierikzee (1973); see section 5.

In addition some results from older measurements are available:

4. Azimuth by Polaris and some circumpolar stars. Ubachsberg (1893) [2].
5. Determination of the longitude difference Leiden–Ubachsberg (1893) [2].
6. Azimuth by Polaris. Leeuwarden (1897), Ameland (1897), Zierikzee (1897) [1].

A summary of the results of the above mentioned measurements is given in table 1.1.

For the Laplace stations Leeuwarden and Ameland the azimuth determinations of 1897 have to be used. The accuracy of these measurements is satisfactory but there is some doubt about the polar motion correction. According to information received from the Director

Table 1.1 Laplace points in The Netherlands

Laplace point (terrestrial direction)	astronomic quantities	$\hat{\sigma}$	year	method/remark
Leeuwarden (C1902) (Hallum, C1897)	$\varphi = 53^{\circ}12'15''.283$	$0''.075$	1897	meridian altitudes (Sterneck)
	$\lambda = - 5^{\circ}47'23''.850$	$0''.090$	1947	meridian transit (Mayer)
	$A = 358^{\circ}31'57''.632$	$0''.35$	1897	Polaris $\hat{\sigma}_{A+\lambda \sin \varphi} = 0''.36$
Ameland (C1897) (Hallum, C1897)	$\varphi = 53^{\circ}27'30''.249$	$0''.026$	1897	meridian altitudes (Sterneck)
	$\lambda = - 5^{\circ}46'56''.550$	$0''.180$	1947	meridian transit (Mayer)
	$A = 179^{\circ}05'52''.922$	$0''.20$	1897	Polaris $\hat{\sigma}_{A+\lambda \sin \varphi} = 0''.25$
Goedereede (C1896) (Zierikzee, C1896)	$\varphi \approx 51^{\circ}49'10''$			
	$\lambda = - 3^{\circ}58'34''.965$	$0''.090$	1950	meridian transit (Mayer)
	$A = 192^{\circ}43'00''.980$	$0''.10$	1969	Polaris $\hat{\sigma}_{A+\lambda \sin \varphi} = 0''.13$
Zierikzee (C1896) (Goedereede, C1896)	$\varphi = 51^{\circ}39'03''.558$	$0''.085$	1897	meridian altitude (Sterneck)
	$\lambda = - 3^{\circ}54'53''.655$	$0''.090$	1949	meridian transit (Mayer)
	$A = 12^{\circ}40'07''.071$	$0''.15$	1973	Polaris $\hat{\sigma}_{A+\lambda \sin \varphi} = 0''.17$
Ubachsberg (C1890) (Tongeren, St.A)	$\varphi = 50^{\circ}50'53''.432$	$0''.41$		
	$\lambda = - 5^{\circ}57'04''.320$	$0''.64$	1968	Black ( $z \approx 60^{\circ}$ )
	$A = 258^{\circ}15'24''.273$	$0''.53$		$\hat{\sigma}_{A+\lambda \sin \varphi} = 0''.17$
Tongeren (St.A) (Ubachsberg, C1890)	$\varphi = 50^{\circ}46'55''.775$	$0''.48$		
	$\lambda = - 5^{\circ}27'48''.570$	$0''.78$	1968	Black ( $z \approx 60^{\circ}$ )
	$A = 77^{\circ}52'43''.958$	$0''.64$		$\hat{\sigma}_{A+\lambda \sin \varphi} = 0''.20$

of the Polar Motion Service at Mizusawa, Mr. S. YUMI, it is not possible to reduce these old measurements to the Conventional International Origin. This means that the final azimuths of OUDEMANS [1] have to be used. These azimuths are only corrected for the annual periodical polar movement, but do of course not refer to the C.I.O. to be used. In order to gain an insight into the possible effect of polar motion, the remeasurement of the azimuth Zierikzee–Goedereede (1973) is compared with the old measurement of 1897:

$$\begin{array}{r}
 \text{azimuth 1897: } 12^{\circ}40'07''.387 \quad \hat{\sigma}_A = 0''.31 \\
 \text{azimuth 1973: } 12^{\circ}40'07''.071 \quad \hat{\sigma}_A = 0''.15 \\
 \hline
 \text{difference} \qquad \qquad \qquad 0''.316
 \end{array}$$

In this case the difference is not significant.

## 2 Instrument set-ups, reduction to the centre, polar motion correction

All the measurements shown in table 1.1 were carried out from a stable observation pillar. The reference marks were measured with the aid of a lamp, placed at a distance of about 10 km from the instrument set-up. The plane rectangular coordinates of the centres, set-ups and lamps with respect to Amersfoort are given in metres in table 2.1.

Table 2.1

station	X	Y	remarks
Leeuwarden Centre 1902	+ 26894.377	+116668.837	
Leeuwarden pillar 1947	+ 26889.663	+116669.740	longitude 1947
Ameland Centre	+ 26177.206	+144988.000	longitude 1947
Hallum Centre 1897	+ 26532.521	+128260.440	terrestrial line azimuths 1896
Goedereede Centre 1896	- 97314.981	- 36522.587	
Goedereede pillar 1950	- 97309.722	- 36527.121	longitude 1950
Goedereede pillar 2 (1969)	- 97312.179	- 36525.790	azimuth 1969
Goedereede pillar 2 (1973)	- 97312.171	- 36525.778	lamp 1973
Zierikzee Centre 1896	-101897.967	- 55142.050	
Zierikzee pillar 1949	-101891.448	- 55148.254	longitude 1949
Zierikzee lamp 1969	-101904.164	- 55132.481	lamp 1969
Zierikzee perm. mark 12	-101890.412	- 55134.008	azimuth 1973
Ubachsberg Centre 1890	+ 39845.615	-145477.288	
Ubachsberg pillar 1966	+ 39774.286	-145558.665	Black, 1968
Tongeren Centre 1892	+ 5433.487	-152902.284	
Tongeren centre Stat. A	+ 5433.945	-152907.075	
Tongeren pillar 1966	+ 5437.230	-152899.507	Black, 1968

The station *Leeuwarden* is a detached unfinished tower with a flat roof, called Oldenhove. The azimuth from this station to Hallum was determined in 1897 [1]. The longitude determination was carried out on pillar 1947. All results were reduced to centre 1902.

The station *Ameland* is situated on top of a high dune, north-northeast of the village of Nes, locally known as the "grey-dune". An observation pillar was built at the centre of the station. The azimuth determination (1897) to Hallum and the longitude determination (1949) were executed at this pillar.

The station *Ubachsberg* is situated in the holiday resort "Vrouweheide" near the village of Ubachsberg. The original centre 1890 proved to be unsuitable for the Black method because of high trees and therefore a new observation pillar (pillar 1966) was erected at a distance of about 100 metres from the old centre. The tower of Schimmert was used as terrestrial reference mark. The 1968-observations were reduced to the 1890-centre.

The observations at *Tongeren* were carried out on the flat roof of the tower of the "Basilique Notre Dame" from an observation pillar built in 1966. The terrestrial reference mark used was a lamp placed on the tower of Herderen. The measurements of 1968 were reduced to centre A.

The station *Zierikzee* is the "St. Lievens Monster", a detached unfinished tower. The observations for the longitude determination (1949) were carried out on pillar 1949. The azimuth measurement (1973) was done from permanent mark 12. For the measurement of the azimuth from Goedereede a lamp was placed on this tower. The results were reduced to centre 1896.

The station *Goedereede* is also a detached unfinished tower. The longitude measurement (1949) were performed on pillar 1949, the azimuth determination (1969) from pillar 2. A lamp was placed on pillar 2 for the azimuth determination from Zierikzee. The results were reduced to centre 1896.

The latitude, longitude and azimuth of an excentric set-up were reduced to the centre applying bearing and distance computed from the coordinates of table 2.1. An example of a reduction is given in [7, p. 17].

The polar motion corrections to the final results were applied using the following formulae:

$$\Delta\varphi'' = -(x \cos \lambda + y \sin \lambda)$$

$$\Delta\lambda'' = -(x \sin \lambda - y \cos \lambda) \tan \varphi$$

$$\Delta A'' = (x \sin \lambda - y \cos \lambda) \sec \varphi$$

in which  $x$  and  $y$  are coordinates of the true pole (in seconds of arc) referring to the Conventional International Origin. The data for  $x$  and  $y$  have been taken from publications of the Bureau International de l'Heure (Paris) or from publications of the International Polar Motion Service (Mizusawa).

### 3 Longitude determinations (Leeuwarden, Ameland, Zierikzee, Goedereede)

#### 3.1 Introduction

Longitude determinations were carried out in the stations Leeuwarden (1947), Ameland (1947), Zierikzee (1949) and Goedereede (1950). The results of Goedereede are included in the present report while the measurements of the other stations were already reported in 1950 by BRUINS [5]. However, at that time the definite time corrections and polar motion corrections were not available yet. Applying these corrections the longitude changes by about  $0^{\circ}.040$ , an amount which may not be neglected. In this section a brief description of the measurements and the computations will be given.

#### 3.2 The method used

The longitude determination is based on the relation:

$$\lambda = GMT - LMT \dots \dots \dots (3.1)$$

in which:

$GMT$  = Greenwich Mean Time; to be determined from radio time signals

$LMT$  = Local Mean Time; to be determined from meridian-transits of stars.

In the present terminology  $UT$  (Universal Time) is used instead of  $GMT$ .

Hence we have:

$$\lambda = UT_1 - LMT_1 = (UT_1 - T) - (LMT_1 - T) \dots \dots \dots (3.2)$$

in which the indices 1 mean that both times are related to the Conventional International Origin (C.I.O.).

In formula (3.2) two chronometer corrections are in fact required. They are determined independently but obviously referring to the same moment ( $T$ ). The chronometer corrections



( $UT1 - T$ ) were determined by rhythmic time signals of the radio stations Pontoise (FYP) or Rugby (GBR) with an accuracy of about  $0^s.001$ . Applying some corrections we then obtain:

$$(UT1 - T) = (\text{Signal} - T) + \Delta T_d + (UT0_{\text{Paris}} - \text{Signal}) + (UT1 - UT0_{\text{Paris}}) \quad . \quad (3.3)$$

in which:

(Signal - T) = chronometer correction determined by the radio time signals; usually referring to the moment of receiving the time signals before the measurement (FYP at  $GMT = 20^h06^m$  or GBR at  $GMT = 20^h00^m$ ).

$\Delta T_d$  = time correction due to the travel time of the radio signal (Leeuwarden and Ameland  $0^s.005$ , Goedereede and Zierikzee  $0^s.004$ ).

( $UT0_{\text{Paris}} - \text{Signal}$ ) = definite time correction according to the Bureau International de l'Heure.

( $UT1 - UT0_{\text{Paris}}$ ) =  $\frac{1}{15}(x'' \sin \lambda_{\text{Paris}} + y'' \cos \lambda_{\text{Paris}}) \tan \varphi_{\text{Paris}} \approx -0.076y''$ .

