INVESTIGATION OF THE ACCURACY OF KRAYENHOFF'S TRIANGULATION (1802-1811) IN BELGIUM, THE NETHERLANDS AND A PART OF NORTH WESTERN GERMANY

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INVESTIGATION OF THE ACCURACY OF KRAYENHOFF'S TRIANGULATION (1802-1811) IN BELGIUM, THE NETHERLANDS AND A PART OF NORTH WESTERN GERMANY

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§ II. ASTRONOMICAL PART OF THE TRIANGULATION

Fig. 1

Cornelis Rudolphus Theodorus Krayenhoff
(1758-1840)

INVESTIGATION OF THE ACCURACY OF KRAYENHOFF'S TRIANGULATION (1802-1811) IN BELGIUM, THE NETHERLANDS AND A PART OF NORTH WESTERN GERMANY

1. Introduction

Krayenhoff's triangulation in a part of Belgium, The Netherlands (with the exception of the province of Limburg), and a part of northwestern Germany, carried out between 1802 and 1811 and published in his Précis Historique [1] was praised to the skies shortly after its completion also because of the appreciative but rash judgments of Delambre [2] and Van Swinden [3] .

The first who, in 1824-1825, in his letters to Schumacher [4] , Bessel [5] , and Olbers [6] criticized Krayenhoff's work was the great German mathematician C. F. Gauss [7] . To Bessel e.g. he writes: "Krayenhoff hat aus vielen Winkelreihen immer nur diejenigen beibehalten die am besten zu passen schienen, ohne anzugeben wieviel die anderen abweichen" [8] , [9] .

In the same strain he writes to his pupil and friend Schumacher: "Entweder muss also Herr Krayenhoff seine Ausgleichungen nicht gehörig gemacht haben oder seine Winkelmessungen involvieren versteckter Weise viel grössere Fehler als man nach der Prüfung durch die Dreiecke und die Gyruswinkel erwarten sollte und im letzten Fall ist man berechtigt zu glauben dass die angegebenen Beobachtungswinkel wenigstens parteiisch gewählt sind um diese Schliessung der einzelnen Dreiecke und Tours d'horizon zu erzwingen" [10] , [9] .

In The Netherlands, Gauss' adverse criticism was borrowed by Verdam; on the pages 206-214 of his "Methode der kleinste quadraten" (Method of the least squares, Groningen, 1850) he reproduces in detail what Gauss had said on the accuracy of the northeastern part of the triangulation network (the surroundings of Drachten, Leeuwarden, and Dokkum). In Jordan's "Handbuch der Vermessungskunde" this same part of the network is discussed [11] .

In 1864 appears, also in The Netherlands, the criticism of Kaiser [12] and Cohen Stuart [13] : "De eischen der medewerking aan de ontworpen graadmeting in Midden Europa voor het Koningrijk der Nederlanden" [14] . As the title: "Requirements for the cooperation of the Kingdom of The Netherlands in the designed Middle European Triangulation" already suggests, the motive for this criticism was a request of the Prussian general Baeyer whether Krayenhoff's observations could be used for such a triangulation.

Baeyer claimed [15] that, if so, the measurements should be recomputed and suggested that, like in other countries, some army officers should be charged with this work under the supervision of Kaiser. The measurements should be completed with new astronomical measurements. The inaccuracy in length between two far distant points in the adjusted network should not exceed the factor 1 to 20,000 [16] . For, such was the reasoning in those days, if the inaccuracy of latitude determination is estimated at about 1/3" (about 10 metres), and the distance between two astronomical stations at about 200 km, then the error in length on account of the astronomical determination is about 1 to 20,000.

Apart from his introduction on the pages 4-16 of the booklet, Kaiser has not collaborated with the investigation laid down in the latter part. As the well-known and elder astronomer he only gave his name to the contents. All the work - and it was a thorough investigation indeed - was done by Cohen Stuart. He concludes that Krayenhoff's measurements should be rejected. His judgment agrees with Gauss' opinion: the far too small closing errors (standard deviation m_1) in the angles around a central point, the also very small closing errors in the sum of the angles of the triangles (standard deviation m_2) and the often considerable closing errors (standard deviation m_3) in the sine equations demonstrate that Krayenhoff made his observations look better than they really are. In reality, according to Cohen Stuart, the standard deviations m_1, m_2, and m_3 should be alike when the observations are independent of each other [17] . By this judgment the sentence on Krayenhoff's triangulation was passed.

After a new but unsuccessful attempt for a triangulation by Stamkart [18] in the years 1865-1881 it has been replaced in The Netherlands by the network of the Rijksdriehoeksmeting. The first order measurements for the network were carried out between 1885 and 1905 by the Rijkscommissie voor Graadmeting en Waterpassing (government commission for Triangulation and Levelling). They have a high accuracy as may be found in the publication of the first order triangulation in "Triangulation des Pays Bas". Thanks to the precautions during the measurements and the system of measuring the angles on a station in all combinations, m_1, m_2, and m_3 are about alike as Cohen Stuart made it already his ideal. These results could be attained by much self-control which made the art of measuring a waiting for the most favourable circumstances. At Finsterwolde, e.g., one of the first order points, the two engineers charged with the measurements, remained for six weeks with the result that not even one angle could be measured [19] . In 1888 at only three stations the measurements could be finished.

Cohen Stuart should have known, however, that even in his time not one triangulation satisfied these conditions and that also his demands do not hold for the very large English triangulation described in "Account of the Principal Triangulation of the Ordnance Survey of Great Brittain and Ireland (London 1858)" .

Twenty five years later - in 1889 - Van der Plaats returned to the subject of Krayenhoff's triangulation in an excellent paper in the Dutch professional journal "Tijdschrift voor Kadaster en Landmeetkunde" [20] . Not, as Gauss and Cohen Stuart did, to condemn the triangulation but to take it under protection because the judgment of its opponents "is partial and based upon wrong principles of justice and wrong considerations" [21] . In an often emotional manner and certainly not free from a theatrical effect he reacts on Cohen Stuart's judgment "that the measurements are far too inaccurate to be used for the new Middle European Triangulation. It is even not possible, neither to judge Krayenhoff's measurements, nor to recompute them as even his registers are not the unchanged results of mutual independent observations" with the words:
"Let us suppose that (what would not have been impossible) this judgment would have been given in 1818. With fervent indignation the then sixty years old general would have answered to the waylayer of his honour: Judge, yes condemn my geodetic work; an honest judgment is welcome to me and I will answer, give information, and correct my work as much as I can. But don't attack my personality. Are you a stranger in the national history of the past twenty years that you think me capable of such a thing. I have concealed nothing in my documents, nothing added to or withheld from the results of the observations. Go and investigate the publications of others; test my work by theirs" [22] . I shall have the opportunity to quote Van der Plaats' work several times. Here follows already such a quotation in which he remarks that "the only effectual means to judge a triangulation is to compare it with a later one with an uncontested higher accuracy" [23] .

Up till now this was never done. This study will be an attempt. It can give an answer to the question whether, according to the requirements of 1864, (a relative length error of 1 to 20,000 between two far distant points in the recomputed network) Krayenhoff's measurements could be used or had to be rejected. Van der Plaats was convinced that they could be used [24] , notwithstanding the imperfection of the triangulation which he admits. For Krayenhoff's rehabilitation as geodesist according to Van der Plaats the following lines of poetry might then be used as an introduction to an eventual new (third) edition of his Précis Historique. They are borrowed from Racine's tragedy Brittannicus (second act, third scene) and they run:

"J'ose dire pourtant que je n'ai mérité
Ni cet excès d'honneur, ni cette indignité". [25] .

2. Krayenhoff's biography

Cornelis Rudolphus Theodorus Krayenhoff was born at Nijmegen on June 2nd, 1758, from the marriage of Cornelis Johannes Krayenhoff (1722-1782) and Clara Jacoba de Man [26, 27, 28] . His father was a military engineer officer who destined his son for a juridical career. Though the young Krayenhoff cared more for his father's profession he submitted to his father's wishes and visited the Latin school in Nijmegen from 1770 till 1776. In 1777, he was matriculated as a student of law in the then university of Harderwijk. He cared, however, more for the study of philosophy and medicine and, with his father's permission, he was matriculated as a philosophical candidate on June 26th, 1779, and as a medical candidate on December 8th, 1783. In 1780 he obtained the degree of doctor of philosophy and in 1784 that of doctor of medicine. He settled as a physician in Amsterdam, where, after some years, he had a flourishing practice. From his marriage to Johanna Geertruida van der Plaat two sons were born, Cornelis Johannes (1788-1865) and Johan (1790-1867).

With his independent character, his vivid mind and his relieved judgment he ranged himself on the side of the Patriots who hated the coarse abuses in the out-of-date Republic. In 1787 he could, just in time, withdraw himself from the defence of Amsterdam against the Prussians. When, however, after its victory, the Orange party appeared to have learned nothing, the Patriots knew that Krayenhoff should not stay behind when the moment would come that the reform-minded people could interfere. That moment came in 1794. After the battle of Fleurus (northeast of Charleroi, Belgium) on June 26th, 1794, when the French armies marched upon the borders of the Republic, the proclamation of the revolution seemed opportune. But the plans trickled out and Krayenhoff and some of his friends had to leave Amsterdam in order to prevent capture. But with the French armies and over the frozen rivers he came back. On January 18th, 1795, he was at the headquarters of his friend Daendels, now general in the French armies at Maarssen, a village between Utrecht and Amsterdam.

Already on the morning of the same day he left for Amsterdam with letters for the military governor and the burgomaster in order to effect that the town would be surrendered without bloodshed. On the same evening he was military commander of Amsterdam and the following morning the civil governing body passed into other hands. At the age of 36 his medical career had come to a sudden stop and a military one began as he had longed for in his youth. It was to last till 1826. The most important commissions to be carried out in those years, besides the triangulation analysed in this paper, were the construction of fortifications and inundations, the levelling along the large rivers and the making of a new map of The Netherlands.

Fig. 1 is one of Krayenhoff's portraits. It was painted by Adriaan de Lelie and it represents the general in civil dress. Next to him is one of his geodetic instruments. The painting is in possession of Mr. Chr. Matthes at Bussum. He was so kind as to make the reproduction for me. He is a great-grandson of Krayenhoff's granddaughter Cornelia Johanna Rudolphina Geertruida Theodora Krayenhoff (daughter of Johan), born May 21st, 1819, at Nijmegen, and died September 8th, 1881, at Baarn. On September 14th, 1843, she married Wouter Karel Willem Matthes, born on March 14th, 1815, in Amsterdam and died June 10th, 1876, in Amsterdam. From this marriage the grandfather of the present Mr. Chr. Matthes, Johan Amile Matthes was born on August 17th, 1846. I owe these details to Mr. Chr. Matthes to whom I am very grateful for his information and for his kind permission to reproduce the portrait.

In 1806 Krayenhoff was aide de camp of king Louis Napoléon who appointed him minister of war in 1809. In 1810, after the incorporation of the kingdom Holland in the French Empire, he asked to be dismissed from his military functions but this was refused three times. The emperor Napoleon appointed him even brigade general of the French Engineers. On November 19th, 1813, two days after The Hague had declared in favour of the return of Orange, he sent in his resignation from all his civil and military functions and on November 24th, at the request of the Commissary-general of the governing body at The Hague, he charged himself with the defence of Amsterdam. King Willem I overloaded him with honour, appointed him lieutenant-general, inspector general of the fortifications in 1814, and raised him to the peerage with the title of baron in 1815.

In 1824 malversations committed by engineer officers under his command came to light. Krayenhoff was held responsible for these malversations and summoned for the high military court of justice on May 10th, 1826. Some months later he was suspended from his functions. The juridical investigation ended April 28th, 1830, with acquittal.

Krayenhoff died, 82 years old, in his native town Nijmegen, on November 24th, 1840. He was buried there in the fort that is named after him. In the year after his death king Willem I did erect a simple monument on the grave. On the gravestone are the words:

"Een man van standvastigheid, beleid en heldenmoed, van ware Bataafsche trouw, mannelijke ervarenheid en eindeloozen arbeid; die door eigene verdiensten tot de hoogste militaire en staatswaardigheden opgeklommen, zich eenen roemrijken naam heeft verworven, boven alle wangunst verheven".

The English translation of the Dutch text runs as follows:

"A man of steadfastness, prudence and heroism, of true Batavian loyalty, manly experience and endless labour, who by own merits risen to the highest military and civil positions, obtained an illustrious name, raised above all jealousy".

3. The motive for the triangulation

On the motive for the triangulation Krayenhoff informs us in detail on the pages 1-7 of his Précis Historique. In 1798, the National Convention solemny declared "that the Batavian Republic should be one and indivisible". Thereafter the first Chamber of the Legislative Assembly appointed a commission charged with the task to divide the territory of the Republic into departments, arrondissements and municipalities. There were, however, no maps available on a convenient scale on which the projected borders could be marked. On October 10th of the same year, Krayenhoff was charged with the making of this map.

His first efforts failed when he tried to join existing maps on different scales to one map on the scale 1 to 115,200 (one Rhineland inch to 800 Rhineland roods = 1 to 800x12x12). He was rightly convinced that the rather serious errors in the mutual distances on such a map could only be rectified if a trigonometric network would cover the territory. In February, 1800, a beginning was made with the measurement of this network. The part of the Zuiderzee (zee=sea) between Volendam and the isle of Marken, about 20 km northeast of Amsterdam, was then frozen over. Krayenhoff made use of these circumstances by measuring a base line over the ice with a length of 1500 Rijnlandse roeden (Rhineland roods) $\simeq$ 5650 metres [29]*. The measurement was done with a surveyor's chain in two opposite directions. After that he measured with a sextant in each of the two terminals of the base line the angles between the other base line point and the towers which could be seen from there. From these observations he computed

* As the sign $\approx$ for "approximately equal to" was not available on the typewriter used, it has been replaced by $\simeq$.

the length of the side Amsterdam (Western tower) - Haarlem (St. Bavo church). The result was 4457.9 roods, about 16,788.5 metres, a very good result indeed, if the primitive sextant measurements from which it was computed are taken into consideration. The exact distance is 16,790.4 metres.

From the length of this new base line Amsterdam-Haarlem and the angles measured with the sextant in other points of his planned trigonometric network other distances could be computed. In this manner the triangulation proceeded and the construction of the map kept step with the progress of the measurements and the computation of the network.

In November, 1800, he showed his results to professor Van Swinden, who regretted that Krayenhoff had not availed himself of the opportunity "of measuring a triangulation network as perfect as performed lately in France for the determination of a part of the arc of the meridian" [30] . As he had computed it, Van Swinden knew all about this triangulation. Krayenhoff saw the importance of Van Swinden's remarks. "They did me see with a certain aversion the imperfections in my previous work and I wished to begin anew. I saw with regret that such an imperfect execution would give an unfavourable impression of the state of science in Holland and would compromise the fame of our nation in the domain of mathematics, astronomy, and geography" [30] .

Though it was difficult to convince the commission, Krayenhoff at last got permission to execute the triangulation as he had proposed. The measurements would be carried out with a large repetition circle (cercle répétiteur) made by Lenoir in Paris at the expenses of the government. He began his observations in the autumn of 1801, at Zierikzee, Bergen op Zoom, Antwerpen (Antwerp), and Hoogstraten (see Fig. 2 in section 4), the most northern stations of an already existing triangulation network between Duinkerken (Dunkirk) and Zierikzee. It had been measured in 1795, by order of the French government by the French astronomer J. Perny de Villeneuve. Krayenhoff hoped that he could build on this triangulation. But provisional computations during the winter of 1801-1802 gave such great differences that Perny's measurements had to be rejected [31] . And so Krayenhoff had to start again, now for the third and last time.

I. GEODETIC PART OF THE TRIANGULATION

4. General survey of the triangulation

The triangulation network is represented in Fig. 2. It extends from the side Duinkerken (Dunkirk) - Mont Cassel in the southwest to the side Jever-Varel in the northeast. It connects the French triangulation between Duinkerken and Barcelona of Méchain and Delambre with the triangulation in the northwestern part of the present Western Germany, executed during the French occupation in the Napoleonic era by the French lieutenant-colonel Epailly. A map of the area of about 1800 serves as underground. For the determination of the form of the network, 505 angles have been measured. With their numbers they are marked on the map with arcs and, in some cases, with double arcs. The network consists of 161 numbered triangles. No. 1 was already measured by Delambre. In general their shape is very good and the sides are neither too long nor too short. The shortest side is Nijmegen-Biesselt (9.6 km) in triangle 53 and the longest is Gent-Antwerpen (50.7 km) in triangle 15. The sides Hulst-Zierikzee (42.2 km, triangles 16 and 17), Hoogstraten-Lommel (43.1 km, triangles 21 and 27), Gorinchem-Rhenen (43.1 km, triangles 49 and 50) and Lemelerberg-Beilen (43.5 km, triangles 103 and 104) are also longer than 40 km. The southern part of the network (triangles 2-10) is a chain. From the side Aardenburg-Gent in triangle 10 it passes into a triangulation network. The former Zuiderzee (zee=sea) within the so called Zuiderzee pentagon Urk-Harderwijk-Naarden-Edam-Enkhuizen made the construction of triangles there impossible (Harderwijk-Enkhuizen $\simeq$ 45.2 km, Harderwijk-Edam $\simeq$42.8 km, Naarden-Enkhuizen $\simeq$46.3 km and Naarden-Urk $\simeq$ 50.2 km).-

If we leave the triangles 79, 129, 156, 162, and 163 with apexes Petten, Schiermonnikoog, Aschendorf, Stolham, and Wangeroge, respectively, out of consideration, then the network consists of 106 angular points. In three of them, as can be seen in Fig. 2, no measurements took place (Herentals No. 104, Biesselt No. 105, and Borkum No. 106). The others are indicated by the same sequence number as they have in tableau I of the Précis Historique. The Western tower in Amsterdam, e.g., is station No. 40, the Weighhouse steeple at Alkmaar is station No. 52, etc. Nearly all the stations were church towers. In some cases, however, Krayenhoff was obliged to build his observation towers with a signal serving as a sighting point. It were the stations Kijkduin (No. 65), Imbosch (No. 43), Hettenheuvel (No. 44), Harikerberg (No. 50), Lemelerberg (No. 60), and Uelsen (No. 63). They were of a simple construction. After the measurements the place of the signal was marked by a long pole driven deep into the ground (Kijkduin) or by a large stone of about 7.5x1x1 (Paris) feet (2.5x0.3x 0.3m). They were placed in such a way that they stuck about one foot out above the ground. On the upper side of the stone the number of the station was carved and the date (Imbosch, Hettenheuvel, Harikerberg, Lemelerberg, and Uelsen). Already during Krayenhoff's life the pole in Kijkduin and the stone in Imbosch were lost.

On the church towers at Harderwijk (No. 46), Rotterdam (No. 28), and Strakholt (No. 99) and on the tower of the castle at Bentheim ("the tower formerly used as powder magazine") (No. 62) he used signals consisting of rather long fir trees with in the top a horizontal cross with great baskets in order to make them better visible.

On the Veluwe, in Krayenhoff's time still an uncultivated area with high trees behind which the few church towers very often disappeared, a high observation tower was necessary (75 feet $\simeq$ 24 m). Notwithstanding the measures taken even the tower shook by a light wind. Measurements could therefore only be done in calm weather. The centre of this observation station - Krayenhoff called it Observatoire (No. 47) - was also marked by a stone of about 2.5x0.3x0.3 m with the inscription "Observatoire 1805". The station was situated in the neighbourhood of the present palace "Het Loo". In 1875 the stone was also used as a triangulation point for Stamkart's unsuccessful triangulation. According to his diary volume II - the diary is in the archives of the Netherlands Geodetic Commission at Delft - the stone was dug up on June 8th, 1875. It had an inclined position and it was heavily damaged. Stamkart placed it anew in a vertical position and surrounded it with some brickwork. During the reconnaissance of the first order triangulation of the R(ijks) D(riehoeksmeting) in 1889 the stone was still present in this position and, as the point "Veluwe", used as a first order triangulation point. Notwithstanding the manipulations with the stone in 1875 I assumed in section 21 the R.D.-point to be identical with Krayenhoff's Observatoire.

