STANDARD BASE “LOENERMARK”

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STANDARD BASE "LOENERMARK"

REPORT ON THE RESULTS OF THE MEASUREMENTS PERFORMED BY T. J. KUKKAMÄKI AND T. HONKASALO OF THE FINNISH GEODETIC INSTITUTE

EDITED BY G. J. BRUINS

1964
RIJKSCOMMISSIE VOOR GEODESIE, KANAALWEG 4, DELFT, NETHERLANDS

PREFACE

It is a privelege to write a preface to this Report on the Standard base "Loenermark", as it gives me an opportunity to express, in the name of the Netherlands Geodetic Commission, sincere thanks to several Finnish colleagues, whose inventiveness, devotion and skill provided our country with a first order standard base line.

In the first instance it was Dr. Heiskanen who informed the Secretary General of the I.A.G. that the Finnish Geodetic Institute had decided to put its Väisälä comparator at the disposal of those countries, which desired to establish a standard base line. In addition, there was the possibility of having experienced Finnish scientists carrying out the principal observations.

It was under this arrangement that, after the necessary preparatory work in the Loenermark had been completed, Dr. Kukkamäki and Dr. Honkasalo were invited to come to the Netherlands to measure the base line, which is situated in one of the most favourable and at the same time most beautiful parts of the country.

Considering that the instrument used is an invention of the Finnish scientist Dr. Väisälä, it is obvious that the Loenermark Standard base is real Finnish import. But, as Dr. Heiskanen generously remarked in a letter to me, it was "a good and important cooperation between the Dutch and the Finnish Geodetic Institutes".

It was indeed Prof. Bruins to whom the Netherlands Geodetic Commission entrusted the actual cooperation with the Finnish colleagues – a close cooperation, continuous during more than four weeks, on a very delicate project, which resulted in a standard base line, the Netherlands Geodetic Commission is proud of.

To my admiration for the care and skill with which Dr. Kukkamäki and Dr. Honkasalo made their observations, I wish to add my gratitude to Prof. Bruins for his important part of the work, which now finds its completion in this report Standard base "Loenermark".

The President of the Netherlands Geodetic Commission,

Prof. R. Roelofs

INTRODUCTION

In 1913 the primary triangulation network of the Netherlands was augmented with a base line. This base line was measured near Stroe and the results of the measurements were published by Prof. Ir. Hk. J. Heuvelink in 1931.*)

The object of this base line measurement was to check the unit of length derived in 1900 from the German base line near Bonn. When the length of the latter proved to be in good agreement with the results of the measurements at Stroe (with an accuracy of $1:2.5 \times 10^6$) the primary triangulation of the Netherlands – including the unit of length determination – was considered to be completed.

Apart from the base line measurement at Stroe the only work done in the field of primary length measurements in those years was an occasional comparison in length between the invarwires No. 89, 90 and 91 of the Netherlands Geodetic Commission and the wires No. 285, 286 and 287 belonging to the Topographic Service of the former Netherlands East Indies during the period 1915–1922. This occurred when the latter were sent via Delft to the Bureau International des Poids et Mesures for calibration.

It was not until 1957 that primary length measurements in the Netherlands drew attention again. In that year in the Loenermark (near the town of Apeldoorn) a standard base with a length of about 576 m and an accuracy of about $1:10^7$ was measured by the Finnish geodesists Prof. Dr. T. J. Kukkamäki and Prof. Dr. T. Honkasalo using the interference method of Väisälä. The results of this measurements are laid down in the following chapters:

Chapter 1, by the editor, contains the history of the base and a detailed description of the preparations and construction of it.

Chapter 2, by Prof. Dr. T. J. Kukkamäki and Prof. Dr. T. Honkasalo, is the most important part of this publication and deals with the measurements itself and the results. The authors preferred to omit from this publication the more technical description of the interference method and the instruments used. For details of the method etc. the reader is referred to the references at the end of this chapter.

Chapter 3 gives a short description of an auxiliary base constructed in 1960 for the calibration of invar wires of which the length was derived from the interference base.

This base makes it possible to calibrate in the Netherlands invar wires belonging to several institutes and survey departments which until 1957 had to be done in other countries. The Loenermark base has in particular proved its usefulness when in 1960 in German-Dutch collaboration the base of the German primary triangulation near Meppen was remeasured. Use has also been made of the base for the calibration of the rapidly increasing number of instruments for electronic distance measurements.

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*) Hk. J. HEUVELINK, Basis bij Stroe 1913. Netherlands Geodetic Commission, Delft, 1931.

Prospective users of the standard base are kindly invited to contact the Netherlands Geodetic Commission, Kanaalweg 4, Delft who will gladly give the necessary information.

This is also the place to thank all who contributed to the realization of this base and the publication of its result.

First of all sincere gratitude may be expressed to Prof. Dr. T. J. Kukkamäki and to Prof. Dr. T. Honkasalo and through them to the Finnish Geodetic Institute for the measurements and computations. The generous offer of contributing their time and instruments to this work is gratefully acknowledged.

The cooperation of the municipality of Apeldoorn who permitted the construction of the base on her territory is greatly appreciated.

Sincere thanks are also due to the engineering firm Grabowski and Poort and to the contractor Mr. Reusken who were always willing to comply with the sometimes unusual requests in the design and construction of the base and the sudden wishes of the geodesists in charge.

Many thanks are due to Ir. G. Bakker and Ir. G. L. Strang van Hees for reading the manuscript and suggesting several improvements.

Last but not least thanks are due to Mr. N. van der Schraaf and Mr. H. C. van der Hoek for correcting the manuscript and preparing the lay-out and to the printer W. D. Meinema N.V., who all contributed to give this edition its present form.

G. J. Bruins,

Member of the Netherlands Geodetic Commission

Standard base "Loenermark".

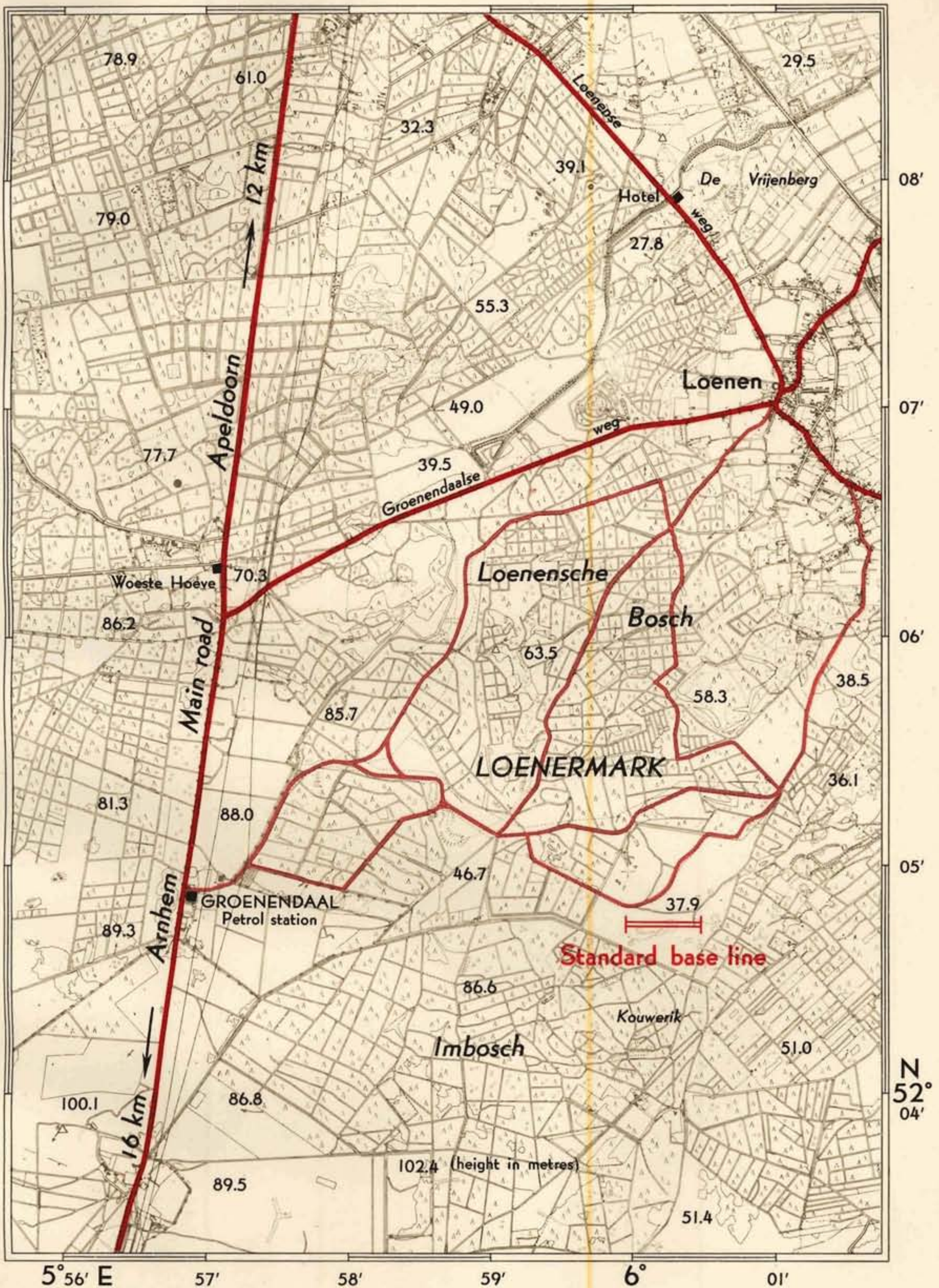


Figure I. Topographic map of the Loenermark.

CONTENTS

HISTORY AND CONSTRUCTION OF THE STANDARD BASE "LOENERMARK"

1 **Introduction**

At the 10th General Assembly of the Union Géodésique et Géophysique Internationale in Rome the following resolution, proposed by Section I, Triangulation, was adopted.

RESOLUTION 2

General adjustment of the European triangulation net

The International Union of Geodesy and Geophysics,

considering that the countries of Europe which have participated in the first phase of the European triangulation adjustment are now determining the best procedures for continuing that work into a second and more scientific phase,

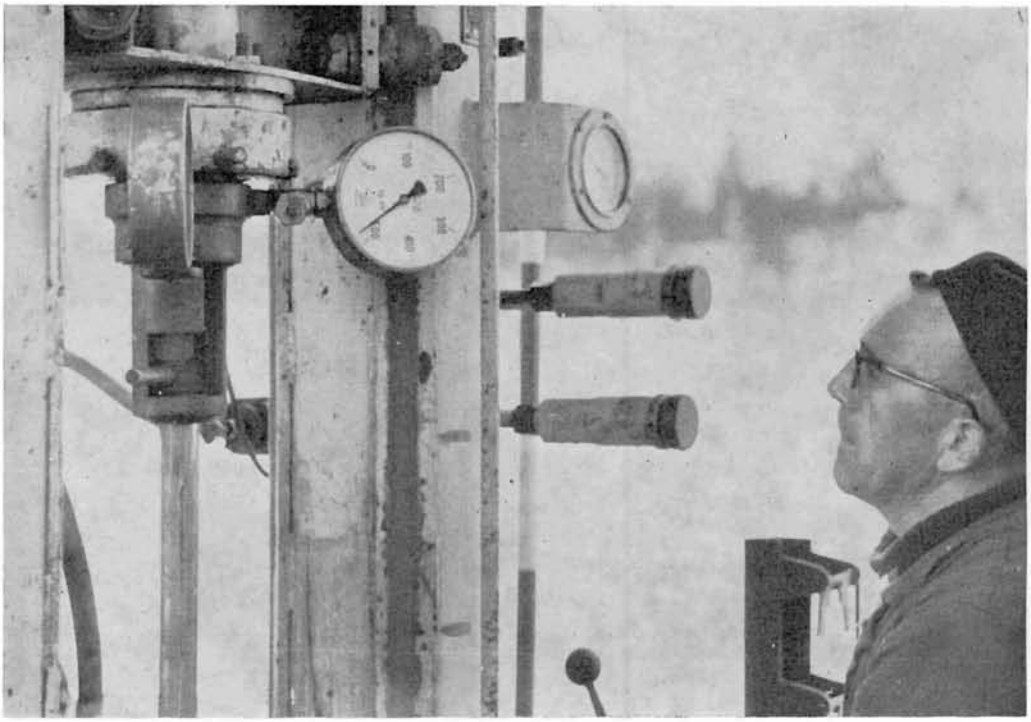
resolves that these member countries adopt the following programme insofar as is possible:

1. Complete the scientific analysis of the results of the first phase.
2. Improve and complete the observations along the borders between the various national nets.
3. Establish a standard base line in each country using the Väisälä method (or similar apparatus) for assuring a uniform scale in all networks and for calibrating invar tapes and geodimeters.
4. Increase the density of Laplace azimuth and base lines (either invar tape or geodimeter) by a uniform European plan so that an over-all accuracy of 1 part in 100,000 may be expected; and
5. Consider supplementing national nets or international connections with high precision Shoran in an effort to obtain greater accuracy in the final adjustment.