The latter corrections were computed in accordance with a suggestion made by Mr. B. GUINOT, Director of the Bureau International de l'Heure in Paris. The polar coordinates  $x, y$  with respect to the C.I.O. were taken from the "Publications of the International Latitude Observatory of Misusawa" (Vol. VII, No. 1, 1969).

The other chronometer corrections in formula (3.2), ( $LMT1 - T$ ) were determined by meridian transits of stars using Mayer's formula. In case of upper culmination of stars this formula reads:

$$\alpha = T + \Delta T + \frac{\sin(\varphi - \delta)}{\cos \delta} a \pm \frac{\cos(\varphi - \delta)}{\cos \delta} b \pm \frac{1}{\cos \delta} c \left( \begin{array}{l} + \text{eye piece east} \\ - \text{eye piece west} \end{array} \right) \quad . \quad (3.4)$$

in which:

- $\varphi$  = latitude of the station
- $\alpha$  = apparent right ascension of the star
- $\delta$  = apparent declination of the star
- $T$  = chronometer time observed
- $\Delta T$  = chronometer correction ( $LMT0 - T$ )
- $a$  = deviation of the line of sight from the meridian
- $b$  = inclination of the rotation axis of the telescope
- $c$  = collimation error

From the relation (3.4) the following observation equations are obtained:

$$\frac{\Delta T}{\cos \delta_i} + \frac{\sin(\varphi - \delta_i)}{\cos \delta_i} a \pm \frac{\cos(\varphi - \delta_i)}{\cos \delta_i} b \pm \frac{1}{\cos \delta_i} c = (\alpha_i - T_i) + \varepsilon_i \quad . \quad . \quad . \quad . \quad . \quad (3.5)$$

$i = 1, 2, \dots s$  (= number of stars).

from which a number of unknowns ( $\Delta T, a, b, c$ ) can be determined by the method of the least squares. It should be noted that for each group of stars observed in the same instru-

ment position, different unknowns for the deviation of the line of sight from the meridian are introduced. In this way we have:

- $a_1$  : for the first group of stars observed with eye-piece east;
- $a_{23}$  : for the second and third group observed with eye-piece west;
- $a_4$  : for the fourth group of stars observed with eye-piece east.

The observation vector  $(\alpha_i - T_i)$  in (3.5) is derived from the right ascension of the star and the time observed. Since a mean time chronometer was used, the right ascension was converted into mean time:  $\alpha = LAST \rightarrow LMT$ , in which *LAST* denotes Local Apparent Sidereal Time and *LMT* Local Mean Time. *LMT* refers to the momentaneous pole, therefore, similar to *UT0*, we can write *LMT0*. In fact, we determine from the adjustment the chronometer correction  $(LMT0 - T)$ . Applying the polar motion correction to the longitude according to section 2 we then obtain:

$$(LMT1 - T) = (LMT0 - T) + (x \sin \lambda - y \cos \lambda) \tan \varphi$$

To the observed star's transit time some corrections were applied (omitting the index *i*):

$$T = \bar{T} + N + G \dots \dots \dots (3.6)$$

in which  $\bar{T}$  is the mean value of contact-times resulting from the self-recording micrometer, *N* and *G* are respectively corrections for the inclination of the horizontal axis and for the rate of the chronometer used.

The mean value  $\bar{T}$  is obtained from:

$$\bar{T} = \frac{1}{N} [T_k + (F_k + \frac{1}{2}\tau)C] \quad k = 1, 2, \dots N \dots \dots \dots (3.7)$$

in which:

- N* = number of contacts used (*N* = 36)
- $T_k$  = contact times
- $F_k$  = wire distances with respect to the centre wire (or collimation point)
- $\tau$  = lost motion of the micrometer
- C* = 7<sup>s</sup>.831 (one revolution of the self-recording micrometer).

It should be noted that the correction for the (different) widths of the 12 contact strips of the micrometer is eliminated by taking the same end of the marks on the chronograph tape.

The correction *N* in (3.6) was computed from levelling the horizontal axis with the suspension level:

$$N = \frac{\cos(\varphi - \delta)}{\cos \delta} (M_0 - M)p^s$$

in which:

- $M_0$  = reference position of the bubble ( $M_0 = 30$ )
- M* =  $\frac{1}{2}(l+r)$ : mean value of the level reading
- $p^s$  = 0<sup>s</sup>.114: level value per division

Finally the correction  $G$  in (3.6) is computed from:

$$G = (\bar{T} - T_0)\Delta_1 T$$

in which:

$\bar{T}$  = the moment of observation

$T_0$  = reference moment (usually  $GMT = 20^h00^m$  or  $20^h06^m$  before starting the measurements)

$\Delta_1 T$  = chronometer rate (sec/hour); determined from time signals for a period of  $12^h$  or  $14^h$  (see appendix I). Pontoise (FYP):  $18^h \rightarrow 8^h$  (next day); Rugby (GBR):  $18^h \rightarrow 10^h$  (next day).

Another remark should be made about the method of least square adjustment applied to the observation equations (3.5). In this adjustment different weights were used depending on the declination of the stars, see table 3.2.1. The data of this table were taken from a publication of the U.S. Coast and Geodetic Survey [6].

Table 3.2.1

$\delta$	$g$
0°	1
10	1
20	0.98
30	0.91
40	0.82
45	0.76
50	0.69
55	0.61
60	0.51
65	0.40
70	0.29
75	0.18
80	0.09

### 3.3 Instruments

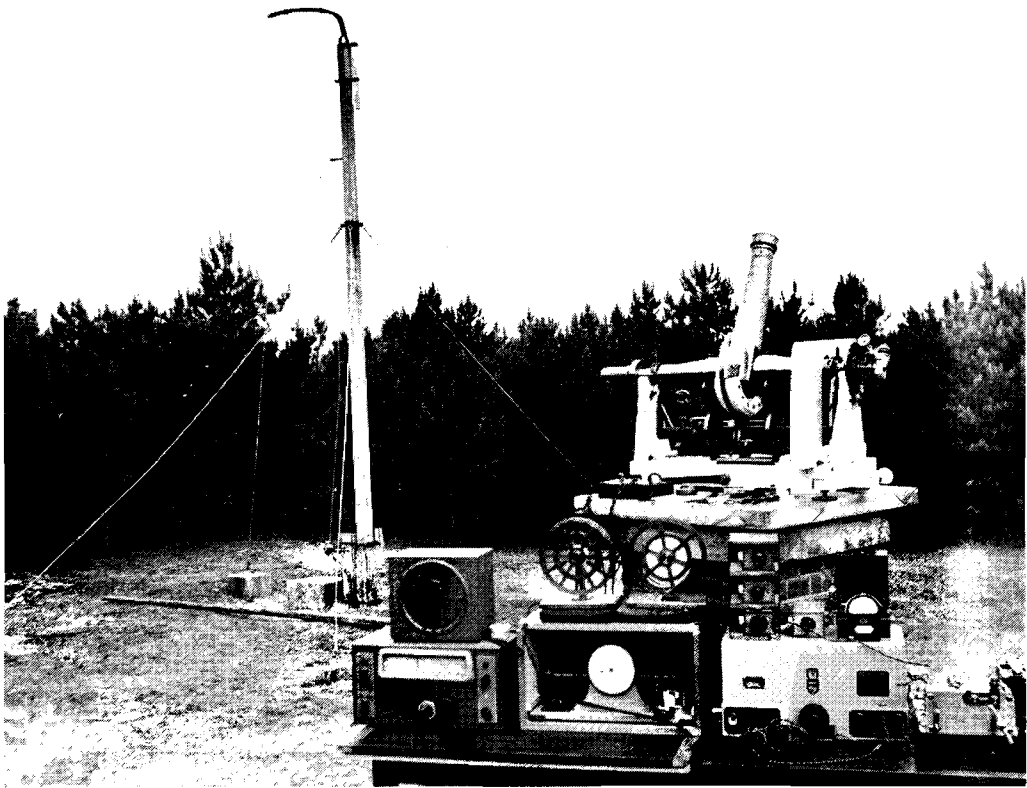
The observations were made using the following equipment:

- a. transit instrument
- b. chronograph
- c. synchron clocks (2)
- d. radio receiver

The meridian transits of stars were observed by a transit instrument of an usual type. This instrument was made in 1869 by Pistor and Martins. The technical data are as follows:

Telescope: magnification  $85 \times$   
 aperture 67.8 mm  
 focal length 861 mm

Suspension level: sensitivity  $0^s.114$  per division



Transit instrument and time recording equipment used for longitude determination. Pictures show set-up at Ameland.

## Self-recording micrometer:

graduation per revolution:	100 divisions
value of one revolution:	$7^{\text{s}}.831$
number of contacts:	12
widths of contact strips:	variable
lost motion:	0.26 div.

The time observations were recorded by an analogue tape chronograph made by the Great Northern Telegraph Cy, Copenhagen. The tape speed of this chronograph is about 24 mm/sec. One pen recorded the observations, the other pen fixed the time scale kept by a quartz clock of the Radio Laboratory of the Post and Telegraph Office in The Hague. The connection between the astronomical station in question and the quartz clock in The Hague was realised by means of a telegraph line. One synchron clock on the astronomical station displayed the time, the other one produced second pulses for the chronograph.

In order to determine the chronometer corrections before and after the measurements, a radio receiver was used for the time signals of the radio stations Pontoise (FYP) or Rugby (GBR).

### 3.4 Observations

In order to obtain an optimal accuracy, a theoretical investigation was carried out with various models of star-programmes. One model consisted of 3 north and 3 south stars in meridian transit having different zenith distances. The weight coefficients of the unknowns ( $\Delta T$ ) and ( $a$ ) were computed and presented in a nomogram [5]. Based on this nomogram the following method of star selection was applied.

One measuring programme consisted in general of 24 stars, observed in 4 groups. The first group of 6 stars was measured with instrument position eye-piece east, the second and the third groups with eye-piece west, and finally the last one with eye-piece east again. In each group 4 north stars were selected with  $\delta \approx 68^\circ$  and 2 south stars with  $\delta \approx 23^\circ$ , all in upper culmination. Only at the station Zierikzee  $4 \times 8$  stars were measured, with an equal number of north and south stars.

Each star was tracked over 3 revolutions by the self-recording micrometer of the transit instrument used, while  $3 \times 12$  contact times were registered on the chronograph.

The observations and different quantities computed are shown in appendix I, in which:

column 1: FK4 number of the stars observed

2: instrument position, eye-piece east or west

3: mean value of suspension level reading (zero of the level always on the opposite side of the eye-piece)

4: the mean time  $\bar{T}$ , reduced according to formula (3.7)

5: estimate of standard deviation of the mean time  $\bar{T}$

6: observation vector ( $\alpha - T$ )

7: weights, used in the adjustment.

