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SELECTION OF STARS FOR THE DETERMINATION OF TIME, AZIMUTH AND LAPLACE QUANTITY BY MERIDIAN TRANSITS

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CONTENTS

SELECTION OF STARS FOR THE DETERMINATION OF TIME, AZIMUTH AND LAPLACE QUANTITY BY MERIDIAN TRANSITS

Summary

A procedure is described and diagrams are given for selecting the most favourable stars for the determination of local time by observing their meridian transits. It is assumed that for the observations and computations the suggestions given in the International Geophysical Year's "Instruction Manual VIII, Longitude and Latitude", are followed.

1 **Introduction**

A discussion, in 1927, in the Baltic Geodetic Commission [1] on the problem of selecting stars for time determination by meridian transits, generated a series of papers on this question, which continued until 1942 [2–19]. After the war the geodetic and astronomic literature recurs to this problem only rarely and incidentally [20–22]; almost everybody concerned with time determinations apparently having adopted a certain system of selection.

After the International Geophysical Year's Instruction Manual VIII for Longitude and Latitude has suggested a reduction method as applied by the Paris Observatory, which may have become more or less general practice by now, there is a case for taking up the problem anew.

2 **Method of observing and reducing**

The I.G.Y-Instruction Manual VIII, Longitude and Latitude, gives the following suggestions for the observations and reductions:

> "If the observations are made with a small transit instrument, the instrument should be reversed during the transit of each star. If the observations are made with a meridian telescope, it should be reversed at least once in the middle of the series of stars that are observed.
>
> The determination of the inclination of the axis of the telescope (the level error) may be made using either a mercury bath or a bubble level. In the first case, the inclination of the axis should be determined at the beginning and at the end of each series. In the second case, the level should be read for each star. If the level of the telescope is stable, the whole series of observations can be reduced using the same value of the level error. If it is found that the level changes progressively during the observations, the level error for each star observed can be found either by calculation or by a graphical method.

If the level changes in an irregular manner, the values of the level error observed for each star should be used.

For the reduction of the astronomical observations either the formula of HANSEN or that of MAYER can be used. It is preferable to use HANSEN's formula when the level error of the telescope is stable; if it is not stable the formula of MAYER should be used. For the calculation of the azimuth of the telescope and of the observed correction to the clock, the method of least squares, or that of CAUCHY, or a simple graphical method can be used."

Accordingly, the following assumptions are made in this paper:

a. Ten stars are observed: five north and five south of the zenith.
b. A portable transit instrument or a first order theodolite is used, the telescope of which is reversed halfway the series of observations of each star.
c. The inclination of the rotation axis of the telescope is determined by means of a bubble level. The level remains untouched in the same position with respect to the rotation axis; it is reversed simultaneously with this axis and read just before and just after the observations of each star.
d. The reduction of the observations is made by the method of the Paris Observatory, which is summarized in the Instruction Manual mentioned and described in more detail in the next paragraph.

3 Computation of the azimuth and the chronometer correction

MAYER's formula reads:

$$\Delta T = \alpha - T + Aa' - Bb \quad \ldots \quad (1)$$

in which:

ΔT = Chronometer correction to Local Sidereal Time
α = Right ascension of the star observed
a' = Azimuthal deviation of the telescope from the meridian. If a designates the star's azimuth, reckoned from north to east, the sign of a' is defined by the following relations:

$$\left.\begin{array}{ll} a' = a & \text{for a north star} \\ a' = a - 180^\circ & \text{for a south star} \end{array}\right\} \quad \ldots \quad (2)$$

b = Inclination of the rotation axis of the telescope (positive if the west end of the axis is higher than the east end)
T = Chronometer time observed ($+12^{\mathrm{h}}$ for a star observed at lower culmination)

$$\left.\begin{array}{l} A = \sin\varphi \mp \cos\varphi \tan\delta \\ B = \cos\varphi \pm \sin\varphi \tan\delta \end{array}\right\} \frac{\text{upper}}{\text{lower}} \text{ culmination}$$

φ = Latitude of the station
δ = Declination of the star

Putting:

$$\alpha - Bb - T = l \quad \ldots \quad (3)$$

the formula becomes:

$$\Delta T = +Aa' + l$$

or for north and south stars respectively:

$$\Delta T = +A_i a' + l_i \quad (i = 1,\ldots,5) \qquad (4)$$

$$\Delta T = +A_j a' + l_j \quad (j = 6,\ldots,10) \qquad (5)$$

These formulae do not of course suggest that the stars should be observed in this sequence.

Summation of both sets of equations (4) and (5) gives:

$$5\Delta T = +[A_i]a' + [l_i]$$
$$5\Delta T = +[A_j]a' + [l_j]$$

from which is solved:

$$\boxed{-a' = \frac{[l_i]-[l_j]}{[A_i]-[A_j]}} \qquad (6)$$

This value of a' is introduced in all ten equations (4) and (5) and the mean of the values of ΔT, thus obtained, computed:

$$\boxed{\Delta T_m = {}^1/_{10}[\Delta T]} \qquad (7)$$

This is the method as used at the Paris Observatory, which is not evidently a solution according to the method of least squares, but a solution which is at the same time very simple and completely satisfactory in nearly all cases including first order work.

4 Variances and covariance of ΔT_m and a

Introduction of (4) and (5) into (7) gives after some reduction:

$$\Delta T_m = \frac{[A_i][l_j]-[A_j][l_i]}{5\{[A_i]-[A_j]\}} \qquad (8)$$

Application of the law of propagation of variances to (6) and (8) gives, referring to (2):

$$m_{a'}^2 = m_a^2 = \frac{[m_{l_i}{}^2]+[m_{l_j}{}^2]}{\{[A_i]-[A_j]\}^2} \qquad (9)$$

$$m_{\Delta T_m}{}^2 = \frac{[A_i]^2[m_{l_j}{}^2]+[A_j]^2[m_{l_i}{}^2]}{25\{[A_i]-[A_j]\}^2} \qquad (10)$$

$$m_{a'\cdot\Delta T_m} = m_{a\cdot\Delta T_m} = \frac{[A_i][m_{l_j}{}^2]+[A_j][m_{l_i}{}^2]}{5\{[A_i]-[A_j]\}^2} \qquad (11)$$

5 Variance of the Laplace quantity

The Laplace condition in a triangulation net reads:

$$(a-a^g)+(\lambda-\lambda^g)\sin\varphi = 0$$

where λ^g is the geodetic value of the longitude of a triangulation point and a^g the geodetic azimuth of a triangulation side at that point.

The astronomical part of the left hand side of this equation may be called the Laplace quantity and designated L:

$$a+\lambda\sin\varphi = L \quad (12)$$

λ is the astronomical longitude computed from:

$$\lambda = \text{GMST}-\text{LMST} = \text{GMST}-(T+\Delta T_m) \quad (13)$$

From (12) and (13) follows the variance of L:

$$m_L{}^2 = m_a{}^2+m_{\Delta T_m}{}^2\sin^2\varphi-2m_{a\cdot\Delta T_m}\sin\varphi \quad (14)$$

The quantities $m_a{}^2$, $m_{\Delta T_m}{}^2$ and $m_{a\cdot\Delta T_m}$ in this formula are given by (9), (10) and (11).

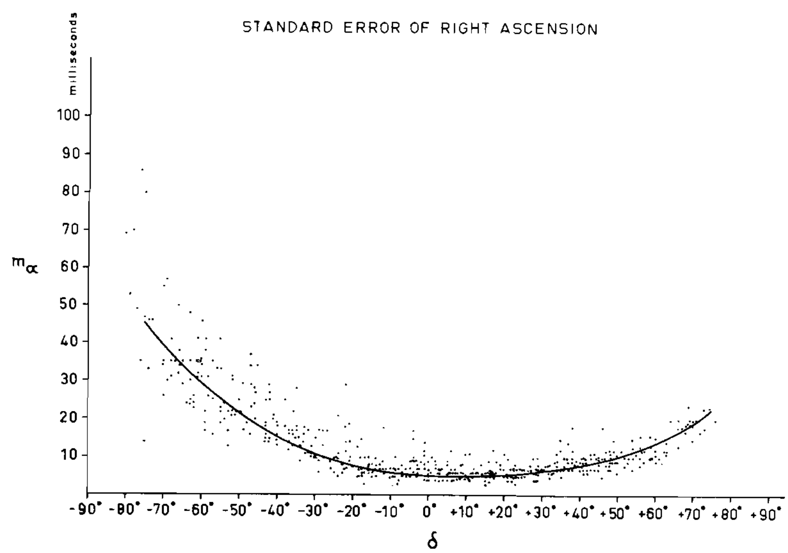
6 Numerical values of $(m_l{}^2)_{i\ or\ j}$

From (3) follows:

$$m_l{}^2 = m_\alpha{}^2+B^2m_b{}^2+m_T{}^2 \quad (15)$$

Numerical values of the variances at the right hand side will be determined in the following paragraphs.

a. The standard deviations in right ascension, m_α, as given in the FK 3, are represented in the diagram below as a function of the declination.



For practical reasons the diagram is restricted to stars whose right ascension is between 0^h and 12^h. It may nevertheless be taken as a representative picture for all stars, contained in the "Apparent Places".

b. The classic formula for the standard deviation of the inclination, determined by means of a bubble level is:

$$\mu_b = 0.20\sqrt{H}$$

in which H is the angular value of the level.

If μ_b and H are given in seconds of time:

$$\mu_b = 0.0517\sqrt{H}$$

Assuming $H = 0^s.115$ (a value often used for levels of portable transit instruments) we have:

$$\mu_b = 0^s.018$$

b in (3) is the mean of two level readings, just before and after the series of observations on each star, whence:

$$m_b = \tfrac{1}{2}\mu_b\sqrt{2} = 0^s.013$$

or rounded off:

$$\boxed{m_b = 15 \text{ milliseconds}} \quad \dots\dots \quad (16)$$

c. The standard deviation of the observation of a star's transit is generally given by the formula:

$$\mu_T^2 = p^2 + q^2 \cdot C^2$$

where $C = \sec\delta$

For the case — which will be assumed here — that an impersonal non-automatic micrometer is used, NIETHAMMER [23] gives for each of the coefficients p and q two values:

$$p = 0^s.057 \text{ and } 0^s.031$$
$$q = 3^s.0 : v \text{ and } 2^s.6 : v$$

in which v is the magnification of the telescope.

We will assume $v = 85$ and take the round values:

$$p = 45 \text{ milliseconds}$$
$$q = 3^s.0 : 85 = 35 \text{ milliseconds}$$

whence:

$$\mu_T^2 = (45)^2 + (35)^2 \cdot C^2$$

If the number of observation-contacts per star is 24:

$$m_T^2 = {}^1/_{24}\mu_T^2 = 84.38 + 51.04C^2$$

or rounded off:

$$\boxed{m_T^2 = 85 + 50C^2} \quad \dots\dots \quad (17)$$

d. Introduction of (16) and (17) into (15) gives:

$$\boxed{m_l^2 = m_a^2+225B^2+85+50C^2} \qquad (\text{msec})^2 \quad \dots\dots\dots \quad (18)$$

where m_a is to be read from the diagram on page 8.