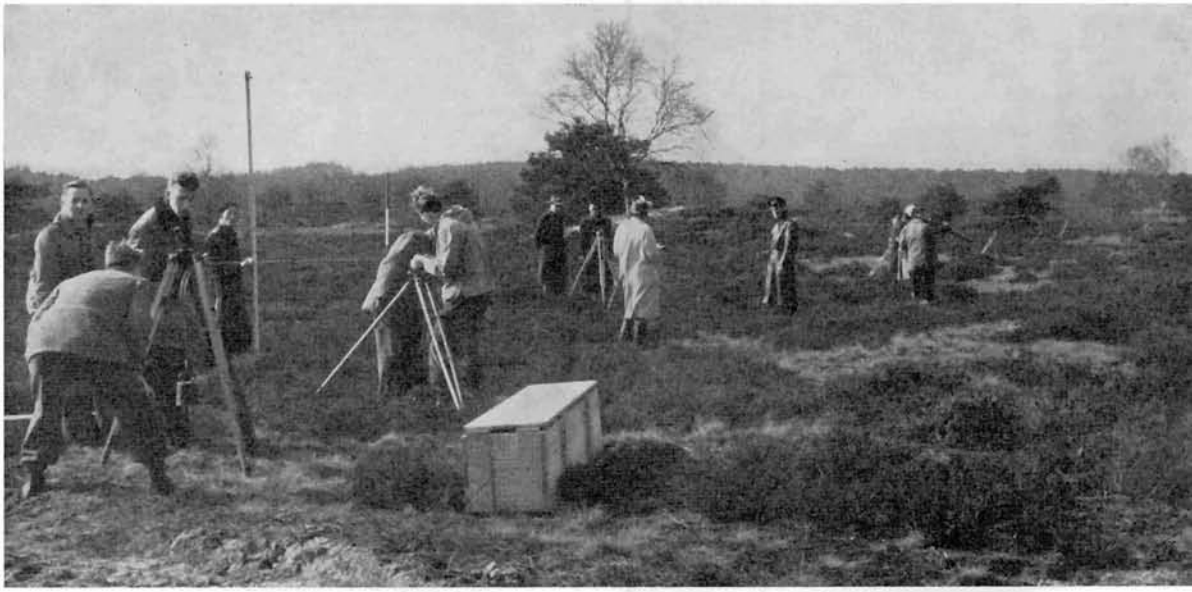
(See Bulletin Géodésique, Nouvelle Série, 1955, nr. 35, p. 96 and 97.)

Referring to point 3 Prof. Dr. W. HEISKANEN, Director of the Finnish Geodetic Institute wrote Prof. P. TARDI, Secretary General of the Association Internationale de Géodésie, a letter dated January 15, 1955 from which the following is quoted.

> "For the realisation of the idea mentioned under point 3 the Association of Geodesy considering that it is not feasible for each country to obtain the special equipment required for carrying out precise geodetic length measurements resolved:
> That the Central Bureau initiates plans for the organization of an international base measurement and geodimeter party, when it is fully determined that the latter method is satisfactory, with the equipment to be supplied by some of the larger countries and the cost of the personel to be contributed by the countries actually engaged in the project."



Testing the soil.
Reading the pressure gauge of the sounding-apparatus.



Staking-out the positions of the concrete pillars.

> "To obtain the best possible standard base line measurements the Finnish Institute informs that its Väisälä comparator will be available for this work. Also when requested the Finnish Geodetic Institute will make arrangements to place experienced scientists of its staff at disposal under the conditions indicated in the resolution above. Also the laboratory of Prof. Väisälä has promised all possible cooperation."

This letter distributed by Prof. P. Tardi on February 3, 1955 as "Lettre Circulaire aux Présidents des Comités Nationaux et des Commissions Géodésiques" was discussed at a meeting of the Netherlands Geodetic Commission on February 11, 1955 and it was decided to accept in principle the Finnish proposal. The following considerations led to this decision.

The existing base of the national triangulation near Stroe was measured in 1913 along the then local road Amersfoort–Apeldoorn. This road however has been widened since and the underground bolts have disappeared under its pavement and consequently are inaccessible now. As a remeasurement is not possible because of the increased traffic, it is therefore necessary to create a new base preferably in the northwest of the country. The establishment of this new base together with bases near Kiel, Meppen and Bonn in Western Germany and Brugge in Belgium will lead to the required density and distribution of bases necessary for the readjustment of the northwestern part of the European network. For the measurement or remeasurement of these bases in the uniform length unit, advocated in the above resolution, it should be convenient to have a standard base *at hand* for the calibration of the invarwires to be used.

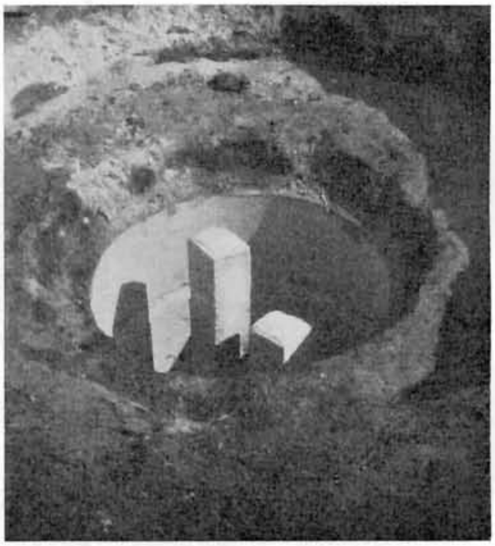
As in 1955 only Finland and Argentine disposed of such a base it was also considered that geodetic institutes of neighbouring countries might profit by a standard base in the Netherlands. That function has now partly been taken over by the standard base near Munich measured in 1958.

Prof. Dr. W. Heiskanen was informed of the decision of the Netherlands Geodetic Commission through Prof. P. Tardi.

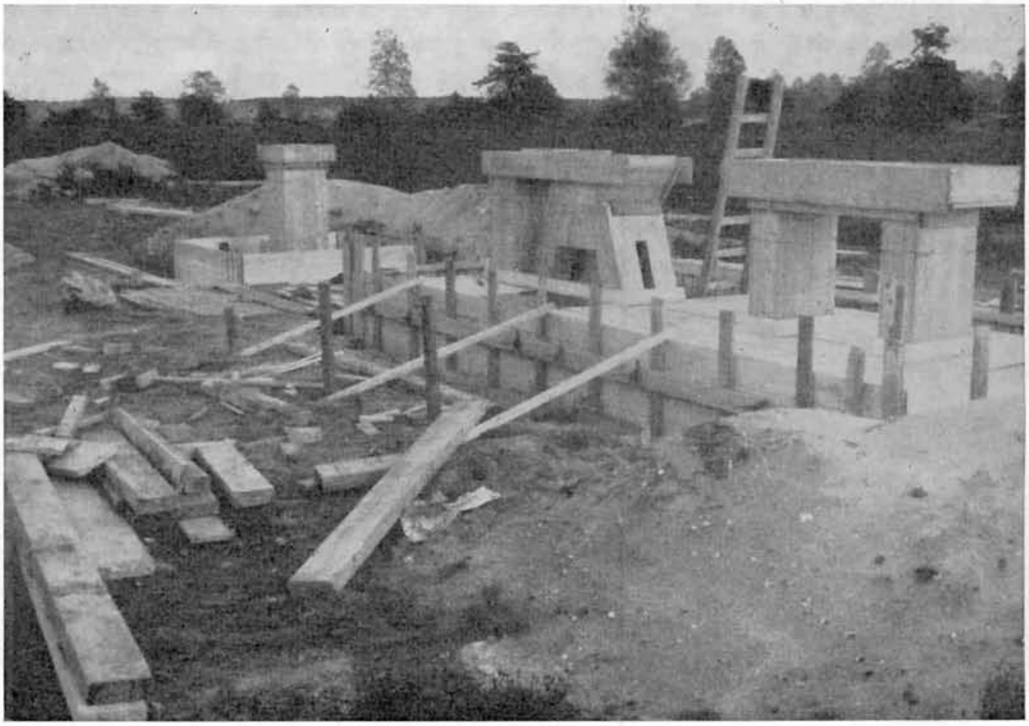
2 **Preparations**

By letter dated March 15, 1955, Prof. Dr. W. Heiskanen mentioned some suggestions for the planning of the standard base.

1. The soil should be as stable as possible from a geological point of view. In Finland the post-glacial moraine gravel appeared to be very satisfactory. In Argentine however the fluvial deposited soil caused some difficulties.
2. The most suitable length of a standard base is about 500 m. This length can be measured with light interference even under less favourable weather conditions and the base is also long enough to make sure that possible slight changes of the underground bolts will have no appreciable effect on the ultimate length of the invarwires to be calibrated.
3. It is recommended to place in addition to the underground bolts at the end points of the base, a similar bolt in the middle of it, in order to be able to check for possible relative changes in the two parts of the base.



Construction of the 0–1 m pillar.



4. In the Netherlands, where 24 m wires are used, the length of the base must be a multiple of 24 m. For the setting up of the mirrors, the pillars must be placed at distances of 0, 1, 6, 24, 72, 216 and 432 m in case of a length of 432 m and at distances of 0, 1, 6, 24, 96, 288 and 576 m in case of a length of 576 m.
5. The pillars and the underground bolts should be built at least one year in advance to be sure that no appreciable aftereffects will occur.

With reference to the first suggestion the secretary of the Netherlands Geodetic Commission Prof. R. ROELOFS consulted Prof. Ir. E. C. W. A. GEUZE, Professor of Soil Mechanics, Dr. A. J. PANNEKOEK, Professor of Geology, en Dr. A. A. THIADENS, at that time Director of the Geological Bureau of the Netherlands Mining Area, on the most favourable terrain in the Netherlands.

Prof. GEUZE thought the moraine landscape of the Veluwe to be the most suitable terrain for the foundations of the pillars. Up to a depth of about 50 m the soil is mainly composed of sand and gravel layers whereas unconsolidated soil layers like clay, loam and peat, which might cause secular deformations, are thin if occurring at all.

The advice given by Dr. PANNEKOEK pointed in the same direction. He too was of the opinion that the glacial pressure ridges of the Veluwe and Utrecht might be very suitable. Because of the pressure and the repeated small thrusting caused by the ice, the consolidation of the steep sand and gravel layers may be even better than normal. Some large bodies of clay, *e.g.* near Hattem, should of course be avoided. From a tectonic point of view little so far is known about the subsurface. The northwestern part of the Veluwe was a slowly subsiding region during the Tertiary and early Quarternary, but this tendency decreased towards the south.

Dr. THIADENS reported the following about the province of Limburg. In the region south of Ransdaal, the solid limestone seems favourable for the foundations of the pillars. However faults may occur in this area and earthquakes may bring about effects which are difficult to assess.

With reference to these reports and as a result of a visit to the national park "De Hooge Veluwe" and the national reserve "De Loenermark" the latter location was chosen for the construction of the standard base at a meeting of the Netherlands Geodetic Commission on October 7, 1955. "De Loenermark" is situated in a quiet part of the territory of Apeldoorn and entirely owned by the municipality. The municipal authorities kindly permitted the establishment of the standard base in the southernmost part of the reserve (see Fig. 1, opposite page 8) and offered their cooperation when needed.

The advice given by Prof. HEISKANEN under point 2 and 4 of his letter led to a choice of a base not much longer than 500 m, a limitation which was also set by the dimensions of the terrain. Eventually a length of 576 m was chosen and on November 11, 1955 the base line was provisionally set out in the field. Along this line holes were drilled to investigate the composition of the upper layers of the soil and the results of this examination appeared to be favourable.

In the meantime provisional sketches for the construction of the pillars were received from Prof. Dr. KUKKAMÄKI who was to carry out the measurements together with Prof. Dr. HONKASALO.

