Robust Shape Reconstruction from Point Clouds

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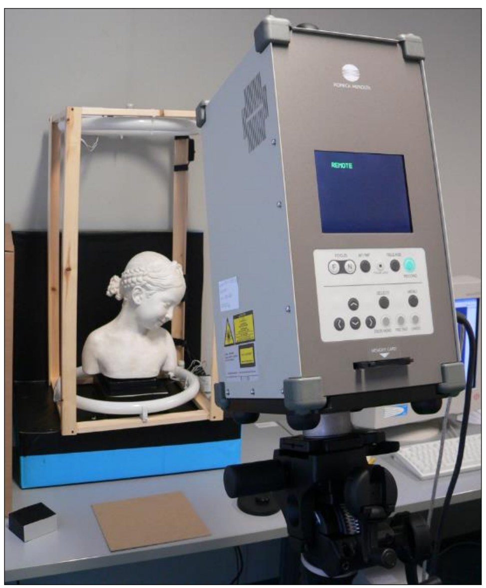
Problem Statement

<u>Input</u>:

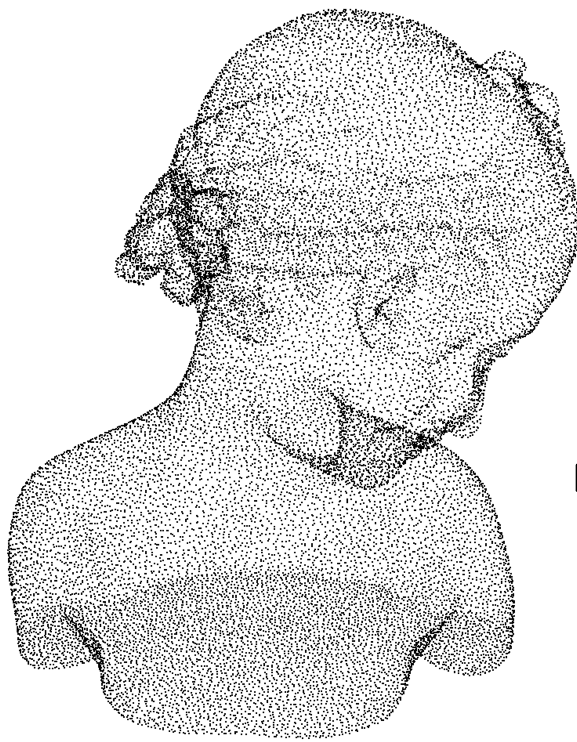
Dense point set *P* sampled over surface *S*

<u>Output</u>:

Surface: Approximation of *S* in terms of topology and geometry



Laser scanning



Point set

Reconstruction

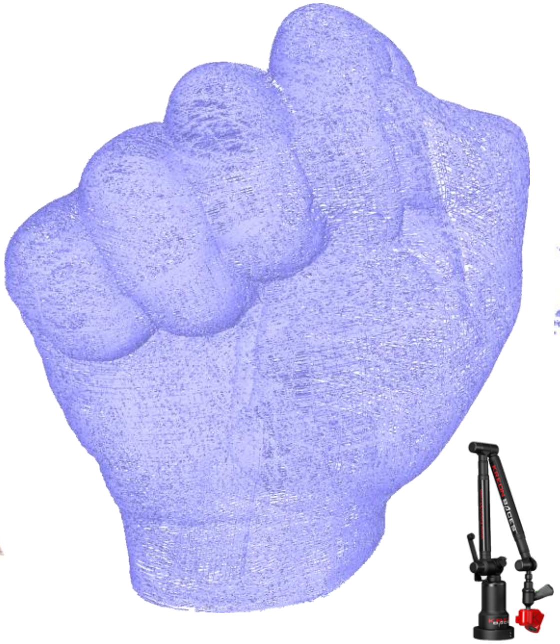
Reconstructed surface

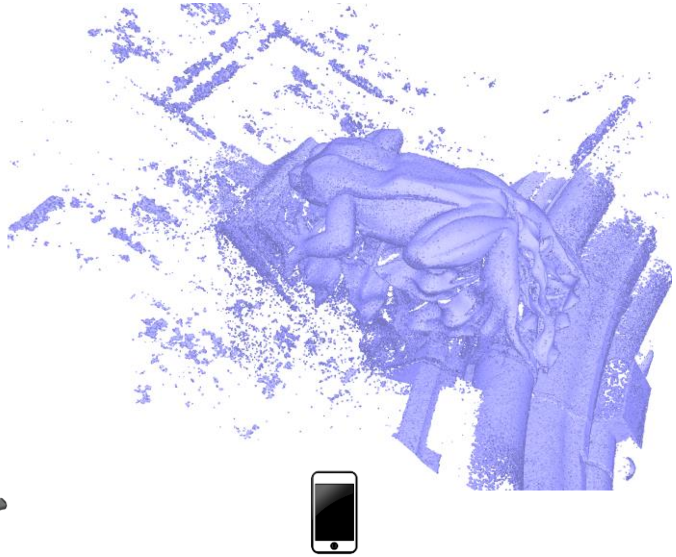
Real-World Problems

<u>Input</u>:

~~Dense~~ point set *P* sampled over surface *S*:

- Imperfect sampling
  - Non-uniform
  - Anisotropic
  - Missing data (holes)
- Uncertainty
  - Noise





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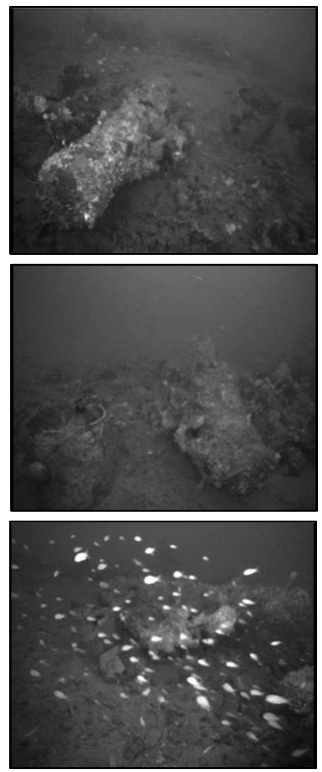

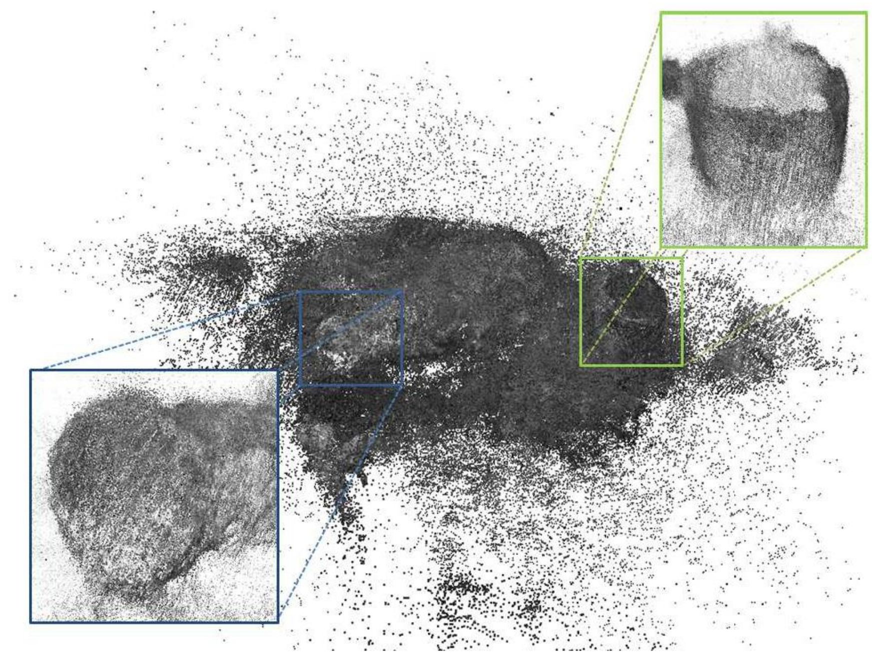

“La lune”: Data from Dassault Systèmes.
Sun King's flagship, sank off the Toulon coastline in 1664.