7 Selection of stars. Diagrams

For the following latitudes:

$$\varphi = 0°, +10°, +20°, +30°, +40°, +50° \text{ and } +60°$$

and a great number of combinations of north and south groups of stars the standard errors $m_{\Delta T_m}$, m_a and m_L were computed by the formulae (10), (9) and (14) taking m_l^2 according to (18).

With a view to coming to general conclusions, some sort of schematizing had to be applied in forming these groups of five stars each. Therefore, each group was assumed to contain stars whose declinations differed by equal amounts β_1 resp. β_2: *)

north group	*south group*
$\delta_1 = \delta_N - 2\beta_1$	$\delta_6 = \delta_S - 2\beta_2$
$\delta_2 = \delta_N - \beta_1$	$\delta_7 = \delta_S - \beta_2$
$\delta_3 = \delta_N$	$\delta_8 = \delta_S$
$\delta_4 = \delta_N + \beta_1$	$\delta_9 = \delta_S + \beta_2$
$\delta_5 = \delta_N + 2\beta_1$	$\delta_{10} = \delta_S + 2\beta_2$

The "amplitude" of each group:

$$\delta_5 - \delta_1$$
$$\delta_{10} - \delta_6$$

was chosen so that the "Apparent Places" contains at least 60 stars within the declination-range concerned.

The resulting amplitudes are shown in the table on page 11.

To cope with the large volume of the computations they had to be programmed for the electronic computer TR-4 of the Delft Technical University.

The result is given in the form of the attached 21 diagrams on which the standard errors to be expected for all possible combinations of north and south groups can be read.

For stations in the southern hemisphere the diagrams can be used as well, if only the sign of the declinations is reversed.

8 Some remarks with the diagrams

In the diagrams for $m_{\Delta T}$ the curves of equal value of $m_{\Delta T}$ converge to the point:

$$\delta_S = \delta_N = \varphi$$

---

*) It is evident that in practice the stars of each group will be distributed rather irregularly and that sometimes the amplitude has to be larger than suggested here. Several check-computations have proved that this has a negligible effect on the accuracy of the final result.

δ_N or δ_S	Amplitude	Number of stars available
$-75°$	16°	65
-70	16	80
-60	12	79
-50	8	64
-40	8	79
-30	8	88
-20	8	101
-10	8	94
0	8	96
$+10$	8	92
$+20$	8	94
$+30$	8	89
$+40$	8	89
$+50$	8	65
$+60$	12	82
$+70$	12	72
$+75$	12	76

that is when both groups coincide in the zenith. In that case $m_{\Delta T}$ reaches a minimum. It is evident however that this is mere theory, an *exact* coincidence of both groups in the zenith never being attainable.

For groups only *approximately* in the zenith the solution of equations (4) and (5) is very unstable, in the sense that for only small deviations from the zenith $m_{\Delta T}$ may attain a very high value, as is clearly demonstrated by the diagrams.

The conclusion is that it should be avoided to choose both groups in the neighbourhood of the zenith. The diagrams show that this has only a minor influence on the accuracy to be expected but improves the stability of the solution.

A suggestion for favourable combinations of δ_S and δ_N read from the diagrams is the following:

φ	δ_S	δ_N	$m_{\Delta T}$ ($0^s.001$)
0°	$-25°$	$+25°$	6.5
$+10$	-15	$+35$	6.5
$+20$	-10	$+50$	7
$+30$	0	$+60$	7.5
$+40$	$+15$	$+65$ U.C.	8.5
$+50$	$+30$	$+70$ U.C.	9.5
$+60$	$+45$	$+75$ U.C.	12
	$+30$	$+75$ L.C.	13

This applies only to the case when the chronometer correction ΔT is the principal quantity to be determined, while the azimuth is just a parasitic unknown in the problem.

If the Laplace quantity is the principal quantity to be determined, the diagrams for m_L should be consulted.

From these diagrams the following suggestion can be made up:

φ	δ_S	δ_N	m_L (0″.01)
0°	−60°	+60°	9
+10	−50	+65	9
+20	−40	+70	9.5
+30	−35	+70	9.5
+40	−30	+60 L.C.	6.5
+50	−30	+55 L.C.	6.5
+60	−10	+45 L.C.	6.5

If both the chronometer correction and the Laplace quantity are required with some precision, the groups of stars have to be chosen by consulting *both* the $m_{\Delta T}$ and the m_L diagrams. It is for that reason that the latter diagrams have been printed in red on top of the former.

The calibration of the two sets of curves makes it evident that, in general, a choice of two groups has a reverse effect on $m_{\Delta T}$ and m_L. This means that a compromise has to be made, depending on the specific accuracy requirements for ΔT and L.

Example: $\varphi = +20°$.

δ_S	δ_N	$m_{\Delta T}$(0^s.001)	m_a(0″.01)
0°	+40°	6,5	25
−10	+50	7	17
−20	+60	7,5	12
−30	+60	8	11
−40	+65	9	10

It is emphasized that the acccuray as given by the diagrams is an *average* value. The actual value to be obtained may differ from this average value, depending on the skill of the observer, the atmospheric circumstances, etc.

At the semi-permanent geodetic astronomic station at Curaçao ($\varphi = +12°10'$), for instance, which was in operation during the International Geophysical Year 1957–1958 [24], the observers, who were very skilled, had a long and continuous experience and worked in ideal circumstances, obtained a higher accuracy than predicted by the diagrams. The groups of stars had average declinations which corresponded with a standard error to be expected of 6.5 msec, whereas the value actually obtained was only 4.4 msec. This is an exceptional case however.

Another point to be emphasized is that the accuracy dealt with in this paper, is the *internal* accuracy, i.e. the accuracy within a single series of observations of ten stars. The *external* accuracy, as revealed by the discrepancies between series observed at different nights, is much lower; it is not unusual that the external standard error is three to four times the internal one.

A final remark concerns the personal error or personal equation, a constant timing error, which is specific to any observer and whose value may, for some observers, and even if a so-called "impersonal" micrometer is used, be rather high as compared with the internal or external accuracy defined above.

The remedy is to determine this personal error by longitude observations at some reference station or in a laboratory by means of an artificial star, and to take account of the personal error thus obtained in the computations for the station to be determined. If the observations at the reference station or in the laboratory are made just before *and* after those at the station to be determined, the personal error at the latter station is computed by interpolation between the two values obtained.