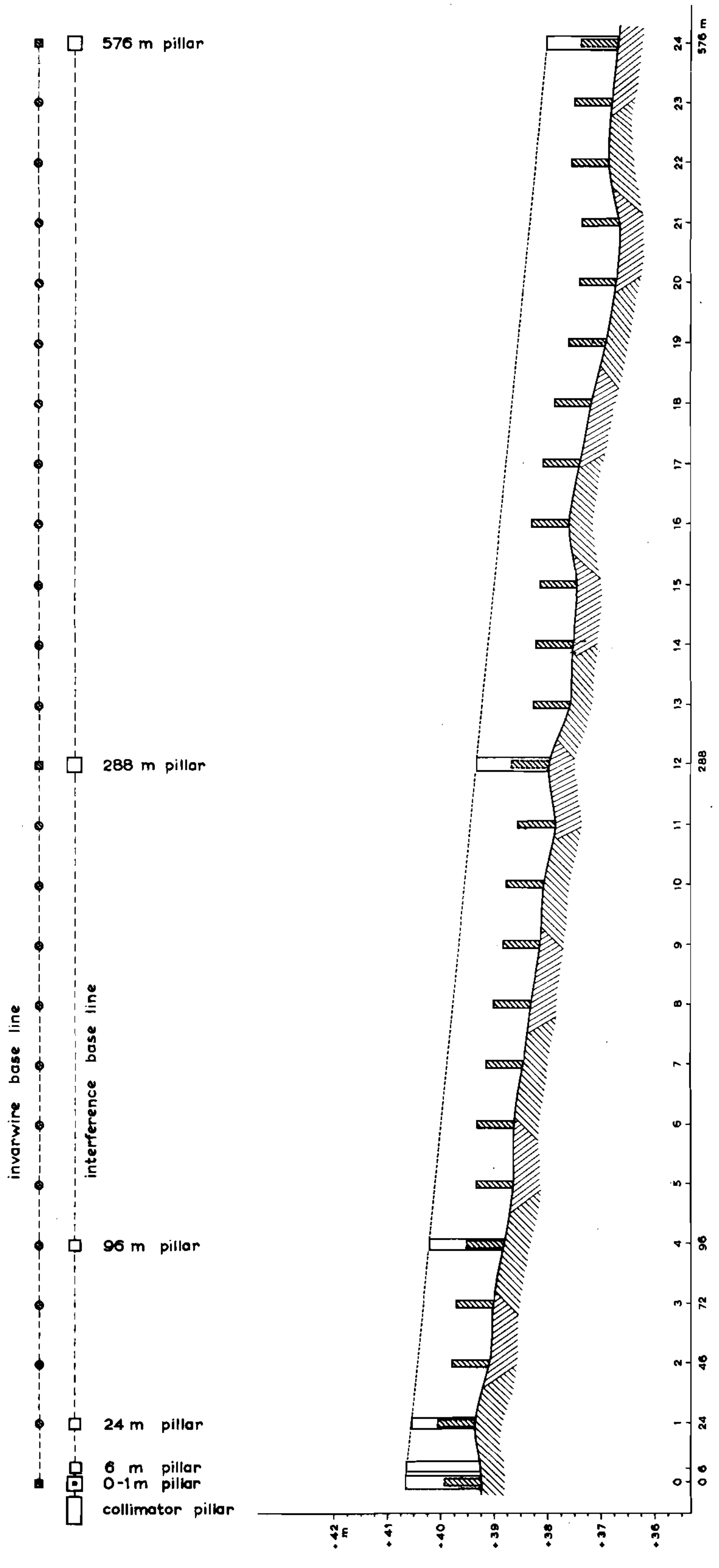
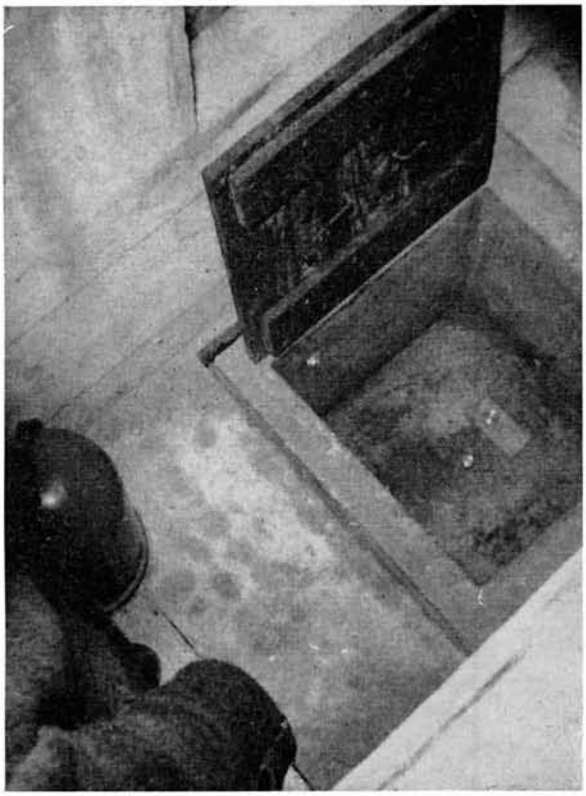


Figure 2. Lay-out of the standard base "Loenermark".

At the meeting of the Netherlands Geodetic Commission on December 16, 1955 the building of the pillars was discussed. Prof. GEUZE informed the members of the Commission that the injecting of the concrete pillars into the ground, although according to him the best method, would cost about *f* 43,000.—, *i.e.* nearly double the amount of the normal foundation method. The Commission decided to abandon this method and to contact "Ingenieursbureau Grabowsky en Poort" at The Hague to design a detailed building plan. The total costs were now estimated at about *f* 25,000.—.

On January 14, 1956 Dr. KUKKAMÄKI visited Delft for a discussion of the plans with some members of the Netherlands Geodetic Commission. Apart from some slight modifications, he approved the plans of "Grabowsky en Poort" and as the pillars could probably not be built before the summer of 1956, it was decided to postpone the measurements, which were initially planned in the autumn of 1956, till the autumn of 1957.

In the meantime much help had already been received from the Survey Department of the municipality of Apeldoorn who had levelled the profile of the base. It appeared to be better to move the base somewhat in order to obtain a more favourable profile.



Cellar with underground bolt.

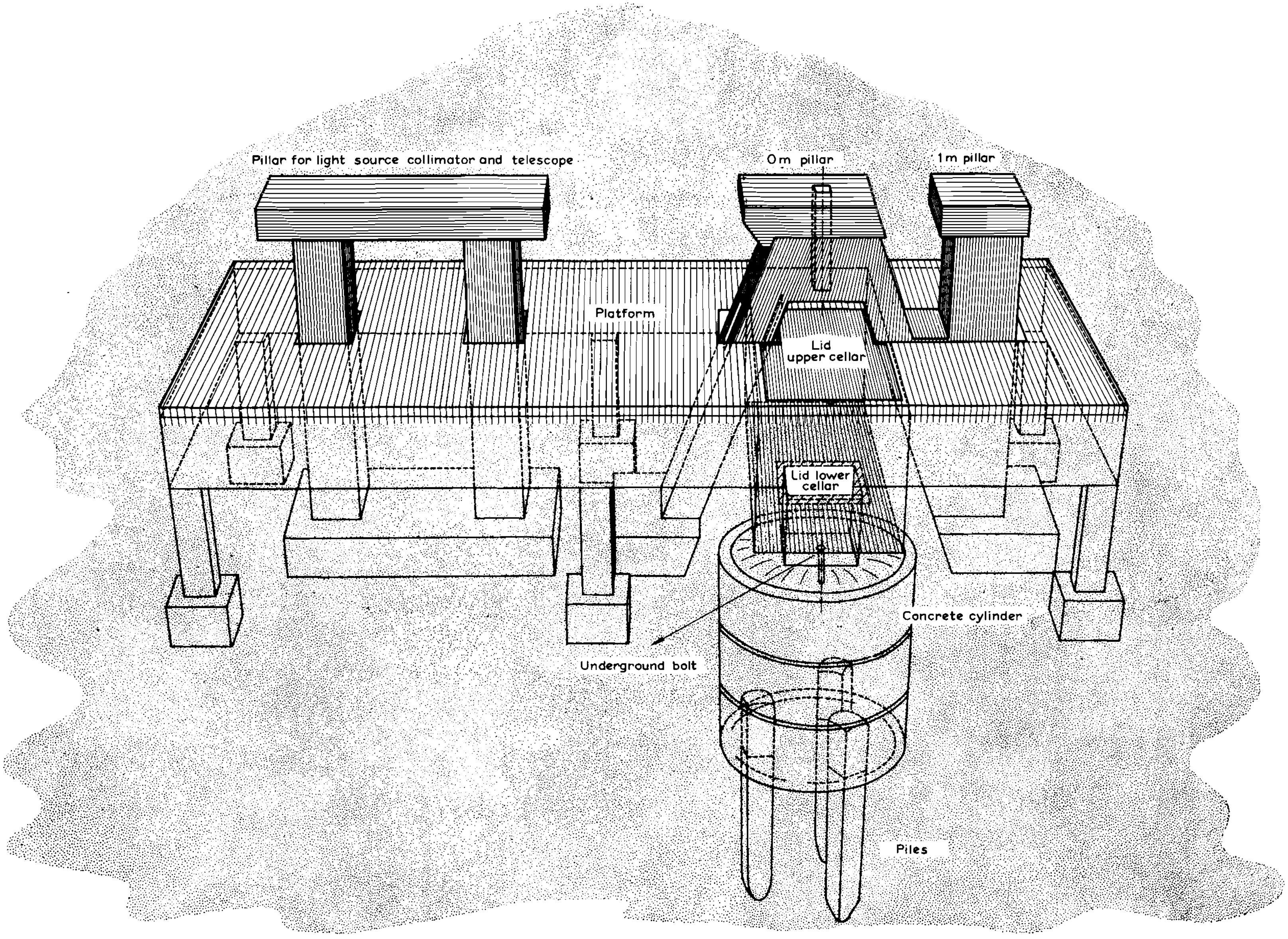


Figure 3. Perspective view of the 0–1 m– and telescope pillar.

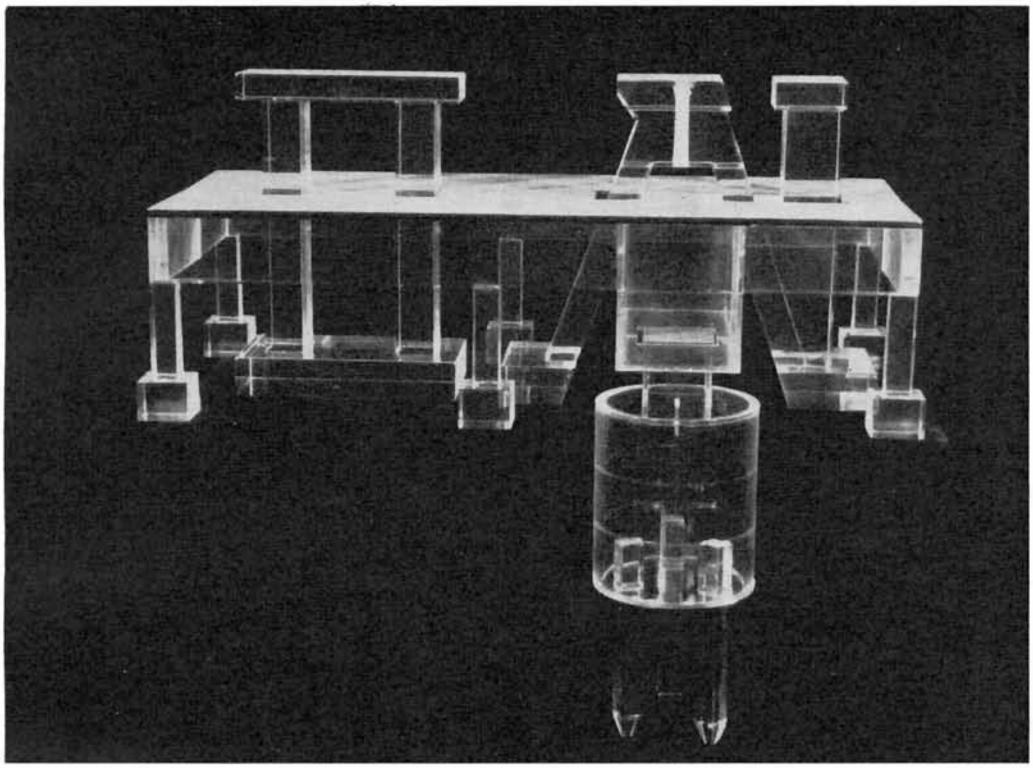


Figure 4. Photograph of a plexiglass model of the 0–1 m– and telescope pillar.

The choice of a base length of 576 m implied that the location of the pillars for the mirrors should be at distances of 0, 1, 6, 24, 96, 288 and 576 m. In the extension of the base, a few meters away from the 0 m pillar a special pillar was to be built for the light source, the collimator and the telescope for the observation of the interference. The underground bolts should be placed at 0, 288 and 576 m in foundations not connected with the pillars itselves.