A very special observation point was Robbezand (No. 72) on the sand bank of the same name in the Waddenzee (Dutch shallows) at a distance of about 18.6 km from Oosterend (No. 71) in the isle of Texel, 17.3 km from Oosterland (No. 66) in the then isle of Wieringen, 24.6 km from Staveren (No. 67), 26.9 km from the lighthouse in the isle of Vlieland (No. 73), and 21.6 km from Harlingen (No. 74). Here too the sighting point was a fir tree with a horinzontal cross with baskets. Moreover the fir tree was surrounded by straw packs in the shape of a pyramid because of the long distances over which had to be pointed. It was also marked with a block of stone of about 1.9x1.3x0.3m on the sand bank. Next to the sighting point an observation platform was built. The depth of the water on the spot ranged from about 8 feet (2.5 m) at low tide to 12 feet (4 m) at high tide. For the observations here too had to be waited for quiet weather as the beating of the waves seriously influenced the accuracy of the observations.

Krayenhoff seems to have spent little time on the reconnaissance of his network and he did not change almost anything in the original plan of his measurements. His great knowledge of the terrain obtained during the measurements for the planned map 1 to 115,200 (see section 3) testifies this. Only by way of exception he visited observation stations which afterwards appeared to be unfit for the purpose. The original station Westerland in Wieringen e.g. was replaced by Oosterland (No. 66) and Neuenhaus by Uelsen (No. 63). Sometimes, e.g. at Leeuwarden (No. 79) and Dokkum (No. 80), some angles measured appeared to be superfluous later on. In general, however, the reconnaissance succeeded very well and the shape of the network is good. Only once - at Haarlem during the measurement of the angle 183 between Alkmaar and Amsterdam - he made a mistake by pointing at the spire of the Laurens church at Alkmaar instead of at the Weighhouse steeple. Later on he corrected this mistake [32].

The weakest part of the network is the chain of triangles in the present Belgium and by this chain the length of the side Duinkerken-Mont Cassel of Delambre's triangulation had to be transmitted to the northern Netherlands. Especially the shape of the triangles 3, 4, 5, and 8 is unfavourable. Some improvement and a welcome check might have been obtained if Krayenhoff had measured some diagonal directions, e.g. the side Duinkerken-Diksmuide in the quadrilateral Duinkerken-Nieuwpoort-Diksmuide-Hondschoote. It might have been better if he had extended his network in Belgium by some triangles south of the chain. I marked them in dotted lines on the sketch in Fig. 2. Van der Plaats already made the same remark [33].

In excuse of his omission, however, can be said that in 1803 Krayenhoff had still the intention to measure a base line of 500 toises (10 km) in the neighbourhood of the villages St. Jacobi Parochie and Vrouwenparochie, situated in triangle 121. Such a base line would have met partly the objections to the chain. In a letter to Freiherr von Zach dated November 30th, 1803, he writes about this intention [34]. At the time, however, that the measurement of the base line had to be done Krayenhoff, according to Van der Plaats [35], was on too bad terms with the French Government to ask with some chance of success for borrowing the base line measurement apparatus. Later on he judged the measurement of the base line no longer necessary as he says in [36] : "This distance (the side Duinkerken-Mont Cassel) has been determined with so much care and accuracy from more than one base line, that we judged it unnecessary to design a special base line for the Dutch triangulation network".

Determinations of latitudes in Amsterdam (No. 40) and at Jever (No. 102) (Duinkerken-Amsterdam $\simeq$ 229 km, Amsterdam-Jever $\simeq$ 243 km) and the measurements of the astronomical azimuths Amsterdam-Utrecht and Jever-Varel form the astronomical part of the triangulation. Some unsuccessful attempts in 1801-1803 were left out of consideration.

It was in 1839 an excellent idea of the then 81 years old general to present to Leiden university the original observations and computations of the several parts of the triangulation. They are kept there as No. 241 in the rubric Binnenlandse Handschriften (Interior Manuscripts). Thanks to the kind collaboration of the Leiden librarian I was able to study them amply in the library of the Delft University of Technology.

If one excludes two volumes with secondary observations the collection consists of:

a. 9 volumes octavo, numbered I up to and including IX. They contain the observations of the angles of the triangulation network,
b. 2 volumes octavo, numbered X and XI with astronomical observations (determinations of latitudes and azimuths),
c. 2 volumes folio with the reduction of the measured angles to centre, horizon, and chords,
d. 1 volume folio with the computation of the provisional and the final lengths of the sides of the network,
e. 1 volume folio with the computations of the latitudes and azimuths,
f. 1 volume folio with the computation of the geographical coordinates of all the points of the triangulation network and the azimuths of all the sides.

All of them are in the French language. Copies, also in French, are at the Dépot de la Guerre de France in Paris and at the Koninklijke Akademie van Wetenschappen (Royal Academy of Sciences) in Amsterdam. A copy in the Dutch language is at the Topografische Dienst (Topographic Institute) at Delft [37] .

5. Description of the instruments used

In his Précis Historique Krayenhoff informs us in detail on the instruments used for the measurement of the angles of his triangulation network. The instrument used in the years 1802, 1803, 1805, and 1807 was a repetition circle (cercle répétiteur), designed by Borda, and, by order of Van Swinden, made by Lenoir in Paris at the expense of the government. It had a circle with a diameter of 16 inches (about 43 cm), two telescopes, four verniers, two by two perpendicular to each other, and two levels. As the name already says one measured the multiple of an angle with it. It seems that still in 1970 it was unknown whether it was lost or preserved. Van der Plaats "does not know where it is and whether it still exists" [38] . In his thesis Triangulaties in Nederland na 1800 (Triangulations in The Netherlands after 1800) [39] Moor can't tell anything more about the instrument than Van der Plaats already did and Van der Schraaf's historical publication [28] only says that Krayenhoff used a repetition circle for his measurements. In the collection of old geodetic instruments of the Sub-Department of Geodesy of the Delft University of Technology, however, is a repetition circle which exactly satisfies the description which Krayenhoff gives of the instrument. It has a circle with a diameter of 433 mm, two telescopes, two levels, four verniers, and the name Lenoir is engraved on it. Because of constructive reasons and just as for Delambre's instruments the lower telescope is eccentric of the centre of rotation of the upper telescope and the centre of the circle. The eccentricity is 40 mm. Though I could not trace the origin of the instrument I suppose that it is Krayenhoff's. The limb is calibrated to the right (clockwise) and not to the left as Moor thinks [40] . Each part of the limb represents 10' = 600". 29 parts coincide with 30 parts of the verniers. The unit of the verniers is therefore 600" : 30 = 20", an amount already estimated by Van der Plaats and Moor.

The instrument is represented in the Figures 3 and 4. A drawing and a description of an analogous instrument made in 1787 and with a limb (diameter 0.33 m) calibrated to the left, can be found in Berthaut: La Carte de France 1750-1798 [41].

U in Figs. 3 and 4 is the upper telescope, L the lower one. O is the object glass, E the compound eyepiece. The cross wires are placed in the focus of the object glass so that, without parallax, only can be pointed at points in the infinite or, in practice, at points far away. In order to obtain this parallax-free image the object glass can be shoven over a small distance (some mm) along the optical axis with the little screw s. The focus length of the object glasses is 610 mm, that of the compound eye pieces 25.4 mm. The magnification is therefore $610:25.4 \simeq 24$. The total length of a telescope is about 640 mm.

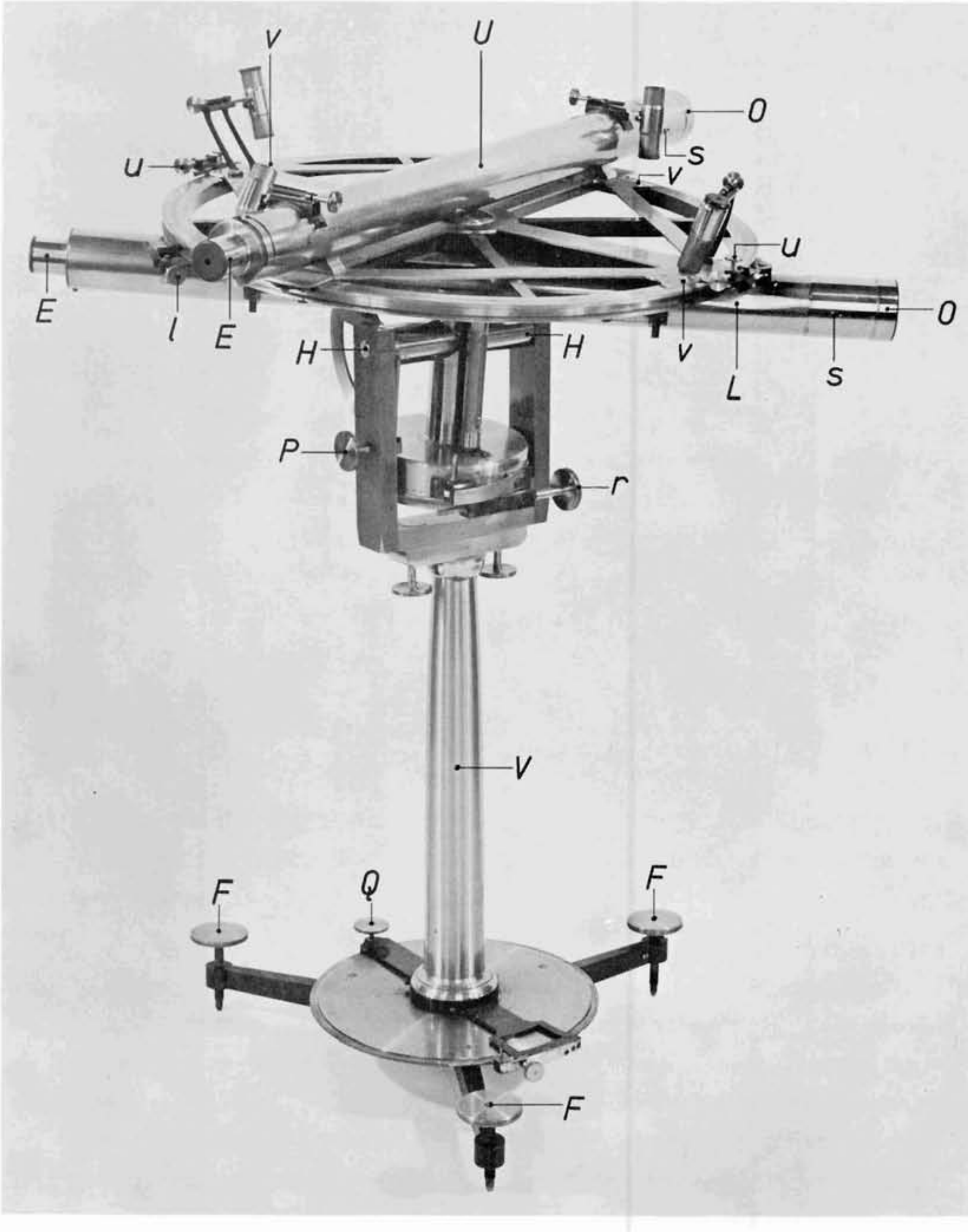


Fig. 3

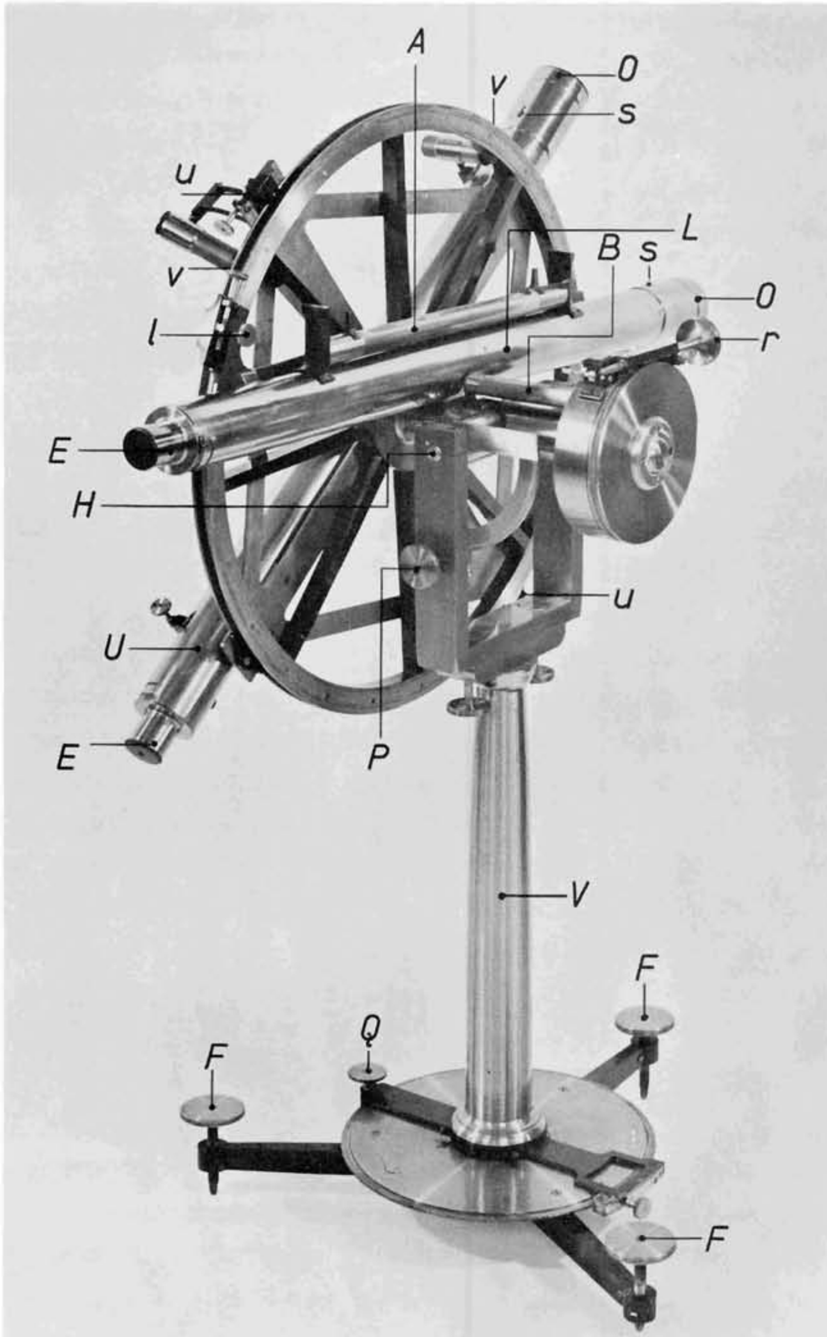


Fig. 4

A is a level mounted on the lower telescope.With the level B the axis V of the instrument can be placed vertically. None of both the levels has a graduation. Two lines on each of them mark very approximately the ends of the bubble when it is centred. In order to trace the sensitivity of the bubbles I stuck a transparent film strip with a graduation of 2 mm on one half of the tube. By this the ends of the bubbles remained clearly visible. After that the sensitivity of the levels was measured with the level trier of the Sub-Department of Geodesy of the Delft University of Technology. That of A appeared to be about 25'' per 2 mm and that of B about 20'' per 2 mm.

As the two telescopes can only be moved parallel to the plane of the circle, this circle must be adjustable. It can turn around the horizontal axis HH with the screw P. HH, connected with the vertical axis V, can be placed in an arbitrary position with the screw Q. The telescopes U and L can be clamped on any arbitrary place to the circle with the clamping and slow motion screws u and l. The repetition screw r enables the user of the instrument to turn the circle with its telescopes over any arbitrary angle in its own plane. The four verniers with which the limb can be read are in v.

Fig. 3 represents the instrument when it is used for the measurement of about horizontal angles. Fig. 4 is the setting up for the measurement of vertical angles. In order to give the reader an idea of the size of the instrument I can mention that the three footscrews F are in the form of an equilateral triangle with sides of 375 mm. When the limb is horizontal its height above the middle of the screw thread of the footscrews is about 700 mm. The weight of the instrument is about 26. 7 kg.

In the northeastern part of the triangulation network, measured in the years 1810 and 1811, the towers were in general much smaller and lower. In order to be able to point still at far distant points the observations had to be done high in the towers on spots which were not easily attainable with the large and heavy instrument. In those years therefore a smaller repetition circle was used. It had a circle with a diameter of 10 inches (about 27 cm) and it was made by Bellet in Paris. It had also smaller telescopes than the bigger one. They had, according to Krayenhoff "an inconvenient parallax which could never fully be removed. By this I lost much time and was obliged to measure every angle in a great number of series. Very often these series gave very different results as can be seen from the observations in 1810 and 1811 and rarely was I satisfied when they agreed, especially when I compared them with those of the large instrument" [42] . From these statements it is clear that the "small" instrument (I don't know where it is and whether it still exists) was much more inaccurate than the other one. There is some doubt whether the observations in 1810 at the stations Leiden (No. 30), Gouda (No. 31), Dordrecht (No. 29), and Den Haag (The Hague) (No. 27) in the western part of the centre of the triangulation network were made with the big instrument or with the smaller one.

Finally a third repetition circle was used for the determination of latitudes at Amsterdam and Jever. It had a circle with a diameter of 14 inches (about 38 cm) [42] and it was made at Krayenhoff's expense by Lenoir. I think that it is the same instrument as pictured in Fig. 1. Krayenhoff praised it very much. Especially he was satisfied with the accuracy of the two levels. As I don't know whether it still exists I can neither confirm this satisfaction nor deny. In any case the accuracy of the level A (Figs. 3 and 4) which determined the accuracy of the measurement of vertical angles should have been much better than the amount of 25" per 2 mm found for the level in Figs. 3 and 4.

6. Execution of the angular measurements

For the measurements of the "horizontal" angles in the mostly eccentric station points one set about as described in Berthaut [41] , page 103.

The instrument was set up in such a way that the line HH (Figs. 3 and 4) was pointed between the left and the right object. "This position is the most favourable to bring the plane of the circle into the plane of the two objects". "This was done by a trial and error method" (the French text says: par tâtonnement). A mathematical solution of this practical problem will be made clear with the aid of Fig. 5.

Let O be the point where the space angle between the left and the right sighting point must be measured. It is supposed to be the centre of a sphere. The rays from O to the two sighting points intersect the sphere in L(eft) and R(ight) respectively. OL'M'R'S is a horizontal plane and Z the zenith point. The inclinations of L and R (the arcs L'L and R'R) are h_1 and h_2 respectively. The projection of the space angle OLR is OL'R' = α . If the inclination of M (the arc M'M) is $\frac{1}{2}$ ($\alpha_1 + \alpha_2$), the angle L'OM' = arc L'M'= x can be computed.