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DIAGRAMS

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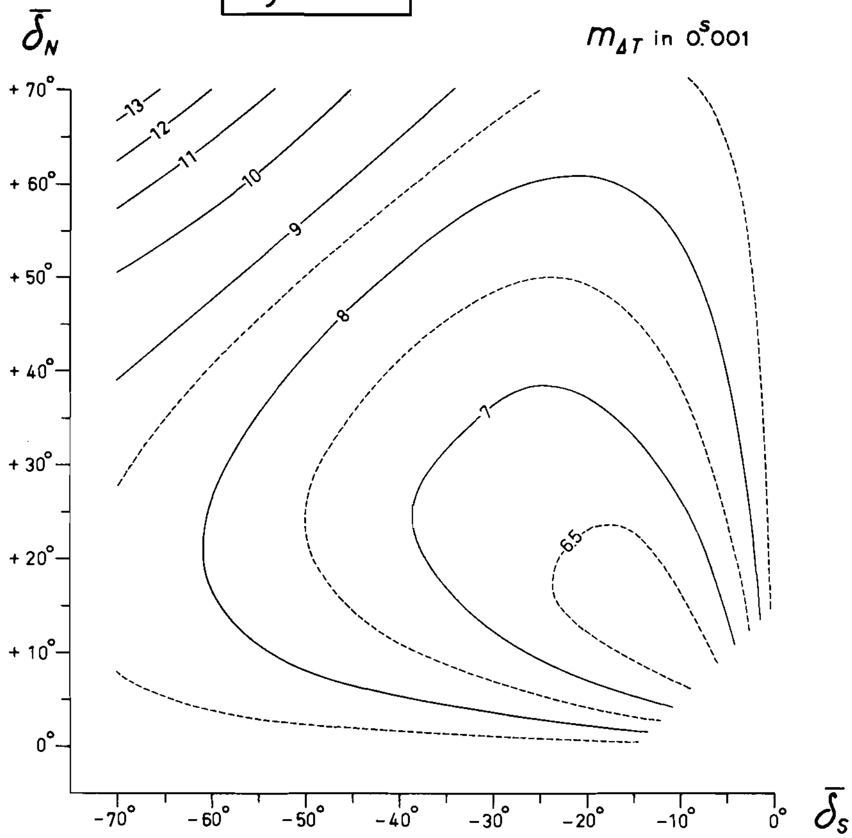
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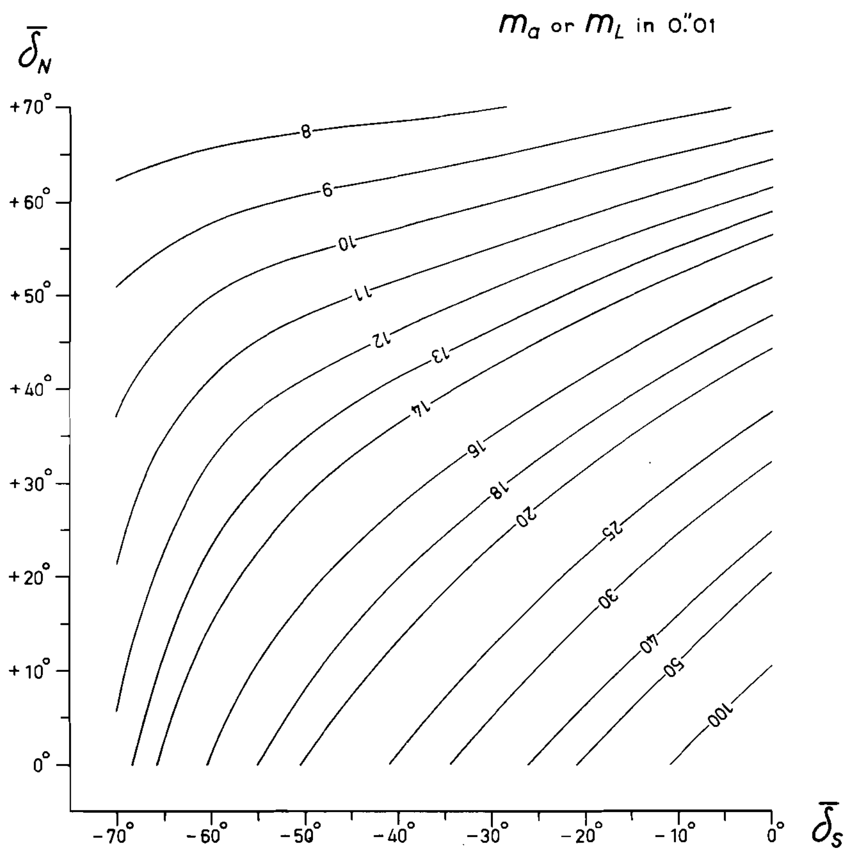
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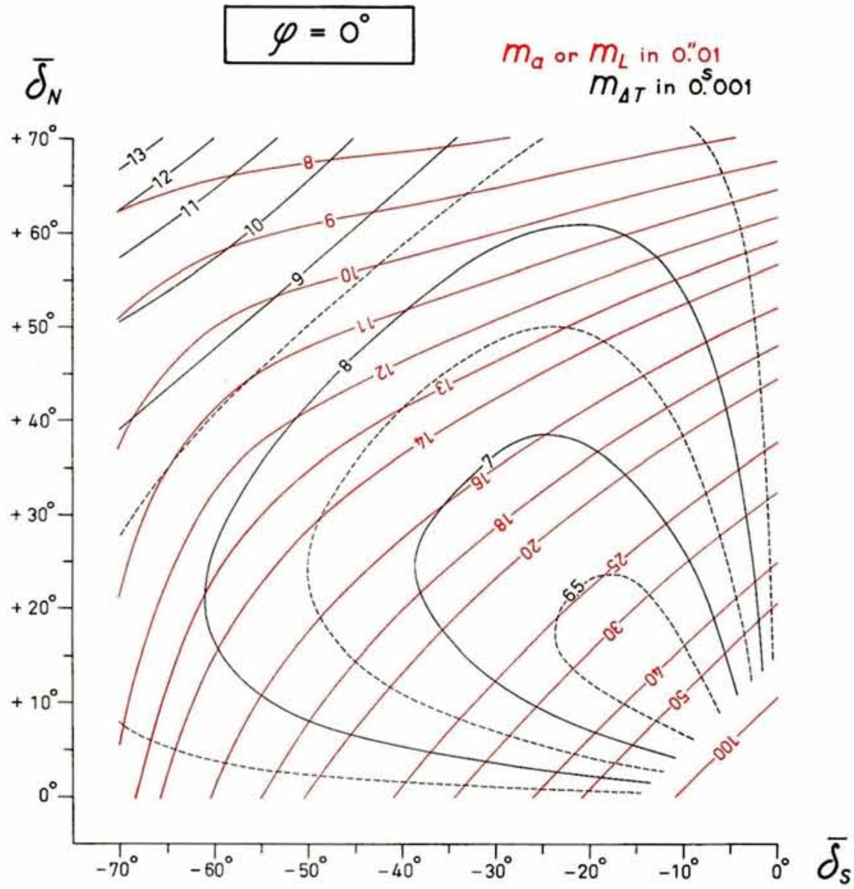
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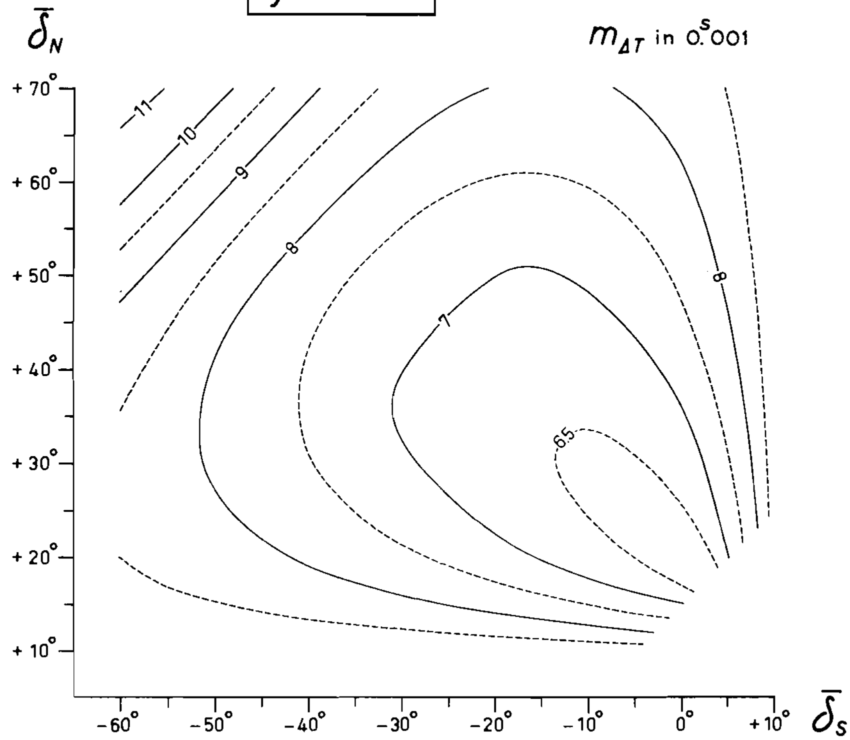
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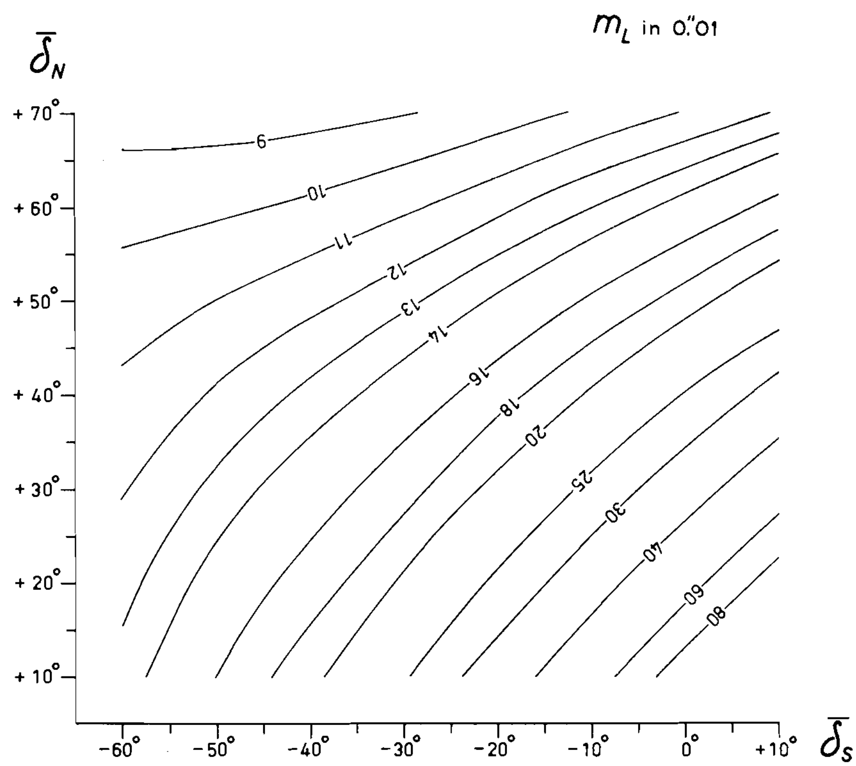
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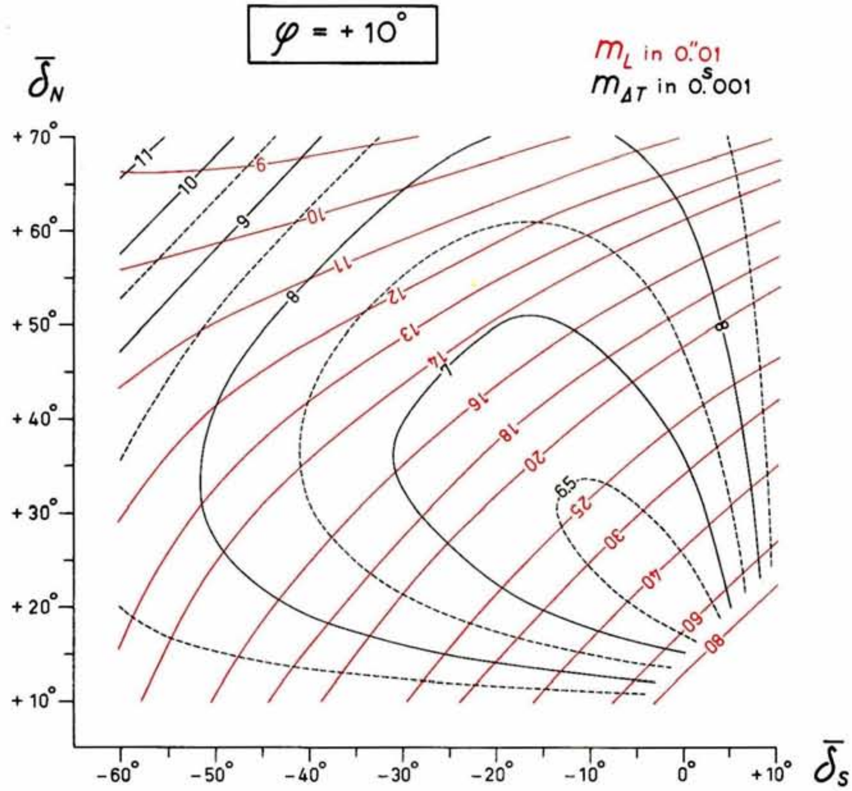
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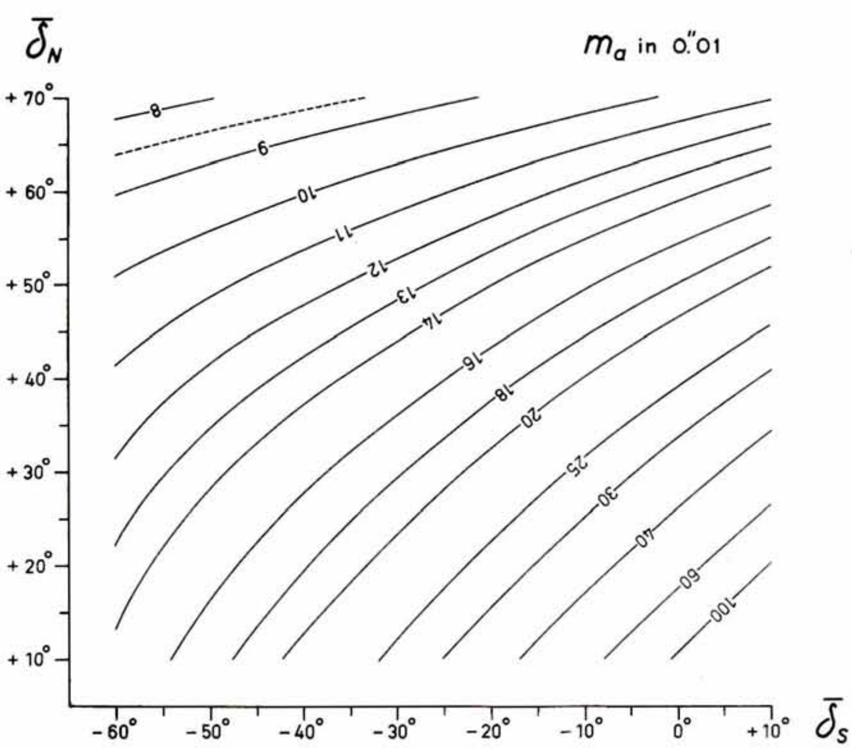
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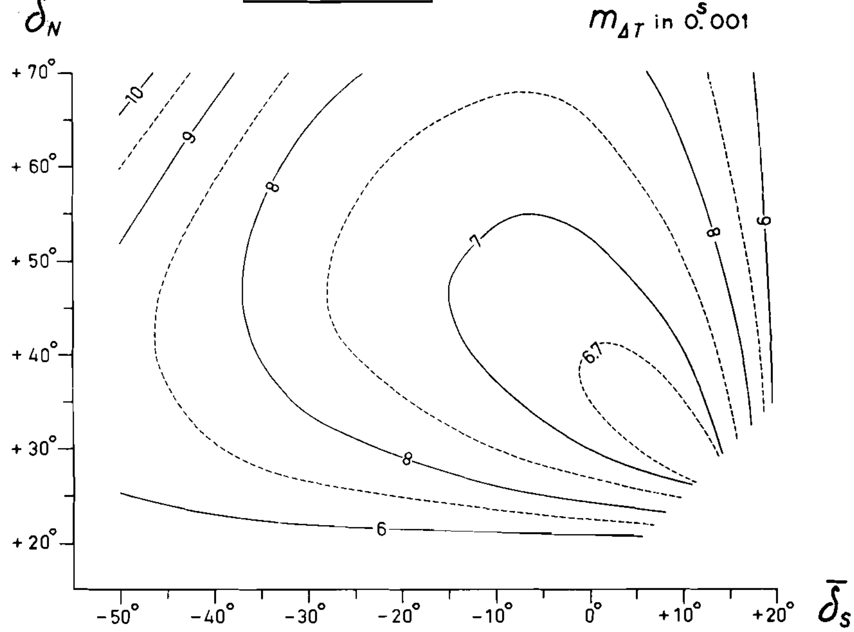
$\varphi = +20°$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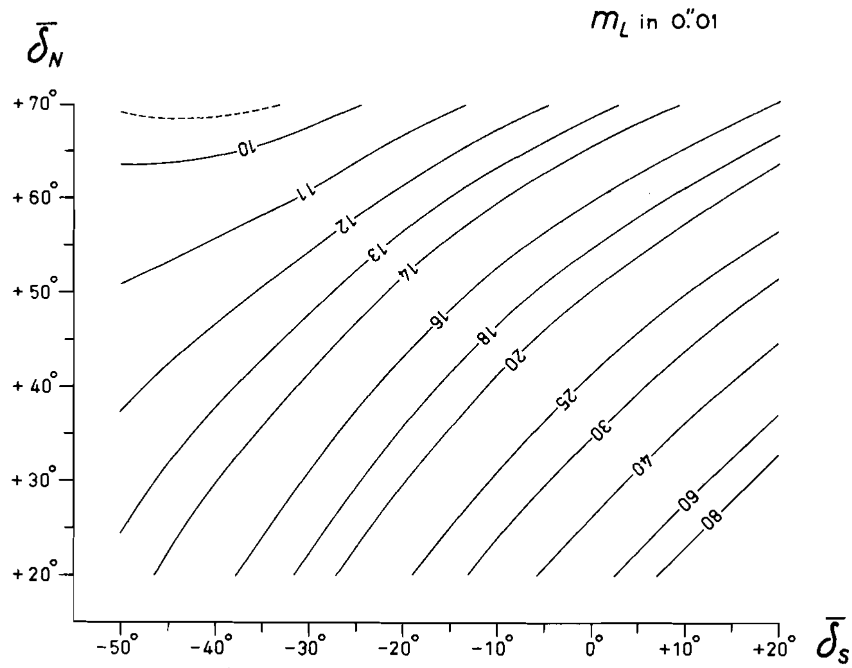
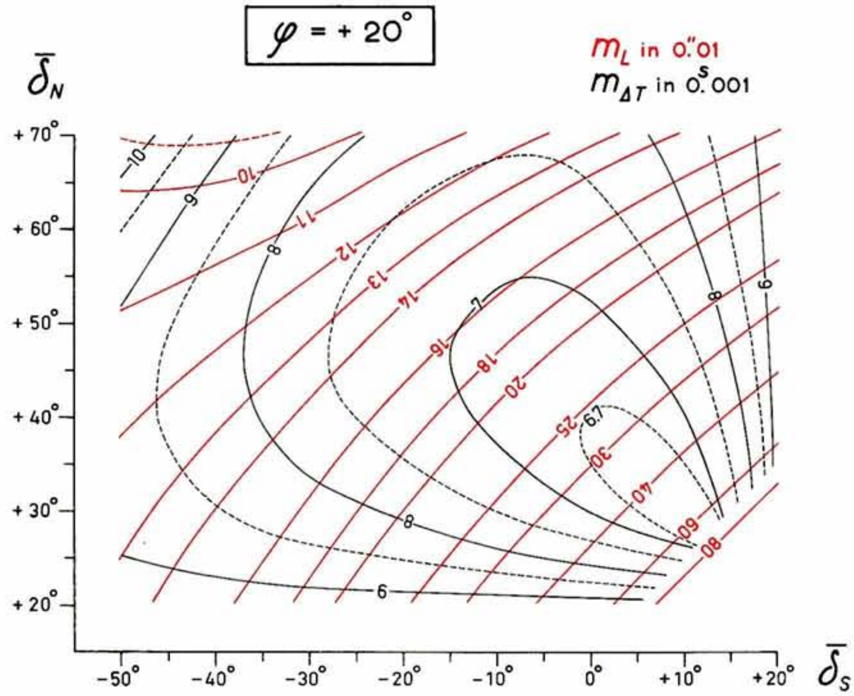
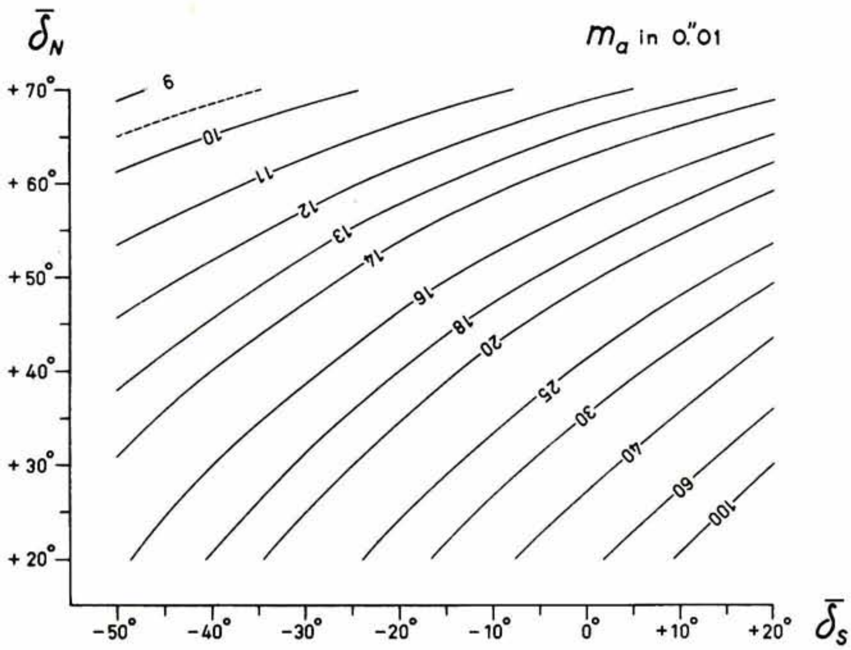
6