### 3.5 Results of the longitude determinations

The definite results of the adjustment applying the formulae of section 3.2 are shown in

Table 3.5.1

station: Leeuwarden		
date: 1947	longitude	$\sigma_\lambda$
May 28-29	-0 <sup>h</sup> 23 <sup>m</sup> 09 <sup>s</sup> .568	0 <sup>s</sup> .012
29-30	9 <sup>s</sup> .578	0 <sup>s</sup> .007
30-31	9 <sup>s</sup> .586	0 <sup>s</sup> .014
31-1	9 <sup>s</sup> .559	0 <sup>s</sup> .012
mean value (pillar 1947)	-0 <sup>h</sup> 23 <sup>m</sup> 9 <sup>s</sup> .573	0 <sup>s</sup> .006
reduction to centre	-	0 <sup>s</sup> .017
Centre 1896	0 <sup>h</sup> 23 <sup>m</sup> 09 <sup>s</sup> .590	0 <sup>s</sup> .006

Table 3.5.3

station: Goedereede		
date: 1950	longitude	$\sigma_\lambda$
June 6-7	-0 <sup>h</sup> 15 <sup>m</sup> 54 <sup>s</sup> .290*	0 <sup>s</sup> .020
15-16	54 <sup>s</sup> .352	0 <sup>s</sup> .011
28-29	54 <sup>s</sup> .353	0 <sup>s</sup> .008
29-30	54 <sup>s</sup> .346	0 <sup>s</sup> .013
mean value (pillar)	-0 <sup>h</sup> 15 <sup>m</sup> 54 <sup>s</sup> .350	0 <sup>s</sup> .006
reduction to centre 1896	+	0 <sup>s</sup> .019
Centre 1896	-0 <sup>h</sup> 15 <sup>m</sup> 54 <sup>s</sup> .331	0 <sup>s</sup> .006

\* rejected

Table 3.5.2

station: Ameland		
date: 1947	longitude	$\sigma_\lambda$
June 16-17	-0 <sup>h</sup> 23 <sup>m</sup> 07 <sup>s</sup> .799	0 <sup>s</sup> .017
17-18	07 <sup>s</sup> .746	0 <sup>s</sup> .015
18-19	07 <sup>s</sup> .766	0 <sup>s</sup> .014
24-25	07 <sup>s</sup> .743	0 <sup>s</sup> .019
25-26	07 <sup>s</sup> .795	0 <sup>s</sup> .014
mean value (centre 1896)	-0 <sup>h</sup> 23 <sup>m</sup> 07 <sup>s</sup> .770	0 <sup>s</sup> .012

Table 3.5.4

station: Zierikzee		
date: 1949	longitude	$\sigma_\lambda$
Sept. 12-13 (I)	-0 <sup>h</sup> 15 <sup>m</sup> 39 <sup>s</sup> .620	0 <sup>s</sup> .010
19-20 (I)	39 <sup>s</sup> .630*	0 <sup>s</sup> .017
19-20 (II)	39 <sup>s</sup> .582	0 <sup>s</sup> .012
20-21 (I)	39 <sup>s</sup> .607	0 <sup>s</sup> .008
20-21 (II)	39 <sup>s</sup> .593	0 <sup>s</sup> .008
Oct. 4-5 (I)	39 <sup>s</sup> .612	0 <sup>s</sup> .009
4-5 (II)	39 <sup>s</sup> .587	0 <sup>s</sup> .006
mean value (pillar 1949)	-0 <sup>h</sup> 15 <sup>m</sup> 39 <sup>s</sup> .600	0 <sup>s</sup> .006
reduction to centre	+	0 <sup>s</sup> .023
Centre 1896	-0 <sup>h</sup> 15 <sup>m</sup> 39 <sup>s</sup> .577	0 <sup>s</sup> .006

tables 3.5.1-3.5.4. The small differences, approximately 0<sup>s</sup>.040, with the results of BRUINS [5] are due to definite time corrections and polar motion corrections now applied (see section 3.1).

The homogeneity of the variances of the different nights was investigated using the Bartlett-test [11]. Based on this test all the results of the stations Leeuwarden and Ameland are acceptable. However the measurements of the second night at the station Zierikzee and those of the first night at Goedereede differ considerably from the rest and should be rejected. Rejection of the measurement of the second night at Zierikzee has no serious consequences because of the large number of observations made. At this station namely, two different star programmes were measured, spread over several nights. However a systematic difference between star programmes (I) and (II), is noted. This may be explained by the influence of the different star selection, i.e. the correlation of the time with the other unknowns.

Rejecting the first measurement at Goedereede is also justified by the fact that just before starting these observations troubles were experienced with the optical part of the telescope. However, with great pains the observer succeeded in rectifying this trouble but it might have had a negative effect on the quality of his work that same night.

The mean values of the longitude per station were computed without the observations

rejected. The external accuracy of the mean value is computed from the spread of the adjusted values. An exception is made with the station Goedereede: instead of 0<sup>s</sup>.002 obtained in this way a value of 0<sup>s</sup>.006 is taken, derived from the precisions of the longitude.

**4 Simultaneous latitude, longitude and azimuth determinations (Ubachsberg, Tongeren)**

In the summer of 1968 geodetic-astronomical observations were carried out at the primary stations Ubachsberg (The Netherlands) and Tongeren (Belgium). The latitude, longitude and azimuth were simultaneously determined applying the Black method. The equipment used was an universal theodolite Wild T4 and an Omega timerecorder. The method of computation is briefly described below; more details are given in a previous paper [7].

With the Black method a number of stars are observed in vertical transit. For theoretical and practical reasons the stars are selected at approximately equal zenith distances (in this case  $z \approx 60^\circ$ ) and regularly distributed in azimuth. The observation equations of the Black method read:

$$\Delta\varphi \sin a_i \cot z_i + \Delta\lambda \cos \varphi_0 \cos a_i \cot z_i - (\Delta A + \Delta\lambda \sin \varphi_0) = l_i + \varepsilon_i \dots \dots (4.1)$$

in which the quantities  $\Delta\varphi$ ,  $\Delta\lambda \cos \varphi_0$  and  $(\Delta A + \Delta\lambda \sin \varphi_0)$  are considered as unknowns. The following relations exist between the approximate values of the latitude, longitude and azimuth and the unknowns:

$$\left. \begin{aligned} \varphi &= \varphi_0 + \Delta\varphi \\ \lambda &= \lambda_0 + \Delta\lambda \\ A &= A_0 + \Delta A \end{aligned} \right\} \dots \dots \dots (4.2)$$

The observation vector  $l_i$  in equation (4.1) is computed from:

$$l_i = A_0 + \psi_i - a_i \dots \dots \dots (4.3)$$

in which  $\psi_i$  is the horizontal angle measured between the terrestrial reference mark (R.M.) and the star in question, and  $a_i$  the azimuth of the star computed from the time observed. In case of a number of series per star, observed in face left and face right of the instrument, the mean value of the observation vector and the mean value of the coefficients of the unknowns is used in (4.1).

The horizontal angle  $\psi_i$  in (4.3) is obtained by applying some corrections:

$$\psi = \psi' - R(\psi') \pm p(M - M_0) \cot z + \Delta a_A + (C'_1 + C'_2) \frac{[F_k^2]}{2Nq''} \dots \dots \dots (4.4)$$

in which

- $\psi'$  = the horizontal angle measured (circle reading: Star - R.M.)
- $R(\psi')$  = periodical horizontal circle error of the angle  $\psi'$ , computed from the circle readings ( $\varphi$ ) according to [7]
- $p$  = level value; No. 434:  $p = 1''$ .03 (if zero of the level on the eye-piece side, the upper sign refers to face left, the lower sign to face right)

- $M$  =  $\frac{1}{2}(r+l)$  mean value of the suspension level reading
- $M_0$  = reference position of bubble
- $\Delta a_A$  =  $-0''.32 \cos \varphi \operatorname{cosec} z \cos a$  (=daily aberration)
- $\frac{[F_k^2]}{2N\varrho''}$  =  $0''.043$  instrumental constant obtained with  $N=27$  contacts of the self-recording micrometer, in which  $F_k$  = distances of the contacts with respect to their centre point in seconds of arc ( $k = 1, 2, \dots, N$ )
- $C'_1$  =  $(\cos z \tan q + \cot a \tan^2 q) \operatorname{cosec}^2 z$
- $C'_2$  =  $-2 \cos z \tan q \operatorname{cosec}^2 z$

The factors  $C'_1$  and  $C'_2$  were derived at the Geodetic Institute of the Delft University of Technology in a similar way as the factors  $C_1$  and  $C_2$  derived by ROELOFS [9] for zenith distance measurement. The factor  $C'_1$  corrects the non-linear relation between time and azimuth;  $C'_2$  must be applied because all the vertical wires except the central wire (i.e. contacts) are in fact non-vertical great circles. Meanwhile these factors were compared with a correction formula given by JORDAN-EGGERT-KNEISSL [8], page 440, which is applied as a time correction to the mean value of the contact-times in the following way:

$$\bar{T} = \frac{[T]}{N} + C_t \frac{[F_k^2]}{2N\varrho''}$$

in which  $C_t = \frac{1}{15} \tan \delta \sec \delta \tan q \sec^2 q$ . In order to correct the star's azimuth instead of the time, this time factor was converted into an azimuth factor. Multiplying with:

$$\frac{da}{dt} = 15 \cos \delta \cos q \operatorname{cosec} z$$

gives:

$$C_a = C_t \times \frac{da}{dt} = \tan \delta \tan q \sec q \operatorname{cosec} z$$

Substituting  $\tan \delta = \cot z \cos q - \operatorname{cosec} z \sin q \operatorname{cosec} z$  gives:

$$C_a = (\cos z \tan q - \cot a \tan^2 q) \operatorname{cosec}^2 z = -(C'_1 + C'_2)$$

Hence it can be concluded that the two corrections are exactly identical (the signs are opposite because the correction with the factors  $(C'_1 + C'_2)$  must be applied to the horizontal angle measured).

The star's azimuth used in (4.3) is computed from:

$$\cot a_i = \frac{\sin \varphi_0 \cos (t_i^G - \lambda_0) - \cos \varphi_0 \tan \delta_i}{\sin (t_i^G - \lambda_0)} \dots \dots \dots (4.5)$$



in which the Greenwich hour angle  $t_i^G$  of the star is determined from the time measurement. To the mean value  $\bar{T}$  of the contact times, obtained by tracking the star with the aid of the vertical wire of the self-recording micrometer, some corrections were applied:

$$(UT1) = \bar{T} + (UTC - \bar{T}) + (UT1 - UTC) + \frac{(\beta'' + \tau'')}{2 \times 0.997 \times 15} |\sec \delta \sec q| \dots (4.6)$$

in which:

- (UT1) = corrected UT1
- (UTC -  $\bar{T}$ ) = chronometer correction determined by radio time signals (usually MSF/Rugby 5 MHz)
- (UT1 - UTC) = correction to the time signal according to the Bureau International de l'Heure
- ( $\beta'' + \tau''$ ) = sum of the width of the contact strips and the lost motion of the self-recording micrometer.