In the right angled spherical triangle L'LS one has, if arc L'S = angle L'OS = y:

$$\cot S \tan h_1 = \sin y,$$

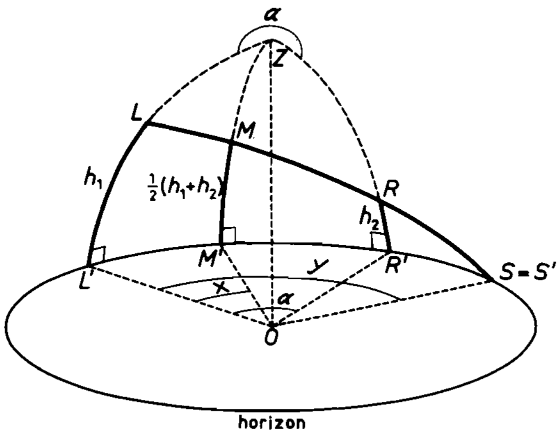


Fig. 5

and in the right angled triangle R'RS:

$$\cot S \tan h_2 = \sin (y - \alpha)$$

whence:

$$\tan h_2 : \tan h_1 = \sin (y - \alpha) : \sin y = \cos \alpha - \sin \alpha \cot y$$

so that:

$$\left.\begin{aligned} \cot y &= \frac{\tan h_1 \cos \alpha - \tan h_2}{\tan h_1 \sin \alpha} \\ \text{and} \quad \tan y &= \frac{\tan h_1 \sin \alpha}{\tan h_1 \cos \alpha - \tan h_2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

In an analogous way:

$$\cot S \tan h_1 = \sin y \qquad \text{and}$$

$$\cot S \tan \tfrac{1}{2} (h_1+h_2) = \sin (y-x)$$

whence:

$$\frac{\tan \frac{1}{2} (h_1+h_2)}{\tan h_1} = \frac{\sin (y-x)}{\sin y} = \cos x - \sin x \cot y$$

Or, in connection with (1):

$$\frac{\tan \frac{1}{2} (h_1+h_2)}{\tan h_1} = \cos x - \frac{\tan h_1 \cos \alpha - \tan h_2}{\tan h_1 \sin \alpha} \sin x$$

Or, for small values of h_1 and h_2:

$$\frac{h_1+h_2}{2h_1} = \frac{h_2-h_1 \cos \alpha}{h_1 \sin \alpha} \sin x + \cos x$$

whence:

$$\sin x + \frac{h_1 \sin \alpha}{h_2 - h_1 \cos \alpha} \cos x - \frac{(h_1 + h_2) \sin \alpha}{2(h_2 - h_1 \cos \alpha)} = 0 \quad \ldots\ldots\ldots \quad (2)$$

From this goniometric equation x can be resolved. x is dependent on α and the ratio $h_1 : h_2$.

For $h_1 : h_2 = R$ (2) changes into:

$$\sin x + \frac{R \sin \alpha}{1-R \cos\alpha} \cos x - \frac{(R + 1) \sin \alpha}{2(1 - R \cos \alpha)} = 0 \quad \ldots\ldots\ldots\ldots \quad (3)$$

x determines the position of the axis HH with respect to the left and right sighting point. It is obvious that for $h_1 = h_2 = 0$ in (2), x is indefinite: every arbitrary positionof HH in the horizontal plane gives the possibility of bringing the plane of the circle through the sighting points.

For $h_1 = - h_2$ ($R = -1$) one finds from (3):

$$\sin x - \frac{\sin \alpha}{1 + \cos \alpha} \cos x = 0 \quad \text{or}$$

$$\tan x = \frac{\sin \alpha}{1 + \cos \alpha} = \tan \tfrac{1}{2}\alpha, \text{whence } x = \tfrac{1}{2}\alpha .$$

HH must therefore coincide with the bisector of α. Its inclination $(h_1 + h_2) : 2$ of course must be zero. The plane of the circle should now only be turned around HH till it goes through the left (right) sighting point. It goes then also through the right (left point).

For $h_1 = +20'$ and $h_2 = +4'$ (or $h_1 = -10'$ and $h_2 = -2'$, etc.), x can be computed from (3). As $R = +5$ one finds:

$$\sin x + \frac{5 \sin \alpha}{1-5 \cos \alpha} \cos x - \frac{6 \sin \alpha}{2\ (1-5 \cos \alpha)} = 0$$

whence, for $\alpha = 60^\circ$, $x \simeq 36^\circ 23'$.

HH can be brought in this position with the aid of the calibration on the circle near the footscrews. If this is done in such a way that the vertical plane through HH coincides with the vertical plane through V and one of the footscrews then the desired result can be obtained by turning the circle around HH and by using only one footscrew. V, however, is then no longer vertical.

In order to give the reader an impression of the values x for various amounts R and for $\alpha = 60^\circ$, I made table 1. As one sees it is rather difficult to obtain a good result with a trial and error method when R is positive. Between $R = +1$ and $R = +10$ e.g. the position of HH alters almost 25°. If this position was wrongly chosen it was impossible to realise the desired result and the manipulation with the screws P and Q and a (the) footscrew(s) had to begin anew. The then geodesists, however, apparently had a great skill to solve this practical problem. It seems that Krayenhoff had never any difficulty with it.

Table 1

R	x ≃	y ≃	R	x ≃	y ≃
0	$25^o40'$	$180^o00'$	0	$25^o40'$	$0^o00'$
+0.1	$24^o44'$	$174^o47'$	-0.5	$28^o30'$	$19^o07'$
+0.2	$23^o37'$	$169^o06'$	-1.0	$30^o00'$	$30^o00'$
+0.3	$22^o15'$	$163^o00'$	-2.0	$31^o30'$	$40^o54'$
+0.4	$20^o41'$	$156^o35'$	-3.0	$32^o13'$	$46^o06'$
+0.5	$18^o35'$	$150^o00'$	-5.0	$32^o56'$	$51^o03'$
+0.6	$16^o04'$	$143^o25'$	-10.0	$33^o34'$	$55^o17'$
+0.7	$12^o47'$	$137^o00'$			
+0.8	$9^o07'$	$130^o54'$			
+0.9	$4^o48'$	$125^o12'$			
+1.0	$0^o00'$	$120^o00'$			
+1.0	$60^o00'$	$120^o00'$			
+2.0	$41^o25'$	$90^o00'$			
+3.0	$38^o11'$	$79^o06'$			
+5.0	$36^o23'$	$70^o54'$			
+10.0	$35^o16'$	$65^o13'$			

For negative R's the problem is much easier. Between R = -1 and R = -10, x (the position of HH) alters but $3\frac{1}{2}^o$. If HH is chosen along the bisector of α a rather good result can be obtained.

It will be clear that the method described and apparently used in those days has the drawback that the line HH of the instrument has the inclination $(\alpha_1 + \alpha_2) : 2$ and that therefore the vertical axis V will never be vertical. Strictly speaking this inconvenience causes errors in the distance of the eccentric point to the centre. For the setting up on "the" eccentric point is not always the same as it is dependent on the inclination of the angles measured there. In a flat country like Holland, however, these errors are negligible. In order to elude the inconvenience of a non-perpendicular vertical axis V, Mr. Pouls of the Sub-Department of Geodesy at Delft suggested the computation of y from formula (1). As (see Fig. 5) S = S' is the intersection point of the great circle LR with the horizontal plane, the inclination of S is zero, HH horizontal, and the axis V vertical.

For small values of h_1 and h_2 and for $h_1 : h_2 = R$ one has:

$$\tan y = R \sin\alpha : (R \cos\alpha - 1).$$

Of course here, too, y is indefinite for $h_1 = h_2 = 0$ and $y = \frac{1}{2}\alpha$ for $R = -1$. In order to have a comparison with the amounts x, I computed in table 1 the y's for the same values R and for the same $\alpha = 60^o$. As one sees the amounts y range much more than the x's. Notwithstanding the attractiveness of this method - for the axis V was vertical - it could not be used and it was not used in practice by trial and error. For every measurement of an angle asked a computation of y from a provisional α and the amounts h_1 and h_2 which had to be measured in advance.

In order to bring the plane of the circle through the station and the two sighting points one could also act as described in J. F. Salneuve: Cours de Topographie et de Géodésie (seconde édition, Paris 1850, page 362). The instrument was set up in such a way that the axis V was vertical and the connecting line of two of the footscrews was pointed at (or was perpendicular to the direction to) the sighting point with the smallest inclination. The upper telescope U was brought parallel to the axis HH and, with the screw Q turned around V till it was also pointed at the said sighting point. The inclination of HH, necessary for this manipulation, could be realised with the two footscrews or - in the case the connecting line of these screws was perpendicular to the direction to the sighting point - with the third footscrew. In this position U remains pointed at the sighting point if the limb was turned around HH with the screw P. The desired position of the limb could therefore be realised by pointing with the lower telescope L and a loosened P at the second sighting point. After fastening P the instrument was ready for the measurement. The method described has the drawback that for $h_1 : h_2 = -1$ the vertical axis has the inclination $90^o - h_1$ ($90^o - h_2$) instead of 90^o as found in the method described before. I can't tell but I doubt whether Krayenhoff used it for his measurements.

The measurement of the angles was done by two observers. One of them used the upper telescope U, the other the lower one L. In contradistinction to Delambre Krayenhoff never tried to eliminate errors in the graduation of the limb by measuring on different parts of it. He always began his measurements with a reading zero when U was pointed at the left object. This facilitated of course his computations and it gave an easy survey of the regular progress of the measurement. U was therefore brought on the reading zero and clamped to the limb with u. With the repetition screw r and its slow motion screw - for the reading should not change - limb and telescope were now turned in such a way in the plane of the circle that L was pointed at the left object. Thereafter the limb was fastened. With the lower telescope L the second observer pointed then at the right object. After clamping the telescope to the limb with l he loosened the repetition screw and turned the circle with the two telescopes clamped to it to the left till L was pointed at the left object. In this position the limb was fastened with r. Then the first observer turned his telescope U over the standing circle, pointed at the right object and clamped his telescope to the limb. Now the four verniers were read and the mean of the readings noted down. It is obvious that, apart from the influence of the eccentricity of L, the angle read will be the double of the angle between left and right object.

In this manner one can proceed and determine the 4-, 6-, 8-, ... 2n-multiple of the angle (n usually 9, 10, 11 or 12). For each of the multiples Krayenhoff computed the single angle in his observation registers. They gave him a check on the regular course of the repetition. An example of such a repetition is given in table 2. It relates to the determination of angle 157, measured in an eccentric point of the station No. 40 (Amsterdam on June 29th, 1803 at 17.45 hours (series 19) [43] .

Table 2

number of rep.	readings	single angle
2	$153^\circ 23' 05''$	$76^\circ 41' 32''.50$
4	$306^\circ 45' 45''$	$26''.25$
6	$460^\circ 08' 45''$	$27''.50$
8	$613^\circ 31' 15''$	$24''.37$
10	$766^\circ 54' 10''$	$25''.00$
12	$920^\circ 17' 00''$	$25''.00$
14	$1073^\circ 39' 45''$	$24''.64$
16	$1227^\circ 02' 45''$	$25''.31$
18	$1380^\circ 25' 00''$	$23''.33$
20	$1533^\circ 48' 00''$	$24''.00$
22	$1687^\circ 10' 30''$	$23''.18$
24	$1840^\circ 33' 30''$	$76^\circ 41' 23''.750$

Nieuwkoop (No. 35) is the left object, Haarlem (No. 39) the right one. On the circumstances during the measurement Krayenhoff remarks: "Objets très visibles; très bonne observation" (objects very well visible; very good observation).

Measurement of the same angle had already taken place on June 14th (series 6), and June 28th (series 16 and 18). The results were $76^{\circ}41'25''.000$, $76^{\circ}41'21''.250$, and $76^{\circ}41'23''.571$, respectively. For the mean from the four series one finds for the eccentric angle, measured in its own plane $76^{\circ}41'23''.393$.

For the determination of vertical angles zenith distances z were measured. The instrument was set up as indicated in Fig. 4 with the axis V in a vertical position. This could be done with the footscrews and the level B.

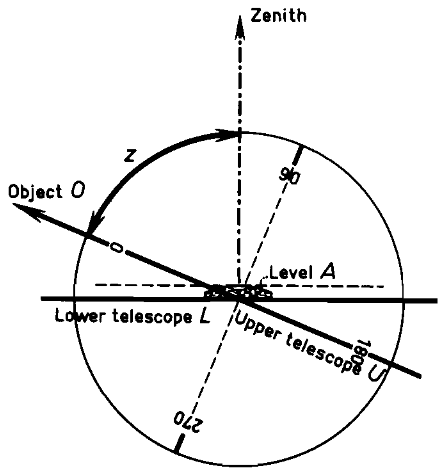


Fig. 6a

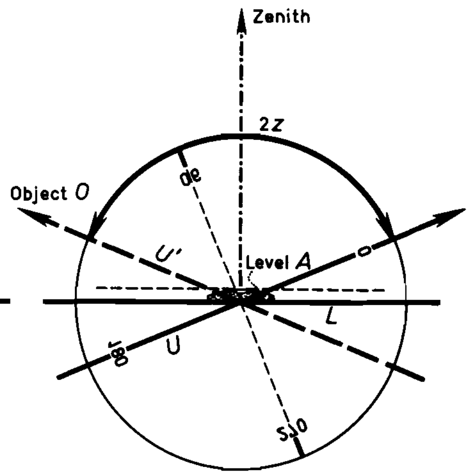


Fig. 6b

The upper telescope U with a reading zero on the circle and the circle to the right of the telescope was now pointed at the object O, the zenith distance of which had to be measured (Fig. 6a). In order that the reading should not alter, this was done with the repetition screw and its slow motion screw. The lower telescope L was set in a horizontal position. For this sort of measurement it served not as a sighting telescope but only as a support of the level A. If its bubble was well centered it remained centered when the instrument was turned 180° around the vertical axis V with the aid of screw Q. U is then in a symmetrical position with respect to the zenith (Fig. 6b). The zero point of the circle is in 0.

With a fastened circle the upper telescope was now pointed again at the object O. Its position was then U'. The circle is then to the left of the telescope. It will be clear that the reading on the circle will be 2z.

The manupilation described could be repeated as much as necessary. The start for the next repetition was circle to the right and a reading 2z. In an analogous way one finds at the end of this repetition (circle to the left of U) a reading 4z, and, proceeding in the same way, 6z, 8z, ... 2nz. For the measurement of the distances z of the sighting points in the triangulation network - in flat Holland they are all about 90^{o} - n was usually 1 or 2. For the determination of latitudes, however, n was taken about 20. It will be clear that for these latter measurements with pointings at moving stars a good cooperation between the two observers was necessary. For a small alteration with the slow motion repetition screw during the pointing at the star by the first observer implicated a small alteration in the centering of the level by the second.

7. Chronological order in which the stations were visited and survey of the number of angles and series measured there

The nine volumes octavo I-IX, mentioned in section 4 under a give an excellent survey of the sequence of the stations which were visited, the number of angles and series measured there, the data for the reduction to centre and the weather conditions during the measurements. Table No. 3 was composed from these volumes. It is arranged in order of the years in which the measurements took place. Columns 1 and 2 give the number and the name of the station, column 3 the number of angles measured there and columns 4, 5, and 6 the total number of series measured rejected and retained. The vertical angular measurements, necessary for the reduction of the space angles to the horizon are not included in these numbers.

The first observation dates from Friday, June 11th, 1802 at four o'clock in the morning. At this early hour in an eccentric point of the station Nieuwkoop (No. 35) the angle between Gouda and Utrecht was measured. Already the same day the observations at Nieuwkoop could be finished. Each of the five angles was measured in only one series. This, however, was a great exception. In his "Instructie voor de geographische ingenieurs" [44] Krayenhoff informs us how the measurements with a repetition circle should be executed. Article 4 of his instruction says that the measurement of a double angle should be repeated at least 10 times. The same measurement had to be repeated in several series, dependent on the circumstances. "It will never be allowed to suffice with only one series unless more series are absolutely impossible". As one sees Krayenhoff apparently kept himself not quite to his own instructions. It must be said, however, that in general the number of series was two or three.

Table 3

Stations			Series			Stations			Series		
No.	Name	angles	measured	rejected	retained	No.	Name	angles	measured	rejected	retained
1	2	3	4	5	6	1	2	3	4	5	6
	1802 Volume I										
35	Nieuwkoop	5	5	0	5	2	Mont Cassel	1	4	2	2
32	Gorinchem	6	11	1	10	3	Hondschoote	3	9	3	6
21	Brielle	3	5	2	3	4	Nieuwpoort	3	11	3	8
16	Zierikzee	4	18	6	12	5	Diksmuide	4	10	2	8
13	Middelburg	3	9	3	6	8	Hooglede	2	6	1	5
23	Breda	7	24	6	18	9	Tielt	2	5	1	4
24	Hilvarenbeek	5	19	1	18	7	Brugge	5	13	0	13
19	Lommel	3	10	3	7	11	Aardenburg	3	6	0	6
18	Hoogstraten	6	19	3	16	10	Gent	5	11	0	11
14	Hulst	6	15	3	12	15	Antwerpen	4	8	0	8
12	Assenede	4	10	1	9	17	Bergen op Zoom	6	12	0	12
6	Oostende	2	5	0	5	22	Willemstad	6	13	3	10
1	Duinkerken	2	6	0	6	28	Rotterdam	6	6	0	6
						26		106	270	44	226
	1803 Volume II										
39	Haarlem	5	10	0	10	56	Medemblik *	3	7	0	7
40	Amsterdam	6	19	5	14	57	Enkhuizen	5	13	2	11
41	Naarden	5	15	1	14	54	Hoorn	5	13	1	12
52	Alkmaar	6	16	1	15	58	Urk	4	9	0	9
53	Edam	5	8	0	8	46	Harderwijk	5	12	0	12
55	Schagen *	4	14	4	10	11		53	136	14	122
							* See also 1807				
	1805 Volume III										
42	Amersfoort	5	24	6	18	33	's-Hertogenbosch	6	15	1	14
47	Veluwe	7	29	9	20	20	Nederweert	2	6	1	5
59	Kampen *	4	10	0	10	25	Helmond	6	15	1	14
48	Zutphen	6	15	0	15	34	Grave	6	18	2	16
43	Imbosch	5	12	0	12	26	Vierlingsbeek	3	8	0	8
37	Rhenen	8	20	3	17	38	Nijmegen	5	11	2	10
	* See also 1807					12		63	183	24	159

Table 3 (continued)

Stations			Series			Stations			Series		
No.	Name	angles	measured	rejected	retained	No.	Name	angles	measured	rejected	retained
1	2	3	4	5	6	1	2	3	4	5	6
	1807 Volumes IV and V										
44	Hettenheuvel	6	25	1	24	56	Medemblik*	2	5	0	5
50	Harikerberg	5	19	4	15	55	Schagen*	2	4	0	4
49	Groenlo	5	15	0	15	66	Oosterland	6	13	0	13
45	Bocholt	2	7	2	5	65	Kijkduin	2	3	0	3
51	Ahaus	4	10	2	8	71	Oosterend	3	7	2	5
61	Oldenzaal	5	12	1	11	73	Vlieland	3	7	1	6
62	Bentheim	3	10	0	10	77	Midsland	2	4	0	4
64	Kirch Hesepe	2	7	0	7	79	Leeuwarden**	3	6	0	6
63	Uelsen	5	11	0	11	74	Harlingen	7	12	2	10
84	Coevorden*	3	11	3	8	67	Staveren	8	11	1	10
60	Lemelerberg	9	16	0	16	75	Sneek	6	15	2	13
70	Meppel	5	14	3	11	59	Kampen***	2	5	1	4
69	Blokzijl	5	12	2	10	36	Utrecht	7	23	1	22
68	Lemmer	5	10	0	10	28		122	304	28	276
72	Robbezand	5	10	0	10		*See also 1803				
	*See also 1810						**See also 1810				
							***See also 1805				
	1810 Volumes VI and VII										
30	Leiden	6	22	9	13	84	Coevorden*	2	9	4	5
31	Gouda	6	18	6	12	88	Sleen	5	22	10	12
29	Dordrecht	5	12	3	9	92	Onstwedde	6	19	9	10
27	Den Haag	2	9	2	7	89	Uithuizermeden**	3	11	5	6
78	Ballum	3	15	7	8	85	Hornhuizen	5	17	9	8
76	Oldeholtpa	7	36	12	24	86	Groningen	9	35	17	18
81	Drachten	6	26	14	12	91	Midwolda**	3	9	2	7
79	Leeuwarden*	2	8	2	6	90	Holwierde	5	19	10	9
80	Dokkum	6	21	7	14	20		98	358	146	212
82	Oosterwolde	5	14	5	9		*See also 1807				
83	Beilen	7	20	7	13		**See also 1811				
87	Rolde	5	16	6	10						
	*See also 1807										

Table 3 (continued)

Stations			Series			Stations			Series		
No.	Name	angles	measured	rejected	retained	No.	Name	angles	measured	rejected	retained
1	2	3	4	5	6	1	2	3	4	5	6
	1811 Volumes VIII and IX										
95	Leer	8	36	20	16	97	Hage	3	12	7	5
96	Barssel	4	10	3	7	102	Jever	7	36	20	16
94	Emden	6	23	11	12	103	Varel	5	18	6	12
93	Pilsum	4	17	8	9	100	Westerstede	7	22	11	11
98	Aurich	6	32	19	13	91	Midwolda*	2	9	5	4
99	Strakholt	4	20	13	7	89	Uithuizermeden*	2	7	3	4
101	Esens	5	21	7	14	13	*See also 1810	63	263	133	130

Table 4

Year	stations visited	angles measured	series			
			measured	rejected	retained	retained (in percents)
1	2	3	4	5	6	7
1802	26	106	270	44	226	84
1803	11	53	136	14	122	90
1805	12	63	183	24	159	87
1807	28	122	304	28	276	91
1810	20	98	358	146	212	59
1811	13	63	263	133	130	49
	110	505	1514	389	1125	77

The second station visited was Gorinchem (No. 32). Six angles were measured there in 11 series. On of these series had to be rejected, etc. The observations in the campaign 1802 ended in Rotterdam. During this working season Krayenhoff and his assistent Jacob de Gelder [45] measured (see tables 3 and 4) 106 angles at 26 stations. The number of series was 270. Fourty four of these series were rejected (not used for the computation of the network). The percentage of series retained was 84.