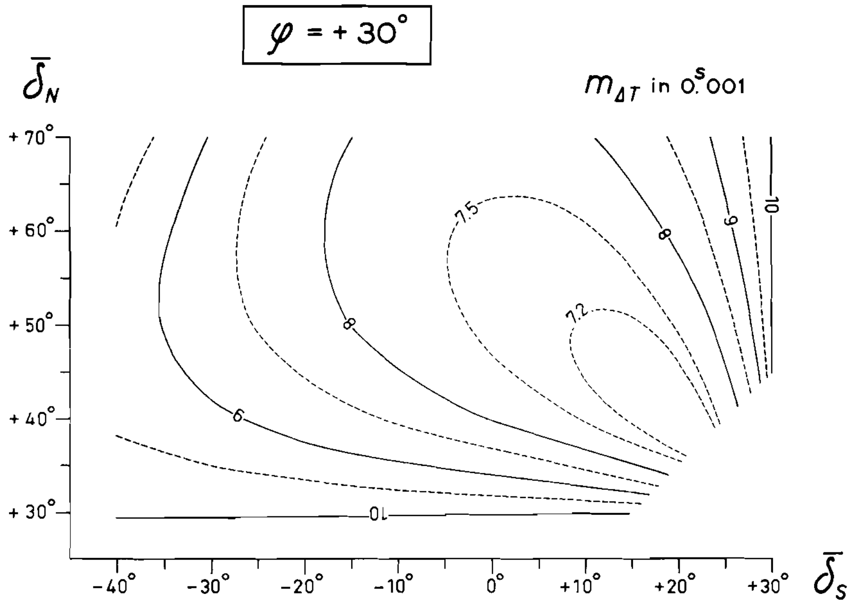

δ̄N
mL in 0.″01
+70°
+60°
+50°
+40°
+30°
+20°
-50°
-40°
-30°
-20°
-10°
0°
+10°
+20°
δ̄S
10
11
12
13
14
16
18
20
25
30
40
60
80



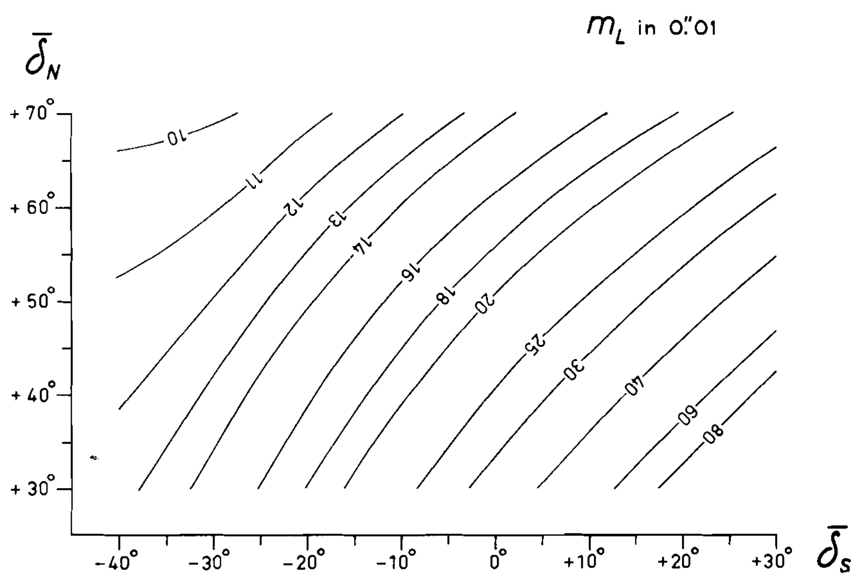
$\varphi = +20°$
m_L in 0."01
$m_{\Delta T}$ in 0.ˢ001
$\bar{\delta}_N$
$\bar{\delta}_S$
+ 70°
+ 60°
+ 50°
+ 40°
+ 30°
+ 20°
−50°
−40°
−30°
−20°
−10°
0°
+10°
+20°