The (UT1) obtained in (4.6) is then converted into Greenwich apparent sidereal time:

$$GAST = (UT1) \times 1.0027379 + GAST(0^b UT) + \Delta e \dots (4.7)$$

in which  $\Delta e$  is the change in the equation of equinoxes during the period of UT.

Finally from GAST the Greenwich hour angle to be used is obtained from:

$$t^G = GAST - \alpha \dots (4.8)$$

The determination of the three unknowns according to the observation equations (4.1) was carried out by least square adjustment. From the observation equations (4.1) written in the form:

$$AX = L + E \dots (4.9)$$

it follows:

$$X = \begin{pmatrix} \Delta\varphi \\ \Delta\lambda \cos \varphi_0 \\ \Delta A + \Delta\lambda \sin \varphi \end{pmatrix} = (A^* A)^{-1} A^* L = Q A^* L \dots (4.10)$$

in which  $Q$  is approximately a diagonal matrix with elements  $Q_{11}$ ,  $Q_{22}$  and  $Q_{33}$ , if the stars are selected at equal zenith distance and regularly distributed in azimuth. The astronomical latitude, longitude and azimuth are then obtained from (4.2).

The estimate of the variance of the observation vector per star follows from:

$$\hat{\sigma}^2 = \frac{E^* E}{s - 3} \dots (4.11)$$

in which  $E$  is determined by substituting the unknowns into (4.9). The standard deviations of the unknowns are then:

$$\left. \begin{aligned} \hat{\sigma}_\varphi &= \hat{\sigma}\sqrt{Q_{11}} \\ \hat{\sigma}_\lambda &= \hat{\sigma}\sqrt{Q_{22}\sec^2\varphi} \\ \hat{\sigma}_{A_g} &= \hat{\sigma}\sqrt{Q_{33}} \\ \hat{\sigma}_A &= \hat{\sigma}\sqrt{Q_{44}} \end{aligned} \right\} \dots \dots \dots (4.12)$$

in which  $A_g$  denotes the geodetic azimuth:  $A_g = A + \Delta\lambda \sin \varphi_0$ . The weight coefficient  $Q_{44}$  of the astronomical azimuth is computed in the following way:

$$Q_{44} = Q_{33} + Q_{22} \tan^2\varphi - Q_{23} \tan \varphi \dots \dots \dots (4.13)$$

In total 36 stars were observed at the station Ubachsberg and 32 stars at Tongeren in different nights. The observation of one star was made in the following sequence:

- face left: reference mark 2 ×
- star 2 × tracking over 27 contacts
- face right: star 2 ×
- reference mark 2 ×

The observations and some additional data are shown in appendix II, in which:

- Column 1: date
- 2: FK4 number of the star observed
- 3: face: (1) = left; (2) = right
- 4: horizontal circle reading reference mark
- 5: horizontal circle reading star
- 6: mean value of the suspension level reading (zero of the level always on the eye-piece side)
- 7: chronometer time  $\bar{T}$ ; the mean value of  $N = 27$  contact times
- 8: chronometer correction:  $(UTC - \bar{T})$  from radio time signals

A common adjustment of all the stars measured per station gives the following matrices of weight coefficients:

$$\text{Ubachsberg: } Q = \begin{pmatrix} 0.1678 & -0.0009 & 0.0000 \\ & 0.1655 & 0.0002 \\ & & 0.0278 \end{pmatrix}$$

$$\text{Tongeren: } Q = \begin{pmatrix} 0.1802 & -0.0019 & -0.0008 \\ & 0.1957 & -0.0000 \\ & & 0.0313 \end{pmatrix}$$

The final results, corrected for the polar motion and reduced to the centre, are shown in table 4.1. It shows a favourable standard deviation of the geodetic azimuth due to the negative correlation between the astronomical azimuth and the astronomical longitude.

Table 4.1

station	quantity	$\sigma$
Ubachsberg Centre 1890	$\varphi = 50^{\circ}50'53''.432$	0".41
	$\lambda = - 5^{\circ}57'04''.320$	0".64
	$A = 258^{\circ}15'24''.273$	0".53
	$Ag = 258^{\circ}15'30''.558$	0".17
Tongeren Station A	$\varphi = 50^{\circ}46'55''.775$	0".48
	$\lambda = - 5^{\circ}27'48''.570$	0".78
	$A = 77^{\circ}52'43''.958$	0".64
	$Ag = 77^{\circ}52'47''.306$	0".20

**5 Azimuth determinations (Goedereede, Zierikzee)**

The astronomical azimuths Goedereede–Zierikzee and Zierikzee–Goedereede were determined by Polaris in 1969 and 1973 respectively. The measurements and the computations were carried out in the usual way, a brief description of which is given below.

The horizontal angle between the reference mark and Polaris was measured using a first order universal theodolite (Wild T4 or Kern DKM3A) in the following sequence:

- face left: – reference mark: 2 × (pointing and reading the horizontal circle)
- Polaris: 2 × (pointing and reading the horizontal circle including the striding level)
- face right: – Polaris: 2 ×
- reference mark: 2 ×

Such a group of four single series is called one set of observations. At each station about 24 sets should be measured, spread over several nights, in order to obtain an external accuracy of approximately 0".2 for the mean azimuth.

For the time keeping an Omega chronograph was used, by which the time of pointing to Polaris was recorded by means of a tap-key. The star's hour angle is computed as follows:

$$\left. \begin{aligned}
 UTC &= T + (UTC - T) \\
 UT1 &= UTC + (UT1 - UTC) \\
 GAST &= 1.0027379 \times UT1 + GAST(0^h UT) \\
 t &= GAST - \alpha - \lambda
 \end{aligned} \right\} \dots \dots \dots (5.1)$$

in which:

- $T$  = the time recorded by the chronograph
- $(UTC - T)$  = chronometer correction determined by radio time signals of HBG (Switzerland)
- $(UT1 - UTC)$  = correction to the time signal according to Bureau International de l'Heure
- $t$  = local hour angle of the star.

The star's azimuth (counting clockwise from the north) is then computed from:

$$\tan a = \frac{\sin t}{\sin \varphi \cos t - \cos \varphi \tan \delta} \dots \dots \dots (5.2)$$

The horizontal angle measured between the reference mark and Polaris was corrected in the following way:

$$\psi = \psi' - R(\psi') \mp p(M - M_0) \cot z + \Delta a_A \dots \dots \dots (5.3)$$

in which:

- $\psi'$  = horizontal angle from the circle reading (Star-R.M.)
- $R(\psi')$  = periodical horizontal circle error of the angle  $\psi'$ , computed from the circle readings ( $\varphi$ ) according to appendix IV and [7], appendix 1.
- $p$  = level value: Wild T4:  $p = 1''.11$  per division of 2 mm (level No. 668)  
 DKM 3A:  $p = 1''.24$  per division of 2 mm (level No. 152002)  
 (if zero of the level on the circle side: upper sign refers to face left, lower sign to face right)
- $M$  =  $\frac{1}{2}(l+r)$  mean value of the level reading
- $M_0$  = reference position of the bubble
- $\Delta a_A$  =  $-0''.32 \cos \varphi \operatorname{cosec} z \cos a$  (= daily aberration)

Finally the azimuth of the reference mark is computed from (5.2) and (5.3):

$$A_{ij} = a_{ij} - \psi_{ij} \quad \left. \begin{array}{l} i = 1, 2, \dots n \quad (\text{number of sets}) \\ j = 1, 2 \quad (\text{series in face left}) \\ j = 3, 4 \quad (\text{series in face right}) \end{array} \right\} \dots \dots \dots (5.4)$$

The mean values of the azimuth obtained with instrument positions left and right are, respectively:

$$\bar{A}_L = \frac{[A_{i,1} + A_{i,2}]}{2n} \quad \bar{A}_R = \frac{[A_{i,3} + A_{i,4}]}{2n} \dots \dots \dots (5.5)$$

from which follows the mean azimuth per night:

$$\bar{A} = \frac{1}{2}(\bar{A}_L + \bar{A}_R) \dots \dots \dots (5.6)$$

The corrections to the single azimuths were computed as follows:

$$\left. \begin{array}{l} \varepsilon_{i,1} = \bar{A}_L - A_{i,1} \quad \varepsilon_{i,3} = \bar{A}_R - A_{i,3} \\ \varepsilon_{i,2} = \bar{A}_L - A_{i,2} \quad \varepsilon_{i,4} = \bar{A}_R - A_{i,4} \end{array} \right\} \dots \dots \dots (5.7)$$

from which the estimate of the variance of the mean azimuth is:

$$\hat{\sigma}_{\bar{A}}^2 = \frac{[\varepsilon_{ij}^2]}{4n(4n-2)} \quad (\text{method A}) \dots \dots \dots (5.8)$$

Another method is preferred for the computation of the variance in case the observations are spread over several nights. In order to eliminate the systematic instrumental errors, first the mean values of face left and face right are computed from:

$$A_{i,13} = \frac{1}{2}(A_{i,1} + A_{i,3})$$

$$A_{i,24} = \frac{1}{2}(A_{i,2} + A_{i,4})$$

Table 5.1

Station: Goedereede (pillar 2)				instr.: Wild T4/1957			
$\varphi = 51^{\circ}49'09''.7$ $\lambda = -0^{\text{h}}15^{\text{m}}54^{\text{s}}.35$				observer: Steur			
reference mark: Zierikzee (lamp 1969)							
date: 1969	number of sets	adjusted azimuth $A$	$\hat{\sigma}$ method A	correction polar motion $\Delta A$	corrected azimuth $A + \Delta A$	$\hat{\sigma}$ method B	
May 22	6	192°45'09".145	0".16	-0".590	192°45'08".555		
May 29	6	08".944	0".27	-0".582	08".362		
June 3	6	08".318	0".28	-0".575	07".743		
June 4	6	08".341	0".23	-0".572	07".769		
June 5	6	08".434	0".19	-0".570	07".864		
June 11	6	08".255	0".27	-0".559	07".696		
mean		192°45'08".573	0".10	-0".575	192°45'07".998	0".10	
					bearing traverse $\delta$	-02'06".900	
					meridian conv. $\gamma$	- 0".118	
azimuth Goedereede							
Centre 1896 - Zierikzee							
					Centre 1896	192°43'00".980	0".10