As one sees from the observations in 1803 the stations Schagen and Medemblik were also visited in 1807. The reason that in both of these stations two angles had to be measured anew was caused by the fact that the original station Westerland in the isle of Wieringen was replaced by Oosterland (No. 66). Two angles measured in 1803 at Kijkduin (No. 65) could not be used because of the substitution of Westerland by Oosterland and the substitution of Oude Schild in the isle of Texel by Oosterend (No. 71). The new measurements at Kijkduin were also done in 1807.

As one sees no measurements took place in 1804, 1806, 1808, and 1809 on account of military and civil duties (minister of war) which had to be fulfilled. Krayenhoff says on this subject "En 1804, mon service ordinaire qu'il ne m'était pas permis de négliger me força à une interruption complète et ce ne fut qu'en 1805 que je pus y employer une partie de mon temps; mais bientôt la guerre allumée en Allemagne vint m'en distraire et me livra à des occupations plus pressantes. Il me fallut faire tous les préparatifs nécessaires à la défence d'Amsterdam et exercer les fonctions de Commissaire-Général du Gouvernement Batave au quartier-général du Prince Français Louis, commandant en chef de l'armée du Nord. Dans l'année suivante, époque de l'avénement de ce Prince au trône de Hollande, S. M. me nomma son aide de camp et me confia des traveaux importants et en grand nombre. Je ne pus donc encore m'occuper de la triangulation. En 1807, il me fut permis d'y travailler pendant cinq mois et j'opérai avec d'autant plus d'ardeur que j'avais été obligé de discontinuer ce travail à différentes fois. Cependant il me fut impossible de le terminer de suite parce qu'ayant reçu le titre d'inspecteur -général de fortifications et de président du commité central je me livrai entièrement à ces nouveaux emplois durant l'année 1808.

En 1809, ayant été nommé ministre de la guerre, toute autre espèce d'occupations me fut interdite par les travaux importants que réclamait ce poste honorable. Je commençai à désespérer de voir se terminer mon travail géodésique malgré le désir que j'avais de le conduire à sa fin. Mais au printemps de l'année 1810 je me vis heureusement rendu à moi-même et dégagé du fardeau de ces fonctions éminentes, plus flatteuses à la vérité que convenables à mes goûts pour des occupations plus simples et pour la culture des sciences" [46] .

I gave this rather ample quotation because, in my opinion, it illustrates in such an excellent manner the enormous energy of the then about 50 years old general and his sincere desire to finish the great work which he had undertaken. An energy and a devotion which also can be seen from his habit to be busy working on his triangulation when he had to wait in the room of the aides de camp till the king could receive him.

It matched with his character that Krayenhoff used to measure on any arbitrary moment that seemed favourable and on any arbitrary day of the week. Any is used here in the most literal signification of the word. Not only do we see him working during the whole Saturday - a very unusual occupation nowadays - but also the Sunday was often considered a normal working day.

Table 5

Date 1803	Hour	Angle	number		Weather conditions
			series	rep.	
1	2	3	4	5	6
Aug. 6 (Saturday)	15.30	274	1*	24	The air full of vapour; the objects, however, clearly visible
	16.15	277	2*	14	Clearly visible objects, good observation
	16.45	274	3*	24	Very good objects; excellent observation
	17.30	277	4*	14	Very good objects, excellent observation
Aug. 7 (Sunday)	7.30	238	5*	20	Clearly visible objects; in the middle of the series the rain interrupted the observations for some minutes
	9.15	238	6	20	Hoorn clearly visible; Edam very faint; doubtful observation.
	9.45	238	7	20	Inconvenient heat shimmer; objects rather visible; doubtful observation
	10.15	238	8*	20	Clearly visible objects; good observation
	10.45	484	9*	24	As the previous series
	11.30	484	10*	24	Clearly visible objects; very good observation
	12.15	235	11*	24	As the previous series
	13.00	235	12*	24	Very strong wind; objects rather visible; doubtful observation
	13.30	235	13*	24	Clearly visible objects; the wind less strong; good observation

Table 5 is an arbitrary example for this impulse for action. It concerns 13 series of repetitions of 5 angles, measured on Saturday, August 6th, and Sunday, August 7th, 1803, in eccentric points of his station Enkhuizen (No. 57). The number of the angle is mentioned in column 3, the number of the series in column 4. I followed here Krayenhoff's custom to indicate the number of the series retained with an asterisk. Column 5 gives the number of repetitions and column 6 the weather conditions during the measurement. With these measurements the operations at Enkhuizen were finished apart of course from the measurement of the vertical angles, necessary for the reduction of the angles to the horizon and those for the reduction to centre. As one sees the number of repetitions ranges between 14 and 24. I don't know why this number for angle 277 is but 14. In general it is considerably higher, in my opinion about 20.

The series for the angles 238, 484, and 235 were measured immediately after each other and those for the angles 274 and 277 almost immediately after each other. This is of course a serious objection: the constant influences of one-sided illumination of the objects on which was pointed and the lateral refraction (the rays to Staveren, Urk, and Edam pass the sea over their full length) are not rendered harmless in a satisfactory way by these measurements.

Column 4 shows that the series 6 and 7 are not used for the computation of the network. According to column 6 the observations in these series are doubtful. Therefore it seems to be justified that they were rejected. Series 12, however, is also doubtful; nevertheless it was retained. Arbitrariness demonstrates here its influence, arbitrariness against which Cohen Stuart objected rightly. Later on (in section 17) I shall have the opportunity of returning to this subject.

As can be seen in column 7 of table 4 the percentage of series retained in the campaigns 1802-1807 is about 88. In the campaigns 1810 and 1811 it falls to 59 and 49, respectively. According to Krayenhoff this low percentage is not only due to the smaller instrument used during these campaigns but also to the lateral refraction and to the smoke of heath fire and peat-moor fire in the northeastern part of the triangulation network. Gauss too complains of the inconvenience of heath and moor fires in the adjoining areas during his measurements in 1825.

8. Accuracy of the angular measurement

In order to get an impression of the internal accuracy of the angular measurement I give in table 6 a survey of the amounts [vv] in a number of angles measured in eccentric points of the stations mentioned in column 2.

In order to make them not too unreliable I computed them for those angles of which the number of "series retained" (column 4) was at least 3. They are arranged in sequence of the numbers of the stations and the years 1802-1807 (first part of the table) and 1810-1811 (second part) in which they were measured

Table 6

Stations No.	Name	angles	series retained	[vv]	Stations No.	Name	angles	series retained	[vv]
1	2	3	4	5	1	2	3	4	5
1	Duinkerken	4	3	0.67	41	Naarden	163	3	0.17
1	"	1	3	1.16	41	"	475	3	0.35
4	Nieuwpoort	10	3	0.29	41	"	192	4	30.17
4	"	8	3	0.29	42	Amersfoort	169	7	66.44
6	Oostende	12	3	0.39	42	"	166	4	20.52
7	Brugge	25	3	2.54	43	Imbosch	175	3	5.22
7	"	22	3	3.89	43	"	180	3	2.47
10	Gent	28	3	0.82	44	Hettenheuvel	181	5	5.63
12	Assenede	36	4	21.05	44	"	200	4	3.51
16	Zierikzee	59	4	55.58	44	"	203	5	27.92
16	"	63	4	131.37	44	"	476	4	0.75
18	Hoogstraten	56	4	9.14	46	Harderwijk	195	4	3.89
24	Hilvarenbeek	75	4	21.80	47	Veluwe	247	3	8.15
24	"	73	4	15.17	47	"	243	5	46.50
24	"	104	4	8.06	49	Groenlo	210	5	16.28
34	Grave	147	4	13.92	52	Alkmaar	225	4	12.60
34	"	113	3	3.62	58	Urk	283	3	15.50
36	Utrecht	164	4	18.57	59	Kampen	242	3	1.29
36	"	168	4	17.31	59	"	288	3	2.79
37	Rhenen	177	3	1.58	62	Bentheim	262	4	6.03
37	"	148	3	49.22	63	Uelsen	303	3	0.50
37	"	145	3	56.51	63	"	304	3	0.67
40	Amsterdam	157	4	7.33	64	Kirch Hesepe	264	4	9.78
40	"	182	3	6.12			47	172	733.53

$m^2 = 733.53 : (172-47) = 5.87$ $\qquad m = \pm 2''.4$

(continued on page 39)

Table 6 (continued)

Stations		angles	series retained	[vv]	Stations		angles	series retained	[vv]
No.	Name				No.	Name			
1	2	3	4	5	1	2	3	4	5
76	Oldeholtpa	364	5	115.8	91	Mildwolda	406	3	60.7
76	"	341	5	1.0	97	Hage	432	3	29.2
78	Ballum	348	4	15.3	98	Aurich	455	4	40.7
80	Dokkum	369	4	53.8	99	Strakholt	458	3	66.7
80	"	373	3	26.0	101	Esens	504	3	78.4
86	Groningen	375	4	121.2	101	"	452	3*	286.2
87	Rolde	384	3	33.5	101	"	450	4	51.2
88	Sleen	393	3	23.2	102	Jever	468	3	32.3
88	"	490	4	35.2					
							17	61	1070.4

$m^2 = 1070.4 : (61-17) = 24.33$ $m = \pm\ 4''.9$

* The very high m^2 must be imputed to the bad observations (I mention only the seconds of arc):

52".500	− 11".042
43.125	− 1.667
28.750	+ 12.708
41".458	0

No.	Name	angles	series retained	[vv]	No.	Name	angles	series retained	[vv]
27	Den Haag	87	4	25.87	30	Leiden	471	3	1.50
30	Leiden	119	4	33.22	31	Gouda	127	3	4.67
							4	14	65.26

$m^2 = 65.26 : (14-4) = 6.53$ $m = \pm\ 2''.6$

with the big and the smaller instrument respectively. As it is not known with certainty whether the angles at the stations Den Haag, Leiden, Gouda, and Dordrecht were measured with the former instrument or with the latter, I mentioned them in a separate (third) part of the table. Dordrecht fails in this part: the angles were measured there in no more than 2 series.

In Amsterdam (station No. 40) e.g. the eccentric angle 157 was measured in 4 (retained) series. The results (see also section 6 page 30 were $76^\circ 41' 25''.000$, $76^\circ 41' 21''.250$, $76^\circ 41' 23''.571$, and $76^\circ 41' 23''.750$ with a mean $76^\circ 41' 23''.393$ and with deviations v from this mean of $-1''.607$, $+2''.143$, $-0''.178$ and $-0''.357$. The amount [vv] = 7.33 and the number of series is a measure for the accuracy of the angle concerned. It is mentioned in column 5 of the first part of table 6.

The 47 angles of this part, measured in 172 series give $[vv] = 733.53$ from which a standard deviation $m = \pm 2''.4$ can be computed. It contrasts very badly with the strongly exaggerated registration of the observations in thousandths of a second of arc. In an analogous way one finds for the measurements in the years 1810 and 1811 (second part) $m = \pm 4''.9$. The results in both groups demonstrate the great difference in accuracy of the measurements in the two periods. The amount $m = \pm 2''.6$ for the third group is not convincing. One might conclude that on the stations mentioned there the big instrument was used but the number of observations is too small for such a conclusion.

As in the years 1802-1807 the mean of the series retained is about 2.28 (see table 4) and that in the years 1810-1811 about 2.12, the standard deviation in the mean of 2.28 series measured with the big instrument is about $M = 2''.4 : \sqrt{2.28} = \pm 1''.6$ and that in the mean of 2.12 series with the smaller one about $M = 4''.9 : \sqrt{2.12} = + 3''.4$. The accuracy of the vertical angular measurements will be discussed in detail in the astronomical part of the triangulation.

9. Influence of the eccentricity of the lower telescope on the results of the angular measurements

As already remarked in section 5 (page 20) the lower telescope of Krayenhoff's repetition circles was eccentric with respect to the centre of the graduated circle. The eccentricity for the big instrument in Figs. 3 and 4 is 40 mm.

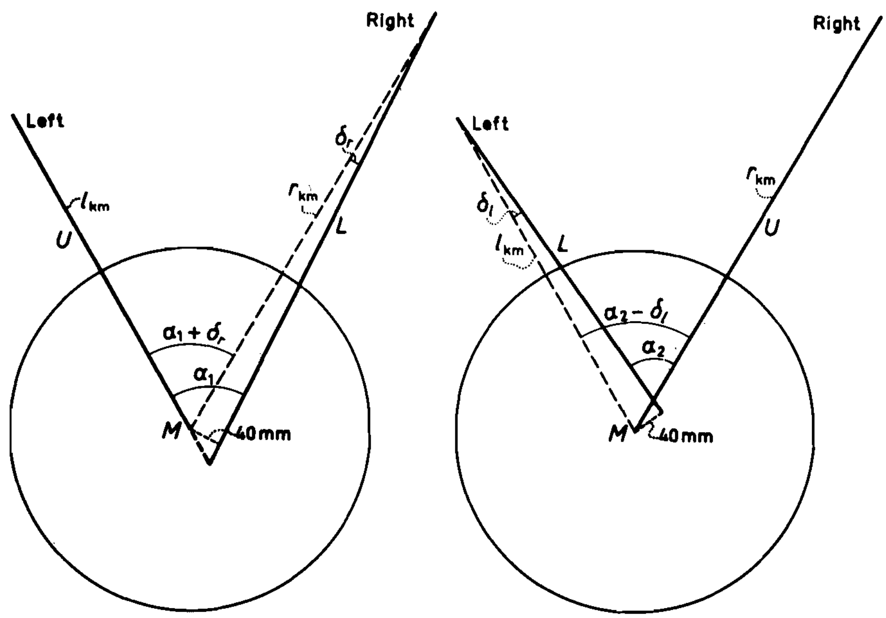


Fig. 7a Fig. 7b

When (see Fig. 7a) the upper telescope U was pointed at the left object and the lower telescope L at the right, one measured the angle α_1 instead of $\alpha_1 + \delta_r$ When U was pointed at the right object and L at the left (Fig. 7b) one measured α_2 instead of $\alpha_2 - \delta_1$. In these values

$$\delta_1 = \frac{40\ \text{mm}}{(10^6 r_{km})\text{mm}}\ 206265'' = \frac{8''.25}{r_{km}} \quad \text{and} \quad \delta_r = \frac{8''.25}{1_{km}}$$

l and r are the distances in km to the left and right sighting point. The mean of the double of the measured angle is $(\alpha_1 + \alpha_2):2$. It should be $(\alpha_1 + \delta_r + \alpha_2 - \delta_1):2=$

$$(\alpha_1 + \alpha_2) : 2 + 4''.13 \left(\frac{1}{r_{km}} - \frac{1}{1_{km}} \right)$$

The measured mean and also the angle computed from the 4-, 6-, ..., 2n-multiple of the angle must therefore have a correction:

$$c = 4''.13 \left(\frac{1}{r_{km}} - \frac{1}{1_{km}} \right)$$

For r = l this correction is zero. For r = 16.79 km (Amsterdam-Haarlem) and l = 25.92 km (Amsterdam-Nieuwkoop) (approximate values of r and l will do) the correction to angle 157 is $c = +0''.087$.

A "large" c can be expected when one of the distances r or l is small and l or r large. The left leg of angle 116 e.g. (Grave-Biesselt) is l = 11.22 km and the right leg (Grave-Vierlingsbeek) r = 26.12 km. c is then $+ 0''.210$. It is in my opinion about the largest c in the whole triangulation network. Neither at Grave nor at any other station of his network, however, Krayenhoff computed the corrections. He does not even mention them.

The errors made are small and of no importance in connection with the standard deviation in the observations (see section 8). If, however, Krayenhoff wished to attach value to his observations in thousandths of a second - and apparently he did so - he was not allowed to neglect the corrections.

As in a central point the sighting points are alternately right and left object, the corrections are of no influence on the closing error around this central point. For the same reason they have no influence on the closing error in the sum of the angles of a triangle. They find expression, however, in the side (sine) equations of a triangulation network.

10. Reduction of the measured space angles to the horizon

As all angles were measured in the plane through the observation station and the two sighting points, these angles had to be reduced to the horizon. Nowhere in his Précis Historique Krayenhoff gives a consideration on the computation of such a reduction. Only in a detailed example relating to the station Amsterdam on the pages 27-29 he informs us amply how the corrections to the space angles must be computed in order to find the horizontal angles.

For the mathematical background one must consult Delambre's work "Méthodes analytiques pour la détermination d'un arc de méridien" (Paris, an VII). In this book Delambre gives the derivation of a great number of formulae used by him for the computation of his triangulation. The formula relating to the reduction of the angles to the horizon can be found on the pages 11 and 12. The derivation can run as follows:

In Fig. 5 (see section 6) the space angle between the left and the right sighting point is the angle LOR. It is the arc LR = φ of the spherical triangle ZLR. In this triangle holds:

$$\cos \varphi = \cos (90 - h_1) \cos (90 - h_2) + \sin (90 - h_1) \sin (90 - h_2) \cos \alpha$$

whence:

$$\cos \alpha = \frac{\cos \varphi - \sin h_1 \sin h_2}{\cos h_1 \cos h_2} \quad \ldots\ldots\ldots\ldots \quad (4)$$

from which α can be computed. An exact computation however, takes up much time. It is therefore easier to compute the difference $\alpha - \varphi = x$ which is very small if h_1 and h_2 are small.