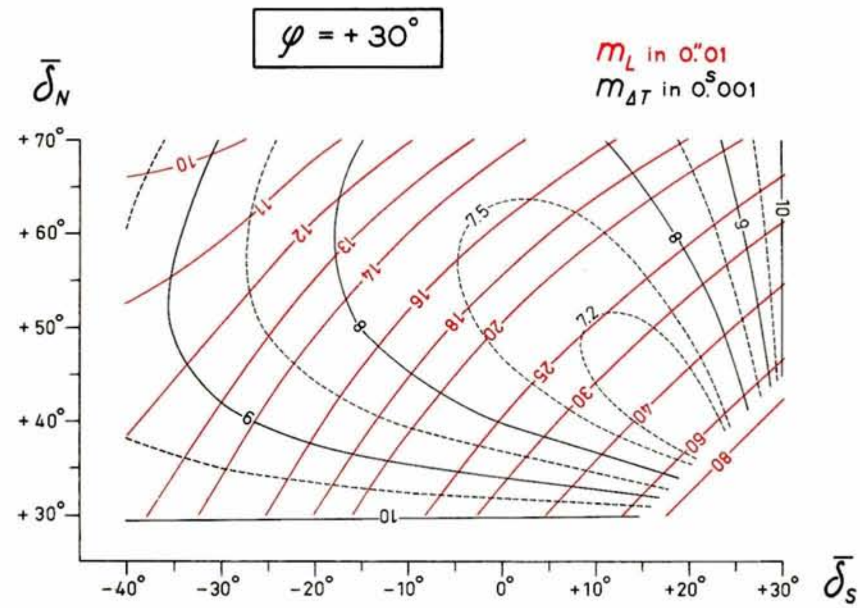
m_a in 0."01
$\bar{\delta}_N$
$\bar{\delta}_S$
+ 70°
+ 60°
+ 50°
+ 40°
+ 30°
+ 20°
−50°
−40°
−30°
−20°
−10°
0°
+10°
+20°

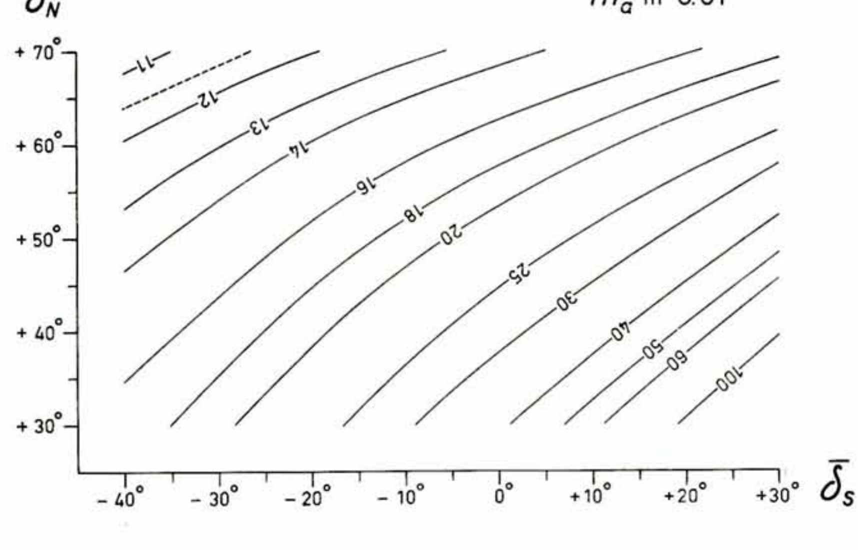


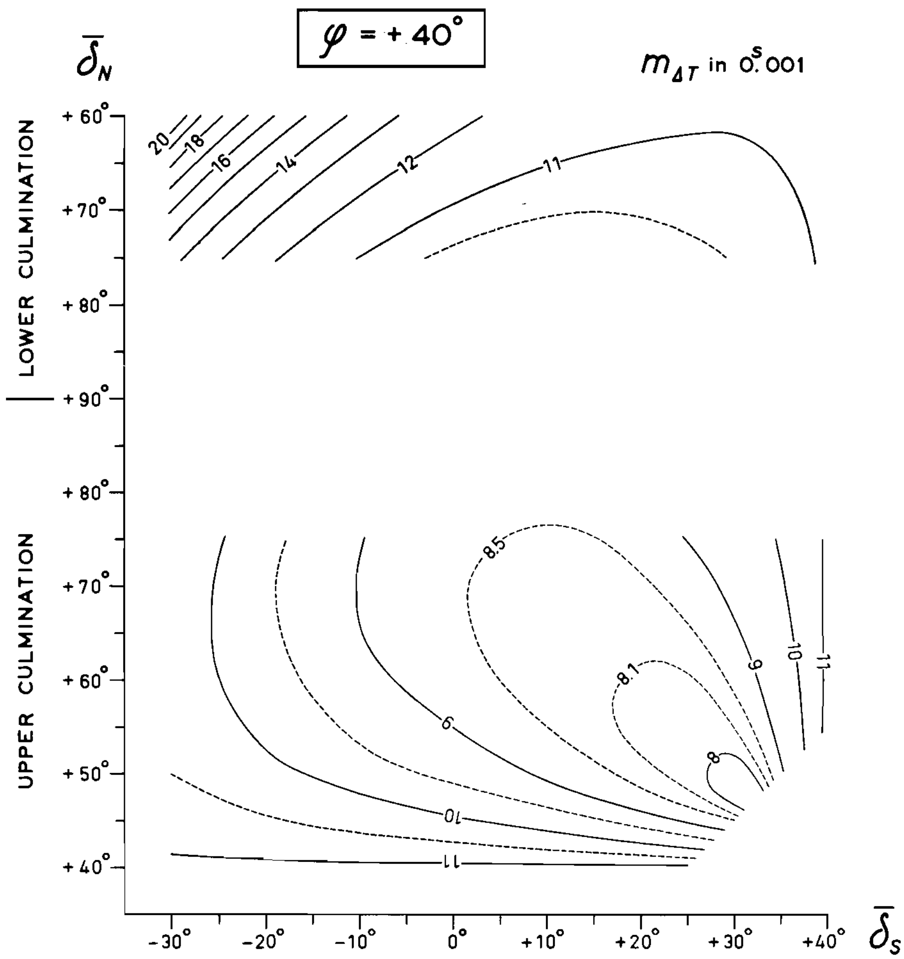
φ = + 30°
$\bar{\delta}_N$
$m_{\Delta T}$ in 0.ˢ001
+ 70°
+ 60°
+ 50°
+ 40°
+ 30°
7.5
7.2
8
9
10
-40°
-30°
-20°
-10°
0°
+10°
+20°
+30°
$\bar{\delta}_S$



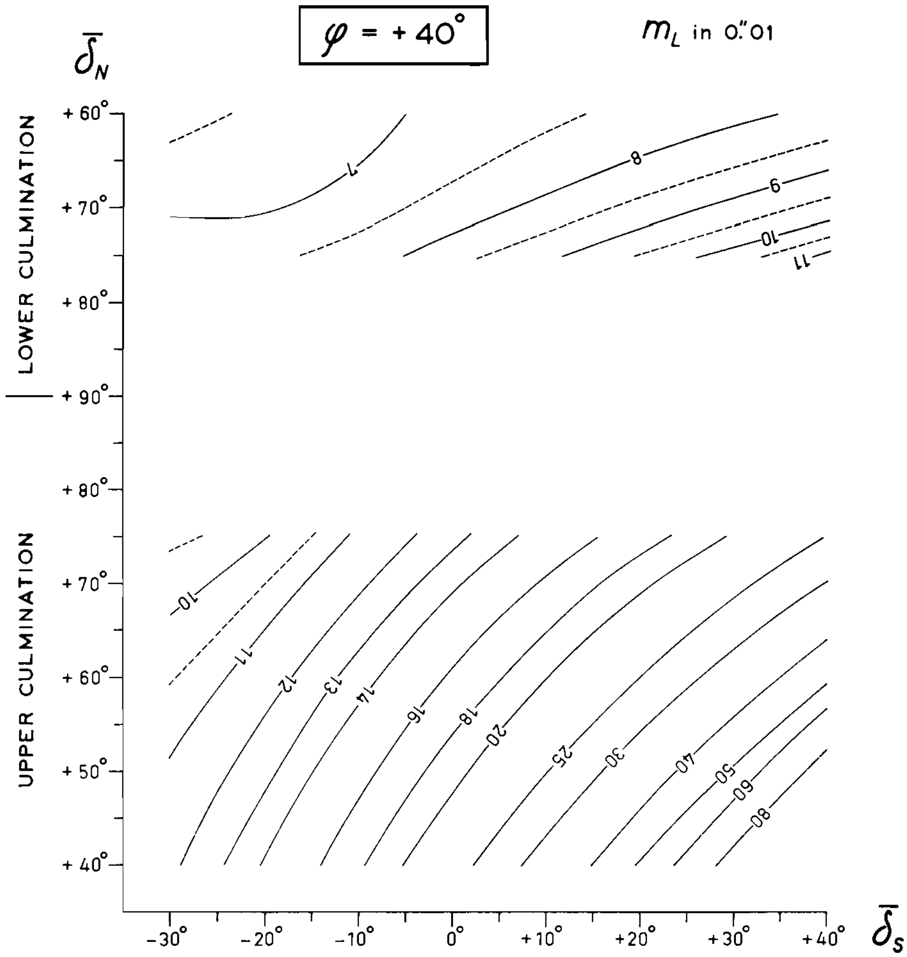
m_L in 0.″01
$\bar{\delta}_N$
+ 70°
+ 60°
+ 50°
+ 40°
+ 30°
10
11
12
13
14
16
18
20
25
30
40
60
80
-40°
-30°
-20°
-10°
0°
+10°
+20°
+30°
$\bar{\delta}_S$