Table 5.2

Station: Zierikzee (perm. mark 12)				instr.: DKM 3A No. 134824			
$\varphi = 51^{\circ}39'04''.6$ $\lambda = -0^{\text{h}}15^{\text{m}}39^{\text{s}}.62$				observer: Steur			
reference mark: Goedereede (pillar 2)							
date: 1973	number of sets	adjusted azimuth $A$	$\hat{\sigma}$ method A	correction polar motion $\Delta A$	corrected azimuth $A + \Delta A$	$\hat{\sigma}$ method B	
Aug. 21	6	12°39'47".850	0".31	-0".542	12°39'47".308		
Aug. 22	6	47".110	0".29	-0".543	46".567		
Aug. 23	9	46".777	0".26	-0".544	46".233		
Aug. 28	3	47".531	0".24	-0".547	46".984		
weighted mean		12°39'47".223	0".15	-0".544	12°39'46".679	0".15	
					bearing traverse $\delta$	+20".694	
					meridian conv. $\gamma$	- 0".302	
azimuth Zierikzee Centre							
1896 - Goedereede Centre							
					1896	12°40'07".071	0".15

The corrections to these azimuths are:

$$\varepsilon_{i,13} = \bar{A} - A_{i,13}$$

$$\varepsilon_{i,24} = \bar{A} - A_{i,24}$$

in which  $\bar{A}$  now denotes the mean value of all observations. Hence the variance of the mean azimuth is:

$$\hat{\sigma}_{\bar{A}}^2 = \frac{[\varepsilon_{i,13}^2 + \varepsilon_{i,24}^2]}{2n(2n-1)} \quad (\text{method B})$$

The observations and some additional data are given in appendix III, in which:

- column 1: set number
- 2: face: (1) = left; (2) = right
- 3: horizontal circle reading reference mark
- 4: horizontal circle reading star
- 5: mean value of the level reading (zero of the level always on the vertical circle side)
- 6: chronometer time:  $T$

The results of the computation are shown in the tables 5.1 and 5.2. The standard deviation of the mean azimuth per night is computed using method A. Applying the Bartlett-test to these standard deviations indicates that the measurements of the various nights are homogeneous. However, some differences between the mean values have little significance. In addition the standard deviation of the station's mean value was computed applying method B; i.e. taking into account all observations with respect to the station's mean azimuth. Method A and B give the same result:

Goedereede:  $\hat{\sigma}_{\bar{A}} = 0''.10$   
 Zierikzee:  $\hat{\sigma}_{\bar{A}} = 0''.15$

The latter values can be considered as the external accuracy.

## 6 Misclosures of the Laplace stations

### 6.1 Method of computation

The misclosure of a Laplace point can be computed from:

$$w = A_g - A_g^* \dots \dots \dots (6.1)$$

in which both  $A_g$  and  $A_g^*$  indicate geodetic azimuths;  $A_g$  determined from the geodetic network, and  $A_g^*$  from geodetic-astronomical observations.

To check the primary network of The Netherlands (adjusted without Laplace points) and at the same time gain an insight into the quality of the six Laplace points, the geodetic azimuth is derived in the following way:

$$A_g = \psi - \varepsilon + \gamma . . . . . (6.2)$$

in which  $\psi$  is the grid bearing,  $\varepsilon$  is the reduction of the direction to the Bessel ellipsoid and  $\gamma$  is the meridian convergence. These quantities can be computed from the plane rectangular coordinates of the network according to [10] and [4].

The geodetic azimuth  $A_g^*$  in (6.1) follows from the Laplace equation:

$$A_g^* = A - (\lambda - \lambda_g) \sin \varphi + \{(\lambda - \lambda_g) \cos \varphi \cos A - (\varphi - \varphi_g) \sin A\} \tan H . . . . (6.3)$$

in which:

- $\varphi, \varphi_g$  = latitude (astronomic or geodetic)
- $\lambda, \lambda_g$  = longitude (in this formula: positive east of Greenwich!)
- $A$  = astronomic azimuth
- $H$  = elevation angle

Geodetic latitude and geodetic longitude, also contained in formula (6.3), can be computed from plane rectangular coordinates applying the formulae given in [10]. The geodetic longitudes obtained in this way refer to the Amersfoort meridian, while the astronomic longitude is usually determined with respect to the Greenwich meridian. In order to have the same reference meridian for the longitudes in (6.3) we can:

- a. reduce the astronomic longitude to the Amersfoort meridian, or
- b. reduce the geodetic longitude to the Greenwich meridian.

In both reductions the astronomic longitude of Amersfoort should be used, so that in (b) in fact a fictitious geodetic longitude is obtained. Both methods give, of course, the same result.

Since the longitude of Amersfoort is not determined with high precision (see [3]), it will have a systematic influence on the misclosures. The systematic error, however, will be eliminated by taking the relative misclosure between two Laplace points.

### 6.2 Misclosures

The misclosures were computed according to the above formulae. The geodetic latitudes and longitudes of the Laplace points are given in table 6.1.

The geodetic azimuths  $A_g$  and  $A_g^*$  computed by formulae (6.2) and (6.3) are given in table 6.2. It should be noted that the last term in (6.3) has been omitted, because in a flat country like The Netherlands, the elevation angle is practically zero.

Table 6.1

station	$\varphi_g$	$\lambda_g$ (Greenwich)
Leeuwarden	53°12'14".750	5°47'24".655
Ameland	53°27'30".999	5°46'54".367
Goedereede	51°49'09".697	3°58'33".549
Zierikzee	51°39'04".346	3°54'53".967
Ubachsberg	50°50'49".180	5°57'12".425
Tongeren	50°46'53".616	5°27'52".892

Table 6.2

station	$A_g$	$\sigma_{A_g}$	$A_g^*$	$\sigma_{A_g^*}$	$w = A_g - A_g^*$
Leeuwarden	358°31'56".063	0".42	358°31'58".277	0".36	-2".214
Ameland	179°05'49".063	0".41	179°05'51".168	0".25	-2".105
Goedereede	192°42'58".386	0".46	192°42'59".867	0".13	-1".481
Zierikzee	12°40'05".980	0".46	12°40'07".316	0".17	-1".336
Ubachsberg	258°15'26".413	0".48	258°15'30".558	0".17	-4".145
Tongeren	77°52'42".590	0".48	77°52'47".306	0".20	-4".716

From the misclosures  $w$  obtained in table 6.2 the following conclusions can be drawn:

- all the misclosures are negative, obviously effected by the approximate longitude of Amersfoort; ( $\lambda = 5^\circ 23' 15''.500$ );
- assuming the condition  $[w] = 0$ , the following longitude of Amersfoort is obtained:  $\lambda = 5^\circ 23' 12''.114$ ;
- the relative misclosures between the different Laplace points are small;
- the largest relative misclosures, average  $2''.6$ , are obtained between the twin Laplace point Ubachsberg-Tongeren and the other points.

### 6.3 The precision of the geodetic part of the Laplace azimuth

In table 6.2 the precisions of the geodetic azimuths  $A_g$  are also given. These data were computed in the following way.

Let us consider the geodetic part of the Laplace quantity:

$$L_i = A_{ik} + \lambda_i \sin \varphi_i \dots \dots \dots (6.4)$$

Expressing  $A_{ik}$  and  $\lambda_i$  in plane rectangular coordinates, (see(6.2)) and ignoring the quantities  $\varepsilon$  and  $\gamma$  we obtain:

$$L_i = \arctan \frac{x_k - x_i}{y_k - y_i} - 2.551 \times 10^{-7} x_i \sin \varphi_i \dots \dots \dots (6.5)$$

Linearisation of (6.5) gives:

$$\Delta L_i = \frac{\sin A_{ik}}{s_{ik}} \Delta y_i - \frac{\cos A_{ik}}{s_{ik}} \Delta x_i - \frac{\sin A_{ik}}{s_{ik}} \Delta y_k + \frac{\cos A_{ik}}{s_{ik}} \Delta x_k - 2.551 \times 10^{-7} \sin \varphi_i \Delta x_i \quad (6.6)$$

in which  $s_{ik}$  is the distance between the Laplace point  $i$  and the azimuth point  $k$ . Denoting the coefficients of the coordinates by  $a, b, c, d$ , we then have:

$$\Delta L_i = a_i \Delta y_i + b_i \Delta x_i + c_i \Delta y_k + d_i \Delta x_k \dots \dots \dots (6.7)$$

Based on equation (6.7) the matrix of weight coefficients of the six Laplace points were computed, using the covariance matrix of the coordinates related to the base Amersfoort-Veluwe (see appendix V). The computations were carried out by Mr. J. J. KOK of the Computing Centre of the Delft Geodetic Institute. The results of this computation, in (sec of arc)<sup>2</sup>, are shown in table 6.3.



Table 6.3 The matrix of weight coefficients of the geodetic part of the Laplace quantity

	L	A	G	Z	U	T
Leeuwarden	+.1736	+.1726	+.0224	+.0225	+.0210	+.0210
Ameland	+.1726	+.1716	+.0223	+.0224	+.0208	+.0209
Goedereede	+.0224	+.0223	+.2100	+.2108	+.0552	+.0557
Zierikzee	+.0225	+.0224	+.2108	+.2116	+.0554	+.0558
Ubachsberg	+.0210	+.0208	+.0552	+.0554	+.2332	+.2336
Tongeren	+.0210	+.0209	+.0557	+.0558	+.2336	+.2340

Since the variance of the coordinates of the azimuth point Hallum are not available, the points Ameland and Leeuwarden were used as azimuth points instead of Hallum.

The standard deviations  $\sigma_{A_p}$  shown in table 6.2 are obtained from the matrix of variance of table 6.3, by taking the square root of the figures in the diagonal. Moreover, the data of tables 6.2 and 6.3 make it possible to compute the standard deviations of the misclosures and those of the relative misclosures. The results of these computations are given in table 6.4.

Table 6.4 Standard deviations of the misclosures and of the relative misclosures

station	L	A	G	Z	U	T
Leeuwarden	0''.55	0''.44	0''.70	0''.71	0''.72	0''.73
Ameland		0''.48	0''.65	0''.66	0''.67	0''.68
Goedereede			0''.48	0''.21	0''.62	0''.62
Zierikzee				0''.49	0''.63	0''.63
Ubachsberg					0''.51	0''.26
Tongeren						0''.52

From table 6.4, for example, the standard deviation of the misclosure of the Laplace point Leeuwarden is 0''.55, and the standard deviation of the relative misclosure between Leeuwarden–Ameland is 0''.44. The latter is favourably influenced by the strong correlation between the geodetic data (see table 6.3). It should be noted that the relative misclosures between Leeuwarden–Ameland–Goedereede–Zierikzee are quite in accordance with their theoretical standard deviations, while the relative misclosures of those stations with Ubachsberg and Tongeren are considerably larger.