For these small values h_1 and h_2:

$$\sin h_1 \simeq h_1, \ \sin h_2 \simeq h_2,$$

$$\cos h_1 \simeq (1 - h_1^2)^{\frac{1}{2}} \simeq 1 - \tfrac{1}{2} h_1^2,$$

$$\cos h_2 \simeq (1 - h_2^2)^{\frac{1}{2}} \simeq 1 - \tfrac{1}{2} h_2^2, \text{ and therefore, according to (4):}$$

$$\cos (\varphi + x) \simeq \frac{\cos \varphi - h_1 h_2}{1 - \frac{1}{2}(h_1^2 + h_2^2)}, \text{ or:}$$

$$\cos \varphi - x \sin \varphi \simeq (\cos \varphi - h_1 h_2) \left\{ 1 + \tfrac{1}{2} (h_1^2 + h_2^2) \right\}$$

whence:

$$x \simeq \frac{h_1 h_2 - \frac{1}{2}(h_1^2 + h_2^2) \cos \varphi}{\sin \varphi}$$

By application of a trick one can write it as follows:

$$x \simeq \left(\frac{h_1 + h_2}{2}\right)^2 \frac{1 - \cos \varphi}{\sin \varphi} - \left(\frac{h_1 - h_2}{2}\right)^2 \frac{1 + \cos \varphi}{\sin \varphi}$$

Or, as $\frac{1-\cos\varphi}{\sin\varphi} = \tan\frac{1}{2}\varphi$ and $\frac{1+\cos\varphi}{\sin\varphi} = \cot\frac{1}{2}\varphi$,

$$x \simeq \left(\frac{h_1+h_2}{2}\right)^2 \tan\tfrac{1}{2}\varphi - \left(\frac{h_1-h_2}{2}\right)^2 \cot\tfrac{1}{2}\varphi$$

In this formula x, h_1, and h_2 are expressed in radians. If they are in seconds of arc it runs:

$$x'' \simeq \rho''\left\{\left(\frac{h_1+h_2}{2\rho}\right)^2 \tan\tfrac{1}{2}\varphi - \left(\frac{h_1-h_2}{2\rho}\right)^2 \cot\tfrac{1}{2}\varphi\right\} \quad \ldots\ldots\ldots \quad (5)$$

with $\rho = 206264.81$.

It is this formula that Delambre and therefore Krayenhoff used for the reduction of the angles φ to the horizon ($\alpha = \varphi + x$). As an example I give underneath the results of the computation of $\alpha - \varphi = x$ for the angle between Nieuwkoop (left object) and Haarlem (right object) measured in an eccentric point of the station Amsterdam. As (see table 2 in section 6) $\varphi \simeq 76^\circ 41' 24''$ ($\frac{1}{2}\varphi \simeq 38^\circ 20' 42''$), $h_1 = -455''.0$, and $h_2 = +172''.5$, $h_1 + h_2 = -282''.5$ and $h_1 - h_2 = -627''.5$, one finds $x = -0''.527$. Krayenhoff finds $-0''.534$ in the same example on page 27 of his Précis Historique. The amount was copied from the first two volumes folio mentioned in section 4 under c. The reductions of all 505 angles of the triangulation network are computed there in sequence of the number of the stations where they were measured. A small error in the small amounts h_1 and h_2 is of little influence on the computation of x from (5). That is why the h's were generally measured in one series with one or two repetitions.

11. Reduction of the measured angles to centre

In the volumes octavo I-IX kept in the library of the Leiden University Krayenhoff does not only give the observations of the angles of the network but also all the measures and angles necessary for the reduction of the measured angles to centre.

In Fig. 8, I give a reproduction of the first gallery of the Western Tower in Amsterdam (station No. 40) where six angles of the network were measured in the four eccentric points D, G, I, and A. It has been borrowed from Instructie voor de geographische ingenieurs [44] . The sketch can also be found on page 17 of Krayenhoff's Précis Historique. The drawing in the concerning octavo volume was not suitable for reproduction.

The position of D, G, I, and A in the square FBHA with sides 26.623 Paris' feet (8.648 m) (1 foot = $\frac{1}{6}$ toise = $\frac{1}{6}$ x 1.94904 m = 0.32484 m) is determined by the lengths bG = bA = 6.936 feet (HG = BA = 6.375 feet = 2.071 m), ED = 5.541 feet (1.800 m) and HI = 6.100 feet (1.982 m). Applying Pythagoras'

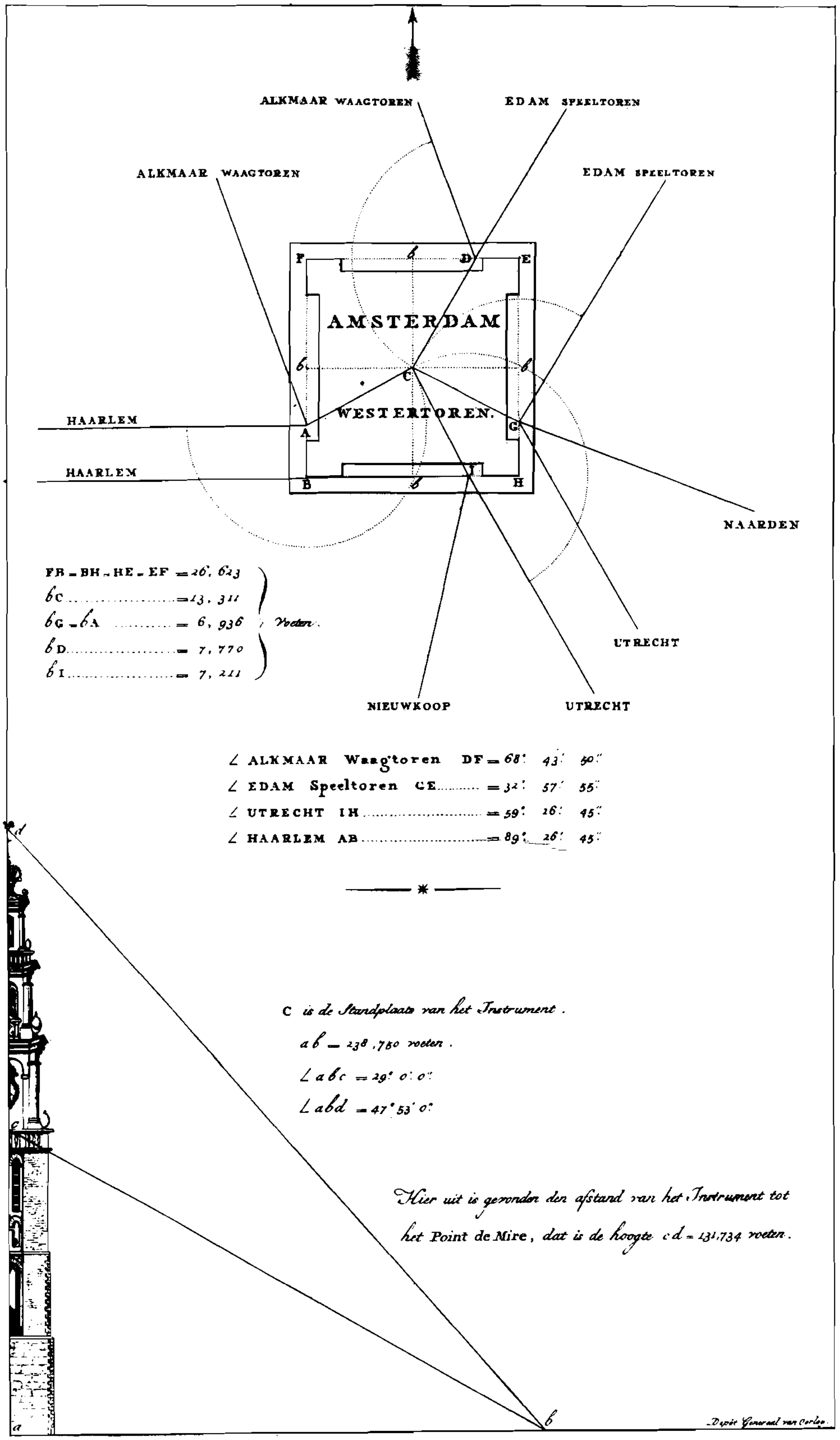

ALKMAAR WAAGTOREN
EDAM SPEELTOREN
ALKMAAR WAAGTOREN
EDAM SPEELTOREN
AMSTERDAM
WESTERTOREN.
HAARLEM
HAARLEM
NAARDEN
FB = BH = HE = EF = 26, 623
bC = 13, 311
bG = bA = 6, 936 Voeten.
bD = 7, 770
bI = 7, 211
UTRECHT
NIEUWKOOP
UTRECHT
∠ ALKMAAR Waagtoren DF = 68° 43′ 50″
∠ EDAM Speeltoren CE = 32° 57′ 55″
∠ UTRECHT IH = 59° 26′ 45″
∠ HAARLEM AB = 89° 26′ 45″
C is de Standplaats van het Instrument.
ab = 238,780 voeten.
∠ abc = 29° 0′ 0″
∠ abd = 47° 53′ 0″
Hier uit is gevonden den afstand van het Instrument tot
het Point de Mire, dat is de hoogte cd = 131,734 voeten.

Fig. 8

theorem one can compute from these measures the distances DC, GC, IC, and AC to the centre C of the square. IC e.g. is 15.139 feet = 4.918 m. The angles between the sides of the square and the sides DC, GC, IC, and AC just mentioned can also be computed. Angle CIH e.g. is $90^{\circ} + \arc\tan \frac{7.211}{13.311} \simeq 118^{\circ}26'45''$. If one finally measures in each of the four eccentric stations the angle that one of the sides of the square makes with one of the rays to the sighting points (e.g. in I the angle between H and Utrecht = $59^{\circ}16'45''$), then one has all data necessary for the reduction of the eccentric angles to centre if at least provisional distances to the surrounding sighting points are known and the assumed centre of the station coincides with the projection of the spire on the horizontal plane through the first gallery.

It should be said that by this method of centring, assuming as centre the middle of the frame of the tower at the height where the measurements were done, a deviation between this centre and the projection of the spire can never be found as the spire was not used in the measurement.

On each station of his network, however, Krayenhoff used this method. He will have seen the objections against it but he could hardly act otherwise. The repetition circle was not suitable for a centring as we do this in our days with a theodolite.

Krayenhoff computed his corrections for reduction to centre in the same way as we do this nowadays. If e is the distance from the eccentric point to "centre", φ_r and φ_l the angles between the right (left) sighting point and the centre and l_r and l_l the distances from the station to the right and left sighting point, then the correction to the measured angle is:

$$\delta_r - \delta_l = \rho \frac{e \sin \varphi_r}{l_r} - \rho \frac{e \sin \varphi_l}{l_l} \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

For $\rho = 206264.81$, $\delta_r - \delta_l$ is expressed in seconds of arc; e and l are expressed in the same unit of length. Krayenhoff used the Paris' foot for this unit. For the computation of $\delta_r - \delta_l$ for the angle between Utrecht (left) and Nieuwkoop (right), measured in I, one finds:

$$\delta_l = \rho \frac{15.139 \sin 177^{\circ}43'30''}{109\ 344} = +1''.134$$

and, in the same way:

$$\delta_r = -25''.649, \text{ whence:}$$

$$\delta_r - \delta_l = -26''.783.$$

The computation can be found as an example on page 23 of the Précis Historique. It was of course borrowed from the computation in the first of the two volumes folio at Leiden, already mentioned before.

It is unknown how Krayenhoff got acquainted with the distances l_r and l_l, necessary for the computation of the δ's. He calls them "distances droites" and "distances gauches" or abbreviates them as D and G (Précis Historique, page 21). Without any comment he mentions them in toises in his computation registers. In his circumstantial paper [20] Van der Plaats does not say a word about this question. Moor supposes [47] that they were "borrowed from former data, e.g. maps", but this cannot be so. I took the trouble to compare the provisional l's between a great number of stations with the distances computed between identical stations in the R. D.-triangulation network. A small arbitrary excerpt is given in table 7.

Table 7

From to	Provisional distances l (toises)	Distances R.D. (toises)	Differences 3 - 2 toises	Differences 3 - 2 metres
1	2	3	4	5
Amsterdam				
Alkmaar	15380	15381.42	+1.42	+2.7
Edam	9715	9716.62 *	+1.62	+3.2
Naarden	10736	10736.45	+0.45	+0.9
Utrecht	18224	18224.75	+0.75	+1.5
Nieuwkoop	13297	13298.70 *	+1.70	+3.3
Haarlem	8614	8614.69	+0.69	+1.3
Rhenen				
Nijmegen	12228	12228.76	+0.76	+1.5
's-Hertogenbosch	17831	17831.84	+0.84	+1.6
Gorinchem	22123	22123.12	+0.12	+0.2
Utrecht	17363	17363.99	+0.99	+1.9
Amersfoort	12916	12916.66	+0.66	+1.3
Veluwe	18921	18921.38	+0.38	+0.7

The two distances marked with an asterisk are not quite reliable because the spires of Edam and Nieuwkoop are not quite the same as those in Krayenhoff's time. The towers, however, remained unchanged.

As one sees the differences in columns 4 and 5 are so small that the distances in column 2 must have been borrowed from a computation, I suppose from a provisional computation of the network in which Krayenhoff did not take into account - and of course could not take into account - the eccentricity of the points where the angles were measured. In my opinion he started for this computation from the length of Delambre's side Duinkerken-Mont Cassel which was also known in toises. If so the table shows for the distance Amsterdam-Haarlem an excellent agreement with the value 4457.9 roods $\simeq$ 16788.5 metres $\simeq$ 8613.7 toises, derived from Krayenhoff's base measurement in 1800 (see section 3). I made attempts to trace the provisional computation of the network but I failed. It is neither in the archives of Topografische Dienst at Delft nor in those of the present family Krayenhoff at Amersfoort.

It will be clear that an error Δl_r and Δl_l in the distances l_r and l_l in (6) will influence the amount $\delta_r - \delta_l$. As:

$$\sin\delta = \frac{e\sin\varphi}{l}, \text{ one has:}$$

$$\cos\delta\,\Delta\delta = -\frac{e\sin\varphi}{l^2}\,\Delta l = -\frac{\sin\delta}{l}\Delta l$$

whence:

$$\Delta\delta = -\frac{\Delta l}{l}\tan\delta, \quad \text{or, as } \delta \text{ is small,} \quad \Delta\delta'' = -\frac{\Delta l}{l}\delta''.$$

For $\Delta l_l = +0.8$ toise, $l_l = 18224$ toises and $\delta_l'' = 1.''134$, $\Delta l_l = 0$

and for $\Delta l_r = +1.7$ toise, $l_r = 13297$ toises, and $\delta_r'' = -25.''649$, $\Delta l_r = +0.''003$.

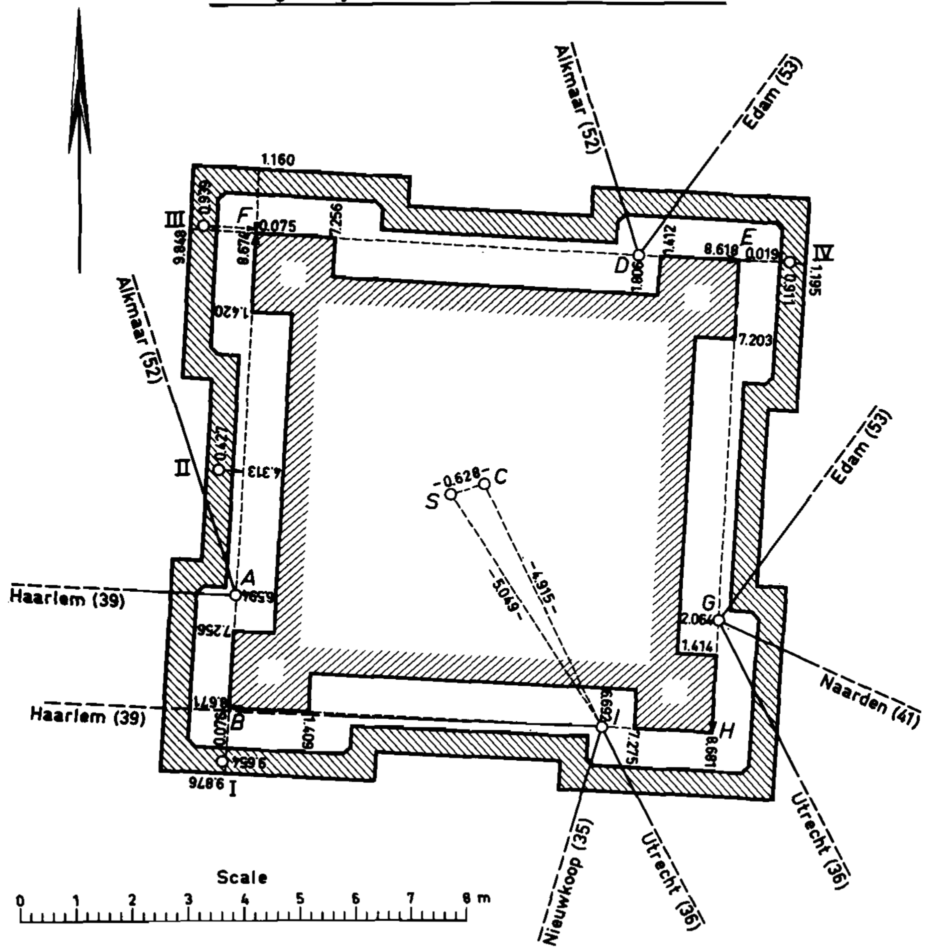
The error in the angle between Utrecht and Nieuwkoop as a result of the errors +0.8 toises and +1.7 toises in the distances Amsterdam-Utrecht and Amsterdam-Nieuwkoop is therefore +0.''003, an amount which can be neglected.

It will be clear that an error in e in consequence of the non-coincidence of the middle C of the frame of the tower and the projection S of the spire is of much more importance.

The error will be greater the more the place where the measurements were carried out is situated beneath the spire of not quite a vertical tower. It appears from the vertical measurements in Fig. 8 that for the station Amsterdam this distance dc = 131.734 Paris feet $\simeq$ 42.8 m. The height of the tower is 264.074 feet $\simeq$ 85.8 m.