Since the two quartzmeters to be used were about 150 μ longer than 1 m, the distances between the 0 m and the 96 m, 288 m and 576 m pillar had to be longer than the whole number of metres by 1.5, 4.5 and 9 cm respectively. After the distances had been set out by graduated students in geodesy with an accuracy of 1 cm, the building of the pillars was started early 1957.

3 Description of the standard base "Loenermark"

The description of the standard base is best given with the help of some photographs and sketches.

Figure 2 gives a view from above and from aside of the interference base and also of the invarwire base. A short description of the latter is given in Chapter 3.

As is shown in Figure 2 the heads of the interference pillars lie in the same plane with an angle of slope of 1 to 230. Figure 3 is a perspective view and Figure 4 a photograph of a model of the 0–1 m pillar with underground bolt and the telescope

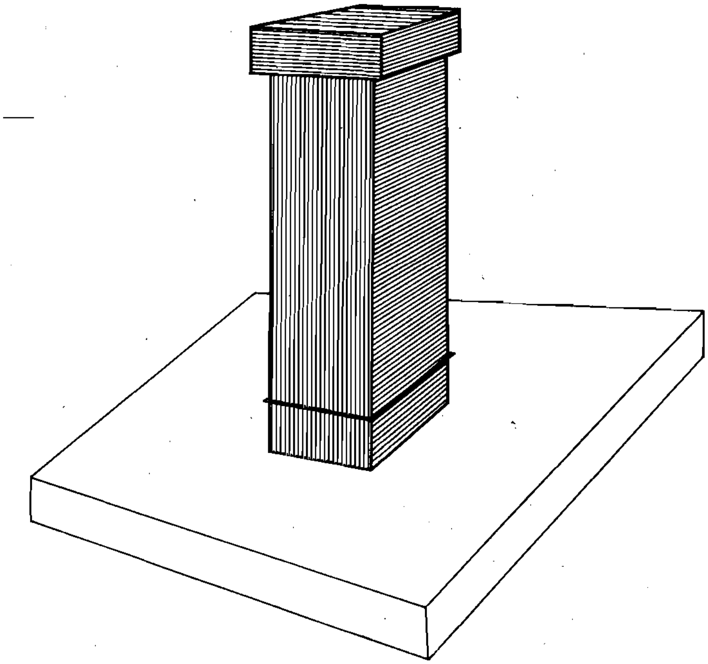


Figure 6. Sketch of an intermediate pillar.

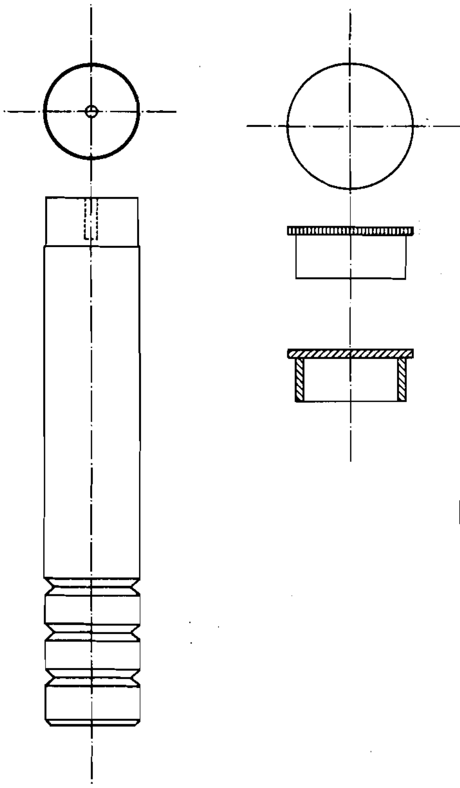


Figure 5. Underground bolt with protective cap.

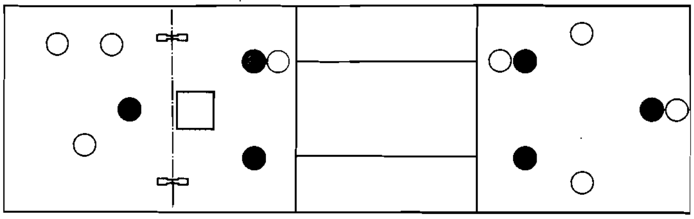


Figure 7a. 0–1 m pillar.

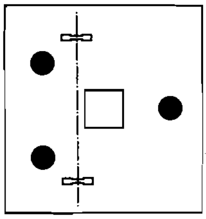


Figure 7b. 288 m and 576 m pillar.

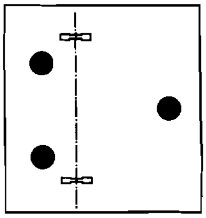


Figure 7c. 6 m, 24 m and 96 m pillar.

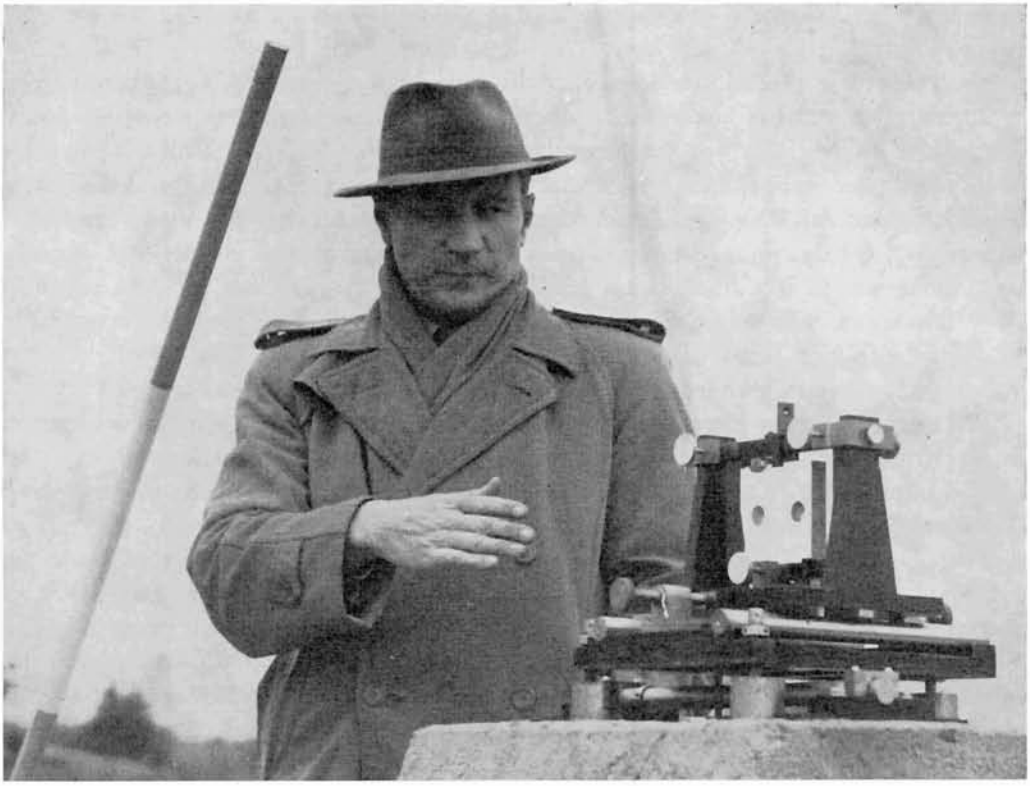
● ⛁	Bolts for the mirror supports.
○	Bolts for equipment.
▭—▭	Supports for the index bars.

pillar. The bolt is placed in a concrete cylinder with a diameter of 1 m and a height of 1.5 m. The walls of the cylinder are formed by three concrete rings, let down during the excavation to prevent disturbance of the soil. Inside the tube formed by these rings piles were driven which met such a resistance of the solid soil that they partly snapped off. The cylinder was then filled up with concrete and the top of it was given a gentle convex shape. A few centimetres above the cylinder and entirely unconnected with it a small cellar has been built in the shape of a cube with sides 70 × 70 × 70 cm. The cellar, with walls of concrete, has no floor and is covered by a wooden lid, which forms part of the floor of another cellar. This upper cellar is larger and covered by an iron lid forming part of the platform (see Figures 3 and 4).

The concrete of the interference pillars and the underground cylinder was casted up to 15 cm from the top. This was done in view of the bolts which were to be placed later by Dr. KUKKAMÄKI and Dr. HONKASALO themselves. The underground bolt is made of bronze and provided with a protective cap (see Figure 5).

The platform and both cellars form one concrete body not connected with the cylinder and the interference pillar which are likewise independent of each other.

Figure 6 gives a picture of the more simple 6 m, 24 m and 96 m pillar and the Figures 7a–c are sketches of the 0–1 m pillar, the 288 m and 576 m pillar and the other pillars showing the bolts of the mirror supports and of the indexbars. The non shaded circles indicate the support bolts for the collimator lens, the quartzmeter stands and the arch.



Dr. Kukkamäki explaining details of the measurements.

Dr. Honkasalo performing the projection measurements.

THE INTERFERENCE AND THE PROJECTION MEASUREMENTS

1 **Programme of measurements and its accomplishment**

The installation of the Väisälä comparator of the Finnish Geodetic Institute and the measurements were carried out together by the Netherlands Geodetic Commission and the Finnish Geodetic Institute during the period October 1 – November 1, 1957. The observations were made by Prof. Dr. T. J. KUKKAMÄKI and Prof. Dr. T. HONKASALO, who were assisted by Ir. M. HAARSMA from October 1 – 14 and from October 28 – November 1, by Ir. J. C. DE MUNCK from October 14–19 and by Mr. H. A. VERHOEF from October 23–28.

The completion of the casting of the pillars and the mounting of the iron supports for the different units of the comparator was performed during the period October 1–5. The telescope and the pillars at 0 and 1 m were sheltered by a tent. The setting up of the instruments and the test measurements were done between October 7 and October 18.

The interference measurements of distance 288 m were performed on October 19, 23, 25, 25, 27, 28 and those of distance 576 m on October 23, 23, 25, 28, 28.

The projection of the mirror indices to the underground bolts were made on the following dates:

October 16 and 29: 0 m,
October 18 and 30: 288 m,
October 19 and 30: 576 m.

2 **Refraction correction**

For measuring the changes of the air refraction index along the base line nineteen thermometers were used. These thermometers were placed as shown in Table I, page 24.

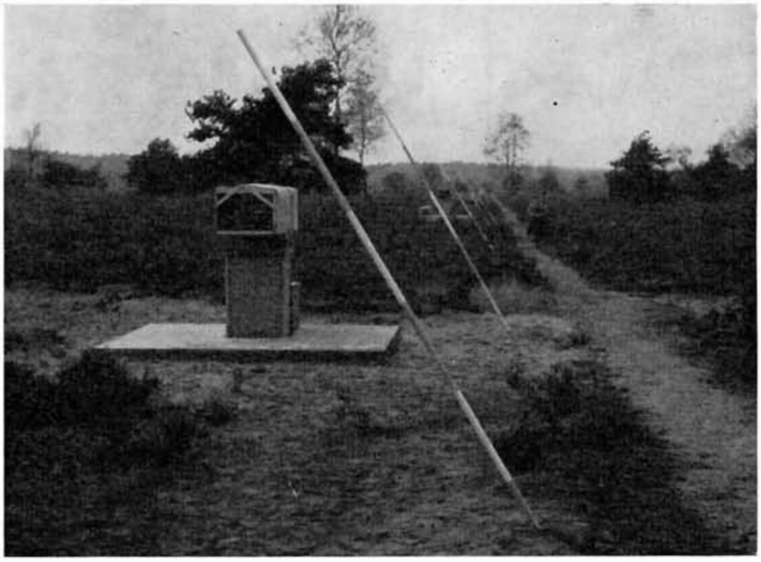
The thermometers indicated in Table I were read by two observers going back and forth simultaneously during the time interference observations were made. The refraction corrections were calculated by the same method as used on former occasions ([3], p. 13).