### Acknowledgements

The geodetic-astronomical observations were carried out by the Netherlands Triangulation Service. The organization of the fieldwork of the recent measurements of the Laplace stations Ubachsberg, Tongeren, Zierikzee and Goedereede has, since 1968, been the responsibility of Mr. M. HAARSMa, who also assisted in many respects with the preparation of the present report. The author also expresses his gratitude to Prof. G. J. BRUINS, in charge of the longitude determinations in 1947–1950, for the helpful information provided. The assistance of Mr. F. RENEMAN who wrote the computer programmes for the IBM 360/65 is gratefully acknowledged.

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LONGITUDE BY MERIDIAN TRANSIT OF STARS.

STATION. . . . . : AMELAND.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : JUNE 16 - 17, 1947.
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(FYP)-T AT 20H 6M: 0 9 17.076
CHRON. RATE DIT. . . . . : -0.041/12H

APPROXIMATE VALUES:
LATITUDE . . . . . : 53 27 30.999
LONGITUDE. . . . . : -0 23 7.624

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Contains 30 rows of star observation data.

STATION. . . . . : AMELAND.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : JUNE 17 - 18, 1947.
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(FYP)-T AT 20H 6M: 0 53 7.682
CHRON. RATE DIT. . . . . : -0.048/12H

APPROXIMATE VALUES:
LATITUDE . . . . . : 53 27 30.999
LONGITUDE. . . . . : -0 23 7.624

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Contains 30 rows of star observation data.

no.1421 is not used in the computation

LONGITUDE BY MERIDIAN TRANSIT OF STARS.

STATION. . . . . : AMELAND.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : JUNE 18 - 19, 1947.
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(FYP)-T AT 20H 6M: 0 1 55.994
CHRON. RATE DIT. . . . . : -0.050/12H

APPROXIMATE VALUES:
LATITUDE . . . . . : 53 27 30.999
LONGITUDE. . . . . : -0 23 7.624

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Contains 30 rows of star observation data.

STATION. . . . . : AMELAND.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : JUNE 24 - 25, 1947.
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(FYP)-T AT 20H 6M: 0 2 3.811
CHRON. RATE DIT. . . . . : -0.043/12H

APPROXIMATE VALUES:
LATITUDE . . . . . : 53 27 30.999
LONGITUDE. . . . . : -0 23 7.624

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Contains 30 rows of star observation data.

no.1421 is not used in the computation

LONGITUDE BY MERIDIAN TRANSIT OF STARS.

STATION. . . . . : AMELAND.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : JUNE 25 - 26, 1947.
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(FYP)-T AT 20H 6M: 0 2 3.750
CHRON. RATE DIT. . . . : -0.048/12H

APPROXIMATE VALUES:
LATITUDE . . . . . : 53 27 30.999
LONGITUDE. . . . . : -0 23 7.624

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Rows include stars like 1396 E, 565, 569, etc.

STATION. . . . . : ZIERIKZEE.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : SEP 12 - 13, 1949 (II).
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(GBR)-T AT 20H 0M: 4 15 24.831
CHRON. RATE DIT. . . . : -0.088/14H

APPROXIMATE VALUES:
LATITUDE . . . . . : 51 39 4.346
LONGITUDE. . . . . : -0 15 39.598

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Rows include stars like 759 E, 760, 765, etc.

LONGITUDE BY MERIDIAN TRANSIT OF STARS.

STATION. . . . . : ZIERIKZEE.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : SEP 19 - 20, 1949 (I)
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(GBR)-T AT 20H 0M: 4 10 30.052
CHRON. RATE DIT. . . . : -0.091/14H

APPROXIMATE VALUES:
LATITUDE . . . . . : 51 39 4.346
LONGITUDE. . . . . : -0 15 39.598

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Rows include stars like 759 E, 760, 765, etc.

STATION. . . . . : ZIERIKZEE.
INSTRUMENT . . . . . : PISTOR AND MARTINS (1869).
DATE . . . . . : SEP 19 - 20, 1949 (III).
OBSERVERS. . . . . : BRUINS/DE VRIES.

CHRON. CORRECTION
SIGNAL(GBR)-T AT 20H 0M: 4 10 30.052
CHRON. RATE DIT. . . . : -0.091/14H

APPROXIMATE VALUES:
LATITUDE . . . . . : 51 39 4.346
LONGITUDE. . . . . : -0 15 39.598

OBSERVATIONS:

Table with columns: STAR, LEVEL, CHRON. TIME, SIGMA, DEC., G, (RA-T). Rows include stars like 875 E, 880, 882, etc.





Appendix II

LATITUDE, LONGITUDE AND AZIMUTH

DETERMINATION BY THE BLACK METHOD.

STATION. . . . . : UBACHSBERG.  
 REFERENCE MARK . . . . . : SCHIMMERT.  
 INSTRUMENT . . . . . : WILD T4/1957.  
 DATE . . . . . : JUNE - JULY, 1968  
 OBSERVER . . . . . : VAN LCON.

APPROXIMATE VALUES:

LATITUDE . . . . . : 50° 50' 50" 700  
 LONGITUDE . . . . . : -0° 23' 48" 000  
 AZIMUTH . . . . . : 303° 5' 5" 000

OBSERVATIONS:

DATE	STAR	FACE	HOR. CIRCLE	R.N.	HOR. CIRCLE	STAR	LEVEL	CHRON. TIME	CHR. CORR. (UTC-T)	
VI.11	1568	1	303	5' 18.920	52° 9'	36.060	47.4	5 <sup>h</sup> 29 <sup>m</sup> 44.788	15 <sup>h</sup> 40 <sup>m</sup> 58.163	
		1	303	5 18.800	52 26	10.300	47.4	5 31 42.371	15 40 58.163	
		2	123	5 22.920	233 33	30.040	51.4	5 39 40.800	15 40 58.165	
		2	123	5 22.820	233 49	53.810	51.0	5 41 37.753	15 40 58.165	
	17	1	303	5 18.500	38 51	6.030	46.1	7 40 51.790	15 40 58.184	
		1	303	5 18.280	39 8	59.830	45.9	7 43 13.939	15 40 58.184	
		2	123	5 22.610	220 5	36.040	51.1	7 50 45.704	15 40 58.186	
		2	123	5 22.700	220 22	16.150	50.8	7 52 59.765	15 40 58.186	
		585	1	348	5 18.580	257 42	36.050	49.4	8 12 26.982	15 40 58.190
			1	348	5 18.480	258 3	23.800	49.5	8 13 42.906	15 40 58.190
	VI.12	441	2	168	5 22.650	80 23	30.040	46.9	8 22 19.337	15 40 58.192
			2	168	5 22.730	80 44	14.430	46.8	8 23 36.561	15 40 58.192
1			33	5 18.540	39 8	36.050	45.4	8 49 26.338	15 40 58.196	
		1	33	5 18.320	39 23	59.510	45.3	8 51 21.056	15 40 58.196	
1574		2	213	5 22.420	220 29	24.030	51.1	8 59 26.464	15 40 58.198	
		2	213	5 22.400	220 44	33.390	50.6	9 1 18.703	15 40 58.198	
		1	78	5 18.300	261 25	18.070	49.7	9 19 27.593	15 40 58.202	
		1	78	5 18.300	261 45	17.400	49.6	9 20 48.782	15 40 58.202	
		2	258	5 22.280	83 56	.040	47.2	9 29 32.858	15 40 58.204	
		2	258	5 22.200	84 18	21.580	47.0	9 31 1.416	15 40 58.204	
VI.14		1480	1	33	5 22.420	227 59	48.040	47.0	5 1 41.700	17 6 2.724
			1	33	5 22.500	228 22	20.310	47.0	5 3 8.455	17 6 2.725
	2		213	5 15.100	50 38	24.060	49.5	5 11 46.751	17 6 2.726	
		2	213	5 14.840	50 59	2.380	49.8	5 13 4.432	17 6 2.726	
	516	1	78	5 20.450	1 34	42.040	48.5	5 30 53.215	17 6 2.730	
		1	78	5 20.630	1 56	20.310	48.6	5 32 16.527	17 6 2.730	
		2	258	5 15.280	184 6	48.020	49.1	5 40 45.010	17 6 2.731	
		2	258	5 15.400	184 28	15.840	48.8	5 42 9.629	17 6 2.731	
		17	1	303	5 20.700	38 51	6.030	47.9	6 3 59.538	17 6 2.735
			1	303	5 20.980	39 5	53.120	47.6	6 5 57.015	17 6 2.735
	585	2	123	5 15.480	220 5	36.070	47.9	6 13 55.112	17 6 2.737	
		2	123	5 15.400	220 20	10.140	47.9	6 15 52.313	17 6 2.737	
1		348	5 20.880	257 42	36.100	47.7	6 35 34.311	17 6 2.741		
1		348	5 20.920	258 2	25.600	47.7	6 36 46.750	17 6 2.741		
2		168	5 15.500	80 23	30.030	49.0	6 45 27.441	17 6 2.742		
2		168	5 15.420	80 43	18.250	49.0	6 46 41.057	17 6 2.742		
VI.15		441	1	33	5 20.890	39 8	36.070	48.0	7 12 33.566	17 6 2.747
			1	33	5 20.760	39 24	8.810	48.0	7 14 29.625	17 6 2.747
			2	213	5 15.380	220 29	24.030	48.9	7 22 35.205	17 6 2.749
			2	213	5 15.420	220 46	55.350	48.3	7 24 44.652	17 6 2.749
		1574	1	78	5 20.790	261 47	7.830	47.4	7 44 3.687	17 6 2.753
			1	78	5 20.880	262 8	17.900	47.5	7 45 29.279	17 6 2.753
2	258		5 15.520	83 56	.040	49.2	7 52 40.582	17 6 2.754		
2	258		5 15.320	84 16	24.400	49.3	7 54 1.737	17 6 2.754		
456	1		348	5 20.820	9 45	54.070	47.5	8 45 56.450	17 6 2.763	
	1		348	5 20.650	10 0	35.250	47.4	8 48 1.745	17 6 2.764	
VI.18	17	2	168	5 15.470	150 56	54.020	48.3	8 55 59.569	17 6 2.765	
		2	168	5 15.620	191 12	1.380	48.0	8 58 6.918	17 6 2.765	
		1	303	5 20.450	38 51	6.030	47.4	0 27 22.519	22 26 56.381	
		1	303	5 20.180	39 5	45.540	47.5	0 29 18.999	22 26 56.382	
	585	2	123	5 15.180	220 5	36.010	50.6	0 37 18.110	22 26 56.383	
		2	123	5 14.980	220 20	27.510	50.6	0 39 17.594	22 26 56.383	
		1	348	5 20.420	257 42	36.030	48.9	0 58 57.104	22 26 56.387	
		1	348	5 20.440	258 2	5.830	49.0	1 0 8.300	22 26 56.387	
		2	168	5 15.140	80 23	30.030	49.2	1 8 50.204	22 26 56.389	
		2	168	5 15.280	80 43	44.140	49.0	1 10 5.609	22 26 56.389	





LATITUDE, LONGITUDE AND AZIMUTH  
 DETERMINATION BY THE BLACK METHOD.