Real-World Problems

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Point set *P* sampled over surface *S*:

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  - Non-uniform
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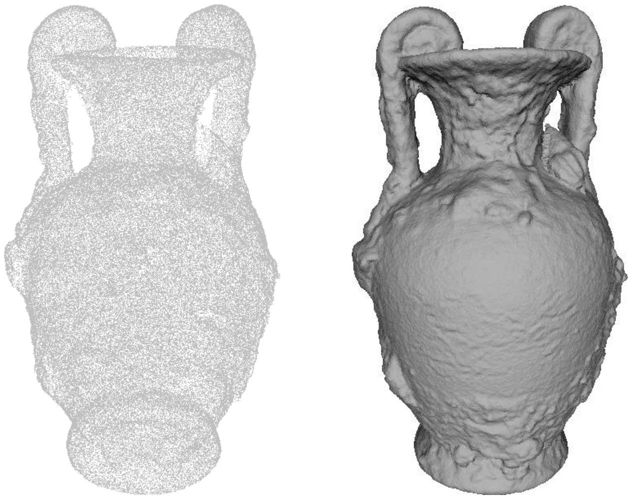
  - Missing data (holes)
- Uncertainty
  - Noise
  - Outliers

Output:

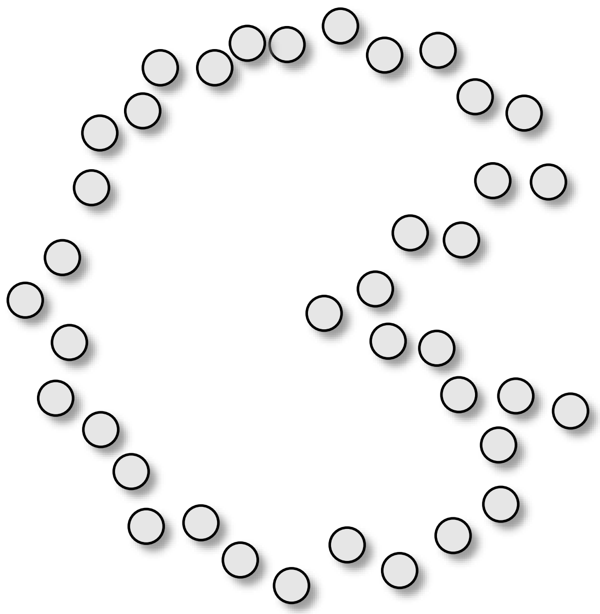
Surface: Approximation of *S* in terms of topology and geometry

Desired properties:

- Watertight
- Intersection free
- Data fitting vs smoothness

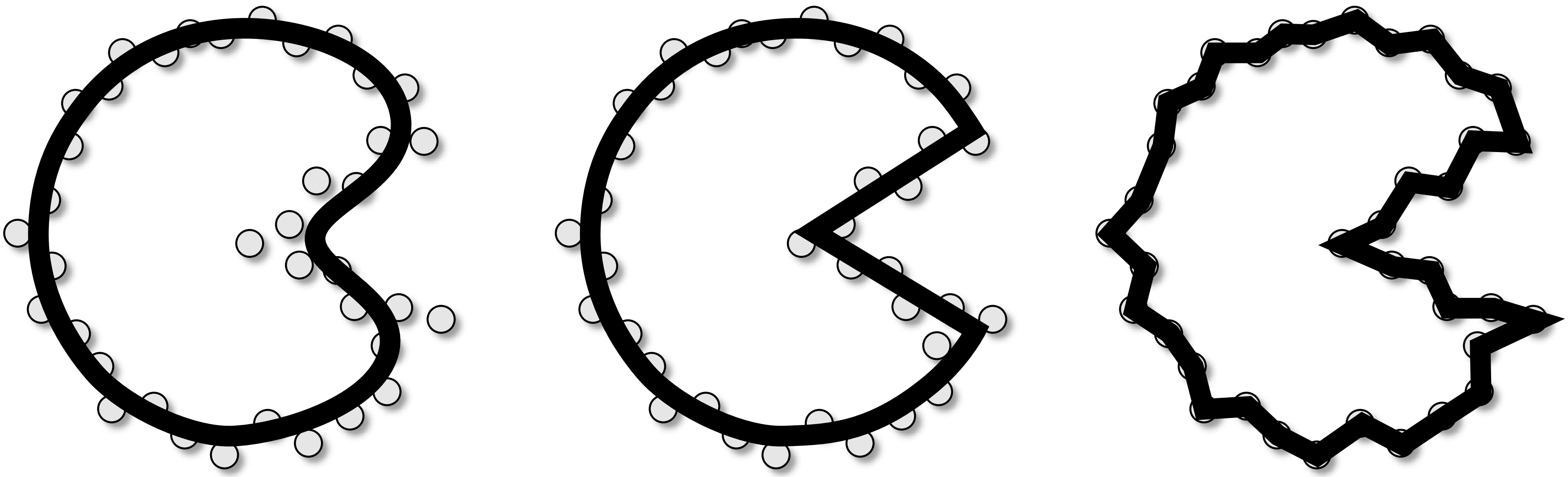


Ill-posed Problem



Many candidate shapes for the reconstruction problem.

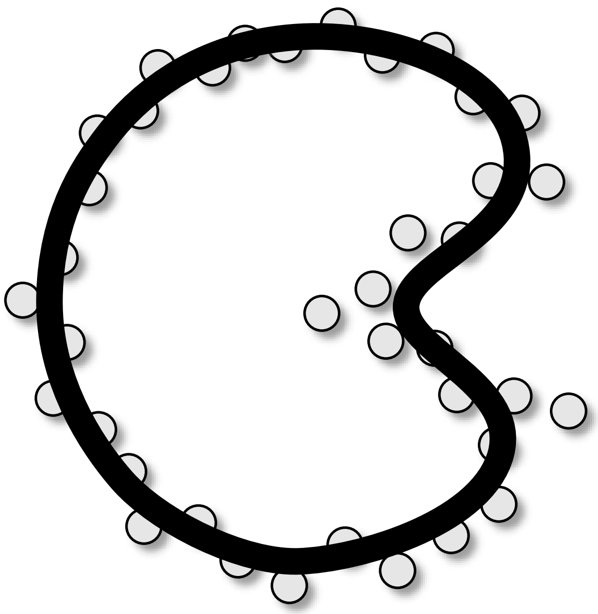
Ill-posed Problem



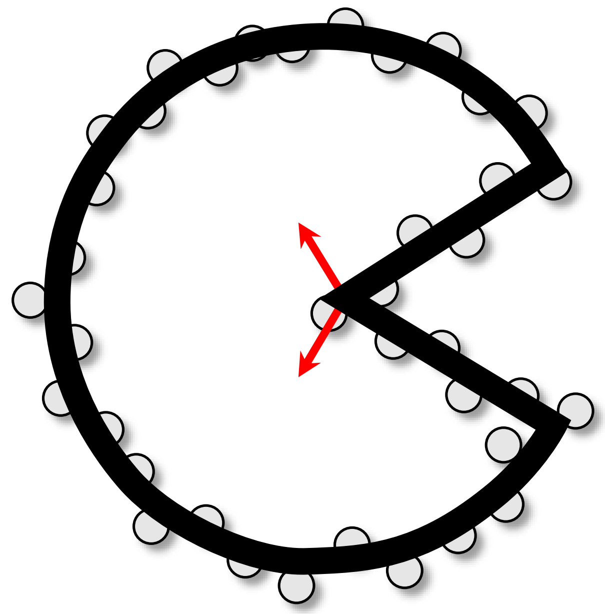
Many candidate shapes for the reconstruction problem.