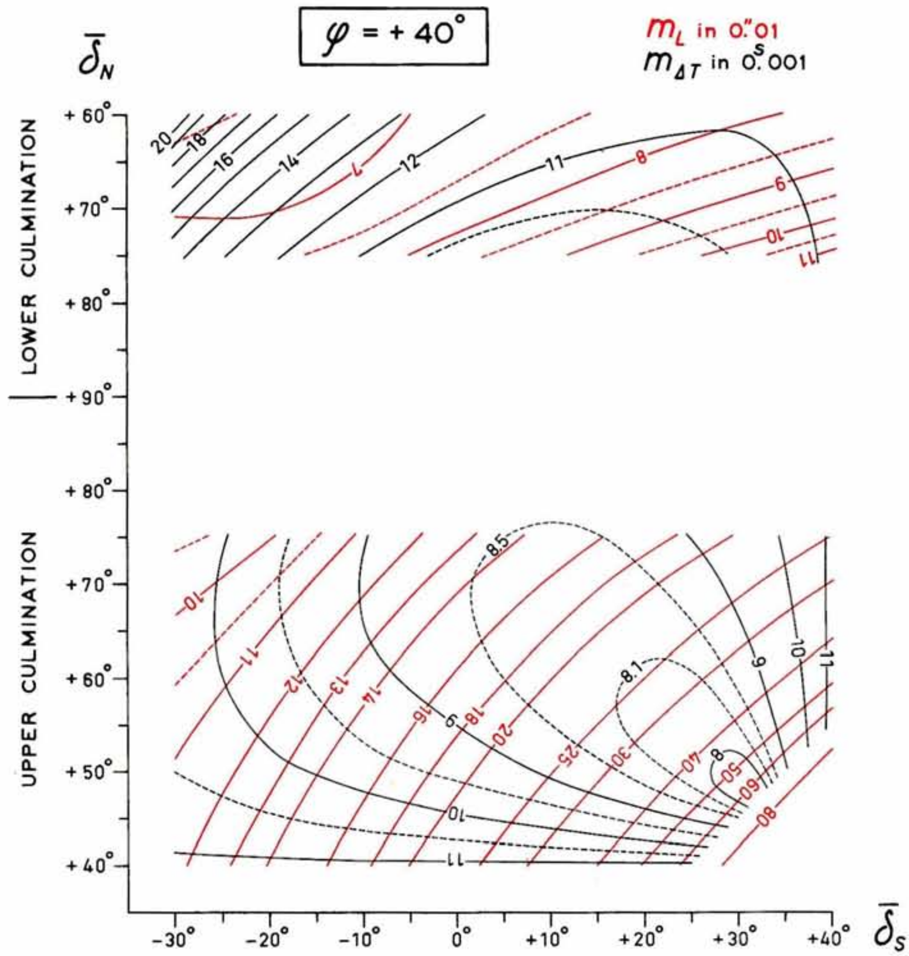

φ = + 30°
m_L in 0."01
$m_{\Delta T}$ in 0.s001
$\bar{\delta}_N$
+70°
+60°
+50°
+40°
+30°
−40°
−30°
−20°
−10°
0°
+10°
+20°
+30°
$\bar{\delta}_S$
10
11
12
13
14
16
18
20
25
30
40
60
80
7.5
7.2
8
9
10


$\bar{\delta}_N$
m_a in 0."01
+70°
+60°
+50°
+40°
+30°
−40°
−30°
−20°
−10°
0°
+10°
+20°
+30°
$\bar{\delta}_S$
11
12
13
14
16
18
20
25
30
40
50
60
100

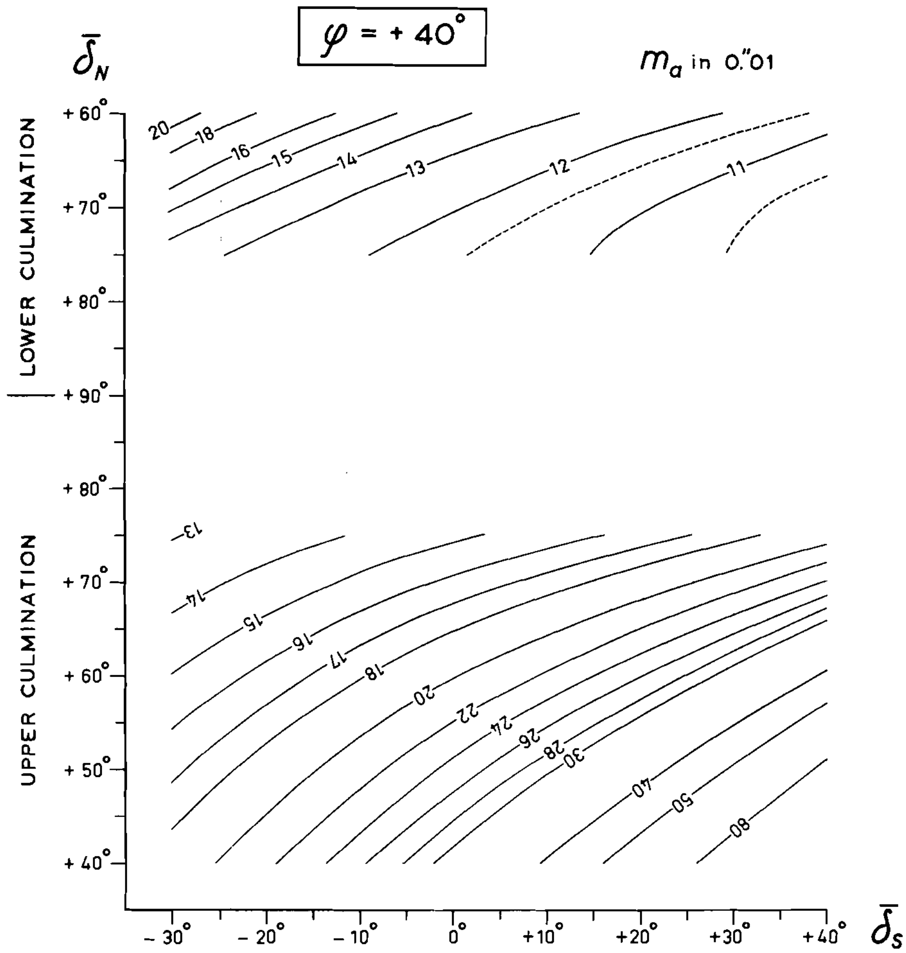


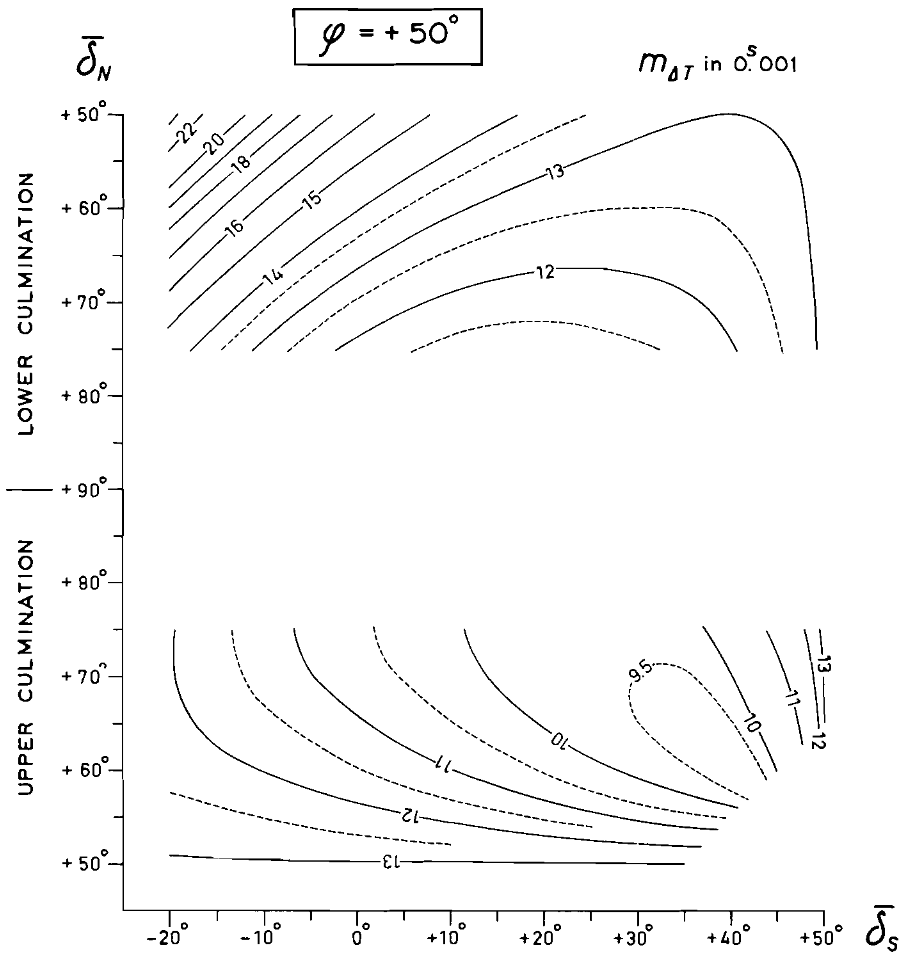
$\varphi = +40°$
$m_{\Delta T}$ in 0.ˢ001
$\bar{\delta}_N$
$\bar{\delta}_S$
LOWER CULMINATION
UPPER CULMINATION
+60°
+70°
+80°
+90°
+80°
+70°
+60°
+50°
+40°
-30°
-20°
-10°
0°
+10°
+20°
+30°
+40°
20
18
16
14
12
11
8.5
8.1
8
9
10
11


φ = +40°
m_L in 0.″01
$\bar{\delta}_N$
LOWER CULMINATION
UPPER CULMINATION
+60°
+70°
+80°
+90°
+80°
+70°
+60°
+50°
+40°
−30°
−20°
−10°
0°
+10°
+20°
+30°
+40°
$\bar{\delta}_S$
7
8
9
10
11
10
11
12
13
14
16
18
20
25
30
40
50
60
80

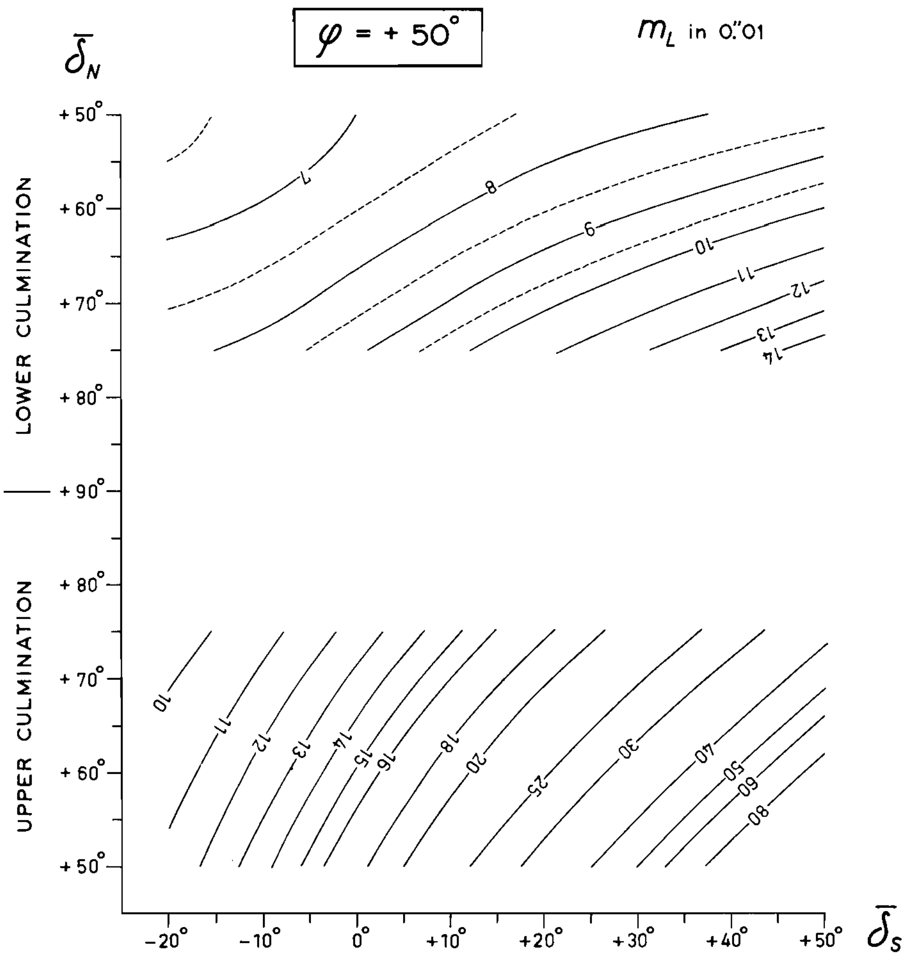


$\varphi = +40^\circ$
m_L in 0.″01
$m_{\Delta T}$ in 0.ˢ001
$\bar{\delta}_N$
LOWER CULMINATION
UPPER CULMINATION
$\bar{\delta}_S$