At my request the Municipal Landsurveying Department in Amsterdam was so kind as to give me a sketch of the first gallery of the tower with its balustrade and to determine the vertical projection of the spire with respect to the square FBHA. This was done by computing the angular points in the coordinate system of the R.D. from the coordinates of four R.D.-marks in the balustrade of the

First gallery Western tower, Amsterdam (40)



Points	X'	Y'	Points	X'	Y'	Points	X'	Y'
I	- 34 303.40	+ 24 520.64	A	- 34 303.143	+ 24 523.687	F	- 34 302.748	+ 24 530.257
II	- 34 303.44	+ 24 525.97	B	- 34 303.267	+ 24 521.617	G	- 34 294.481	+ 24 523.174
III	- 34 303.68	+ 24 530.40	C	- 34 298.674	+ 24 525.678	H	- 34 294.600	+ 24 521.114
IV	- 34 293.19	+ 24 529.68	D	- 34 295.901	+ 24 529.829	I	- 34 296.586	+ 24 521.229
			E	- 34 294.101	+ 24 529.717	S	- 34 299.277	+ 24 525.501

Fig. 9

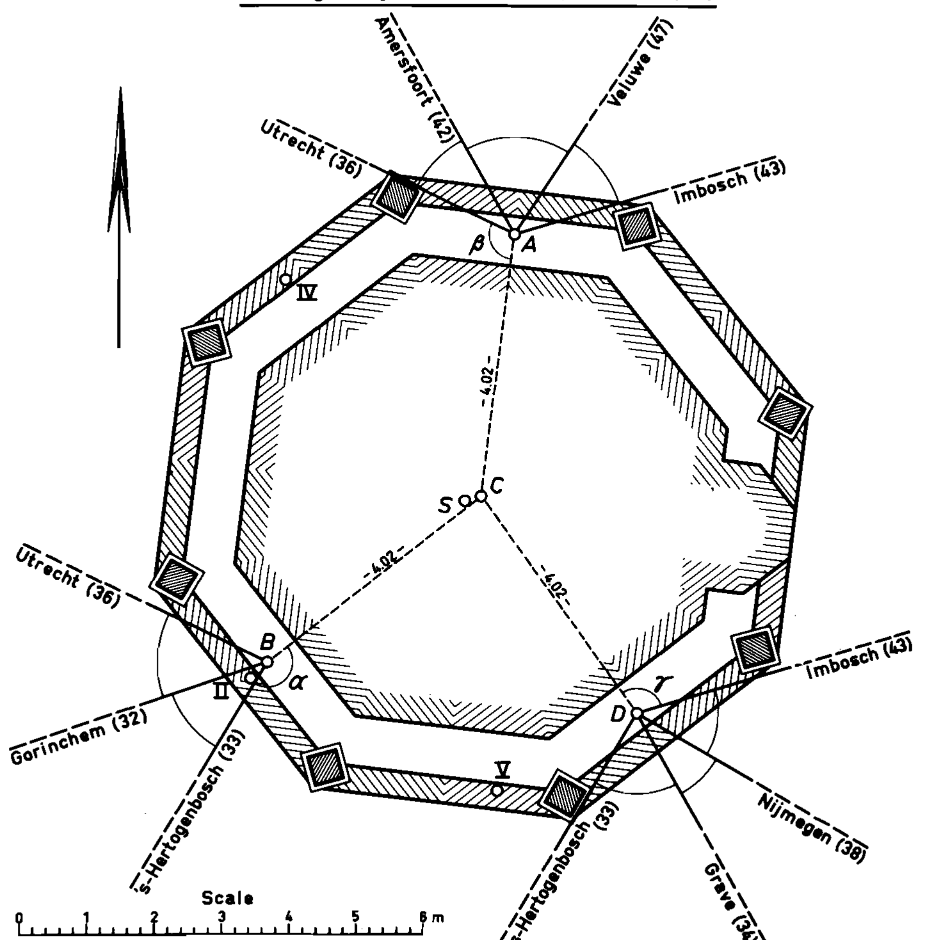
gallery. The results of this computation are mentioned in the table shown in Fig. 9. In the figure one also sees the measures (in metres) of the quadrilateral which is apparently not quite a square. The points D, G, I, and A, interpolated between the angular points of the sides on which they lie, were also computed in the coördinate system. C, Krayenhoff's "centre", was computed twice. Once as the intersection point of the diagonals, once as that of the middles of the junction lines of the opposite sides. The mean of these computations is mentioned in the table. The coordinates of S are those of the spire, determined in 1898. If one assumes that it has not changed since 1803 - there is no alternative - then the distance SC is almost 0.63 m and the eccentricity IS = 5.049 m = 15.543 feet instead of the value 15.139 feet mentioned by Krayenhoff. The angle SIC which determines φ in (6) is about $7^{\circ}04'$. In consequence of these alterations the reduction to centre for the angle between Utrecht and Nieuwkoop is $-29.''867 + 2.''451 = -27.''416$ instead of the amount $\delta_r - \delta_l = -26.''783$ found by Krayenhoff and mentioned before in this section. The difference $0.''633$ demonstrates again the disharmony between the accuracy of the measurement and the use of thousandths of a second in the computation.

As the circle through Utrecht, Nieuwkoop and C passes about through S the difference found there is still rather small. The large error in the eccentricity will find better expression in the reduction to centre of the angle measured in A between Haarlem (left) and Alkmaar (right). Krayenhoff finds for this reduction $-33.''310 + 27.''166 = -6.''144$. It should be $-29.''093 + 24.''729 = -4.''364$, a difference of $1.''780$.

From the observations follows that the sum of the three angles H-I-Utrecht. Utrecht-I-Nieuwkoop and Nieuwkoop-I-Haarlem is about $179^{\circ}11'23''$, about $48'37''$ less than 180°. One computes from it that the ray I-Haarlem is about 0.09 m free from the angular point B of the tower. From the difference of the grid-bearings I-B and I-Haarlem (Haarlem is an identical point, determined by the R.D. in 1898) I found 0.04 m. The two amounts with a mean of about 0.06 m show that the accuracy of the angle between Nieuwkoop and Haarlem will have been influenced in a serious manner by lateral refraction because during the measurement of the series in the afternoon (June 14th, 18.15 hours, June 28th, 19.00 hours, and June 29th, 17.45 hours) the southern wall of the tower will have radiated the heat of the sun.

Figure 10 is another example of reduction to centre applied by Krayenhoff. It concerns his measurements on the third gallery of the Cunera tower at Rhenen (station No. 37), 49.08 feet (about 15.9 metres) beneath the spire. In the eccentric stations A, B, and D the angles indicated with arcs were measured there on August 4th, 5th, and 6th, 1805. In order to reduce them to centre Krayenhoff measured in B also the angle α between the "centre" C and 's-Hertogenbosch, in A the angle β between C and Utrecht and in D the angle γ between C and Imbosch. The amounts for these angles and the distances

Third gallery Cunera tower, Rhenen (37)



Points	X'	Y'	Points	X'	Y'	CA = CB = CD = 12.375 Paris feet = 4.02 m
II	+ 12160.468	- 22041.648	A	+ 12164.38	- 22034.93	
IV	+ 12160.964	- 22035.614	B	+ 12160.71	- 22041.40	α = 158°35'00"
V	+ 12164.139	- 22043.350	C	+ 12163.88	- 22038.92	β = 109°04'00"
S	+ 12163.650	- 22038.969	D	+ 12166.23	- 22042.18	γ = 108°43'00"

Fig. 10.

CA=CB=CD can of course be found in the volumes octavo (volume III) mentioned in section 4 under a (see table 3) and in the first volume folio (page 319) mentioned under c. I have copied them in Fig. 10. Krayenhoff's sketch in the two volumes is very plain, even without measures where the points A, B, and D lie with respect to the sides of the regular octagon of the frame of the tower. From this sketch it appears that the centre of the octagon is supposed to coincide with the projection of the spire. The lines CA, CB, and CD on the sketch are drawn perpendicular to the relating sides of the octagon. Though the tower was partially destroyed in the world war 1940–1945, I could reconstruct the situation of the points C, A, B, and D on a map at the scale 1 to 50 and compute these points in the R. D.-coordinate system with an error estimated at maximum 2 cm. The data available for this reconstruction were a map 1 to 100 of the R. D., dated 1898, with measures and coordinates of some marks (copper bolts) in the balustrade of the third gallery and a very detailed map 1 to 20 of "Monumentenzorg" (Netherlands State Service for the preservation of historical monuments). The coordinates of the bolts II, IV, and V and those of the spire in 1895 are mentioned in the left part of the table in Fig. 10, those of C, A, B, and D borrowed from my reconstruction on the scale 1 to 50 in the right part.

As can be seen, the difference between Krayenhoff's assumed centre C and the projection of the spire is about 24 cm, much less than the corresponding difference in Amsterdam. If we assume that the spire of 1895 was also Krayenhoff's sighting point - and here too is no alternative - then the error in the reduction to centre in consequence of a wrong e will be of the most importance for the two angles measured in B. For the angle between Utrecht and Gorinchem the error is about $1.''0$, for that between Gorinchem and 's-Hertogenbosch about $0.''9$. A more accurate computation is senseless because of the small uncertainty of the reconstruction of the points C and B. As already remarked in connection with table 7 the influence on the reduction because of an error in the distances to the several sighting points is of little importance.

As in D the rays to Imbosch and 's-Hertogenbosch and in B that to Utrecht pass close along the pinnacles on the corners of the balustrade, influence of lateral refraction on the results of the angular measurement might be possible. As in A the ray to Utrecht passes along the northern side of such a pinnacle lateral refraction seems less probable though the angle between Utrecht and Amersfoort (167 eccentric) is very bad indeed (two series measured but only one retained). The differences in the measurements of the series of the angle in D between Imbosch and Nijmegen (177 eccentric), however, are small (see the small amount [vv] in table 6 of section 8). Those in D between Grave and 's-Hertogenbosch (145 eccentric) and in B between Gorinchem and Utrecht (140 eccentric; 2 series retained and 1 rejected), on the contrary, are large.

12. Reduction of the spherical angles, reduced to horizon and centre, to angles between the chords on the sphere

As all the angles measured in the several eccentric stations are angles on the curved earth, the angles reduced to horizon and centre are also angles on the geoid or, approximately, angles on the ellipsoid which, in its turn, can be replaced by a sphere that, in the area where the triangulation is situated, touches the ellipsoid as well as possible. In my further computations I therefore assumed that the network lies on the osculating sphere at Amersfoort to Bessel's ellipsoid, the ellipsoid on which the R. D.-triangulation network was computed. This sphere has a radius of 6382646 m. In this admissible assumption angles between the geodesics on the ellipsoid are replaced by angles between great circles on the sphere.

For the computation of the lengths of the sides of Krayenhoff's network one can set about as follows:

a One computes the sides with spherical trigonometry. The objection to this working method is that one finds their lengths in arc measure which must be converted into metres,

b One computes according to Legendre's method [48] , that is to say, one diminishes each of the angles of a spherical triangle with one third of its spherical excess and computes with plane trigonometry from the length of a spherical side the lengths of the other spherical sides,

c One reduces the spherical angles to angles of plane triangles, the sides of which are the chords on the sphere. From the length of one chord all the chords can be computed with plane trigonometry.

For the computation of Delambre's triangulation all three methods were used, especially that mentioned under a. For present computations that under b is mostly used. That under c is no more used nowadays but Krayenhoff applied it for the computation of his triangulation.

The theoretical background of the reduction of spherical angles to plane angles between the chords can be found on page 40 of Delambre's "Méthodes analytiques", already mentioned before. Another derivation is the the following:

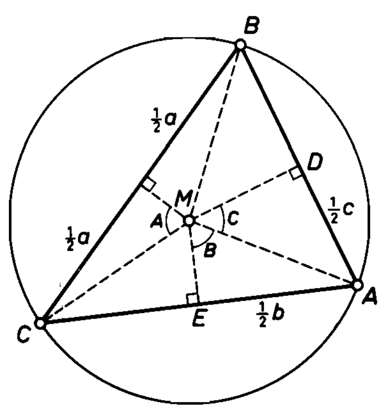


Fig. 11a

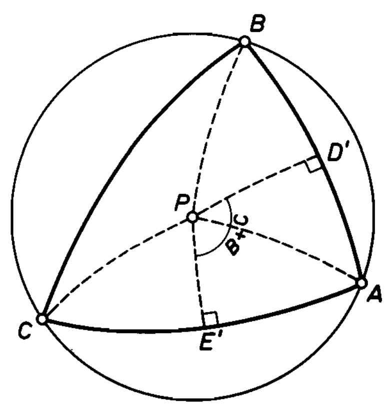


Fig. 11b

In Fig. 11a M is the centre of the circumscribed circle of triangle ABC. A, B, and C are the angles of the plane triangle; the sides are chords. In Fig. 11b P is the pole of M, the point where the line in M perpendicular to ABC in Fig. 11a intersects the sphere. PD' and PE' are parts of great circles perpendicular to the spherical sides AB and AC. MD and ME in Fig. 11a are perpendicular to AB and AC respectively.

As the tangents in P to the sides PD' and PE' of the spherical quadrilateral PD'AE' are parallel to MD and ME respectively, angle D'PE' = angle DME. As also D' = D = 90° and E' = E = 90° the difference between the angle A on the sphere and the angle A of the plane triangle is equal to the spherical excess E of the quadrilateral PD'AE'. This spherical excess can be expressed by the formula $E_{rad} = O{:}R^2$ or $E'' = \rho O{:}R^2$ with $\rho = 206264.81$ in which O is the area of PD'AE' and R the radius of the sphere.

O is approximately equal to that of the plane quadrilateral MDAE. The area last mentioned can be easily computed. For:

$$2\,O_{MDAE} = O_{ABC} - O_{BMC} = \tfrac{1}{2}\,bc \sin A - \tfrac{1}{4}\,a^2 \cot A$$

In this formula a can be expressed with the cosine rule in b, c, and A:

$$2\,O_{MDAE} = \tfrac{1}{2}bc \sin A - \tfrac{1}{4}\cot A\,(b^2 + c^2 - 2bc \cos A), \text{ or:}$$

$$16\,O_{MDAE} = \frac{4bc \sin^2 A - 2 \cos A\,(b^2 + c^2) + 4\,bc \cos^2 A}{\sin A}$$

$$= \frac{4bc - 2(b^2 + c^2) \cos A}{\sin A}$$

$$= \frac{4bc - 4bc \cos A - 2(b^2 + c^2 - 2bc) \cos A}{2 \sin \frac{1}{2}A \cos \frac{1}{2}A}$$

$$= \frac{2bc\,(1 - \cos A) - (b - c)^2 \cos A}{\sin \frac{1}{2}A \cos \frac{1}{2}A}$$

$$= \frac{4\,bc \sin^2 \frac{1}{2}A - (b - c)^2 (\cos^2 \frac{1}{2}A - \sin^2 \frac{1}{2}A)}{\sin \frac{1}{2}A \cos \frac{1}{2}A}$$

$$= \frac{\{4\,bc \sin^2 \frac{1}{2}A + (b-c)^2 \sin^2 \frac{1}{2}A\} - (b-c)^2 \cos^2 \frac{1}{2}A}{\sin \frac{1}{2}A \cos \frac{1}{2}A}$$

$$= \frac{(b+c)^2 \sin^2 \frac{1}{2}A - (b-c)^2 \cos^2 \frac{1}{2}A}{\sin \frac{1}{2}A \cos \frac{1}{2}A}$$

$$= (b+c)^2 \tan \tfrac{1}{2}A - (b-c)^2 \cot \tfrac{1}{2}A,$$

so that:

$$E''_{MDAE} = \frac{\rho''}{16R^2} \left\{ (b+c)^2 \tan \tfrac{1}{2}A - (b-c)^2 \cot \tfrac{1}{2}A \right\}$$

or:

$$y'' = -E''_{MDAE} = -\rho'' \left\{ \left(\frac{b+c}{4R}\right)^2 \tan \tfrac{1}{2}A - \left(\frac{b-c}{4R}\right)^2 \cot \tfrac{1}{2}A \right\} \quad \ldots\ldots\ldots \quad (7)$$

y in this formula is the correction to the spherical angle A in order to find the angle A between the chords. It resembles very much formula (5), found in section 10 for the computation of the reduction x of a space angle to the horizon:

$$x'' = \rho'' \left\{ \left(\frac{h''_1 + h''_2}{2\rho''} \right)^2 \tan \tfrac{1}{2}\varphi - \left(\frac{h''_1 - h''_2}{2\rho''} \right)^2 \cot \tfrac{1}{2}\varphi \right\}$$

Krayenhoff computed therefore both amounts x and y immediately after each other. For this computation he used three tables, already designed by Delambre for his own triangulation:

a a table for $\left(\frac{b \pm c}{4R}\right)^2$ (Krayenhoff calls b and c, P and Q respectively),

b a table for $\left(\frac{h''_1 \pm h''_2}{2\rho''}\right)^2$

c a table for $\rho \tan \frac{1}{2}\varphi$ (A) and $\rho \cot \frac{1}{2}\varphi$ (A)

As one sees the table under c serves the computation of x as well as that of y. Using these tables the computation of the two corrections was reduced to two multiplications and two subtractions.

In table a the amounts b $\pm$ c and R were expressed in toises. As according to Delambre's triangulation one degree of the meridian was about 57020 toises, R for the French triangulation was about 3267000 toises (6367500 m). Krayenhoff used the same value though he should have known that, as a result of the flattening of the earth, it had to be taken larger. For the osculating sphere at Amersfoort to Bessel's ellipsoid it is, as already remarked, 6382646 m. Because of this error Krayenhoff's amounts y, mentioned in tableau I of his Précis Historique, will have the tendency to be too large.

The number of decimals in Delambre's tables was insufficient for the correct computation of the hundredths of a second. Delambre knew this. It seems, however, that Krayenhoff has not realised it in his computation in thousandths of a second. Moreover he made in his computations a great number of small errors which he could have found if he had compared the sum of the y's in a triangle with the amount $-E''_{ABC} = -\frac{1}{2}\rho''\text{bc} \sin A : R^2$, the opposite of the spherical excess of triangle ABC in Fig. 11. For all his triangles these spherical excesses are mentioned in tableau III of the Précis Historique.

Table 8

Station		Angle		Précis Historique		correct
No.	Name	No.	between	page	amount y	amounts y
1	2	3	4	5	6	7
23	Breda	103	Hilvarenbeek-'s-Hertogenbosch	53	-0''443	-0''424
24	Hilvarenbeek	104	's-Hertogenbosch-Breda	53	-0''953	-0''937
33	's-Hertogenbosch	105	Breda-Hilvarenbeek	56	-0''416	-0''420
			Triangle 37		-1''812	-1''781
32	Gorinchem	138	Rhenen-Utrecht	56	-0''745	-0''729
36	Utrecht	139	Gorinchem-Rhenen	57	-1''190	-1''178
37	Rhenen	140	Utrecht-Gorinchem	57	-0''731	-0''715
			Triangle 49		-2''666	-2''622
91	Midwolda	424	Leer-Emden	75	-0''310	-0''367
94	Emden	425	Midwolda-Leer	76	-0''566	-0''562
95	Leer	426	Emden-Midwolda	77	-0''319	-0''371
			Triangle 148		-1''195	-1''300

Some arbitrary examples may make this clear (see table 8). They relate to the triangles 37, 49, and 148. Columns 1 and 2 give the number and the name of the station, columns 3 and 4 the angles with their numbers and column 5 the reference to the Précis Historique where the amounts y in column 6 can be found. Column 7 finally gives the correct amounts y. Their sum in every triangle agrees with the opposite of the spherical excess. In column 6, however, this is not the case. For triangle 37 ([y]= -1''812). Krayenhoff finds on page 122: -E = -1''778. For triangle 49 [y] = -2''666 and (on page 125): -E = -2''618. For triangle 148 Krayenhoff's amount [y]= -1''195 agrees with -E on page 145 but this must be chance; the real E of the triangle is 1''300.

As one sees the correct y's in column 7 differ in some cases considerably from those in column 6, considerably if one sees them - it must be said again - in the light of a computation in thousandths of a second. It will be clear that for sharp-angled triangles y is always negative. If, however, in Fig. 11a M lies outside the triangle - it is then obtuse-angled - then the correction y to a spherical angle can be positive. In triangle 155 e.g. (angle 447 $\simeq 122^\circ 42'$) the correction to angle 446 in Westerstede is + 0''071. Krayenhoff finds on page 78: + 0''074.

The simultaneous computation of the corrections x and y involves that - in contradistinction to the logical sequence in the sections 10 and 11 - Krayenhoff computed first the reduction to centre and thereafter the reduction x to the horizon. The objection to this working method is more of a theoretical than of a practical character: because of the small amounts x in (5) (h_1 and h_2 are small) the influence of the errors in φ_r and φ_l on the computation of $\delta_r - \delta_l$ in (6) is of hardly any importance.

13. Conditions the angles of the triangulation network have to comply with

In section 4 (see Fig. 2) I already said that in the first order triangulation network 505 angles were measured. If the points Petten, Schiermonnikoog, Aschendorf, Stolham, and Wangeroge are left out of consideration (there is no check on these points), then the network consists of 106 angular points, inclusive of the stations Duinkerken and Mont Cassel from which Krayenhoff started his computation. In three of them - Herentals, Biesselt, and Borkum - no measurement took place. Inclusive of the "base line" Duinkerken-Mont Cassel the network has 251 sides (l=251) which border polygons, all the angles of which were measured. The angular points of these polygons are the 106-3=103 stations just mentioned (p=103). For that reason the number of polygon conditions is l-p+1=251-103+1=149.

The number of station equations is 73. For in each of the stations 12, 14, 17, 18, 22-25, 28-44, 46-50, 52-57, 59-61, 63, 66-70, 72, 74-76, 79-96, and 98-103 the sum of the spherical angles, reduced to horizon and centre must be 360°.

As the network has 263 sides (L=263) and 106 angular points (P=106) the number of side equations is L-2P+3=263-212+3= 54. That's why there are 149+73+54=276 conditions the angles have to comply with.