The mean air temperatures in the space between the mirrors were calculated assuming the temperature change being linear from one thermometer to the other. In the first mirror space, where the thermometers hung near the concrete pillars, the mean of the two thermometers was taken, though the thermometers were not exactly at the mirrors 0 and 1 m. Thus we get the following slightly simplified formulas for the mean temperatures (see page 25):

TABLE I. Location of the thermometers

Thermometers read at interference observations						Corrections to the thermometer readings at temperatures		
Distance from 0-mirror in metres	0–1–6	0–6–24	0–24–96	0–96–288	0–288–576	0°	+10°	+20°
0*)	×	×	×			+0°.02	+0°.01	+0°.02
1*)	×	×				−0 .02	+0 .01	+0 .01
3	×	×		×	×	0 .00	+0 .01	−0 .05
6	×	×	×			0 .00	+0 .02	−0 .01
16		×	×			−0 .02	+0 .02	0 .00
24		×	×	×		0 .00	+0 .04	−0 .03
48			×	×	×	−0 .01	+0 .05	+0 .02
72			×	×		0 .00	+0 .05	0 .00
96			×	×	×	0 .00	+0 .06	−0 .02
144				×	×	0 .00	+0 .06	−0 .02
192				×	×	0 .00	−0 .02	−0 .02
240				×	×	−0 .02	0 .00	+0 .02
288				×	×	−0 .02	+0 .05	−0 .03
336					×	0 .00	−0 .02	0 .00
384					×	0 .00	−0 .02	+0 .02
432					×	+0 .01	+0 .09	−0 .01
480					×	−0 .02	+0 .02	−0 .02
528					×	−0 .01	+0 .06	−0 .02
576					×	−0 .04	+0 .05	0 .00

*) Because of practical possibilities these thermometers were placed at distances 0.25 and 0.75 m.



Temperature measurements.

Mirror space	Mean air temperature
0–1	$^1/_2(t_0+t_1)$
0–6	$^1/_8(t_0+t_1+4t_3+2t_6)$
0–6	$^1/_8(t_0+t_1+4t_3+2t_6)$
6–24	$^1/_4(t_6+2t_{16}+t_{24})$
0–24	$^1/_6(t_0+2t_6+2t_{16}+t_{24})$
24–96	$^1/_6(t_{24}+2t_{48}+2t_{72}+t_{96})$
0–96	$^1/_8(t_3+2t_{24}+2t_{48}+2t_{72}+t_{96})$
96–288	$^1/_8(t_{96}+2t_{144}+2t_{192}+2t_{240}+t_{288})$
0–288	$^1/_{12}(t_3+2t_{48}+2t_{96}+2t_{144}+2t_{192}+2t_{240}+t_{288})$
288–576	$^1/_{12}(t_{288}+2t_{336}+2t_{384}+2t_{432}+2t_{480}+2t_{528}+t_{576})$

On the basis of the pairs of formulas above the air temperature differences (Δt) between the paths of the beam reflected once and of the beam reflected many times were calculated by using the following formulas:

$$
\begin{aligned}
&(0\text{–}1\text{–}6): && \Delta t = -0.375t_0-0.375t_1+0.500t_3+0.250t_6\\
&(0\text{–}6\text{–}24): && \Delta t = -0.094t_0-0.094t_1-0.375t_3+0.375t_{16}+0.188t_{24}\\
&(0\text{–}24\text{–}96): && \Delta t = -0.125t_0-0.250t_6-0.250t_{16}+0.250t_{48}+0.250t_{72}+0.125t_{96}\\
&(0\text{–}96\text{–}288): && \Delta t = -0.083t_3-0.167t_{24}-0.167t_{48}-0.167t_{72}+0.167t_{144}+0.167t_{192}+\\
& && \quad +0.167t_{240}+0.083t_{288}\\
&(0\text{–}288\text{–}576): && \Delta t = -0.042t_3-0.083t_{48}-0.083t_{96}-0.083t_{144}-0.083t_{192}-0.083t_{240}+\\
& && \quad +0.083t_{336}+0.083t_{384}+0.083t_{432}+0.083t_{480}+0.083t_{528}+0.042t_{576}
\end{aligned}
$$

On the basis of these temperature differences the refraction corrections ($r_{\Delta t}$) were calculated by the formula

$$r_{\Delta t} = s\cdot\frac{\mathrm{d}n_L}{\mathrm{d}t}\cdot\Delta t$$

where s indicates the distance. The change of the air refraction index ($\mathrm{d}n_L/\mathrm{d}t$) was taken from the following table (cf. [5, page 498], [6]).

Table for $(\mathrm{d}n_L/\mathrm{d}t)\cdot 10^6$

t \ B	700	730	760	790
0°	0.986	1.029	1.071	1.113
10°	0.916	0.957	0.996	1.035
20°	0.854	0.891	0.928	0.964
30°	0.796	0.831	0.865	0.900

B = atmospheric pressure mm Hg. Relative humidity 90%. Wave length of light used 0.570μ.

All other atmospheric factors have such a slight influence on the refraction index that they can be ignored. However the systematic difference in air pressure caused by the inclination of the base line has been taken into consideration as shown on page 38.

3 Interference measurements

Quartzmeters No. VIII and XI were used for these measurements. Their absolute lengths had been determined in terms of the wave length of light (^{198}Hg green) at the International Bureau of Weights and Measures in 1953, [4]. Under normal conditions the lengths were 1 m $+149.89\ \mu$ and 1 m $+135.94\ \mu$ respectively. Before and after this determination these meters were compared with several other similar quartzmeters in the laboratories of Y. Väisälä. After the measurement of the Loenermark base line the quartzmeters were again compared with the same Väisälä-meters. Assuming that the mean length of all these meters has not changed, Väisälä gives the following formulas for the meters:

No. VIII:

$$l = 1\text{ m} + \{149.90+0.430(t-20°)+0.00159(t-20°)^2-0.00347(B\text{–}760)\}\ \mu$$

No. XI:

$$l = 1\text{ m} + \{135.97+0.434(t-20°)+0.00159(t-20°)^2-0.00477(B\text{–}760)\}\ \mu$$

These temperature and atmospheric pressure coefficients were determined by Prof. Dr. T. J. Kukkamäki ([2], p. 83).

Because earlier experience has shown that the accuracy of the whole base line mainly depends on the accuracy of the interference measurements at short range (0–1–6) and (0–6–24), these distances have been measured four times in every determination of distance 288 m. Thus, one series of observations contains the following interference measurements

(0–96–288)
(0–24– 96)
(0– 6– 24)
(0– 1– 6)
(0– 1– 6)
(0– 6– 24)
(0– 6– 24)
(0– 1– 6)
(0– 1– 6)
(0– 6– 24)
(0–24– 96)
(0–96–288)

The interference at the longest distance (0–288–576) was observed separately.

The results of the interference measurements (0–1–6) and (0–6–24) are given in Table II. Distance (0,1) is computed by the same method as used previously ([1], pp. 17–18). Distance (0,6) is computed by multiplying by 6 the value (0,1) obtained above and adding to this product the compensator and the refraction corrections. Distance (0,24) is computed from the above distance (0,6) in a similar way. So each series of four measurements has given distance (0,6) with a standard deviation of $\pm 0.7\ \mu$ and distance (0,24) with a standard deviation of $\pm 2.9\ \mu$. In Table III the following interferences are calculated in the same way starting with the arithmetical mean of distance (0,24).

The discrepancies between the measurements back and forth show that the standard deviation of the multiplication of distance (0,24) by 4 in one set of two measurements is $\pm 2.5\ \mu$ and the standard deviation of multiplying (0,24) by 12 is $\pm 10\ \mu$. The error of one 288 m interference observation series, when computed from

Table II. Results of the interference measurements (0–1–6) and (0–6–24).

Date	Quartzmeter No.	Observer	Distance (0,1)	Compensator correction	Refraction correction	Distance (0,6)	Compensator correction	Refraction correction	Distance (0,24)
Oct.			1m +	μ	μ	6m +	μ	μ	24m +
19	VIII	T.H.	150.45 μ	−72.7	−0.29	829.7 μ	−6.1	0.0	3312.7 μ
		T.H.	146.86	−48.3	−0.41	832.4	−8.7	−0.6	3320.3
		T.J.K.	148.16	−57.2	−0.73	831.0	−9.3	−1.5	3313.2
		T.J.K.	147.51	−55.5	−0.67	828.9	−11.8	−0.5	3303.3
					Mean	830.5±0.8 μ		Mean	3312.4±3.5 μ
23	XI	T.H.	134.68	+48.0	+0.41	856.5	−100.9	+1.2	3326.3
		T.H.	134.69	+46.2	+0.36	854.7	−101.7	+0.6	3317.7
		T.J.K.	134.36	+48.8	+0.06	855.0	−101.5	+1.0	3319.5
		T.J.K.	133.78	+50.7	+0.18	853.6	−101.0	+1.4	3314.8
					Mean	855.0±0.6 μ		Mean	3319.6±2.4 μ
25	XI	T.H.	136.37	+13.5	+1.17	832.9	−20.9	+0.6	3311.3
		T.H.	135.41	+19.1	+1.17	832.7	−10.2	+0.5	3321.1
		T.J.K.	135.50	+17.4	+0.61	831.0	−11.5	+1.4	3313.9
		T.J.K.	136.62	+12.1	+0.79	832.6	−8.5	+0.7	3322.6
					Mean	832.3±0.4 μ		Mean	3317.2±2.7μ
25	XI	T.J.K.	135.78	+25.9	+0.32	840.9	−51.8	+1.0	3312.8
		T.J.K.	135.78	+26.4	+0.09	841.2	−52.7	+0.8	3312.9
		T.H.	135.81	+27.8	+0.09	842.8	−53.3	+0.9	3318.8
		T.H.	135.79	+26.5	+0.03	841.3	−53.4	+0.3	3312.1
					Mean	841.6±0.4 μ		Mean	3314.2±1.6 μ
27	VIII	T.H.	147.94	−12.5	−0.71	874.4	−171.0	+0.3	3326.9
		T.H.	147.09	−9.4	−1.06	872.1 *)	−169.5	−1.2	3317.7
		T.J.K.	149.26	−58.9	−0.32	836.3	−29.1	+0.3	3316.4
		T.J.K.	147.64	−49.4	−0.35	836.1	−28.5	−0.1	3315.8
						±0.6 μ		Mean	3319.2±2.6 μ
28	VIII	T.J.K.	149.80	−68.6	−0.06	830.1	−4.8	−0.2	3315.4
		T.J.K.	149.30	−66.7	−0.35	828.8	−5.7	−0.6	3308.9
		T.H.	148.07	−55.5	−0.62	832.3 **)	−5.7	−0.7	3322.8
					Mean	830.4±1.0 μ		Mean	3315.7±4.0 μ

*) 6 m mirror moved.

**) Fourth observation impossible because of weather conditions.

the standard deviation ±2.9 μ of distance (0,24) and the above multiplying error of ±10 μ, is equal to

$$\sqrt{(12\times 2.9)^2+10^2}\ \mu = \pm 36\ \mu$$

For the mean of all six series we get a standard deviation of ±15 μ for the 288 m distance.

For this computation only the consistency of observation of one and the same series has been considered. In order to get the possible divergence between different observation series, caused by movement of pillars and some other sources of errors, the distances between the index bars on top of the pillars are computed. The differences between the mirror surfaces and the index bars were measured with the aid of a special transferring device. The reading of this device was taken after every interference observation. The distances of the index bars of every interference series have been calculated in Table IV, where

I_ν = measured distance between mirrors at 0 and ν m
ΔL_ν = difference of the transferring device readings on ν- and 0-pillars
$B_\nu = I_\nu - \Delta L_\nu$
B_ν + the thickness of the 0-mirror equals the distance between the index bars on pillars 0 and ν (see Figure 10, p. 39).

The standard deviation ±19 μ of the distance between the index bars at 0 and 288 m, obtained in Table IV, is only slightly greater than the standard deviation ±15 μ of the corresponding interference measurement. This shows that the movements of pillars 0 and 288 m are can not be considerable.

TABLE III. Results of the interference measurements (0–24–96) and (0–96–288).

Date	Quartzmeter No.	Observer	Distance (0,24)	Compensator correction	Refraction correction	Distance (0,96)	Compensator correction	Refraction correction	Distance (0,288)
Oct.			24 m +	μ	μ	96 m +	μ	μ	288 m +
19	VIII	T.H.	3312.4 μ	+ 7.3	−0.6	13,256.3 μ	+ 82.4	− 0.5	39,850.8 μ
				+ 8.1	−3.9	13,253.8	+ 84.0	− 5.6	39,839.8
					Mean	13,255.0±1.2 μ		Mean	39,845.3±5.5 μ
23	XI	T.J.K.	3319.6	−26.0	+4.4	13,256.8	+144.5	−24.8	39,890.1
				−21.0	+1.8	13,259.2	+143.6	+ 1.3	39,922.5
					Mean	13,258.0±1.2 μ		Mean	39,906.3±16.2 μ
25	XI	T.H.	3317.2	−32.1	+1.6	13,238.3	+234.3	−18.1	39,931.1
				−37.0	+7.3	13,239.1	+237.9	−11.6	39,943.6
					Mean	13,238.7±0.4 μ		Mean	39,937.4±6.2 μ
25	XI	T.J.K.	3314.2	−42.9	+2.9	13,216.8	+232.4	− 1.1	39,881.7
				−39.9	−0.8	13,216.1	+232.8	+ 3.7	39,884.8
					Mean	13,216.4±0.4 μ		Mean	39,883.2±1.6 μ
27	VIII	T.H.	3319.2	− 6.1	+1.1	13,271.8	+ 16.6	+ 2.4	39,834.4
				+ 5.6	−1.1	13,281.3	+ 14.8	− 0.7	39,858.0
					Mean	13,276.6±4.8 μ		Mean	39,846.2±11.8 μ
28	VIII	T.J.K.	3315.7	−12.1	+0.1	13,250.8	+ 38.8	− 0.9	39,790.3
				− 3.5	−2.0	13,257.3	+ 43.2	+ 1.1	39,816.2
					Mean	13,254.0±3.2 μ		Mean	39,803.2±13.0 μ

The standard deviations of the distances between index bars at 0 and 6, 24 and 96 m, respectively, are in good proportion to the distances, with the exception of the 6 m, where movement of the pillar is evident in comparison with the internal error of interference measurement. (See Table II, column 7, page 27).