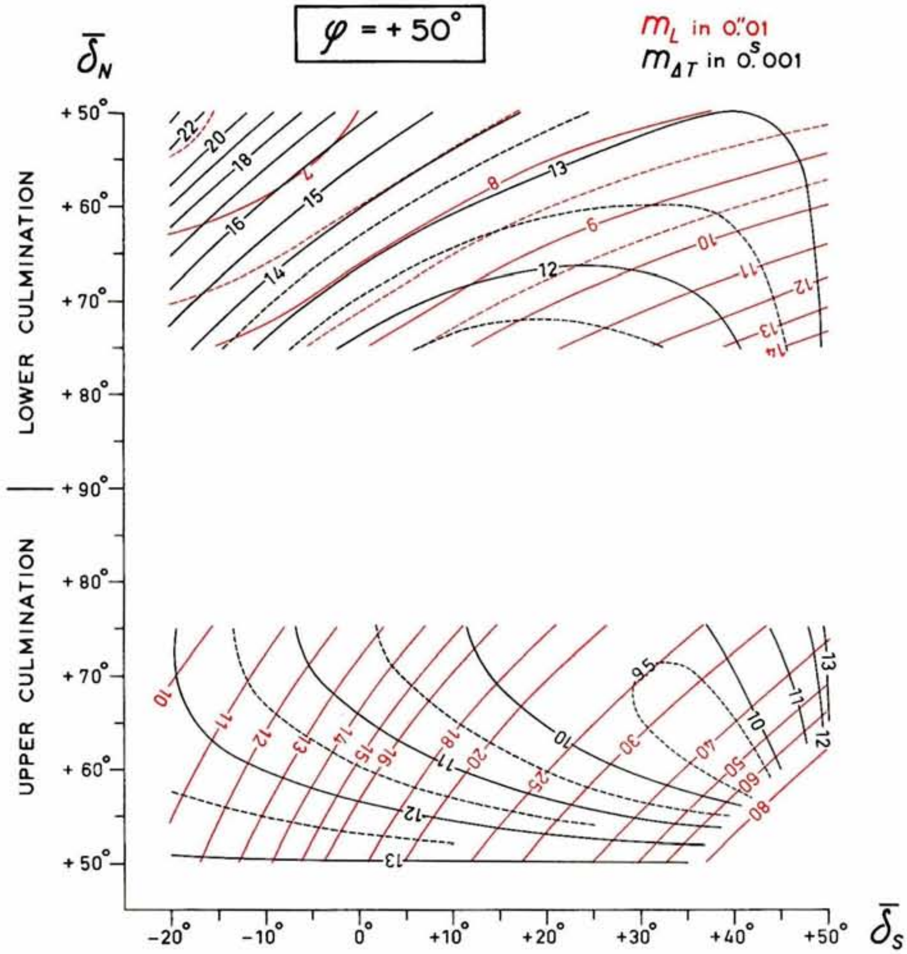

φ = + 40°
$\bar{\delta}_N$
m_a in 0."01
LOWER CULMINATION
UPPER CULMINATION
+ 60°
+ 70°
+ 80°
+ 90°
+ 80°
+ 70°
+ 60°
+ 50°
+ 40°
20
18
16
15
14
13
12
11
13
14
15
16
17
18
20
22
24
26
28
30
40
50
80
– 30°
– 20°
– 10°
0°
+ 10°
+ 20°
+ 30°
+ 40°
$\bar{\delta}_S$



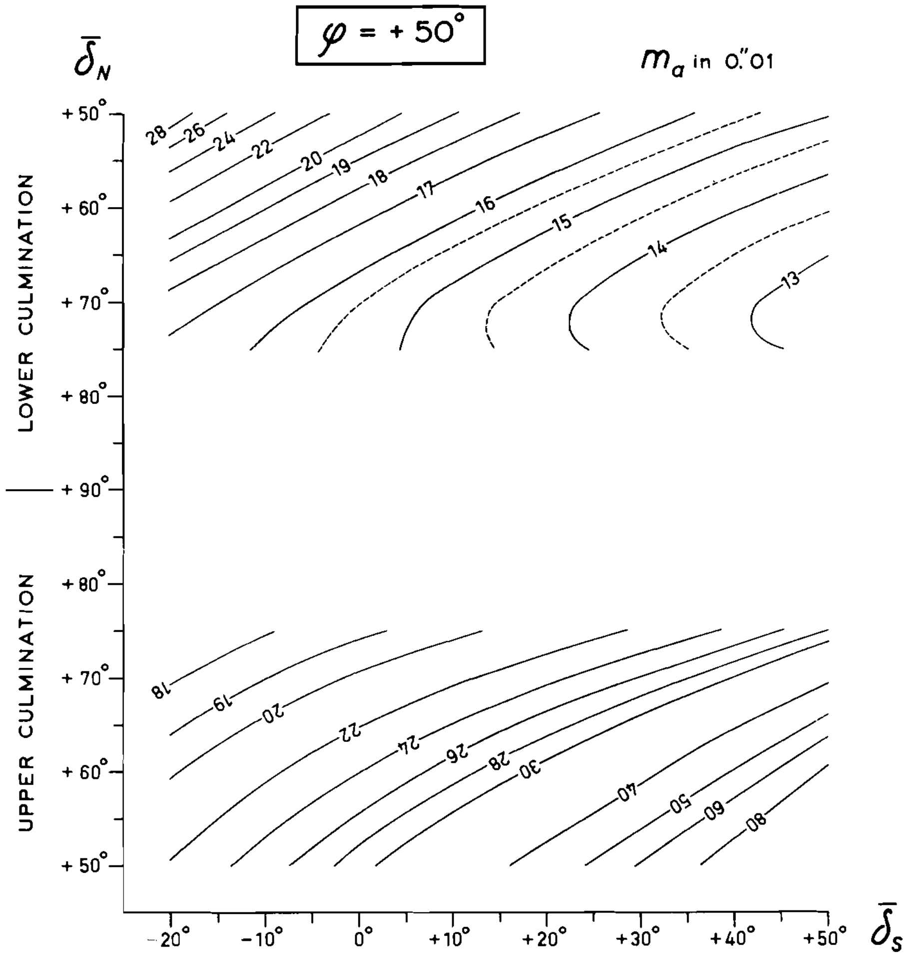
φ = + 50°
$\bar{\delta}_N$
$m_{\Delta T}$ in 0.s001
LOWER CULMINATION
UPPER CULMINATION
+50°
+60°
+70°
+80°
+90°
+80°
+70°
+60°
+50°
22
20
18
16
15
14
13
12
9.5
10
11
12
13
-20°
-10°
0°
+10°
+20°
+30°
+40°
+50°
$\bar{\delta}_S$

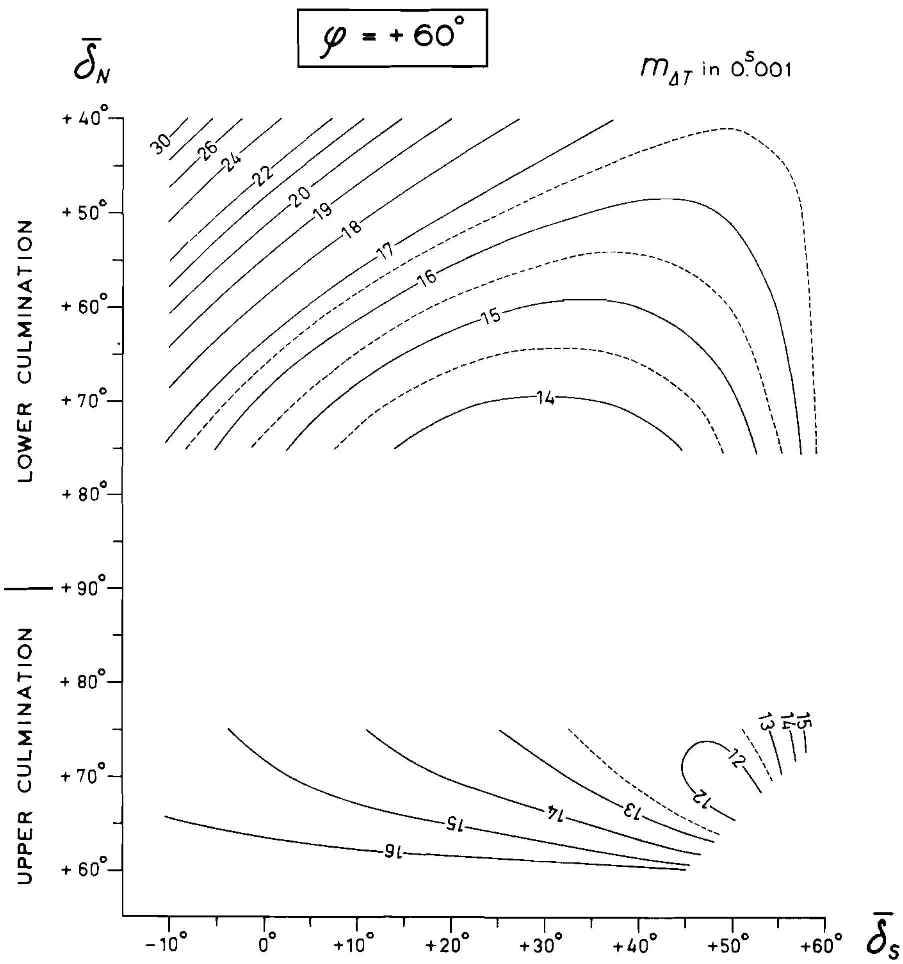


φ = + 50°
m_L in 0."01
δ̄_N
LOWER CULMINATION
UPPER CULMINATION
+ 50°
+ 60°
+ 70°
+ 80°
+ 90°
+ 80°
+ 70°
+ 60°
+ 50°
7
8
9
10
11
12
13
14
10
11
12
13
14
15
16
18
20
25
30
40
50
60
80
−20°
−10°
0°
+10°
+20°
+30°
+40°
+50°
δ̄_S