This number can be checked as follows:
A number of the 505 angles of the network was only measured in order to form a "tour d'horizon" in a station. All these stations lie along the borders of the network. At Varel e.g. (station No. 103) not only was the angle 462 of triangle 161 measured but also the angles 463 up to and including 466. As Stolham, Zandstedt, and Neuenburg, however, are no points of the network, the results of the computation would have been the same if angle 466 would have been replaced by an angle alike to the sum of the angles 463-466. For the determination of the number of redundant angles in the network the number of 505 should therefore, as far as Varel is concerned, be diminished with 3 (the angles 463, 464, and 465). In Fig. 2 they are marked with a double instead of a single arc. In the same way the number of angles with a double arc at Westerstede (No. 100), Barssel (No. 96),

and Leer (No. 95) is 2, 2 and 1 respectively, etc. Their total number is 21. As the number of stations that must be determined is 106-2=104 and every new station is determined by two angles the number of redundant angles is 505-21-208=276, of course the same as the number of conditions just found.

148 out of 149 polygon conditions are triangle conditions: in each of the spherical triangles 2-19, 22-28, 30-40, 42-52, 54-78, 80-83, 85-128, 130-136, 138-143, 146-155, and 157-161 the sum of the spherical angles reduced to horizon and centre must be 180° plus the spherical excess of the triangle. In Krayenhoff's system of conditions one finds that in every triangle the sum of the angles reduced to horizon, centre, and chords is 180°.

If p_i (i= 1, 2, ..., 505) are the corrections to the spherical angles one finds e. g. for triangle 49:

$$(51^\circ 15' 01''.187+p_{138}) + (83^\circ 32' 16''.923+p_{139}) + (45^\circ 12' 44''.843+p_{140}) = 180^\circ 00' 02''.622$$

or:

$$p_{138} + p_{139} + p_{140} + 0''.331 = 0$$

for the spherical excess of the triangle is 2''.622 (see table 8).

The 149th polygon condition can be found from the spherical polygon Naarden-Edam-Enkhuizen-Urk-Kampen-Harderwijk around the former Zuiderzee (see Fig. 2). The sum of its angles must be 720° plus its spherical excess E. As the spherical excess of the triangles Naarden-Edam-Enkhuizen, Naarden-Enkhuizen-Urk, Naarden-Urk-Harderwijk, and Harderwijk-Urk-Kampen is 1''.438, 2''.441, 2''.758, and 1''.887, respectively, E = 8''.524. From Krayenhoff's observations in tableau I of his Précis Historique, arranged in table 9 (column 8) one finds then:

$$p_{240} + p_{239} + p_{478} + p_{475} + p_{481} + p_{484} - p_{279} - p_{280} - p_{283} - p_{286} - 7''.706 = 0$$

In the columns 5-7 of table 9, I give for every angle also a survey of the series measured, rejected and retained. The totals of these series were already mentioned in table 3.

The spherical angles in columns 11 and 12 are borrowed from Krayenhoff's Tableau définitif des triangles in part III (pages 115-148) of his book. They are the results of an "adjustment" of the angles in column 8. The amounts p' in column 9 (in seconds of arc) are the corrections to the observations in order to find the "adjusted" angles. Krayenhoff does not mention them in his book. They give, however, an excellent survey of the size of the corrections. Sometimes the amount of an adjusted angle does not occur in tableau III of the Précis Historique.

Table 9

Stations				Series						adj. spherical angles		
No.	Name	No. angle	No. triangle	measured	rejected	retained	Observed sph. angles reduced to horizon and centre	Corrections (sec. of arc)		Précis Historique		least squares
								p'	p	° '	"	"
1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
1	Duinkerken (main tower)	4	3	3	0	3	43 51 34.238	———	+0.657	4351	34.238	34.895
		1	2	3	0	3	51 07 03.361	———	-0.464	5107	03.361	02.897
		2		6	0	6						
2	Mont Cassel (tower Notre Dame)	2	2	4	2	2	35 21 23.420	-0.697	-0.464	3521	22.723	22.956
3	Hondschoote (tower)	7	4	3	1	2	38 12 04.083	+0.203	+0.203	3812	04.286	04.286
		5	3	4	2	2	102 48 10.174	+0.985	+0.657	10248	11.159	10.831
		3	2	2	0	2	93 31 35.472	-0.696	-0.464	9331	34.776	35.008
		3		9	3	6						
4	Nieuwpoort (tower)	10	5	4	1	3	96 31 19.610	+0.384	+0.385	3631	19.994	19.995
		8	4	3	0	3	72 06 23.004	+0.202	+0.203	7206	23.206	23.207
		6	3	4	2	2	33 20 14.407	+0.984	+0.656	3320	15.391	15.063
		3		11	3	8						
5	Diksmuide (tower)	9	4	2	0	2	69 41 32.944	+0.203	+0.203	6941	33.147	33.147
		11	5	2	0	2	47 00 49.849	+0.384	+0.384	4700	50.233	50.233
		13	6	3	1	2	42 07 50.732	+0.010	+0.011	4207	50.742	50.743
		16	7	3	1	2	59 38 07.335	-0.248	-0.247	5938	07.087	07.088
		4		10	2	8						
6	Oostende (tower)	14	6	2	0	2	93 57 52.236	+0.011	+0.011	9357	52.247	52.247
		12	5	3	0	3	36 27 49.930	+0.384	+0.384	3627	50.314	50.314
		2		5	0	5						
7	Brugge (main tower)	25	10	3	0	3	50 54 24.760	+0.684	+0.229	5054	25.444	24.989
		22	9	3	0	3	46 46 43.116	-0.462	-0.308	4646	42.654	42.808
		19	8	2	0	2	38 38 56.617	+0.056	+0.056	3838	56.673	56.673
		17	7	2	0	2	31 12 30.031	-0.248	-0.247	3112	29.783	29.784
		15	6	3	0	3	43 54 18.195	+0.010	+0.011	4354	18.205	18.206
		5		13	0	13						
8	Hooglede (tower)	18	7	3	1	2	89 09 24.522	-0.247	-0.247	8909	24.275	24.275
		20	8	3	0	3	60 28 54.848	+0.056	+0.056	6028	54.904	54.904
		2		6	1	5						

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
9	Tielt (tower)	21	8	2	0	2	80 52 09.417	+0.056	+0.057	8052	09.473	09.474
		23	9	3	1	2	95 11 15.090	-0.463	-0.308	9511	14.627	14.782
		2		5	1	4						
10	Gent (main tower)	24	9	2	0	2	38 02 04.464	———	-0.307	3802	04.464	04.157
		26	10	2	0	2	25 13 03.257	———	+0.228	2513	03.257	03.485
		28	11	3	0	3	43 56 12.180	———	-0.060	4356	12.180	12.120
		38	14	2	0	2	36 30 14.930	———	+0.578	3630	14.930	15.508
		42	15	2	0	2	26 10 39.576	———	-0.122	2610	39.576	39.454
		5		11	0	11						
11	Aardenburg (reformed church)	31	12	2	0	2	78 29 46.076	———	-0.282	7829	46.076	45.794
		29	11	2	0	2	38 11 30.503	+1.192	+1.030	3811	31.695	31.533
		27	10	2	0	2	103 52 32.617	———	+0.229	10352	32.617	32.846
		3		6	0	6						
12	Assenede (catholic church)	36	13	4	0	4	91 53 46.218	-0.345	-0.049	9153	45.873	46.169
		39	14	3	1	2	111 17 53.274	———	-0.476	11117	53.274	52.798
		30	11	2	0	2	97 52 17.551	-0.355	-0.131	9752	17.196	17.420
		33	12	1	0	1	58 56 04.360	-0.703	-0.747	5856	03.657	03.613
		4		10	1	9	360 00 01.403	-1.403	-1.403	36000	00.000	00.000
13	Middelburg (abbey-tower)	44	16	4	2	2	77 21 50.324	———	+0.286	7721	50.324	50.610
		34	13	2	0	2	33 52 06.765	-0.344	+0.028	3352	06.421	06.793
		32	12	3	1	2	42 34 11.773	———	+0.328	4234	11.773	12.101
		3		9	3	6						
14	Hulst (Willebrordus church)	43	16	4	2	2	38 05 46.579	+0.272	+0.835	3805	46.851	47.414
		46	17	2	0	2	47 29 05.400	———	+0.502	4729	05.400	05.902
		49	18	3	1	2	71 01 48.216	———	-0.387	7101	48.216	47.829
		40	15	2	0	2	116 57 17.281	———	+0.086	11657	17.281	17.367
		37	14	2	0	2	32 11 52.788	+0.010	-0.090	3211	52.798	52.698
		35	13	2	0	2	54 14 07.427	+2.027	+1.363	5414	09.454	08.790
		6		15	3	12	359 59 57.691	+2.309	+2.309	36000	00.000	00.000
15	Antwerpen (main tower)	51	18	2	0	2	60 00 37.149	-0.896	-0.139	6000	36.253	37.010
		53	19	2	0	2	65 40 32.739	+0.821	-0.072	6540	33.560	32.667
		41	15	2	0	2	36 52 05.744	-0.670	-0.631	3652	05.074	05.113
		55	20	2	0	2	48 05 13.808	+1.000	+0.582	4805	14.808	14.390
		4		8	0	8						

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
16	Zierikzee (new church)	59	22	5	1	4	51 16 31.236	+0.750	-0.138	5116	31.986	31.098
		63	23	5	1	4	41 11 24.955	+1.542	+0.637	4111	26.497	25.592
		47	17	3	1	2	43 01 13.763	-1.409	-1.635	4301	12.354	12.128
		45	16	5	3	2	64 32 24.164	+1.237	+0.392	6432	25.401	24.556
		4		18	6	12						
17	Bergen op Zoom (reformed church)	50	18	2	0	2	48 57 37.261	—	-0.368	4857	37.261	36.893
		48	17	2	0	2	89 29 44.515	—	-0.272	8929	44.515	44.243
		64	23	2	0	2	81 16 25.064	-0.151	+0.728	8116	24.913	25.792
		66	24	2	0	2	47 15 25.087	—	-0.418	4715	25.087	24.669
		68	25	2	0	2	34 51 50.312	—	-0.420	3451	50.312	49.892
		52	19	2	0	2	58 08 57.352	+0.560	+1.159	5808	57.912	58.511
		6		12	0	12	359 59 59.591	+0.409	+0.409	36000	00.000	00.000
18	Hoogstraten (catholic church)	56	20	5	1	4	63 13 31.656	—	-0.483	6313	31.656	31.173
		54	19	4	2	2	56 10 30.861	—	+0.298	5610	30.861	31.159
		70	25	3	0	3	74 47 34.387	-1.750	-1.267	7447	32.637	33.120
		72	26	3	0	3	67 31 20.833	—	+0.077	6731	20.833	20.910
		74	27	2	0	2	46 10 29.207	-0.898	-0.624	4610	28.309	28.583
		57	21	2	0	2	52 06 35.704	—	-0.649	5206	35.704	35.055
		6		19	3	16	360 00 02.648	-2.648	-2.648	36000	00.000	00.000
19	Lommel (catholic church)	58	21	3	1	2	36 38 22.186	-1.623	-0.598	3638	20.563	21.588
		76	27	3	1	2	39 54 38.875	—	-0.095	3954	38.875	38.780
		78	28	4	1	3	63 50 11.437	-0.600	+0.068	6350	10.837	11.505
		3		10	3	7						
20	Nederweert (catholic church)	81	29	3	1	2	85 10 07.255	-1.500	-0.512	8510	05.755	06.743
		83	30	3	0	3	44 23 21.748	+1.315	+0.546	4423	23.063	22.294
		2		6	1	5						
21	Brielle (Catherina church)	85	31	2	1	1	57 30 31.318	+5.242	+2.630	5730	36.560	33.948
		88	32	2	1	1	56 21 12.969	—	-0.728	5621	12.969	12.241
		61	22	1	0	1	70 34 24.678	-1.707	-0.598	7034	22.971	24.080
		3		5	2	3						
22	Willemstad (reformed church)	65	24	3	1	2	89 20 33.430	—	-0.090	8920	33.430	33.340
		62	23	3	1	2	57 32 10.481	—	+0.028	5732	10.481	10.509
		60	22	1	0	1	58 09 07.407	—	-0.216	5809	07.407	07.191
		90	32	2	0	2	46 10 29.862	—	+0.991	4610	29.862	30.853
		93	33	2	0	2	41 06 44.271	+0.017	-1.054	4106	44.288	43.217
		94	34	2	1	1	67 40 54.532	—	+0.358	6740	54.532	54.890
		6		13	3	10	359 59 59.983	+0.017	+0.017	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
23	Breda (great church)	71	26	3	0	3	67 56 48.937	+1.378	+0.642	6756	50.315	49.579
		69	25	3	1	2	70 20 38.816	———	-0.061	7020	38.816	38.755
		67	24	3	0	3	43 24 04.399	-1.319	-0.809	4324	03.080	03.590
		96	34	4	1	3	46 10 10.812	-1.723	-1.084	4610	09.089	09.728
		98	35	3	0	3	44 32 14.595	+0.389	+0.326	4432	14.984	14.921
		100	36	4	2	2	46 10 45.883	+1.906	+1.530	4610	47.789	47.413
		103	37	4	2	2	41 25 15.627	+0.300	+0.386	4125	15.927	16.013
		7		24	6	18	359 59 59.069	+0.931	+0.931	36000	00.000	00.000
24	Hilvarenbeek (catholic church)	106	38	3	0	3	63 16 04.874	+1.630	+0.400	6316	06.504	05.274
		77	28	3	0	3	65 27 16.041	———	-0.211	6527	16.041	15.830
		75	27	5	1	4	93 54 55.522	-0.531	-0.708	9354	54.991	54.814
		73	26	4	0	4	44 31 50.208	———	+0.661	4431	50.208	50.869
		104	37	4	0	4	92 49 53.184	-0.928	+0.029	9249	52.256	53.213
		5		19	1	18	359 59 59.829	+0.171	+0.171	36000	00.000	00.000
25	Helmond (St. Lambertus church)	108	38	2	0	2	42 44 45.790	———	+0.537	4244	45.790	46.327
		110	39	3	0	3	56 21 03.842	+1.285	+1.118	5621	05.127	04.960
		112	40	3	0	3	51 20 22.098	———	-0.217	5120	22.098	21.881
		82	30	2	0	2	101 29 46.907	-1.344	-0.196	10129	45.563	46.711
		80	29	2	0	2	57 21 25.713	———	-0.846	5721	25.713	24.867
		79	28	3	1	2	50 42 35.777	-0.068	-0.523	5042	35.709	35.254
		6		15	1	14	360 00 00.127	-0.127	-0.127	36000	00.000	00.000
26	Vierlingsbeek (catholic church)	84	30	2	0	2	34 06 52.917	———	-0.376	3406	52.917	52.541
		114	40	3	0	3	72 13 43.931	+0.826	+0.939	7213	44.757	44.870
		115	41	3	0	3	23 57 11.703	-0.880	-1.145	2357	10.823	10.558
		3		8	0	8						
27	Den Haag (St. Jacobs tower)	118	**42**	3	0	3	89 24 20.957	+2.143	+1.066	8924	23.100	22.023
		87	31	6	2	4	62 23 14.662	———	+1.387	6223	14.662	16.049
		2		9	2	7						
28	Rotterdam (St. Laurens church)	89	32	1	0	1	77 28 19.415	-0.829	-1.090	7728	18.586	18.325
		86	31	1	0	1	60 06 09.816	———	+1.226	6006	09.816	11.042
		117	42	1	0	1	36 09 43.225	+0.306	+0.334	3609	43.531	43.559
		120	43	1	0	1	56 16 44.929	———	-1.920	5616	44.929	43.009
		124	44	1	0	1	77 22 21.592	+0.611	+0.935	7722	22.203	22.527
		91	33	1	0	1	52 36 40.963	-0.028	+0.575	5236	40.935	41.538
		6		6	0	6	359 59 59.940	+0.060	+0.060	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
29	Dordrecht (great church; centre)	92	33	2	0	2	86 16 35.683	-0.029	+0.440	8616	35.654	36.123
		125	44	2	1	1	54 15 22.676	-1.700	-1.328	5415	20.976	21.348
		126	45	2	0	2	76 18 37.923	+1.416	+1.259	7618	39.339	39.182
		97	35	4	2	2	77 00 26.407	———	+0.312	7700	26.407	26.719
		95	34	2	0	2	66 08 56.624	+1.000	+0.005	6608	57.624	56.629
		5		12	3	9	359 59 59.313	+0.687	+0.688	36000	00.000	00.001
30	Leiden (tower of the former "Saaihal")	470	—	3	1	2	63 14 46.155	. . . .	-0.938	6314	. . . .	45.217
		153	54	3	1	2	70 05 48.204	———	+0.920	7005	48.204	49.124
		130	46	3	2	1	45 21 05.907	-2.580	-0.828	4521	03.327	05.079
		121	43	5	4	1	43 58 05.908	+1.600	+2.884	4358	07.508	08.792
		119	42	5	1	4	54 25 55.896	-1.700	-0.650	5425	54.196	55.246
		471	—	3	0	3	82 54 17.480	. . . .	-0.938	8254	. . . .	16.542
		6		22	9	13	359 59 59.550	+0.450	+0.450	36000	00.000	00.000
31	Gouda (St. John church)	129	46	3	1	2	59 43 08.897	+0.650	+1.471	5943	09.547	10.368
		132	47	3	1	2	55 23 26.199	+0.649	+0.299	5523	26.848	26.498
		135	48	3	1	2	65 46 31.432	———	-0.821	6546	31.432	30.611
		127	45	3	0	3	50 59 27.052	-1.044	-0.636	5059	26.008	26.416
		123	44	3	1	2	48 22 15.785	+1.806	+1.112	4822	17.591	16.897
		122	43	3	2	1	79 45 10.672	-2.098	-1.462	7945	08.574	09.210
		6		18	6	12	360 00 00.037	-0.037	-0.037	36000	00.000	00.000
32	Gorinchem (reformed church)	138	49	1	0	1	51 15 01.187	-1.369	-0.353	5114	59.818	60.834
		142	50	2	0	2	53 31 47.872	———	-0.508	5331	47.872	47.364
		101	36	2	0	2	83 02 43.733	+0.800	+1.252	8302	44.533	43.985
		99	35	2	1	1	58 27 20.846	-0.837	-1.084	5827	20.009	19.762
		128	45	2	0	2	52 41 57.129	-1.300	-1.548	5241	55.829	55.581
		136	48	2	0	2	61 01 09.789	+2.150	+2.685	6101	11.939	12.474
		6		11	1	10	360 00 00.556	-0.556	-0.556	36000	00.000	00.000
33	's-Hertogen-bosch (St. John church)	105	37	2	0	2	45 44 52.895	+0.700	-0.340	4544	53.595	52.555
		102	36	3	0	3	50 46 30.588	-0.800	+0.128	5046	29.788	30.716
		141	50	3	0	3	86 06 43.267	+1.336	+1.123	8606	44.603	44.390
		144	51	2	0	2	44 19 53.092	-0.221	-0.538	4419	52.871	52.554
		109	39	3	1	2	59 02 50.019	-0.662	-0.717	5902	49.357	49.302
		107	38	2	0	2	73 59 10.573	-0.787	-0.090	7359	09.786	10.483
		6		15	1	14	360 00 00.434	-0.434	-0.434	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
34	Grave (tower)	147	52	4	0	4	69 36 06.705	-0.330	-0.738	6936	06.375	05.967
		150	53	3	0	3	46 20 50.424	-0.564	-0.528	4620	49.860	49.896
		116	41	2	0	2	47 02 19.299	-0.745	-0.469	4702	18.554	18.830
		113	40	3	0	3	56 25 55.298	-0.400	-0.293	5625	54.898	55.005
		111	39	3	1	2	64 36 07.766	———	+0.225	6436	07.766	07.991
		146	51	3	1	2	75 58 41.229	+1.318	+1.082	7558	42.547	42.311
		6		18	2	16	360 00 00.721	-0.721	-0.721	36000	00.000	00.000
35	Nieuwkoop (tower of the former abbey)	131	46	1	0	1	74 55 45.056	+2.853	+0.280	7455	47.909	45.336
		152	54	1	0	1	67 22 13.986	-1.000	-1.452	6722	12.986	12.534
		155	55	1	0	1	36 32 08.209	+0.716	+2.599	3632	08.925	10.808
		159	56	1	0	1	89 55 42.181	-2.430	-1.806	8955	39.751	40.375
		133	47	1	0	1	91 14 12.929	-2.500	-1.982	9114	10.429	10.947
		5		5	0	5	360 00 02.361	-2.361	-2.361	36000	00.000	00.000
36	Utrecht (tower of the cathedral)	158	56	3	1	2	46 51 34.789	+1.380	+0.954	4651	36.169	35.743
		161	57	3	0	3	34 06 09.301	-1.211	-0.048	3406	08.090	09.253
		164	58	4	0	4	61 23 56.660	———	+0.731	6123	56.660	57.391
		168	59	4	0	4	47 31 22.466	-2.501	-2.380	4731	19.965	20.086
		139	49	3	0	3	83 32 16.923	———	-1.712	8332	16.923	15.211
		137	48	3	0	3	53 12 20.865	-2.394	-2.105	5312	18.471	18.760
		134	47	3	0	3	33 22 20.827	+2.895	+2.729	3322	23.722	23.556
		7		23	1	22	360 00 01.831	-1.831	-1.831	36000	00.000	00.000
37	Rhenen (Cunera tower)	167	59	2	1	1	34 59 33.730	+1.838	+2.244	3459	35.568	35.974
		170	60	2	0	2	61 42 57.299	-0.296	-2.210	6142	57.003	55.089
		173	61	2	1	1	40 06 15.236	———	+1.841	4006	15.236	17.077
		177	62	3	0	3	47 24 02.641	———	-2.401	4724	02.641	00.240
		148	52	3	0	3	30 31 27.217	———	+0.087	3031	27.217	27.304
		145	51	3	0	3	59 41 27.650	-1.170	-0.614	5941	26.480	27.036
		143	50	2	0	2	40 21 30.338	-0.360	+0.365	4021	29.978	30.703
		140	49	3	1	2	45 12 44.843	+1.034	+1.734	4512	45.877	46.577
		8		20	3	17	359 59 58.954	+1.046	+1.046	36000	00.000	00.000
38	Nijmegen (St. Stevens tower)	176	62	3	1	2	82 34 23.739	+0.324	+1.196	8234	24.063	24.935
		179	63	2	0	2	48 58 05.286	-2.300	-1.166	4858	02.986	04.120
		472	—	2	0	2	91 05 21.210	+1.704	-0.296	9105	22.914	20.914
		151	53	2	0	2	57 29 42.299	+0.564	+0.235	5729	42.863	42.534
		149	52	2	0	2	79 52 27.174	———	+0.322	7952	27.174	27.496
		5		11	1	10	359 59 59.708	+0.292	+0.291	35959	60.000	59.999