TABLE IV. Distances of the mirrors at positions with equal interval from mirror surface to index bar.

Date	Quartz-meter	l_6	$\varDelta L_6$	B_6	l_{24}	$\varDelta L_{24}$	B_{24}	l_{96}	$\varDelta L_{96}$	B_{96}	l_{288}	$\varDelta L_{288}$	B_{288}
Oct.		6 m +		6 m +	24 m +		24 m +	96 m +		96 m +	288 m +		288 m +
		mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
19	VIII	+0.830	+1.580	−0.750	+3.312	+9.381	−6.069	+13.255	−1.905	+15.160	+39.845	+2.387	+37.458
23	XI	0.855	1.617	−0.762	3.320	+9.391	−6.071	+13.258	−1.907	+15.165	+39.906	+2.403	+37.503
25	XI	0.832	1.602	−0.770	3.317	+9.399	−6.082	+13.239	−1.926	+15.165	+39.937	+2.412	+37.525
25	XI	0.842	1.608	−0.766	3.314	+9.396	−6.082	+13.216	−1.931	+15.147	+39.883	+2.408	+37.475
27	VIII	0.873	1.606	−0.733	3.319	+9.396	−6.077	+13.277	−1.871	+15.148	+39.846	+2.407	+37.439
27	VIII	0.836	1.569										
28	VIII	0.830	1.568	−0.738	3.316	+9.398	−6.082	+13.254	−1.873	+15.127	+39.803	+2.407	+37.396
			Mean	−0.753			−6.077			+15.152			+37.466
				±0.006			±0.002			± 0.006			± 0.019

In Table IV the observations are taken with equal weights.

As the observation conditions varied on different days, weights were calculated on the basis of the temperature changes during each observation series. ([1], p. 75). The quadratic mean values of the observed maximum differences of temperatures and the weights p, computed from these, were:

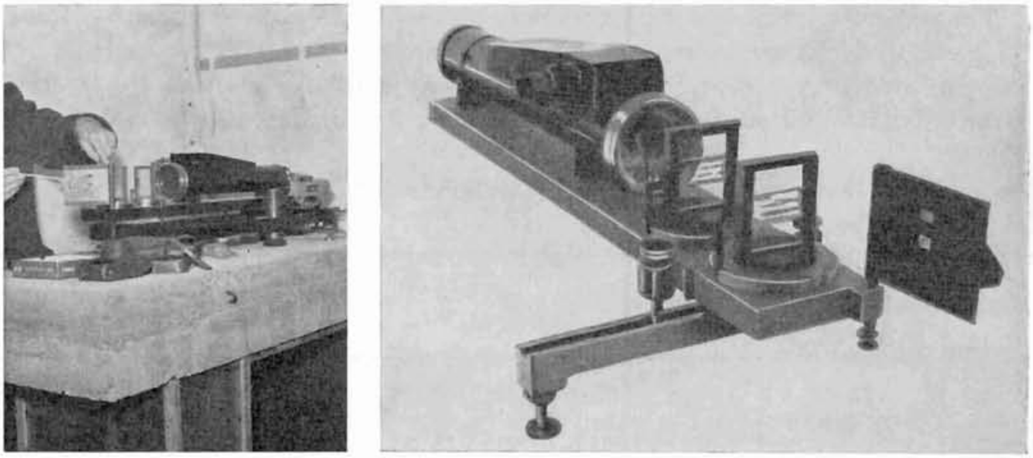
TABLE V. Weights of observations.

Date	Interferences				$\varSigma \varDelta t$	p
	(0–1–6)	(0–6–24)	(0–24–96)	(0–96–288)		
Oct. 19	0°.20	0°.22	0°.22	0°.17	0.81	1.23
23	0 .13	0 .12	0 .18	0 .40	0.83	1.20
25	0 .26	0 .19	0 .20	0 .24	0.89	1.12
25	0 .14	0 .14	0 .11	0 .17	0.56	1.79
27	0 .21	0 .19	0 .07	0 .16	0.63	1.59
28	0 .12	0 .16	0 .10	0 .12	0.50	1.50

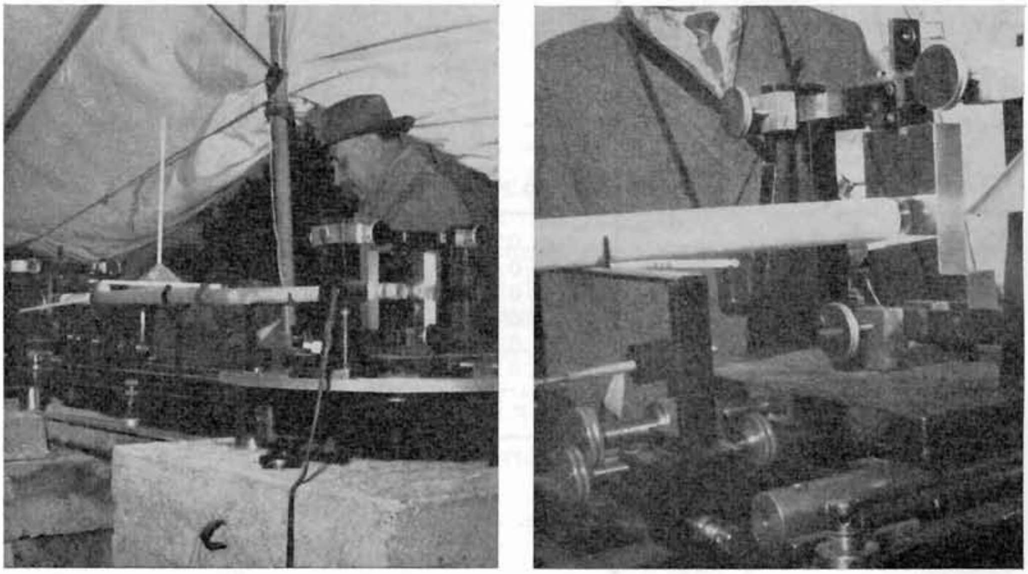
The weights are computed using the formula

$$p = \frac{1}{\varSigma \varDelta t}$$

and in the weight of the observations on October 28 the fact that there were only three measurements of short range instead of four is taken into consideration. With



Set-up of the instruments on the telescope pillar.



Set-up of the instruments on the 0–1 m pillar.

these weights we get the following results for the mirror distances at positions with equal interval from mirror to index bar:

$$B_6 = 6\text{ m} - 0.752 \pm 0.006\text{ mm}$$
$$B_{24} = 24\text{ m} - 6.078 \pm 0.002\text{ mm}$$
$$B_{96} = 96\text{ m} + 15.150 \pm 0.006\text{ mm}$$
$$B_{288} = 288\text{ m} + 37.462 \pm 0.018\text{ mm}$$

Before computing the final results for the distances 6,24,96 and 288 m the possible sources of systematic errors, the refraction correction, the difference between the two quartzmeters, the difference between the two observers are still studied.

Experience has shown that even though the thermometers are similar and have similar radiation shields there is usually a systematic difference of the readings of the first thermometers. The thermometers t_0 and t_1 are hung above the concrete pillars. After a temperature change the great heat capacity of the pillars has a radiation effect on these two thermometers and we get a too large temperature difference. If we now compute how much we should reduce the refraction correction of the two shortest interferences (0–1–6) and (0–6–24) to get the best possible agreement for the results on different days, we see that only 9±34% of the refraction corrections should be used. Because of the small number of observations the standard deviation is large. Similar computations at the other interference base lines (five measurements at Nummela base line in Finland, one measurement at Buenos Aires in Argentina, four measurements at Ebersberg in Germany and one measurement at Mata das Virtudes in Portugal) show the same effect. On the average only one third of the refraction correction of the two shortest interferences should be taken into consideration. This has been verified also with special studies with differential thermometer [7]. If we now reduce the refraction correction of the two shortest interferences to 33% and compute with equal weights and with the weights in Table V we get the following results:

TABLE VI. Reduction of the refraction correction.

Date	Quartzmeter No.	B_{288} from Table IV	B_{288} with reduced refr. correction	Weight
		288 m +	288 m +	
Oct. 19	VIII	+37.458 mm	+37.480 mm	1.23
23	XI	+37.503	+37.486	1.20
25	XI	+37.525	+37.489	1.12
25	XI	+37.475	+37.465	1.79
27	VIII	+37.439	+37.460	1.59
28	VIII	+37.396	+37.411	1.50
Arithmetical mean		+37.466 mm	37.465 mm	
Standard deviation		± 0.019 mm	± 0.012 mm	
Weighted mean		+37.462 mm	37.463 mm	
Standard deviation		± 0.018 mm	± 0.012 mm	

In order to study the possible systematic differences between different meters and between different observers we have reduced the refraction corrections in Table II to 33% and then computed the distance (0,24) for each observer. On the basis of Tables III and IV we get for the indexbar distance (0, 288) the following four values:

Observer	Quartzmeter No. VIII	Quartzmeter No. XI
T.J.K.	288 m +37.405±0.022 mm	+37.471±0.019 mm
T.H.	+37.512±0.005 mm	+37.489±0.012 mm

Any significant difference between the results with the different meters cannot be established. The results of observer Kukkamäki with meter No. VIII seem to deviate from the others. The observations of the two observers are, however, not independent from each other. Because of the symmetry of the observation series the expansion or contraction of the concrete pillar (0,1) owing to the temperature changes is for the greatest part eliminated from the mean value of the back and forth measurements. Since the series are not symmetric for two observers this can affect ostensible personal differences. The large quantity of older independent observations also show no personal differences.

On the basis of the above investigation the value

$$288 \text{ m} + 37.463 \pm 0.012 \text{ mm}$$

is taken as final for the distance (0,288).

The whole length of the base (0,576) is computed on the basis of the above value assuming that the pillars 0 and 288 m have been immovable during these measurements. The weights of the observations were computed as before (see Table V, p. 29).

Table VII. Results of the interference measurements (0–288–576).