MAIN APPROACHES

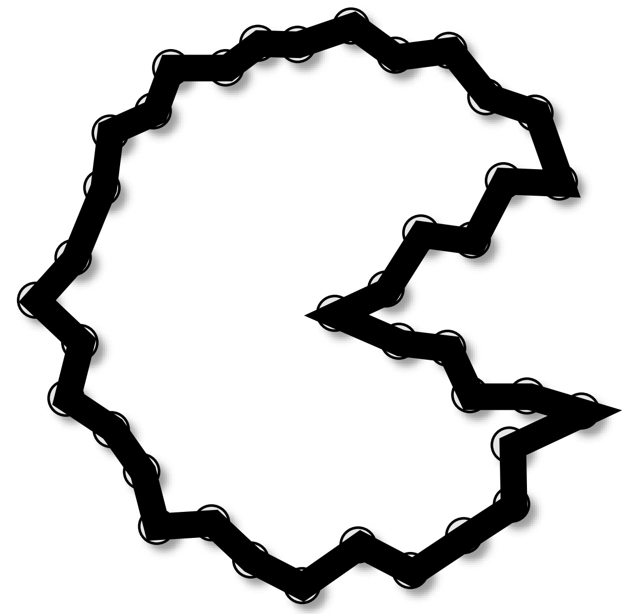
Priors



Smooth



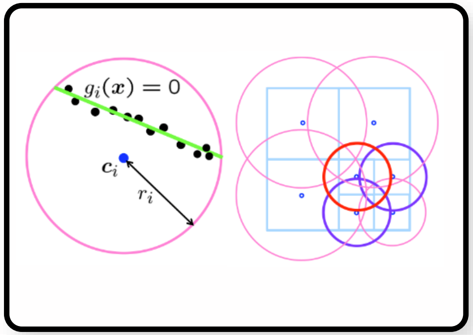
Piecewise Smooth



“Simple”

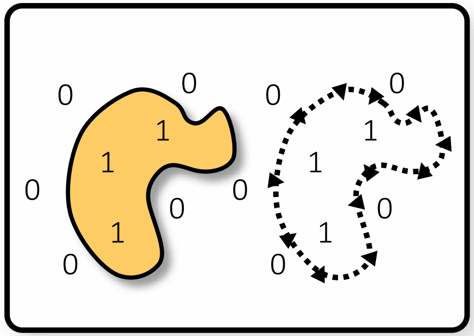
Surface Smoothness Priors

Local Smoothness



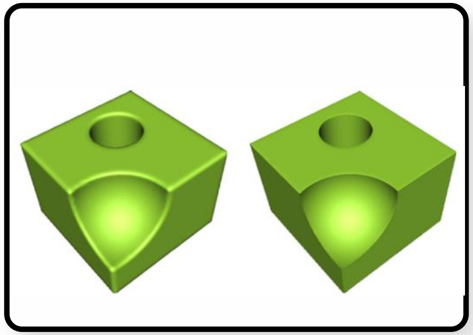
Local fitting
No control away from data
Solution by interpolation

Global Smoothness



Global: linear, eigen, graph cut, …
Robustness to missing data

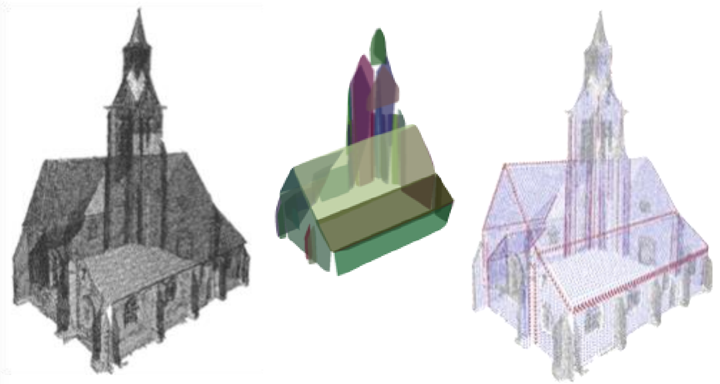
Piecewise Smoothness



Sharp near features
Smooth away from features

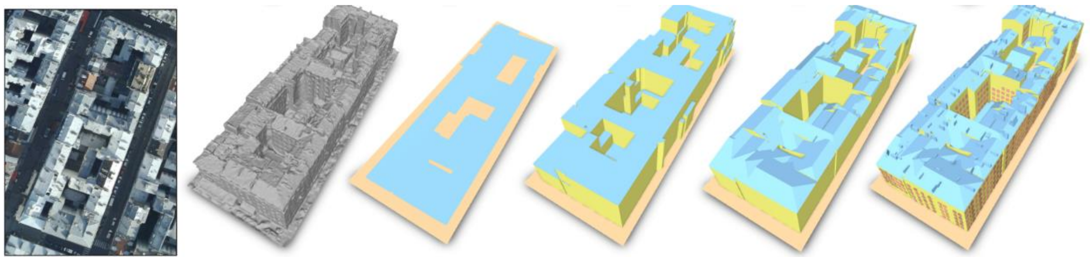
Domain-Specific Priors

Surface Reconstruction
by Point Set Structuring



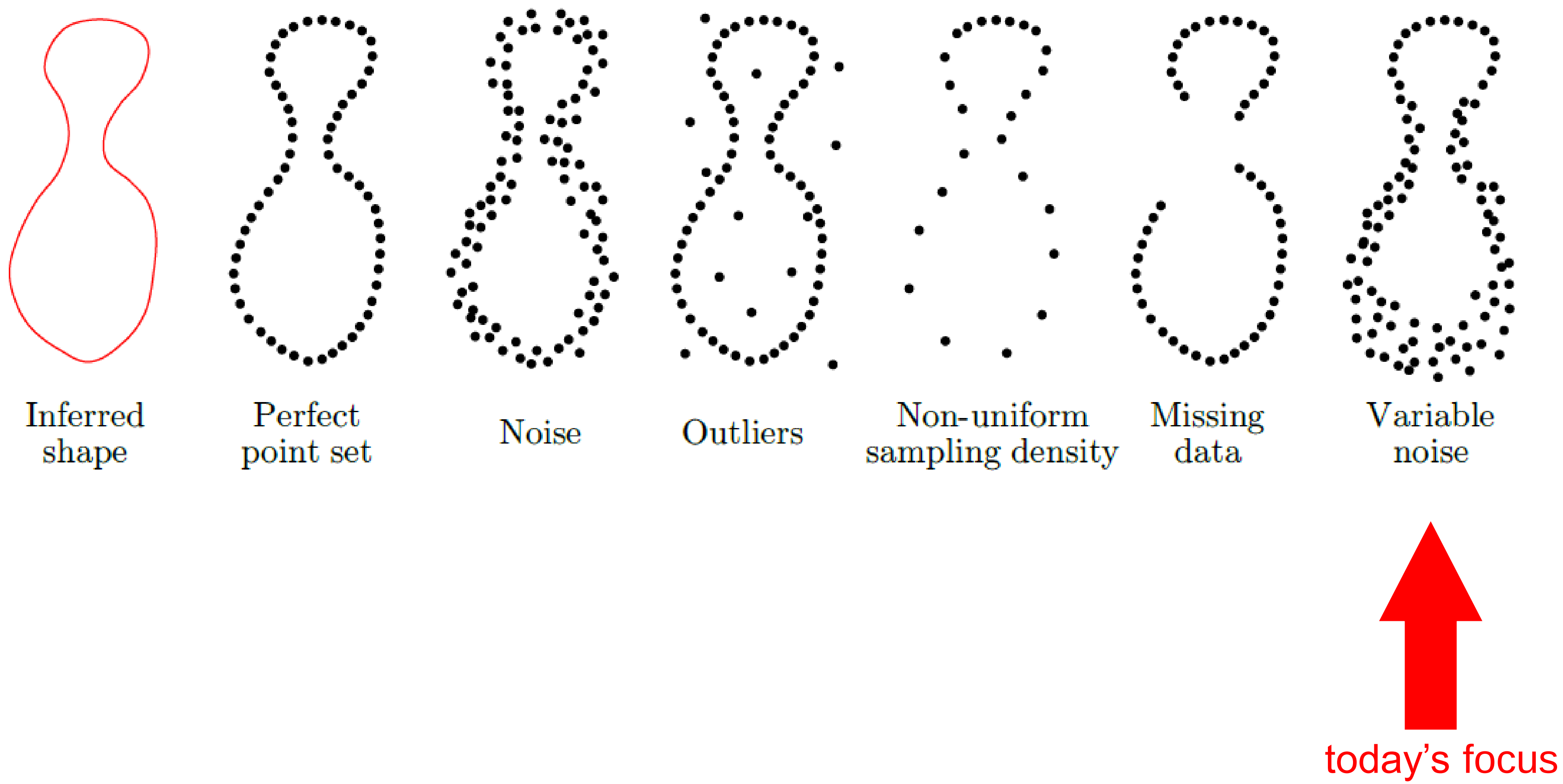
[**Lafarge - A**. EUROGRAPHICS 2013]