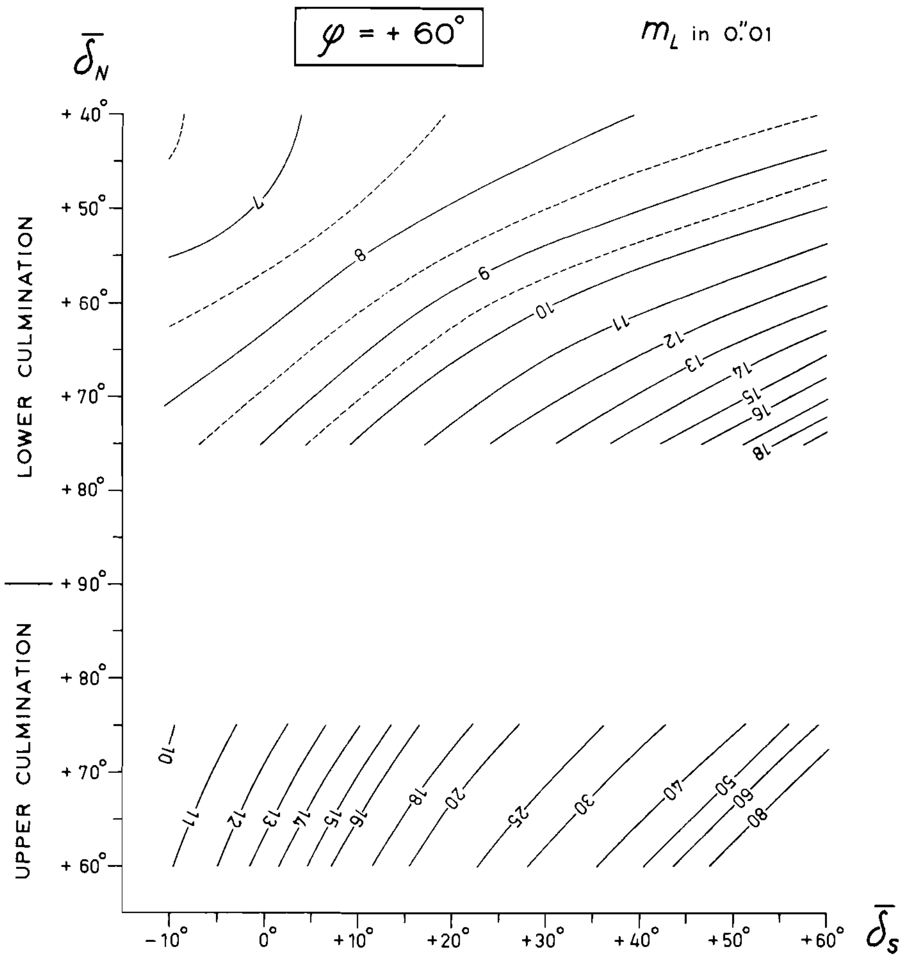


$\varphi = +50°$
m_L in 0.″01
$m_{\Delta T}$ in 0.ˢ001
$\bar{\delta}_N$
$\bar{\delta}_S$
LOWER CULMINATION
UPPER CULMINATION
+50°
+60°
+70°
+80°
+90°
+80°
+70°
+60°
+50°
−20°
−10°
0°
+10°
+20°
+30°
+40°
+50°


φ = + 50°
δ̄N
m_a in 0.″01
LOWER CULMINATION
UPPER CULMINATION
+ 50°
+ 60°
+ 70°
+ 80°
+ 90°
+ 80°
+ 70°
+ 60°
+ 50°
28
26
24
22
20
19
18
17
16
15
14
13
18
19
20
22
24
26
28
30
40
50
60
80
-20°
-10°
0°
+10°
+20°
+30°
+40°
+50°
δ̄S



$\varphi = +60°$
$\bar{\delta}_N$
$m_{\Delta T}$ in $0^s.001$
LOWER CULMINATION
UPPER CULMINATION
+40° +50° +60° +70° +80° +90° +80° +70° +60°
−10° 0° +10° +20° +30° +40° +50° +60°
$\bar{\delta}_S$
30 26 24 22 20 19 18 17 16 15 14
16 15 14 13 12 12 13 14 15



φ = + 60°
m_L in 0.''01
δ_N
LOWER CULMINATION
UPPER CULMINATION
+ 40°
+ 50°
+ 60°
+ 70°
+ 80°
+ 90°
+ 80°
+ 70°
+ 60°
7
8
9
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11
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15
16
18
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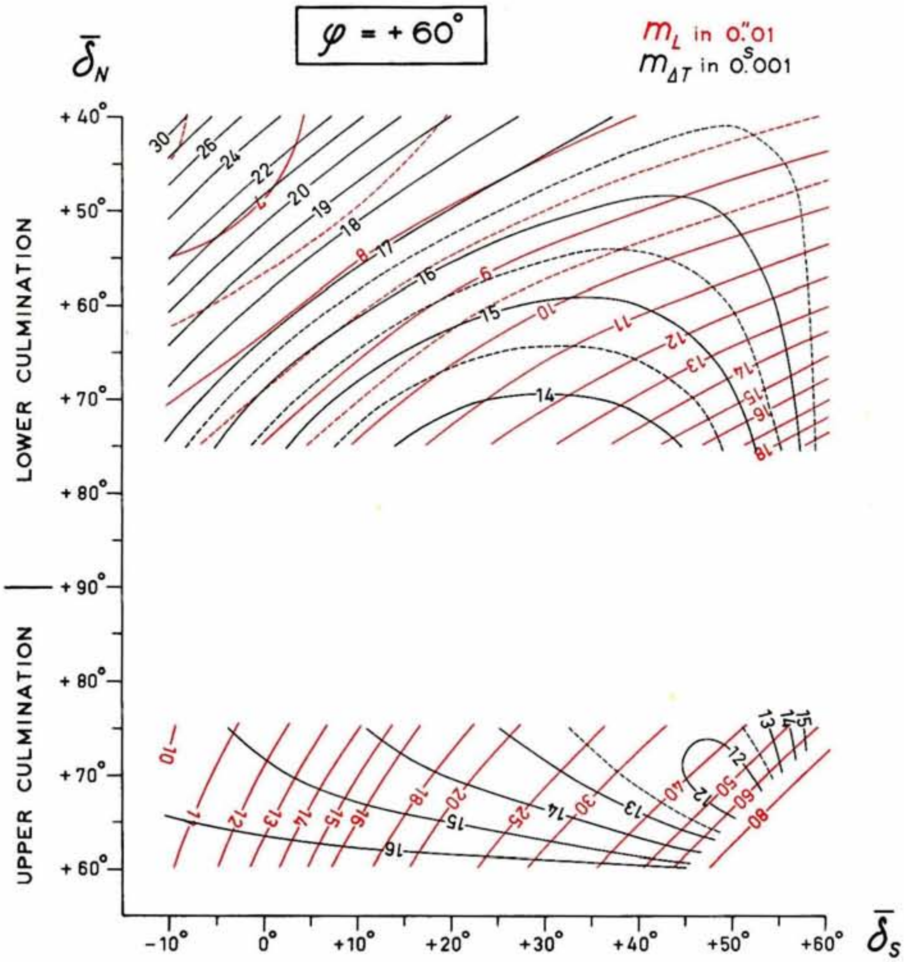
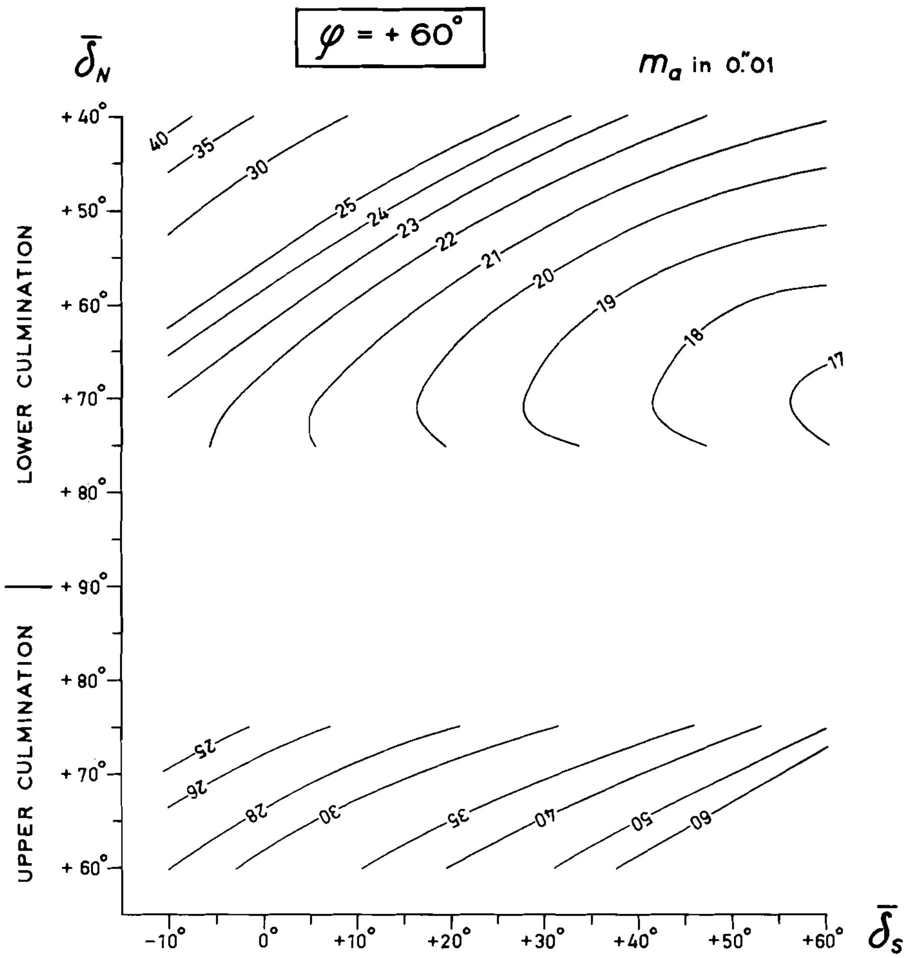
11
12
13
14
15
16
18
20
25
30
40
50
60
80
- 10°
0°
+ 10°
+ 20°
+ 30°
+ 40°
+ 50°
+ 60°
δ_S



$\varphi = +60°$
m_L in 0."01
$m_{\Delta T}$ in 0.^{s}001
$\bar{\delta}_N$
$\bar{\delta}_S$
LOWER CULMINATION
UPPER CULMINATION
+40°
+50°
+60°
+70°
+80°
+90°
+80°
+70°
+60°
-10°
0°
+10°
+20°
+30°
+40°
+50°
+60°



φ = + 60°
$\bar{\delta}_N$
m_a in 0."01
LOWER CULMINATION
UPPER CULMINATION
+40°
+50°
+60°
+70°
+80°
+90°
+80°
+70°
+60°
40
35
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19
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17
25
26
28
30
35
40
50
60
-10°
0°
+10°
+20°
+30°
+40°
+50°
+60°
$\bar{\delta}_S$