Date	Time	Observer	ΔL_{288}	l_{288}	Compensator correction	Refraction correction	l_{576}	ΔL_{576}	B_{576}	Weight
Oct.				288 m mm			576 m mm		576 m mm	
23	20	T.J.K.	+2.405	+39.868	−37.6	−5.4	+79.693	+4.889	+74.804	1.0
	22	T.H.	+2.406	+39.869	−38.2	+9.4	+79.709	+4.891	+74.818	1.0
25	21	T.H.	+2.409	+39.872	−38.9	+4.1	+79.709	+4.893	+74.816	0.9
28	16	T.J.K.	+2.407	+39.870	−59.1	+5.6	+79.686	+4.894	+74.792	2.3
	19	T.J.K.	+2.404	+39.867	−51.3	−0.3	+79.682	+4.886	+74.796	1.1

Weighted mean +74.802±0.005 mm

In addition to the duplicating error given in the Table V the error ±0.012 mm of the distance (0,288) must be taken into consideration. Thus we get for the interference measurement the standard deviation:

$$\sqrt{(2\times0.012)^2+0.005^2}\ \mathrm{mm} = \pm0.025\ \mathrm{mm}$$

and the final result of the interference measurement, the distance between index bars at pillars 0 and 576 m added with the thickness of the 0-mirror is:

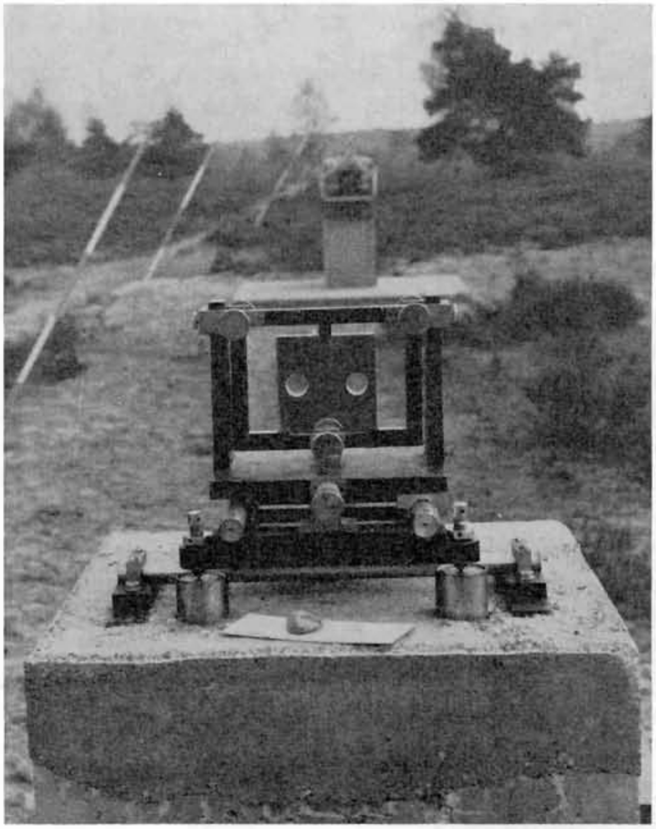
$$\boxed{576\ \mathrm{m} + 74.802 \pm 0.025\ \mathrm{mm}}$$

4 Projection measurements

The projection of the measured distances to the underground bolts was easy and accurate on this base line as compared with the Nummela and Buenos Aires base lines. The underground bolts in the Loenermark were nearly vertically below the mirrors in contrast with the bases at Nummela and Buenos Aires where they were a few metres aside. On the underground bolt a similar plumbing bar was set up as was used at the Buenos Aires base line ([3], p. 22). The lower end of it is conical and it centers the bar into the hole of the underground bolt. Two levels with sensitivities of 6″.56 and 24″ per scale division were fastened to the bar perpendicularly to each other. 1532 mm above the bolt, the upper end of the vertical bar had a similar sighting index as the mirrors ([1], p. 68). The bar was adjusted with the aid of two perpendicular screws fastened to the pillar.

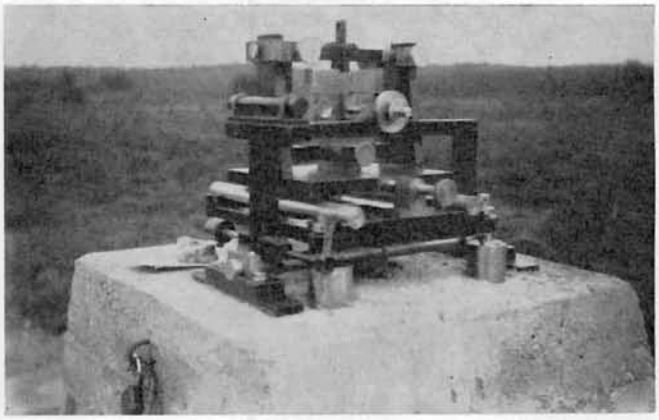
The projection observations were made with Wild T3 theodolite No. 26590 from a distance of about 5 m perpendicular to the base line. In Figure 8 *T3* indicates the

0–1 m pillar.



Mirror without transferring device on the 6 m pillar.

Transferring device.



Transferring device set to mirror.

theodolite Wild T3, *M* the sighting index of the mirror, *u* the underground bolt, *D* the centre of a distant mirror. The angels α and ν were measured by the theodolite in eight sets. The distance *d* was measured with a steel tape and the distance *b* with a theodolite Wild T2 (No. 45924) set up in the base line and with a millimeter scale set at the mirror index *M*. The distance *i* is known from the interference measurement. The small angle β is computed from ν, *d* and *i*. The projection correction *p* is computed with the formula:

$$p = (d-b)\ \mathrm{tg}\ \alpha + b \cdot \mathrm{tg}\ \beta$$

In order to eliminate the excentricity of the mirror index the observation series was repeated in reversed position of the mirror. For this a levelling instrument was set up in the base line and adjusted in such a way that the image of an electric bulb in front of the objective was reflected by the mirror in the centre of the cross wires.

The projecting of the three mirrors, 0, 288 and 576 to the underground bolts was made twice, on October 16th, 18th and 19th, before the interference measurements and on October 29th and 30th, after the interference measurements. Each time the position of the mirror index with respect to the index bar was measured with the aid of the transferring device.

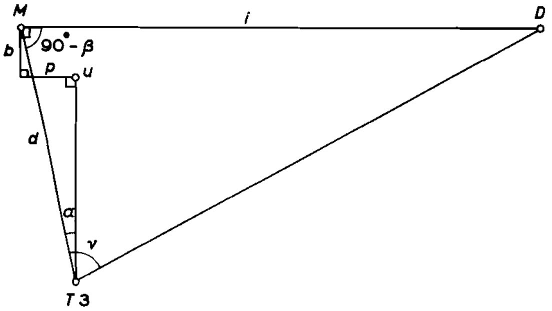


Figure 8. Diagram of the projection measurements.

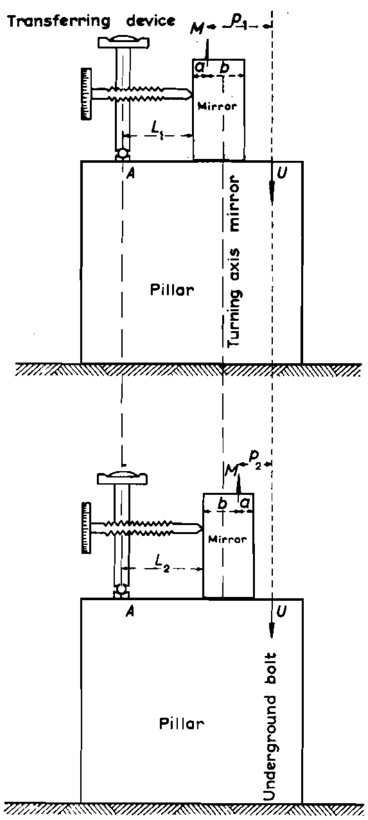


Figure 9. Projection measurement. Transferring device with mirror in position 1 (above) and 2 (below).

The observations and the computations of the projection measurements are compiled in Table VIII, in which the columns α, β, *d*, *b* and *p* are the elements, given in Figure 8. The column *L* gives the reading of the transferring device during the actual projection. So *L* is the distance of the index bar on the pillar to the surface of the mirror. In Figure 9 the situation is given for the two mirror positions 1 and 2. *A* represents the index bar on the pillar, *M* the index on the mirror and *U* the vertical projection of the underground bolt. From Figure 9 follows

$$AU = L_1 + a + p_1$$
$$AU = L_2 + b + p_2$$

so $$AU = \tfrac{1}{2}(L_1+p_1)+\tfrac{1}{2}(L_2+p_2)+\tfrac{1}{2}(a+b)$$

If we put for:

$$L_1+p_1 = L_{v_1}$$

$$L_2+p_2 = L_{v_2}$$

$$\tfrac{1}{2}(L_{v_1}+L_{v_2}) = L_v$$

and $$(a+b) = T$$

we get $$AU = L_v+\tfrac{1}{2}T$$

The quantities L_{v_1}, L_{v_2} and L_v are given in Table VIII.

The quantity $T = (a+b)$ is the thickness of the mirror. For the mirrors 0, 288 and 576 the values of these thicknesses are given in 5.1.

Column $\Delta L_{v_{1,2}}$ gives the differences of the values $L_{v_{1,2}}$ after and before the interference measurements, in both mirror positions 1 and 2.

From the theodolite observations we can compute the standard deviation of α: $m_\alpha = 0.38''$. The resulting standard deviation of p is then $m_p = 9\ \mu$.

On the other hand we can compute the standard deviation of L_{v_1} and L_{v_2} from ΔL_{v_1} and ΔL_{v_2}. We get $m_{L_{v_1}} = m_{L_{v_2}} = 10\ \mu$.

Because this standard deviation has been computed from measurements before and after the interference measurements, it includes possible pillar movements during that time, which does not affect m_p. The standard deviation of $L_{v_{1,2}}$ is only slightly higher than m_p, so we can not conclude that any pillar movements have taken place during the interference measurements. The standard deviations of L_v and L_v(mean) can be computed in the following way:

$$L_v = \tfrac{1}{2}(L_{v_1}+L_{v_2}), \qquad \text{so:} \quad m_{L_v} = \frac{10}{\sqrt{2}} = 7\mu.$$

$$L_v(\text{mean}) = \tfrac{1}{2}\{L_v(\text{before})+L_v(\text{after})\}, \qquad \text{so:} \quad m_{L_v(\text{mean})} = \frac{1}{\sqrt{2}}\cdot 7 = 5\ \mu.$$

5 Length of the base line

In Section 3 the distance between the mirror surfaces has been computed and in Section 4 these mirrors have been projected to the underground bolts. Before calculating the horizontal distance between the bolts some corrections are still needed.

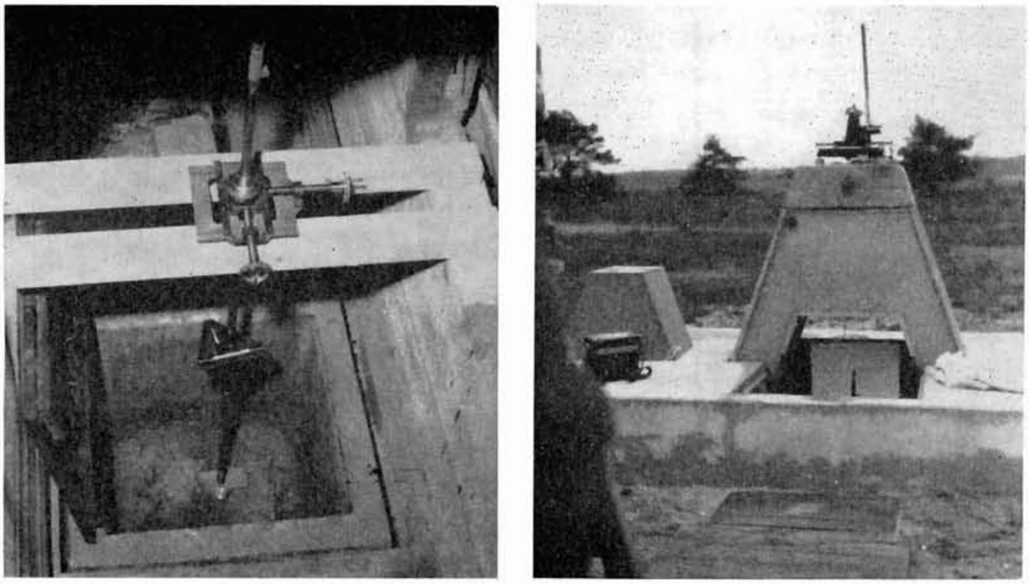
5.1 According to VÄISÄLÄ's information the thicknesses of the mirrors at the mean measuring temperature were as follows

Pillar	Mirror No.	Thickness at + 12 °C
0	40	19.984 mm
288	41	19.958 mm
576	37	19.982 mm

Half these values were used in computing the distances between the mirror centres from the distances between the mirror surfaces.

TABLE VIII. Projection measurements.

Date	Mirror position	α_1 α_2	β_1 β_2	d_1 d_2	b_1 b_2	p_1 p_2	L_1 L_2	$L_{v_1}=L_1+p_1$ $L_{v_2}=L_2+p_2$	ΔL_{v_1} ΔL_{v_2}	L_v	L_v (mean)	$L_{v_{288}}-L_{v_0}$ $L_{v_{576}}-L_{v_0}$
Oct.				mm	mm	mm	mm	mm				
16	0_1	+232.17″	−103.7′	5179.3	+1.8	+5.773	26.708	32.481		31.594		
	0_2	+ 92.99	−106.0	5177.1	+1.2	+2.296	28.411	30.707				
18	288_1	− 15.33	+ 3.7	5293.6	+0.1	−0.393	29.087	28.694		28.820		
	288_2	+ 19.72	+ 4.2	5294.6	+0.7	+0.507	28.440	28.947				
19	576_1	+132.05	+ 5.2	5512.4	−1.6	+3.527	31.582	35.109		35.084		
	576_2	+203.85	+ 6.4	5513.9	+0.5	+5.450	29.609	35.059				
						(Interference measurements)						
29	0_1	+244.25	− 32.2	4924.8	+1.7	+5.814	26.691	32.505	+24	31.614	31.604	
	0_2	+ 94.86	− 34.7	4925.0	+1.4	+2.250	28.472	30.722	+15			−2.787
30	288_1	− 19.21	+ 9.6	4501.8	−0.1	−0.420	29.099	28.679	−15	28.813	28.817	
	288_2	+ 21.29	+ 10.3	4502.3	+0.6	+0.467	28.480	28.947	0			+3.485
30	576_1	+163.06	+ 27.3	4488.8	−1.5	+3.538	31.583	35.121	+12	35.094	35.089	
	576_2	+260.78	+ 28.9	4491.5	+0.6	+5.683	29.384	35.067	+ 8			



Projection measurements. Plumbing bar.