Reconstruction of Urban Scenes



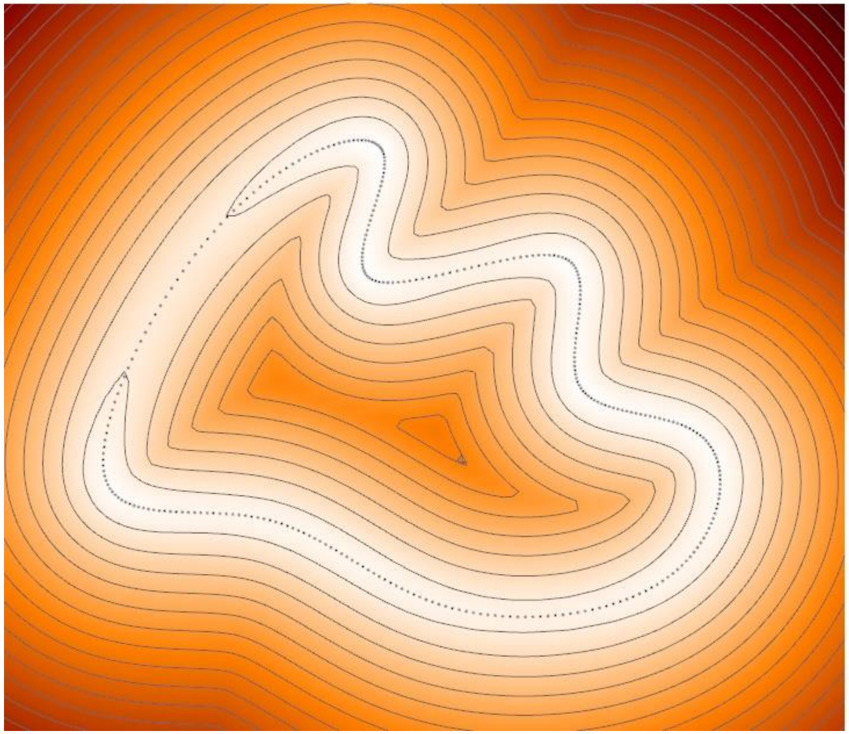
[**Verdie, Lafarge - A**. ACM Transactions on Graphics 2015]

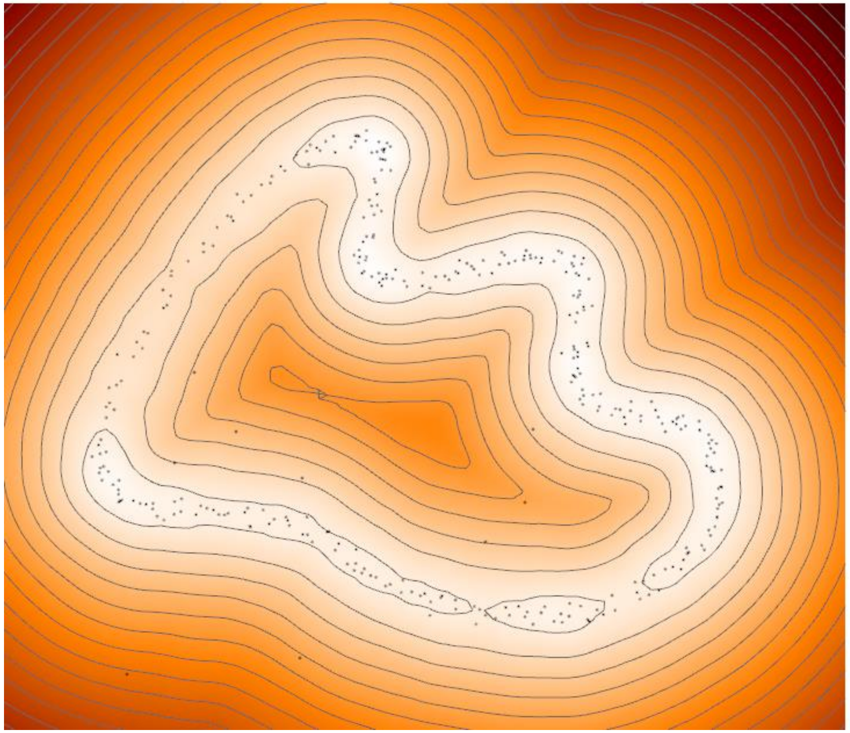
Quest for Robustness



RECONSTRUCTION

Robust Distance Function





[Chazal, Cohen-Steiner, Mérigot 11] Outlier and noise robust. Based on optimal transport distance between geometric measures (W_2 Wasserstein distance, stable)