STATION . . . . . : TONGEREN.  
 REFERENCE MARK . . . . . : HERDEREN.  
 INSTRUMENT . . . . . : WILD T4/1957.  
 DATE . . . . . : MAY - JUNE, 1968  
 OBSERVER . . . . . : VAN LOON.

APPROXIMATE VALUES:

LATITUDE . . . . . : 50° 46' 56" 100  
 LONGITUDE . . . . . : -0° 21' 51" 160  
 AZIMUTH . . . . . : 68° 46' 30" 000

OBSERVATIONS:

DATE	STAR	FACE	HOR.CIRCLE	R.M.	HOR.CIRCLE	STAR	LEVEL	CHRON. TIME	CHR.CCRR.(UTC-T)
V.14	182	1	68° 46'	27.320	330° 43'	4.280	46.8	2 <sup>h</sup> 21 <sup>m</sup> 11.194	18 <sup>h</sup> 53 <sup>m</sup> 12.345
		1	68 46	27.380	330 59	13.740	46.7	2 23 34.766	18 53 12.346
		2	248 46	20.960	151 41	30.020	51.6	2 29 50.064	18 53 12.348
	740	2	248 46	20.920	151 57	19.180	51.5	2 32 8.912	18 53 12.348
		1	113 46	27.300	109 56	24.020	47.0	3 1 18.689	18 53 13.539
		1	113 46	27.320	110 15	9.540	46.6	3 3 17.690	18 53 13.539
	418	2	293 46	20.880	291 30	48.160	51.2	3 11 18.238	18 53 13.540
		2	293 46	20.850	291 51	51.680	50.8	3 13 31.771	18 53 13.541
		1	158 46	27.040	329 42	12.030	51.0	3 29 39.563	18 53 13.544
		1	158 46	26.980	330 5	22.120	51.0	3 31 15.158	18 53 13.544
		2	338 46	20.340	152 4	0.080	47.0	3 39 31.252	18 53 13.545
		2	338 46	20.280	152 27	39.830	47.0	3 41 11.246	18 53 13.546
1445	1	203 46	27.320	279 28	24.120	53.0	4 10 56.111	18 53 13.551	
	1	203 46	27.280	279 51	36.190	52.7	4 12 23.223	18 53 13.551	
	2	23 46	20.680	102 10	36.100	46.2	4 21 .635	18 53 13.552	
	2	23 46	20.650	102 32	6.500	46.1	4 22 19.845	18 53 13.552	
525	1	90 46	26.940	225 58	6.080	54.5	4 50 28.355	18 53 13.557	
	1	90 46	26.920	226 18	22.660	54.1	4 51 40.788	18 53 13.557	
	2	270 46	20.180	48 43	12.060	45.6	5 0 22.741	18 53 13.599	
	2	270 46	20.320	49 4	39.580	45.2	5 1 40.542	18 53 13.559	
	1	135 46	27.290	354 16	54.030	48.3	5 27 55.199	18 53 13.564	
V.15	412	1	135 46	27.300	354 35	45.800	48.0	5 29 49.239	18 53 13.564
		2	315 46	20.680	176 4	58.600	51.7	5 38 51.342	18 53 13.565
		2	315 46	20.740	176 22	19.900	51.7	5 40 36.975	18 53 13.566
	63	1	180 46	26.880	132 56	18.100	43.5	6 2 40.704	18 53 13.570
		1	180 46	26.820	133 14	16.440	43.3	6 5 23.119	18 53 13.570
		2	0 46	20.180	314 1	42.090	53.5	6 12 36.361	18 53 13.571
V.23	741	2	0 46	20.280	314 15	36.080	53.0	6 14 43.660	18 53 13.571
		1	203 46	27.580	244 39	54.020	48.3	1 42 44.209	21 44 22.450
	1	203 46	27.780	245 0	23.340	48.1	1 44 15.810	21 44 22.450	
	2	23 46	21.000	66 56	36.100	48.7	1 52 50.168	21 44 22.451	
V.24	17	2	23 46	21.320	67 17	37.220	48.8	1 54 21.914	21 44 22.451
		1	68 46	28.030	38 53	24.140	48.0	2 54 36.797	21 44 22.461
		1	68 46	27.900	39 9	59.680	47.9	2 56 48.757	21 44 22.461
	585	2	248 46	21.300	220 7	48.080	48.7	3 4 33.063	21 44 22.463
		2	248 46	21.200	220 23	48.150	48.4	3 6 41.869	21 44 22.463
		1	113 46	27.380	257 53	24.040	47.5	3 26 16.839	21 44 22.466
	441	1	113 46	27.500	258 15	55.830	47.6	3 27 39.238	21 44 22.466
		2	293 46	20.880	80 34	6.080	47.2	3 36 10.030	21 44 22.468
		2	293 46	20.830	80 57	58.600	47.2	3 37 38.934	21 44 22.468
		1	158 46	27.380	39 7	36.080	47.4	4 2 21.659	21 44 22.472
		1	158 46	27.480	39 25	13.920	47.5	4 4 33.405	21 44 22.472
		2	338 46	20.680	220 28	8.050	48.2	4 12 22.865	21 44 22.474
1157	2	338 46	20.700	220 49	15.780	48.0	4 14 59.266	21 44 22.474	
	1	68 46	27.980	322 2	42.040	48.3	1 32 51.118	19 9 54.825	
	1	68 46	27.920	322 21	36.600	48.2	1 35 29.293	19 9 54.825	
	2	248 46	21.320	143 15	6.080	50.6	1 42 55.131	19 9 54.827	
		2	248 46	21.280	143 32	40.640	50.8	1 45 20.187	19 9 54.828



Appendix III

AZIMUTH BY POLARIS.

STATION. . . . . : GOEDEREDEE (PILLAR 2)
REFERENCE MARK . . . . . : ZIERIKZEE (LAMP '69)
INSTRUMENT . . . . . : WILD T4 /1957
DATE . . . . . : 22-5 1969.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51° 49' 9.700
LONGITUDE . . . . . : -0° 15' 54.350
CHRONOMETER CORRECTION (UTC-T) : 12 3 33.546
REFERENCE MOMENT . . . . . : 6 25 26.454
CHRONOMETER RATE (SEC/HOUR) . . : .008

OBSERVATIONS:

Table with columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR LEVEL, CHRON. TIME. Contains multiple rows of observation data for sets 1 through 6.

AZIMUTH BY POLARIS.

STATION. . . . . : GOEDEREDEE (PILLAR 2)
REFERENCE MARK . . . . . : ZIERIKZEE (LAMP '69)
INSTRUMENT . . . . . : WILD T4 /1957
DATE . . . . . : 3-6 1969.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 49 9.700
LONGITUDE . . . . . : -0 15 54.350
CHRONOMETER CORRECTION (UTC-T) : 9 59 4.365
REFERENCE MOMENT . . . . . : 8 8 55.635
CHRONOMETER RATE (SEC/HOUR) . . : .008

OBSERVATIONS:

Table with columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR LEVEL, CHRON. TIME. Contains multiple rows of observation data for sets 1 through 6.

STATION. . . . . : GOEDEREDEE (PILLAR 2)
REFERENCE MARK . . . . . : ZIERIKZEE (LAMP '69)
INSTRUMENT . . . . . : WILD T4 /1957
DATE . . . . . : 29-5 1969.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 49 9.700
LONGITUDE . . . . . : -0 15 54.350
CHRONOMETER CORRECTION (UTC-T) : 9 35 33.825
REFERENCE MOMENT . . . . . : 18 51 .000
CHRONOMETER RATE (SEC/HOUR) . . : .009

OBSERVATIONS:

Table with columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR LEVEL, CHRON. TIME. Contains multiple rows of observation data for sets 1 through 6.

STATION. . . . . : GOEDEREDEE (PILLAR 2)
REFERENCE MARK . . . . . : ZIERIKZEE (LAMP '69)
INSTRUMENT . . . . . : WILD T4 /1957
DATE . . . . . : 4-6 1969.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 49 9.700
LONGITUDE . . . . . : -0 15 54.350
CHRONOMETER CORRECTION (UTC-T) : 13 25 38.199
REFERENCE MOMENT . . . . . : 5 6 21.801
CHRONOMETER RATE (SEC/HOUR) . . : .008

OBSERVATIONS:

Table with columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR LEVEL, CHRON. TIME. Contains multiple rows of observation data for sets 1 through 6.

AZIMUTH BY POLARIS.

STATION. . . . . : GOEDEREDEE (PILLAR 2)
REFERENCE MARK . . . . . : ZIERIKZEE (LAMP '69)
INSTRUMENT . . . . . : WILD T4 /1957
DATE . . . . . : 5-6 1969.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 49 9.700
LONGITUDE . . . . . : -0 15 54.350
CHRONOMETER CORRECTION (UTC-T) : 16 19 .863
REFERENCE MOMENT . . . . . : 2 11 59.131
CHRONOMETER RATE (SEC/HOUR) . . : .008

OBSERVATIONS:

Table with 10 columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR, LEVEL, CHRON. TIME. Contains multiple rows of observation data.

STATION. . . . . : GOEDEREDEE (PILLAR 2)
REFERENCE MARK . . . . . : ZIERIKZEE (LAMP '69)
INSTRUMENT . . . . . : WILD T4 /1957
DATE . . . . . : 10 AND 11-6 1969.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 49 9.700
LONGITUDE . . . . . : -0 15 54.350
CHRONOMETER CORRECTION (UTC-T) : 15 45 8.370
REFERENCE MOMENT . . . . . : 3 18 51.630
CHRONOMETER RATE (SEC/HOUR) . . : .008

OBSERVATIONS:

Table with 10 columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR, LEVEL, CHRON. TIME. Contains multiple rows of observation data.

AZIMUTH BY POLARIS.

STATION. . . . . : ZIERIKZEE (PERM. MARK 12)
REFERENCE MARK . . . . . : GOEDEREDEE (PILLAR 2)
INSTRUMENT . . . . . : DKM3A NR. 134824
DATE . . . . . : 21-8 1973.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 39 4.610
LONGITUDE . . . . . : -0 15 39.620
CHRONOMETER CORRECTION (UTC-T) : 19 58 .730
REFERENCE MOMENT . . . . . : 0 2 .000
CHRONOMETER RATE (SEC/HOUR) . . : -.001

OBSERVATIONS:

Table with 10 columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR, LEVEL, CHRON. TIME. Contains multiple rows of observation data.