5.2 By measuring the mirror distances (0,1) with the aid of the quartzmeter, we get the distance between the glass surfaces. The interference multiplication takes place between the aluminium coatings of the mirrors. The thicknesses of the aluminium coatings were measured with the same interference method as described in [1] p. 70. The thicknesses measured were:

0 m mirror: 0.050 μ
1 m mirror: 0.110 μ

The difference in the thicknesses was due to the fact that the 1 m mirror was coated in Helsinki and the 0 m mirror in Delft. Thus we obtained the following correction for the distances:

(0,1) m: 0.160 μ
(0,288) m: 46 μ
(0,576) m: 92 μ

5.3 The correction to horizontality was computed on the basis of the levellings made during the installation of the comparator and during the projection measurements. Starting from the 0-bolt we got the following heights of the mirror indices and underground bolts:

0-bolt: 0.000 m
0-mirror index: +2.942 m
288-bolt: −1.282 m
288-mirror index: +1.657 m
576-bolt: −2.589 m
576-mirror index: +0.367 m

The height difference of 1285 mm between the 0 and 288 m mirror indices gave for the measured distance of 288.060 m an inclination correction of −2.866 mm and the height difference of 2575 mm for the measured distance of 576.100 m a correction of −5.755 mm.

5.4 Because of the inclination of the base line the atmospheric pressure at the 576 m end was higher than at the 0 m end. The refraction corrections for the half and the whole base line were −0.005 mm and −0.022 mm respectively.

5.5 The light beam, which was adjusted in such a way that it passed at the height of the centres of mirrors 0 m and 288 m, appeared to reflect an average of 8 mm above the centre of the 576 m mirror. The geometrical straight line ran, according to the levelling data and taking the earth curvature into consideration, 18 mm above the centre of the 576 m mirror.

The difference of 10 mm was due to refraction, and on the basis of this value we can compute that the 576 m mirror deviates 14″ and the 288 m mirror 7″ from the direction of the 0-mirror. Because of this fact the distances between the mirror indices at 0,288 m and 0,576 m were 0.003 mm and 0.006 mm greater respectively than the distances between the mirror centres.

5.6 The measured distance between the mirror indices still had to be reduced to the level of the 0 m underground bolt.

A height difference of 1 m causes a correction of 90 μ at the distance of 576 m.

According to the levelling data above, the corrections were −0.104 mm and −0.149 mm for the distances of 288 m and 576 m respectively.

The final results are computed as follows (see also Figure 10).

Figure 10. Diagram showing index bar on pillar (A), underground bolt (U), and thickness of mirror (T).

$$
\begin{aligned}
I_{288} &= (C_0C_{288}),\ \text{the measured distance between the mirrors at 0 and 288 m;}\\
(A_0A_{288}) &= I-(L_{288}-L_0)+T_0 = B_{288}+T_0;\\
(A_0U_0) &= L_{v_0}+\tfrac{1}{2}T_0;\ (A_{288}U_{288}) = L_{v_{288}}+\tfrac{1}{2}T_{288};\\
(U_0U_{288}) &= (A_0A_{288})+(A_{288}U_{288})-(A_0U_0) = B_{288}+(L_{v_{288}}-L_{v_0})+\tfrac{1}{2}T_0+\tfrac{1}{2}T_{288}.
\end{aligned}
$$

	0–288 m	0–576 m
Result of interference measurements, B_{288} and B_{576} (see Section 3):	288 m +37.463 mm	576 m +74.802 mm
$(L_{v_{288}}-L_{v_0})$ and $(L_{v_{576}}-L_{v_0})$; (Table VIII):	− 2.787 mm	+ 3.485 mm
1. Half of the thickness of the 0 m mirror:	+ 9.992 mm	+ 9.992 mm
Half of the thickness of the terminal mirror:	+ 9.979 mm	+ 9.991 mm
2. Correction due to mirror coatings:	− 0.046 mm	− 0.092 mm
3. Inclination correction:	− 2.866 mm	− 5.755 mm
4. Atmospheric pressure correction:	− 0.005 mm	− 0.022 mm
5. Correction for deviation from parallellism of the mirrors:	+ 0.003 mm	+ 0.006 mm
6. Correction to the level of the 0 m underground bolt:	− 0.104 mm	− 0.149 mm
Distance of the underground bolts in the level surface of the 0 m underground bolt:	*288* m +51.629 mm	*576* m +92.258 mm

6 Accuracy of the measurements

The errors of the interference measurements are computed in Section 3. These errors include the error of the interference observations, the readings of the transferring device, the influence of the pillar movements and the possible personal errors of the two observers. The error of a single reading of the transferring device is easy to estimate from repeated readings. These errors seldom deviate more than 2 μ from each other. Consequently the effect of this error is significant only for the shortest distances.

The errors of the projection measurements have been calculated in Section 4. We computed the standard deviation of L_v(mean) at 5 μ, so we get the standard deviation of $(L_{v_{288}} - L_{v_0})$ and $(L_{v_{576}} - L_{v_0})$ at 7 μ.

The reduction to horizontality is made on the basis of levellings. The error in the height difference of the mirror indices is evaluated at ±1 mm which results in an error of ±5 μ in the inclination reduction.

All other sources of error are so small that they have no effect on the accuracy with which the base line is determined.

On the basis of the preceding calculations and estimations, the accuracy of the distances between the underground bolts 0,288 and 0,576 may be summarized as follows:

	288 m	576 m
1. Standard deviation of interference measurements:	12 μ	25 μ
2. Standard deviation of projection:	7 μ	7 μ
3. Standard deviation of inclination correction:	5 μ	5 μ
Standard deviation of the distance between the underground bolts:	15 μ	26 μ

The relative accuracy in the Väisälä quartzmeter system is thus 1 : 19,000,000 and 1 : 22,000,000 respectively.

7 Final results

The length of the Loenermark base line at the level of the 0 m underground bolt is:

First half:	**288051.63±0.02 mm**
Second half:	**288040.63±0.02 mm**
Total length base line:	**576092.26±0.03 mm**

When estimating these standard deviations the lengths of the quartzmeters have been assumed errorless. As the standard base lines Nummela, Buenos Aires and Loenermark were all measured with the same Finnish quartzmeters No. VIII and No. XI, the above errors can be considered as real errors of the Loenermark base line in the system of these three standard base lines.

The standard deviation of the absolute determination of the lengths of quartzmeters was 0.03 μ according to J. Terrien ([4], p. 160). This determination had been suspected to include some systematic errors. Therefore Terrien estimated the standard deviation of the quartzmeter at 0.1 μ. This gives a standard deviation of 0.06 mm at 576 m. So the absolute accuracy of the Loenermark base line is to be estimated at 0.08 mm, *i.e.* 1 : 7,000,000.

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SUMMARY OF THE DATA OF THE STANDARD BASE LINE AND THE INVARWIRE BASE LINE

At Nummela, Buenos Aires and Ebersberg the underground bolts of the standard base are placed sideways of the interference pillars in such a way that they can serve as begin-, mid- and endpoint of an invarwire calibration. At Loenermark however these bolts are placed underneath the interference pillars. This arrangement made the calibration of invarwires on the standard base itself a cumbersome procedure. Therefore it was in 1960 decided to construct an invarwire base parallel to the standard base and 4.5 m north of it. (See Figure 2 p. 16).

This invarwire base consists of three small concrete pillars, provided with Jäderin bolts, at 0 m, 288 m and 576 m. In between wooden pegs are placed at intervals of 24 m in which Jäderin bolts can be screwed just before a calibration takes place. In order to facilitate the calibration these pillars and pegs have been made of equal height, *viz.* 70 cm above the ground. The height differences between subsequent bolts are small and the required levelling accuracy can easily be obtained. The determination of the length of the invarwire base starting from the known length of

Invarwire base.

the standard base was a problem in itself.*) The data of both the interference and the invarwire base line are summarized hereunder (see Figure 11).

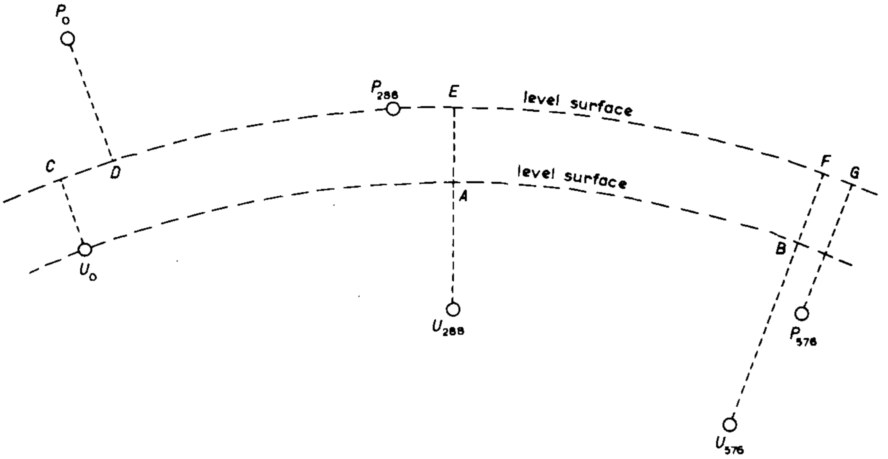

Figure 11. Diagram showing position of invarwire base with respect to interference base. U_0, U_{288}, U_{576} are the underground bolts of the interference base line. P_0, P_{288}, P_{576} are the bolts of the invarwire base line.

1 Underground bolts of the interference base line

Height

U_0: +38.0312 m +N.A.P.
U_{288}: +36.7491 m +N.A.P.
U_{576}: +35.4423 m +N.A.P.

$U_{288}-A$: − 1.2821 m
$U_{576}-B$: − 2.5889 m

Distance of the underground bolts in the level surface of the underground 0–bolt U_0.

0–288 m: $U_0-A = 288\ 051.63 \pm 0.02$ mm
288–576 m: $A-B = 288\ 040.63 \pm 0.02$ mm
0–576 m: $U_0-B = 570\ 092.26 \pm 0.03$ mm

Because of the convergence of the plumblines, a 1 m height difference introduces a length correction of 0.045 mm for a distance of 288 m and 0.090 mm for a distance of 576 m.

2 Invarwire base line

The data of the invarwire base were obtained in 1960. The bolts in the pillars of this base are not as stable as the underground bolts of the standard base. Therefore it must be emphasized that these data and in particular the hereunder mentioned quantities C–D, E–P_{288} and F–G should be redetermined every time an invarwire

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*) A detailed description of the transfer measurements will be published.

calibration is undertaken as the underground bolts are the only values which may be relied upon.

Height

P_0: $+39.9523$ m +N.A.P.
P_{288}: $+36.6904$ m +N.A.P.
P_{576}: $+37.3783$ m +N.A.P.

$P_0 - D$: 1.2619 m
$P_{576} - G$: 1.3121 m

Distances in the levelsurface of P_{288}, which is about the mean level surface of the invarwire base line.

0–288 m: $D - P_{288} = 288\ 045.0 \pm 0.1$ mm
288–576 m: $P_{288} - G = 288\ 044.8 \pm 0.1$ mm
0–576 m: $D - G = 576\ 089.8 \pm 0.1$ mm

Correction of the measured invarwire distance to the level surface of P_{288} (1960):

0–288 m: $DP_{288} - P_0P_{288} = -\ 6.36$ mm
288–576 m: $P_{288}G - P_{288}P_{576} = -\ 9.92$ mm
0–576 m: $DG - P_0P_{576} = -16.28$ mm

3 Projection measurements

Height

$U_0 - C = A - E = B - F = 0.6592$ m

Distance

0 m: $C - D = +3.66$ mm
288 m: $E - P_{288} = -3.00$ mm
576 m: $F - G = +1.13$ mm

4 Heights of the bench marks in the concrete pillars

Pillar at	Bench mark	Height +N.A.P. 1958 and 1960
0 m	RB 3	+39.573 m
	RB 4	+39.561 m
6 m	RB 5	+39.540 m
24 m	RB 6	+39.463 m
96 m	RB 7	+39.133 m
288 m	RB 8	+38.276 m
576 m	RB 9	+36.970 m