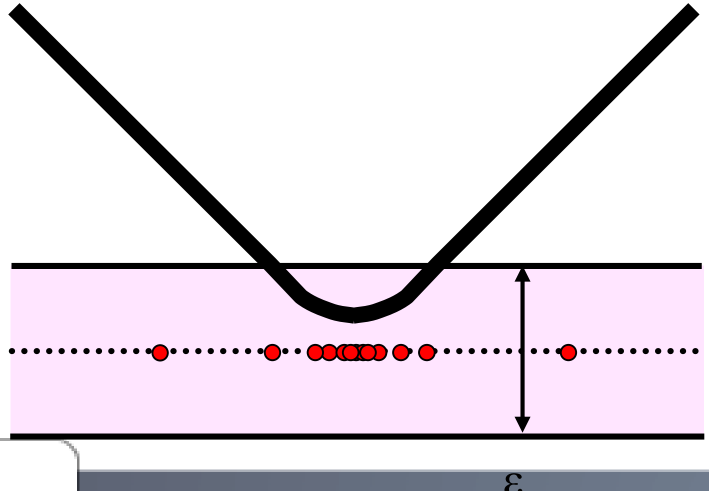
Robust Distance Function

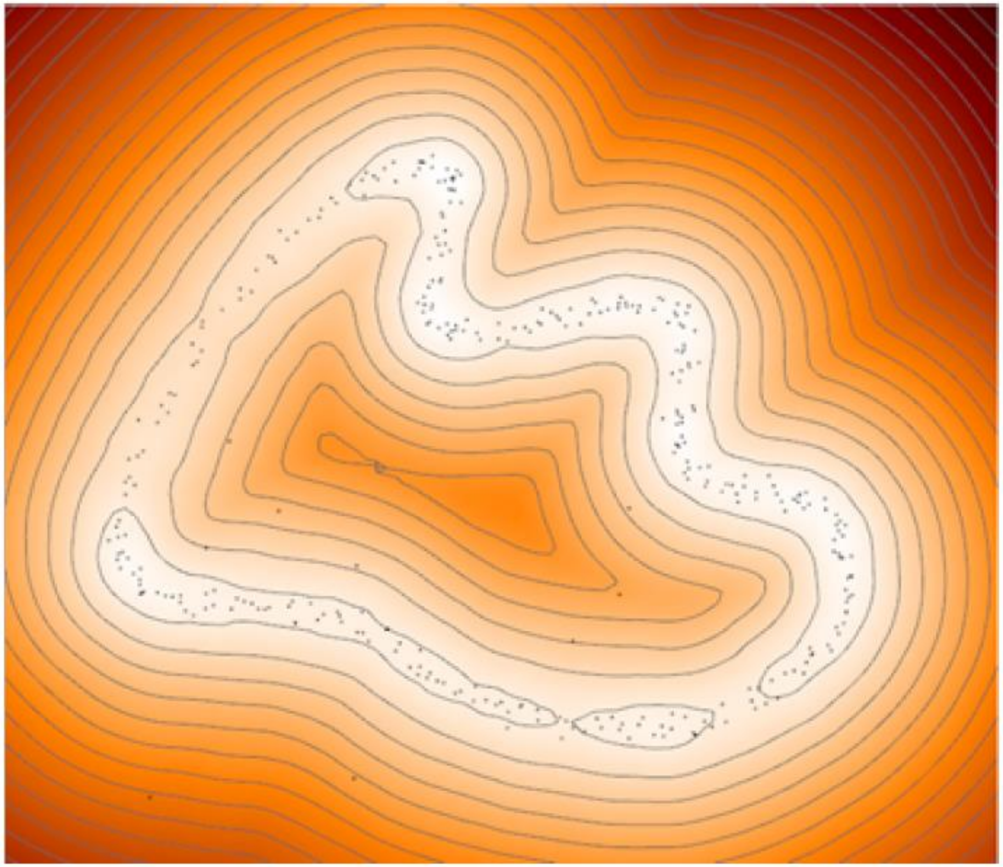
Pros

- Noise and outlier robust
- Efficient evaluation

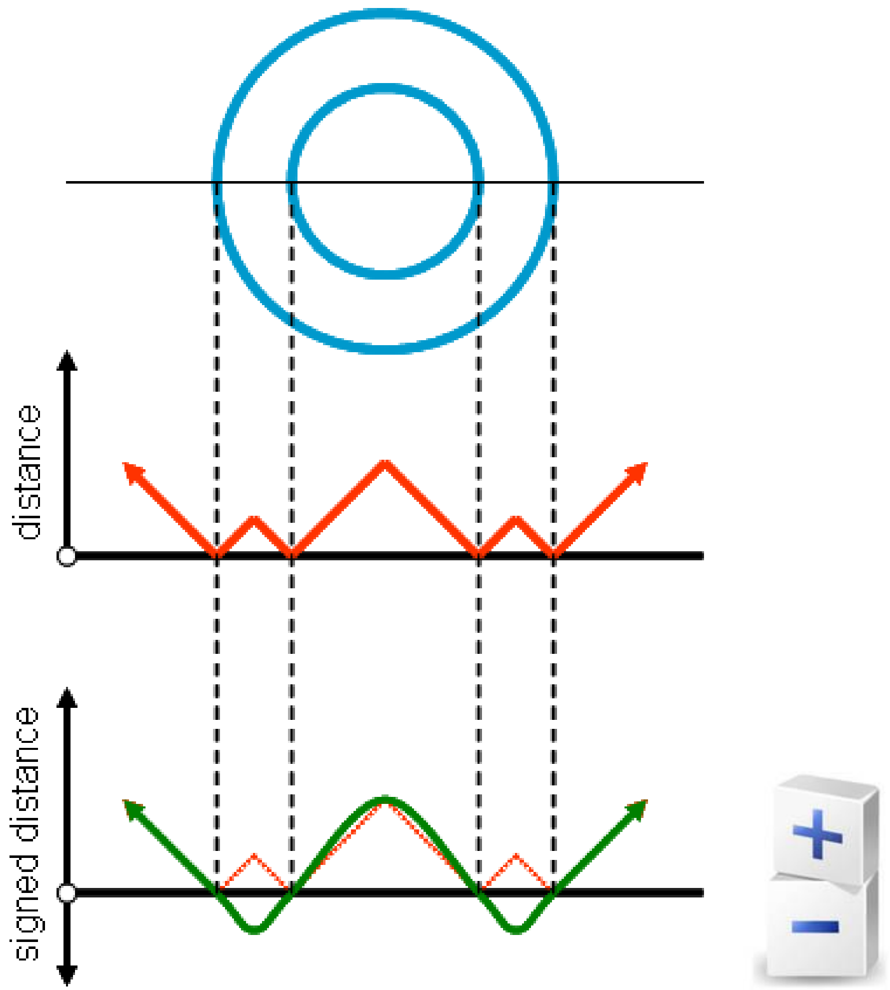
Cons

- Sensitive to variable sampling
- Inaccurate at inferred surface
- Does not reach zero





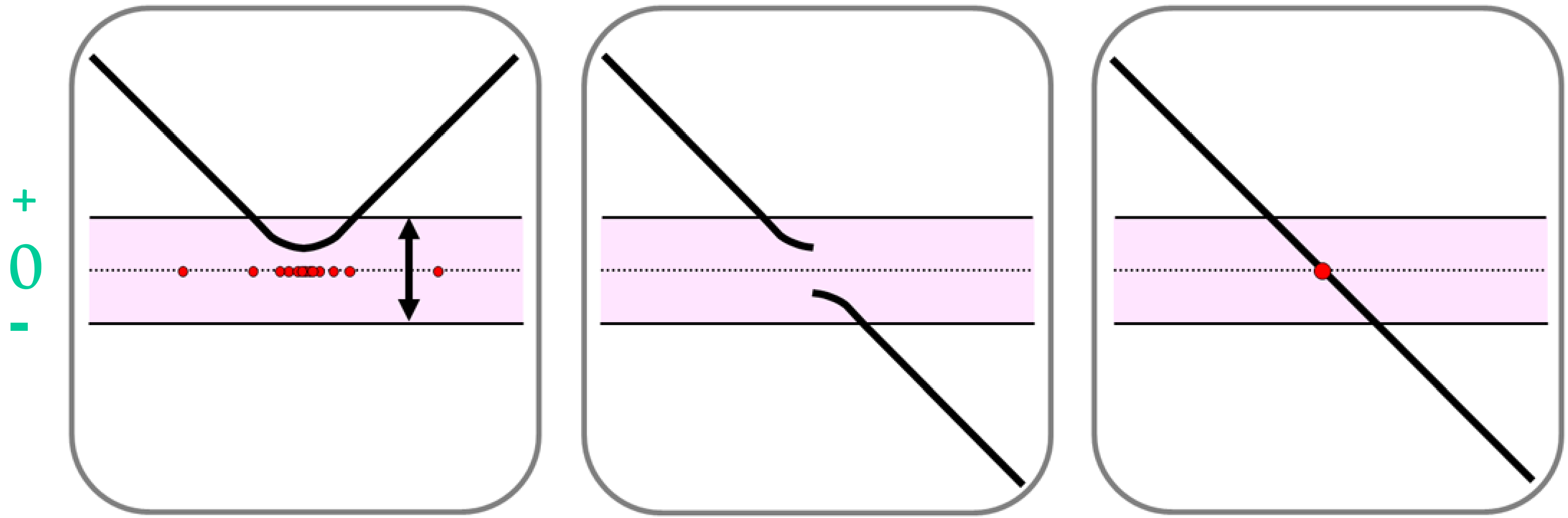
Signing an Unsigned Distance Function?



Robust Approach

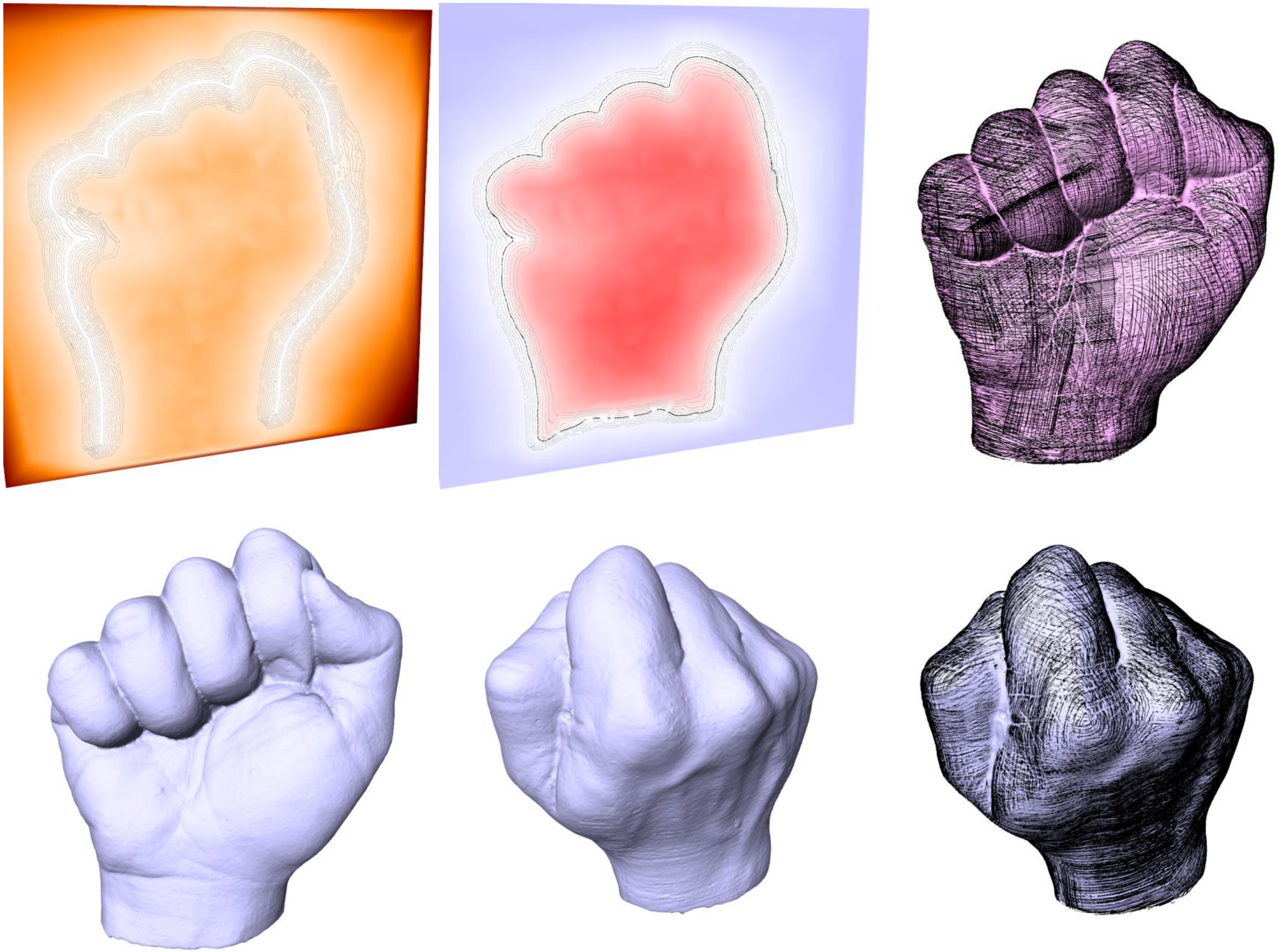
Signing an unsigned distance function

- signed functions are smoother
- closed surfaces



Signing the Unsigned: Robust Surface Reconstruction from Raw Pointsets.
Mullen, de Goes, Cohen-Steiner, A., Desbrun
SGP 2010.