STATION. . . . . : ZIERIKZEE (PERM. MARK 12)
REFERENCE MARK . . . . . : GOEDEREDEE (PILLAR 2)
INSTRUMENT . . . . . : DKM3A NR. 134824
DATE . . . . . : 22-8 1973.
OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 39 4.610
LONGITUDE . . . . . : -0 15 39.620
CHRONOMETER CORRECTION (UTC-T) : 17 0 59.935
REFERENCE MOMENT . . . . . : 0 10 .000
CHRONOMETER RATE (SEC/HOUR) . . : -.001

OBSERVATIONS:

Table with 10 columns: SET FACE, HOR.CIRCLE, R.M., HOR.CIRCLE, STAR, LEVEL, CHRON. TIME. Contains multiple rows of observation data.

## AZIMUTH BY POLARIS.

STATION. . . . . : ZIERIKZEE (PERM. MARK 12)  
 REFERENCE MARK . . . . . : GOEDEREDE (PILLAR 2)  
 INSTRUMENT . . . . . : DKM3A NR. 134824  
 DATE . . . . . : 23-8 1973.  
 OBSERVER . . . . . : STEUR.

LATITUDE . . . . . : 51 39 4.610  
 LONGITUDE. . . . . : -0 15 39.620  
 CHRONOMETER CORRECTION (UTC-T): 18 59 58.897  
 REFERENCE MOMENT . . . . . : 0 13 .000  
 CHRONOMETER RATE (SEC/HOUR). . : -.002

## OBSERVATIONS:

SET	FACE	HOR.CIRCLE	R.M.	HOR.CIRCLE	STAR	LEVEL	CHRON. TIME
1	1	42 37	56.600	31 14	23.200	19.4	1 13 25.560
	1	42 37	55.500	31 14	33.400	19.4	1 14 30.210
	2	222 37	56.400	211 14	41.700	18.2	1 16 25.370
	2	222 37	55.400	211 14	50.200	18.6	1 17 20.150
2	1	102 37	55.900	91 15	50.400	19.0	1 24 18.080
	1	102 37	57.600	91 15	58.300	18.9	1 25 13.000
	2	282 37	53.900	271 16	6.000	18.2	1 27 8.930
	2	282 37	52.000	271 16	14.600	18.4	1 28 5.630
3	1	162 37	52.100	151 17	10.400	18.6	1 36 .600
	1	162 37	51.300	151 17	15.600	18.6	1 36 57.820
	2	342 37	46.800	331 17	22.600	18.4	1 38 55.150
	2	342 37	46.700	331 17	27.400	18.4	1 39 36.110
4	1	222 37	47.400	211 18	9.900	18.0	1 46 18.660
	1	222 37	46.000	211 18	15.700	18.0	1 47 3.540
	2	42 37	44.900	31 18	15.800	17.9	1 48 34.660
	2	42 37	44.600	31 18	23.300	18.0	1 49 37.560
5	1	282 37	44.400	271 20	23.900	19.1	2 18 28.580
	1	282 37	44.800	271 20	27.400	19.0	2 19 19.780
	2	102 37	40.400	91 20	26.400	18.3	2 20 50.550
	2	102 37	41.400	91 20	27.800	18.4	2 21 39.440
6	1	342 37	40.400	331 20	44.600	18.2	2 28 4.810
	1	342 37	40.200	331 20	45.600	18.2	2 28 47.870
	2	162 37	38.900	151 20	44.400	18.0	2 30 30.400
	2	162 37	39.000	151 20	43.800	18.1	2 31 12.760
7	1	57 37	39.500	46 20	25.600	18.0	3 16 6.590
	1	57 37	40.700	46 20	24.400	18.1	3 16 57.320
	2	237 37	38.100	226 20	11.600	18.0	3 18 45.570
	2	237 37	37.200	226 20	6.900	18.2	3 19 27.590
8	1	117 37	5.900	106 18	44.700	18.6	3 33 49.150
	1	117 37	6.400	106 18	41.900	18.5	3 34 31.700
	2	297 37	4.700	286 18	28.200	19.0	3 36 19.810
	2	297 37	4.500	286 18	26.100	19.1	3 37 2.420
9	1	177 37	.800	166 17	30.300	18.4	3 47 47.130
	1	177 37	1.600	166 17	26.600	18.4	3 48 29.910
	2	357 36	56.800	346 17	10.000	19.2	3 50 24.480
	2	357 36	56.300	346 17	5.400	19.3	3 51 6.960

STATION. . . . . : ZIERIKZEE (PERM. MARK 12)  
 REFERENCE MARK . . . . . : GOEDEREDE (PILLAR 2)  
 INSTRUMENT . . . . . : DKM3A NR. 134824  
 DATE . . . . . : 28-8 1973.  
 OBSERVER . . . . . : STEUR.

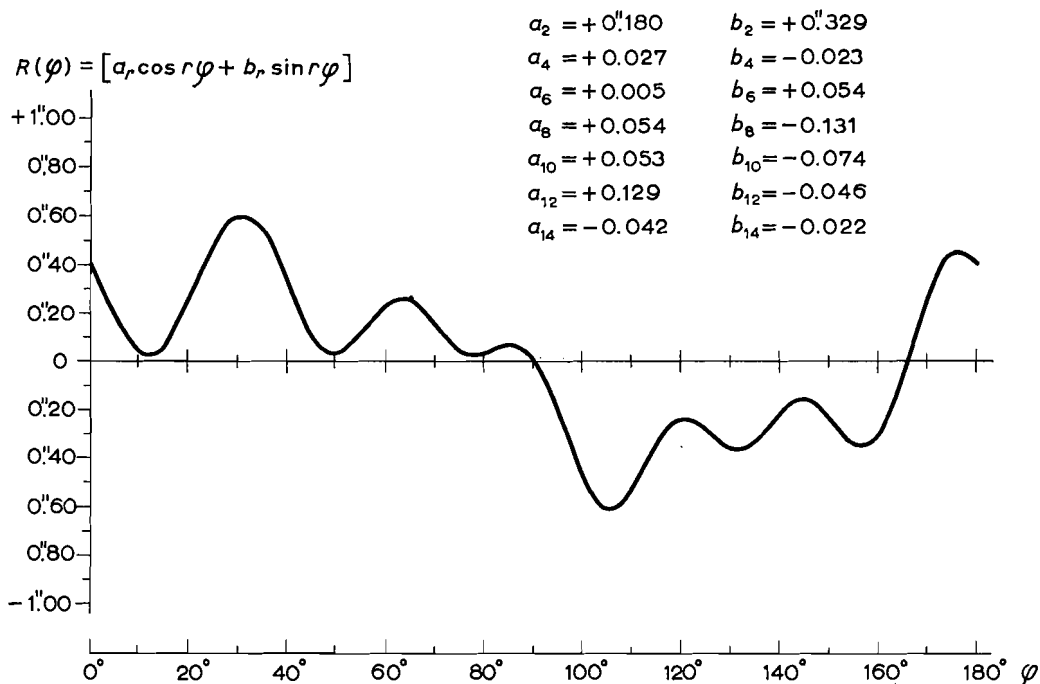
LATITUDE . . . . . : 51 39 4.610  
 LONGITUDE. . . . . : -0 15 39.620  
 CHRONOMETER CORRECTION (UTC-T): 18 17 58.676  
 REFERENCE MOMENT . . . . . : 0 5 .000  
 CHRONOMETER RATE (SEC/HOUR). . : -.010

## OBSERVATIONS:

SET	FACE	HOR.CIRCLE	R.M.	HOR.CIRCLE	STAR	LEVEL	CHRON. TIME
1	1	192 37	13.400	181 4	16.200	19.6	0 43 50.279
	1	192 37	12.500	181 4	25.400	19.5	0 44 36.449
	2	12 37	10.200	1 5	13.500	20.2	0 48 45.697
	2	12 37	9.600	1 5	30.200	20.2	0 50 7.915
2	1	207 37	1.600	196 7	22.200	19.2	0 59 57.050
	1	207 37	2.600	196 7	32.300	19.3	1 0 45.010
	2	27 37	1.800	16 7	42.500	20.2	1 2 6.490
	2	27 37	.700	16 7	55.900	20.4	1 3 13.900
3	1	222 37	5.100	211 9	18.400	18.6	1 9 52.860
	1	222 37	5.200	211 9	25.900	18.6	1 10 41.480
	2	42 37	4.300	31 9	35.400	20.0	1 12 8.810
	2	42 37	2.900	31 9	44.400	20.0	1 13 2.190

Appendix IV

SYSTEMATIC ERRORS IN THE HORIZONTAL CIRCLE DIVISION OF  
THE THEODOLITE DKM 3A No. 134824



Appendix V Covariance matrix of the coordinates

		+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	
LEEUWARDEN	Y	+1	+0.2036	+0.0183	+0.2558	+0.0149	-0.0493	-0.1096	-0.0671	-0.1114	-0.1394	+0.0036	-0.1396	-0.0215
	X	+2	+0.0183	+0.1697	+0.0259	+0.2140	+0.0703	-0.0707	+0.0659	-0.0862	-0.0569	-0.1074	-0.0336	-0.1188
AMELAND	Y	+3	+0.2558	+0.0259	+0.3422	+0.0225	-0.0612	-0.1345	-0.0827	-0.1366	-0.1709	-0.0009	-0.1697	-0.0308
	X	+4	+0.0149	+0.2140	+0.0225	+0.2868	+0.0850	-0.0873	+0.0792	-0.1057	-0.0638	-0.1290	-0.0365	-0.1415
GOEDERHEDE	Y	+5	-0.0493	+0.0703	-0.0612	+0.0850	+0.1857	+0.0153	+0.1989	-0.0199	-0.0213	-0.1588	+0.0173	-0.1763
	X	+6	-0.1096	-0.0707	-0.1345	-0.0873	+0.0153	+0.2575	+0.0638	+0.2786	+0.2368	-0.0144	+0.2485	+0.0351
ZIERIKZEE	Y	+7	-0.0671	+0.0659	-0.0827	+0.0792	+0.1989	+0.0638	+0.2382	+0.0339	+0.0079	-0.1843	+0.0542	-0.1981
	X	+8	-0.1114	-0.0862	-0.1366	-0.1057	-0.0199	+0.2786	+0.0339	+0.3171	+0.2702	+0.0059	+0.2779	+0.0647
UBACHSBERG	Y	+9	-0.1394	-0.0569	-0.1709	-0.0638	-0.0213	+0.2368	+0.0079	+0.2702	+0.4978	+0.0307	+0.4903	+0.1733
	X	+10	+0.0036	-0.1074	-0.0009	-0.1290	-0.1588	-0.0144	-0.1843	+0.0059	+0.0307	+0.3316	-0.0700	+0.3267
TONGEREN	Y	+11	-0.1396	-0.0336	-0.1697	-0.0365	+0.0173	+0.2485	+0.0542	+0.2779	+0.4903	-0.0700	+0.5403	+0.0791
	X	+12	-0.0215	-0.1188	-0.0308	-0.1415	-0.1763	+0.0351	-0.1981	+0.0647	+0.1733	+0.3267	+0.0791	+0.4212