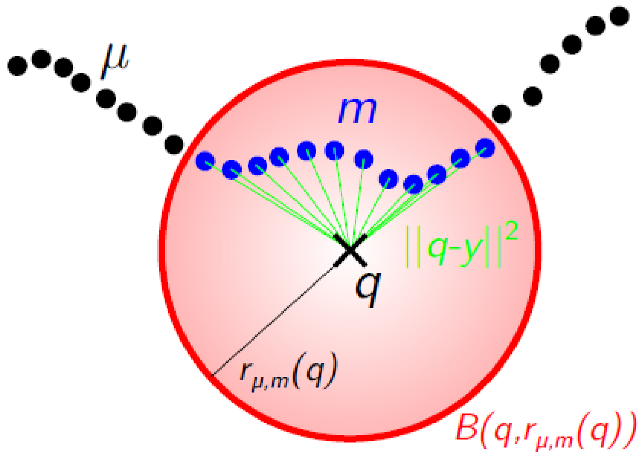
Holes



Robust Distance Function

Unsigned distance function to a measure [Chazal et al., 2011]

$$d^2_{\mu,m} : \mathbb{R}^n \to \mathbb{R},\ q \mapsto \frac{1}{m} \int_{B(q, r_{\mu,m}(q))} \|q - y\|^2 \mathrm{d}\mu(y)$$



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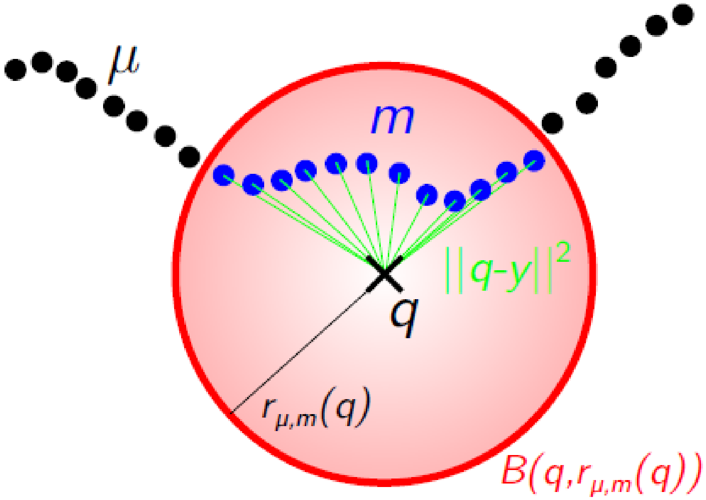
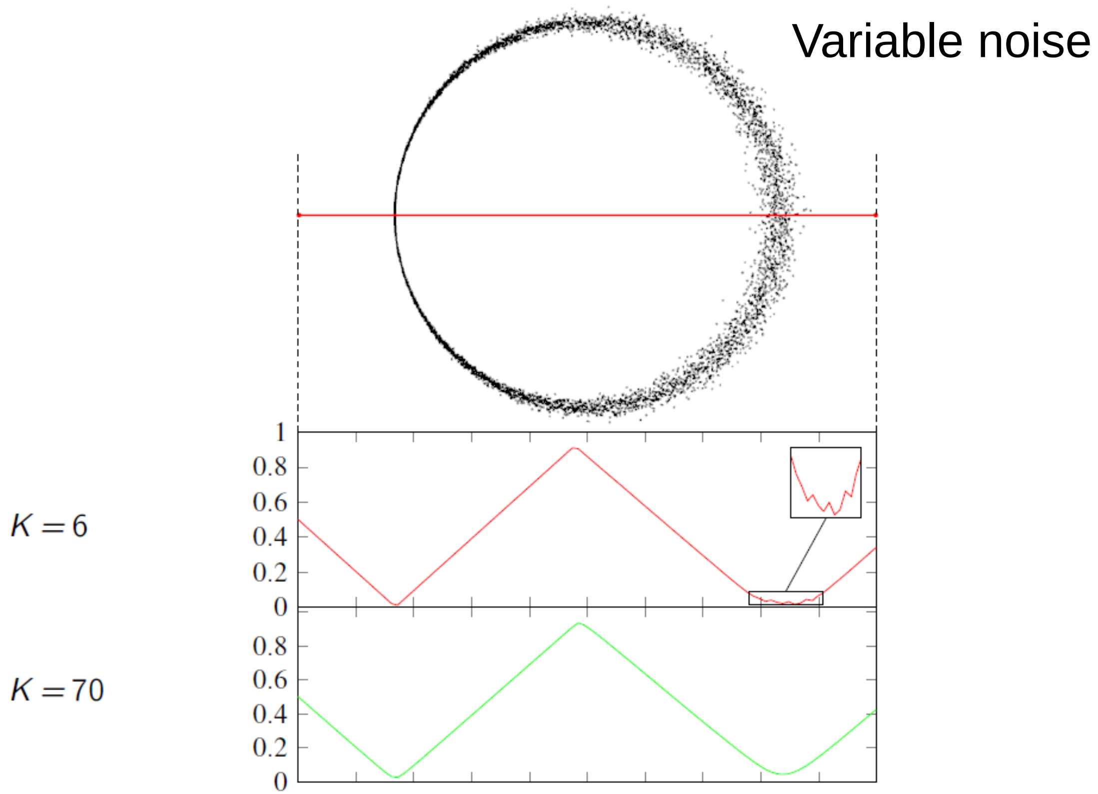


Note: scale parameter m

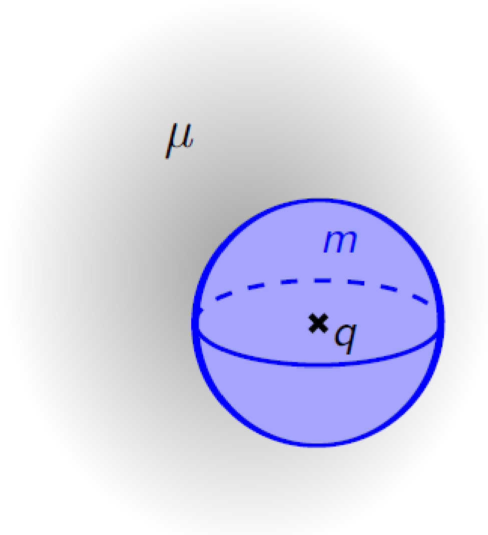
- User-specified
- Depends on point set properties
- Global: not noise-adaptive

Non-adaptive Distance Function



Case of Ambient Noise

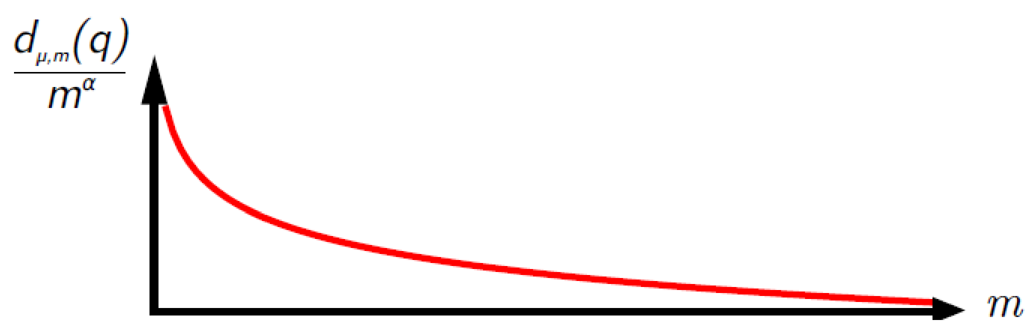
Uniform measure in d-dimensional space



$$d^2_{\mu,m}(q) = c \cdot m^{\frac{2}{d}}$$

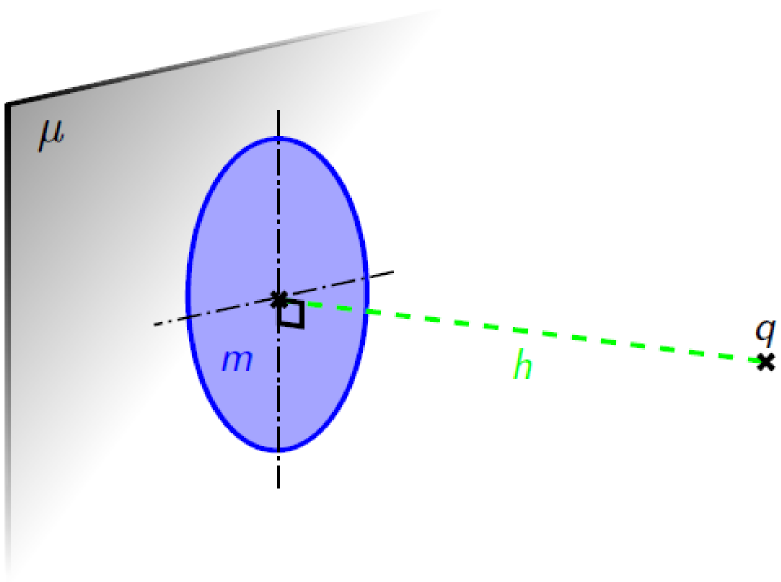
$$d_{\mu,m}(q) \propto m^{\frac{1}{d}} \text{ for } q \text{ fixed}$$

$$\frac{d_{\mu,m}(q)}{m^{\alpha}} \text{ decreasing for } \alpha > \frac{1}{d}$$



Case of Submanifold

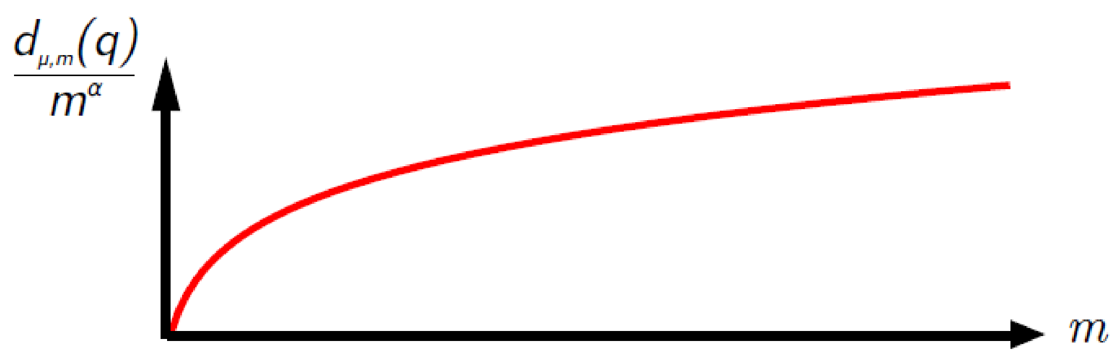
Uniform measure on k-submanifold



$$d^2_{\mu,m}(q) = c \cdot m^{\frac{2}{k}} + h^2$$

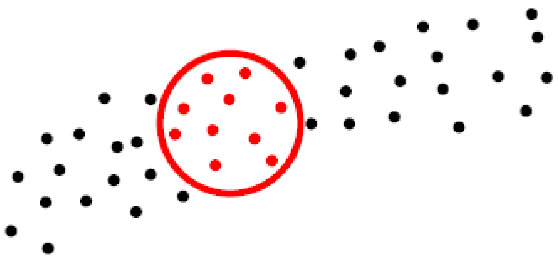
$$d_{\mu,m}(q) \propto m^{\frac{1}{k}} \text{ for } q \text{ fixed}$$

$$\frac{d_{\mu,m}(q)}{m^\alpha} \text{ increasing for } \alpha < \frac{1}{k}$$



Noisy Case

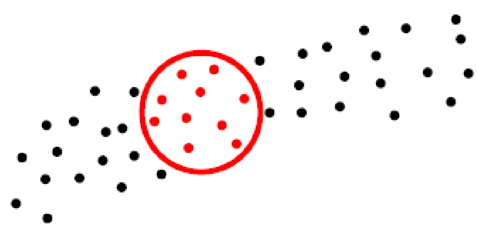
Scale $m = 10$ nearest neighbors



- Apparent dimension $= 2$
- Ambiant noise in $2D$
- $\frac{d_{\mu,m}(q)}{m^\alpha}$ decreasing for $\alpha > \frac{1}{2}$

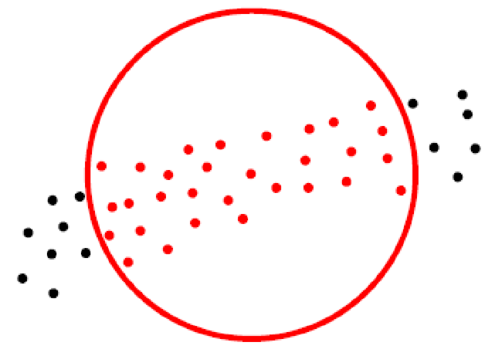
Noisy Case

Scale $m = 10$ nearest neighbors

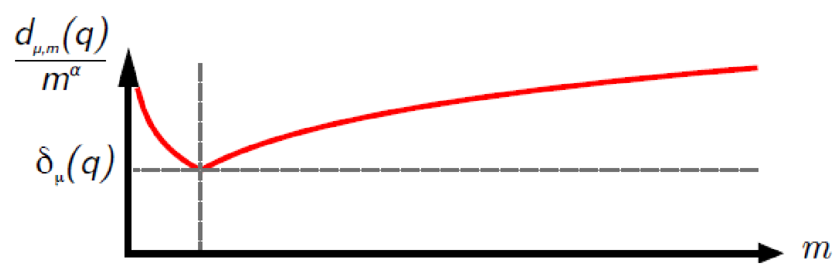


- Apparent dimension $= 2$
- Ambiant noise in $2D$
- $\frac{d_{\mu,m}(q)}{m^{\alpha}}$ decreasing for $\alpha > \frac{1}{2}$

Scale $m = 30$ nearest neighbors



- Apparent dimension $= 1$
- 1-submanifold in $2D$
- $\frac{d_{\mu,m}(q)}{m^{\alpha}}$ increasing for $\alpha < 1$



Noise-adaptive Distance Function

Dimension prior

Assumption

Inferred shape is a submanifold of known dimension

For a *k*-submanifold in *d*-dimensional space:

$$\delta_\mu = \inf_{m>0} \frac{d_{\mu,m}}{m^\alpha},$$

with $\alpha \in [\frac{1}{d}; \frac{1}{k}]$

Noise-adaptive Distance Function

$$\delta_\mu = \inf_{m>0} \frac{d_{\mu,m}}{m^\alpha}$$

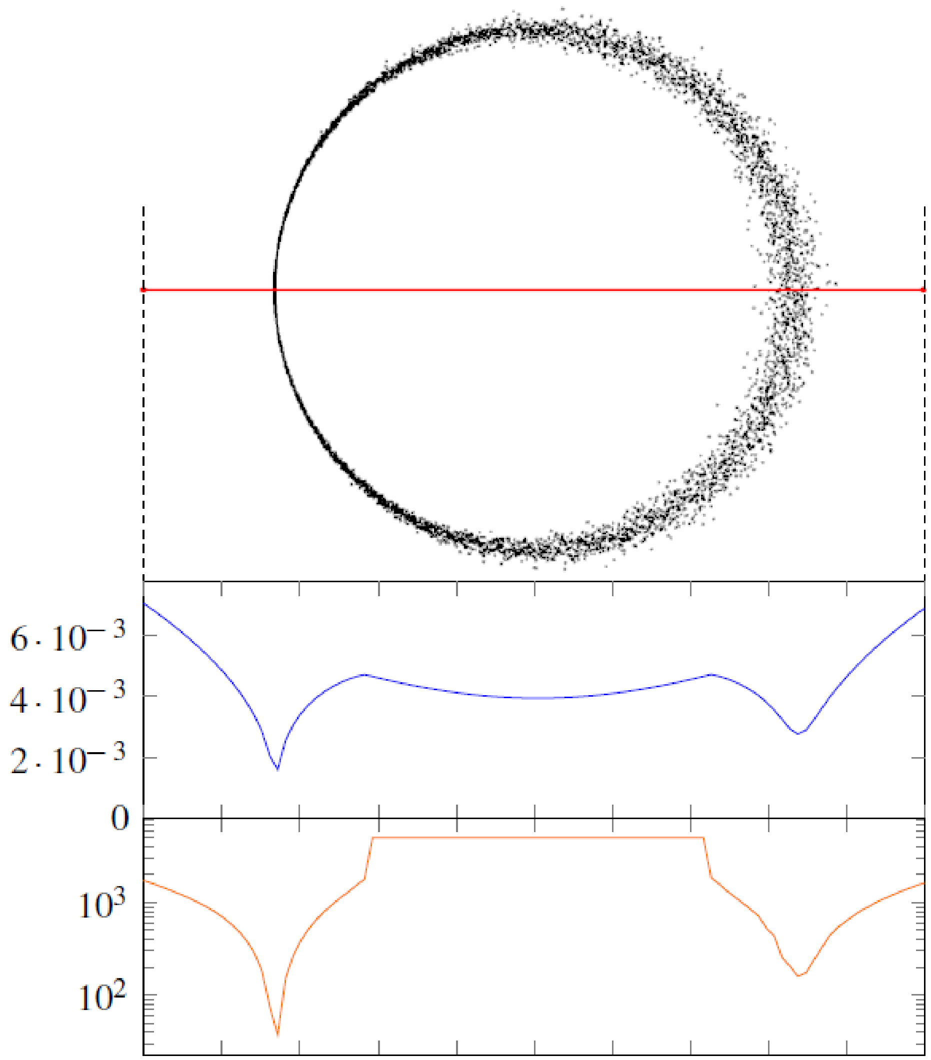
Infimum:

1. m as small as possible $\rightarrow$ no oversmoothing
2. m large enough $\rightarrow$ point subset appears as k-submanifold

Setting α ($\alpha \in [\frac{1}{d}; \frac{1}{k}]$)

- Curve in $2D$: $\alpha = \frac{3}{4}$ to satisfy $\alpha \in [\frac{1}{2}; 1]$
- Surface in $3D$: $\alpha = \frac{5}{12}$ to satisfy $\alpha \in [\frac{1}{3}; \frac{1}{2}]$

Noise-adaptive Distance Function

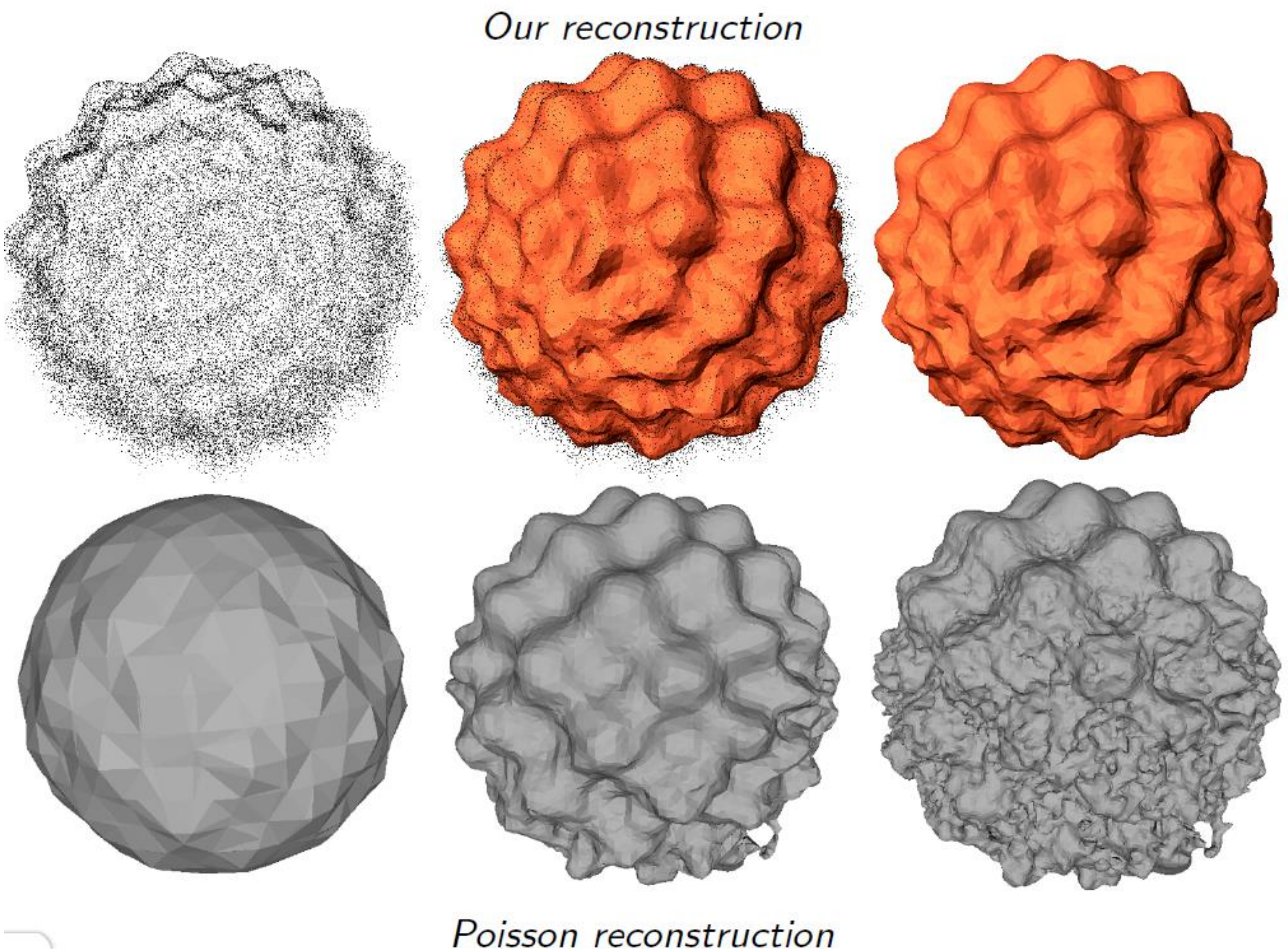


Function δ_μ

Local scale m

Result

Noise-Adaptive Shape Reconstruction from Raw Point Sets. EUROGRAPHICS Symposium on Geometry Processing 2013.
Giraudot, Cohen-Steiner, A.

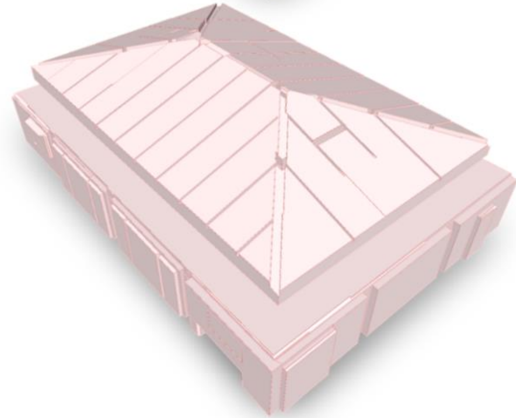


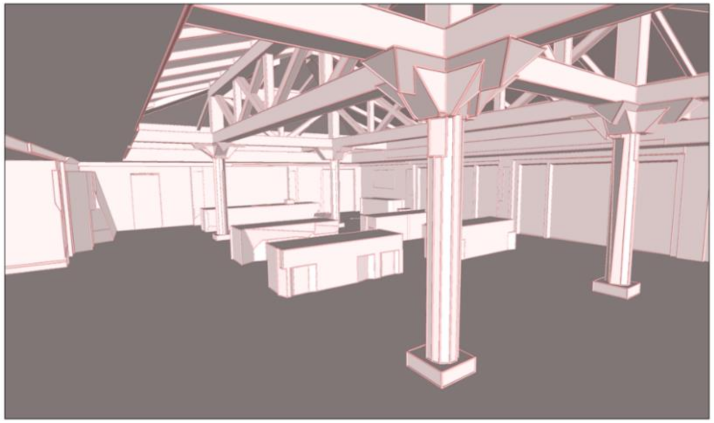
RECENT WORK?

Kinetic Shape Reconstruction

3M points

1.5K facets





Kinetic Shape Reconstruction
Jean-Philippe Bauchet and **Florent Lafarge**
ACM Transactions on Graphics, 2020

presented at SIGGRAPH THINK BEYOND 2020 S2020.SIGGRAPH.ORG

Inria

Thank you.

Survey:

A Survey of Surface Reconstruction from Point Clouds. Berger, Tagliasacchi, Seversky, Alliez, Guennebaud, Levine, Sharf and Silva. Computer Graphics Forum, 2016.

Software:

CGAL

https://www.cgal.org/

erc

"**IRON**" CoG
2011-2015
Robust Geometry Processing

"**TITANIUM**" PoC
2017-2018
Software Components for Robust Geometry Processing

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