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
39	Haarlem (St. Bavo church)	473	—	2	0	2	113 23 43.095	....	-0.989	11323	....	42.106
		183	64	2	0	2	77 02 28.593	+3.558	+2.160	7702	32.151	30.753
		156	55	2	0	2	66 46 54.724	+0.500	+0.146	6646	55.224	54.870
		154	54	2	0	2	42 31 59.378	+0.696	+0.229	4232	00.074	59.607
		474	—	2	0	2	60 14 53.652	....	-0.988	6014	....	52.664
		5		10	0	10	359 59 59.442	+0.558	+0.558	36000	00.000	00.000
40	Amsterdam (Western-tower)	185	65	3	1	2	53 24 03.079	———	+0.056	5324	03.079	03.135
		189	66	3	2	1	78 48 23.130	+0.374	-0.230	7848	23.504	22.900
		162	57	3	1	2	38 01 09.588	+0.342	+0.172	3801	09.930	09.760
		160	56	3	1	2	43 12 46.973	-1.300	-1.495	4312	45.673	45.478
		157	55	4	0	4	76 40 54.984	+1.938	+0.410	7640	56.922	55.394
		182	64	3	0	3	69 52 42.395	-1.503	+0.938	6952	40.892	43.333
		6		19	5	14	360 00 00.149	-0.149	-0.149	36000	00.000	00.000
41	Naarden (reformed church)	163	57	3	0	3	107 52 42.537	+0.600	-0.391	10752	43.137	42.146
		188	66	2	0	2	47 07 13.651	-2.482	-1.970	4707	11.169	11.681
		475	—	3	0	3	96 57 58.153	*-4.211*	-1.698	9657	*53.942*	56.455
		192	67	5	1	4	56 25 38.079	+3.892	+1.640	5625	41.971	39.719
		165	58	2	0	2	51 36 31.081	-1.300	-1.082	5136	29.781	29.999
		5		15	1	14	360 00 03.501	-3.501	-3.501	36000	00.000	00.000
42	Amersfoort (Our Lady tower)	191	67	5	2	3	80 17 59.800	———	+0.668	8017	59.800	60.468
		194	68	2	0	2	38 33 41.746	-1.441	-0.122	3833	40.305	41.624
		171	60	2	0	2	76 39 40.413	-0.778	-1.291	7639	39.635	39.122
		169	59	8	1	7	97 29 06.032	-0.330	-0.855	9729	05.702	05.177
		166	58	7	3	4	66 59 32.774	+1.784	+0.835	6659	34.558	33.609
		5		24	6	18	360 00 00.765	-0.765	-0.765	36000	00.000	00.000
43	Imbosch (signal)	178	62	2	0	2	50 01 34.664	———	+1.531	5001	34.664	36.195
		175	61	3	0	3	83 40 28.485	+0.450	-0.926	8340	28.935	27.559
		197	69	2	0	2	73 10 53.214	-0.257	-1.434	7310	52.957	51.780
		201	70	2	0	2	77 36 32.560	———	+0.688	7736	32.560	33.248
		180	63	3	0	3	75 30 30.884	———	+0.335	7530	30.884	31.219
		5		12	0	12	359 59 59.807	+0.193	+0.194	36000	00.000	00.000
44	Hetten-heuvel (signal)	181	63	6	1	5	55 31 23.184	+4.120	+2.652	5531	27.304	25.836
		200	70	4	0	4	45 38 33.689	-2.460	-1.807	4538	31.229	31.882
		203	71	5	0	5	69 01 23.529	+1.700	+0.048	6901	25.229	23.577
		207	72	3	0	3	46 35 39.391	———	-0.260	4635	39.391	39.131
		476	—	4	0	4	94 31 54.166	......	-0.450	9431	......	53.716
		477	—	3	0	3	48 41 06.308	......	-0.450	4841	......	05.858
		6		25	1	24	360 00 00.267	-0.267	-0.267	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
45	Bocholt (tower)	206	72	4	2	2	71 06 29.541	+2.081	+1.410	7106	31.622	30.951
		209	73	3	0	3	44 43 11.632	-0.180	-1.052	4443	11.452	10.580
		2		7	2	5						
46	Harderwijk (signal on Great church)	241	85	2	0	2	86 38 05.072	-2.469	-2.926	8638	02.603	02.146
		195	68	4	0	4	88 28 12.125	+4.314	+1.319	8828	16.439	13.444
		193	67	2	0	2	43 16 21.185	-1.500	+0.086	4316	19.685	21.271
		478	—	2	0	2	98 05 34.514	*-4.601*	-1.828	9805	*29.913*	32.686
		239	84	2	0	2	43 31 47.020	+4.340	+3.433	4331	51.360	50.453
		5		12	0	12	359 59 59.916	+0.084	+0.084	36000	00.000	00.000
47	Veluwe (stone)	196	68	5	3	2	52 58 04.658	———	+1.678	5258	04.658	06.336
		172	60	6	3	3	41 37 26.029	-0.600	+1.831	4137	25.429	27.860
		174	61	4	2	2	56 13 18.524	-0.843	-1.306	5613	17.681	17.218
		198	69	4	1	3	42 29 06.914	+2.700	+2.230	4229	09.614	09.144
		247	87	3	0	3	59 59 58.832	———	-1.327	5959	58.832	57.505
		244	86	2	0	2	48 13 56.330	+0.773	-1.393	4813	57.103	54.937
		243	85	5	0	5	58 28 06.683	———	+0.317	5828	06.683	07.000
		7		29	9	20	359 59 57.970	+2.030	+2.030	36000	00.000	00.000
48	Zutphen (St. Walburgs-tower)	199	69	3	0	3	64 19 62.273	-3.808	-2.160	6419	58.465	60.113
		249	87	3	0	3	86 29 12.047	+0.740	+0.119	8629	12.787	12.166
		250	88	2	0	2	44 14 33.385	-1.200	-0.707	4414	32.185	32.678
		213	74	2	0	2	44 51 14.201	-0.626	-0.808	4451	13.575	13.393
		204	71	2	0	2	63 20 07.051	-1.200	-1.199	6320	05.851	05.852
		202	70	3	0	3	56 44 53.203	+3.934	+2.595	5644	57.137	55.798
		6		15	0	15	360 00 02.160	-2.160	-2.160	36000	00.000	00.000
49	Groenlo (reformed church)	215	75	2	0	2	95 45 59.600	+0.600	-0.593	9545	60.200	59.007
		210	73	5	0	5	98 50 31.745	-0.160	-0.249	9850	31.585	31.496
		208	72	3	0	3	62 17 50.492	———	+0.933	6217	50.492	51.425
		205	71	3	0	3	47 38 32.910	-2.281	-0.628	4738	30.629	32.282
		212	74	2	0	2	55 27 04.992	+2.102	+0.798	5527	07.094	05.790
		5		15	0	15	359 59 59.739	+0.261	+0.261	36000	00.000	00.000
50	Harikerberg (signal)	216	75	3	0	3	47 07 15.812	+0.150	+0.371	4707	15.962	16.183
		214	74	4	1	3	79 41 42.538	-1.780	-0.292	7941	40.758	42.246
		252	88	5	2	3	95 25 52.981	+0.800	+0.456	9525	53.781	53.437
		253	89	3	0	3	90 49 25.721	+1.192	-0.075	9049	26.913	25.646
		219	76	4	1	3	46 55 41.986	+0.600	+0.501	4655	42.586	42.487
		5		19	4	15	359 59 59.038	+0.962	+0.961	35959	60.000	59.999

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
51	Ahaus (tower)	211	73	3	1	2	36 26 18.268	+0.222	+1.186	3626	18.490	19.454
		217	75	2	0	2	37 06 46.771	-1.433	-0.459	3706	45.338	46.312
		218	76	2	0	2	49 16 03.562	-1.295	-1.081	4916	02.267	02.481
		221	77	3	1	2	33 34 42.066	+1.320	+0.482	3334	43.386	42.548
		4		10	2	8						
52	Alkmaar (Weighhouse tower)	228	80	3	0	3	76 56 19.088	+2.734	-0.887	7656	21.822	18.201
		225	78	4	0	4	36 06 37.800	+2.719	+0.837	3606	40.519	38.637
		186	65	3	1	2	39 07 37.587	+1.753	+0.457	3907	39.340	38.044
		184	64	2	0	2	33 04 48.800	-0.648	-1.689	3304	48.152	47.111
		480	—	2	0	2	141 59 14.134	*-9.014*	-0.070	14159	*05.120*	14.064
		479	79	2	0	2	32 45 24.013	+1.034	-0.070	3245	25.047	23.943
		6		16	1	15	360 00 01.422	-1.422	-1.422	36000	00.000	00.000
53	Edam (Chimes tower)	481	—	2	0	2	124 01 02.685	*+5.965*	+1.422	12401	*08.650*	04.107
		190	66	2	0	2	54 04 25.662	+0.648	+0.741	5404	26.310	26.403
		187	65	2	0	2	87 28 20.879	-2.146	-0.904	8728	18.733	19.975
		224	78	1	0	1	60 01 39.912	-2.553	-0.502	6001	37.359	39.410
		236	83	1	0	1	34 24 30.351	-1.403	-0.246	3424	28.948	30.105
		5		8	0	8	359 59 59.489	+0.511	+0.511	36000	00.000	00.000
54	Hoorn (tower)	227	80	3	1	2	45 08 18.486	-2.793	-0.569	4508	15.693	17.917
		230	81	3	0	3	58 45 27.585	———	+1.230	5845	27.585	28.815
		233	82	2	0	2	54 32 14.155	+0.963	-1.355	5432	15.118	12.800
		237	83	2	0	2	117 42 17.226	+1.503	+0.536	11742	18.729	17.762
		226	78	3	0	3	83 51 42.875	———	-0.168	8351	42.875	42.707
		5		13	1	12	360 00 00.327	-0.327	-0.326	36000	00.000	00.001
55	Schagen (tower)	265	93	2	0	2	57 06 16.779	+3.076	+1.333	5706	19.855	18.112
		268	94	2	0	2	52 13 23.621	+4.263	+0.589	5213	27.884	24.210
		231	81	4	1	3	38 16 46.153	-0.060	-1.972	3816	46.093	44.181
		229	80	4	1	3	57 55 25.418	-2.025	-0.626	5755	23.393	24.792
		483	79	4	1	3	66 25 55.084	-2.067	+1.266	6625	53.017	56.350
		482	—	2	1	1	88 02 11.090	*-1.332*	+1.265	8802	*09.758*	12.355
		6		18	4	14	359 59 58.145	+1.855	+1.855	36000	00.000	00.000
56	Medemblik (reformed church)	275	96	2	0	2	66 07 38.243	+1.503	-0.517	6607	39.746	37.726
		271	95	3	0	3	74 28 04.814	———	+0.183	7428	04.814	04.997
		270	94	2	0	2	66 13 04.686	-0.696	+0.289	6613	03.990	04.975
		232	81	2	0	2	82 57 48.191	-1.083	-0.400	8257	47.108	47.791
		234	82	3	0	3	70 13 24.339	+0.003	+0.172	7013	24.342	24.511
		5		12	0	12	360 00 00.273	-0.273	-0.273	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
57	Enkhuizen (Southern church)	274	96	2	0	2	71 43 32.073	-2.093	-1.721	7143	29.980	30.352
		277	97	2	0	2	89 42 24.939	+0.128	+0.894	8942	25.067	25.833
		484	—	2	0	2	115 26 25.149	+*5.855*	+2.756	11526	*31.004*	27.905
		238	83	4	2	2	27 53 13.893	-1.019	-1.208	2753	12.874	12.685
		235	82	3	0	3	55 14 22.578	-1.503	+0.647	5514	21.075	23.225
		5		13	2	11	359 59 58.632	+1.368	+1.368	36000	00.000	00.000
58	Urk (reformed church)	279	97	2	0	2	44 26 31.201	———	-1.222	4426	31.201	29.979
		280	98	2	0	2	53 59 14.365	-1.255	-0.213	5359	13.110	14.152
		283	99	3	0	3	52 21 02.584	———	-0.414	5221	02.584	02.170
		286	100	2	0	2	43 54 46.072	+0.800	+0.129	4354	46.872	46.201
		4		9	0	9						
59	Kampen (New tower)	245	86	2	0	2	80 27 56.965	———	-0.579	8027	56.965	56.386
		242	85	3	0	3	34 53 52.053	+0.290	+0.432	3453	52.343	52.485
		240	84	2	0	2	76 38 47.519	———	+1.900	7638	47.519	49.419
		288	100	3	0	3	71 19 42.104	+1.979	+0.585	7119	44.083	42.689
		289	101	2	0	2	41 00 58.629	-1.000	-1.049	4100	57.629	57.580
		292	102	3	1	2	55 38 40.461	+1.000	+0.980	5538	41.461	41.441
		6		15	1	14	359 59 57.731	+2.269	+2.269	36000	00.000	00.000
60	Lemelerberg (signal)	246	86	1	0	1	51 18 09.445	-0.371	+2.379	5118	09.074	11.824
		248	87	1	0	1	33 30 52.529	-1.598	+0.354	3330	50.931	52.883
		251	88	2	0	2	40 19 37.920	-2.069	-2.215	4019	35.851	35.705
		254	89	2	0	2	44 32 21.135	———	+0.178	4432	21.135	21.313
		257	90	2	0	2	34 43 02.479	———	-0.588	3443	02.479	01.891
		301	105	2	0	2	34 46 26.178	+2.885	+0.347	3446	29.063	26.525
		298	104	2	0	2	37 21 55.759	+2.885	+0.811	3721	58.644	56.570
		295	103	2	0	2	40 10 24.576	+0.621	-0.207	4010	25.197	24.369
		294	102	2	0	2	43 17 10.477	-2.851	-1.556	4317	07.626	08.921
		9		16	0	16	360 00 00.498	-0.498	-0.497	36000	00.000	00.001
61	Oldenzaal (catholic church)	255	89	3	1	2	44 38 14.838	-0.922	+0.170	4438	13.916	15.008
		256	90	2	0	2	52 56 09.399	-0.545	-0.446	5256	08.854	08.953
		259	91	2	0	2	104 06 20.671	+0.698	+0.045	10406	21.369	20.716
		222	77	2	0	2	74 30 58.834	———	-0.425	7430	58.834	58.409
		220	76	3	0	3	83 48 17.027	———	-0.113	8348	17.027	16.914
		5		12	1	11	360 00 00.769	-0.769	-0.769	36000	00.000	00.000
62	Bentheim (Gunpowder tower of the castle)	223	77	3	0	3	71 54 20.278	-1.476	-0.211	7154	18.802	20.067
		261	91	3	0	3	46 04 42.605	———	+0.773	4604	42.605	43.378
		262	92	4	0	4	48 15 55.697	+0.630	+0.200	4815	56.327	55.897
		3		10	0	10						

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
63	Uelsen (signal)	260	91	1	0	1	29 48 57. 592	-0. 699	-0. 818	2948	56. 893	56. 774
		258	90	2	0	2	92 20 50. 480	—	+0. 492	9220	50. 480	50. 972
		303	105	3	0	3	69 40 26. 484	-2. 337	+0. 247	6940	24. 147	26. 731
		304	106	3	0	3	91 16 24. 539	+2. 570	+0. 239	9116	27. 109	24. 778
		263	92	2	0	2	76 53 21. 043	+0. 328	-0. 298	7653	21. 371	20. 745
		5		11	0	11	360 00 00. 138	-0. 138	-0. 138	36000	00. 000	00. 000
64	Kirch Hesepe (tower)	264	92	4	0	4	54 50 45. 189	-0. 800	+0. 259	5450	44. 389	45. 448
		306	106	3	0	3	33 28 55. 993	+0. 400	+0. 714	3328	56. 393	56. 707
		2		7	0	7						
65	Kijkduin (signal)	308	107	2	0	2	61 54 32. 966	+3. 830	-1. 778	6154	36. 796	31. 188
		266	93	1	0	1	68 19 09. 784	-4. 001	-1. 993	6819	05. 783	07. 791
		2		3	0	3						
66	Oosterland (reformed church)	314	109	2	0	2	70 37 25. 140	+1. 127	+2. 081	7037	26. 267	27. 221
		272	95	3	0	3	57 23 02. 881	+3. 740	+1. 349	5723	06. 621	04. 230
		269	94	2	0	2	61 33 33. 953	-4. 928	-2. 237	6133	29. 025	31. 716
		267	93	2	0	2	54 34 35. 391	-0. 158	-0. 423	5434	35. 233	34. 968
		307	107	2	0	2	55 01 17. 974	-1. 127	-2. 045	5501	16. 847	15. 929
		310	108	2	0	2	60 50 06. 007	—	-0. 071	6050	06. 007	05. 936
		6		13	0	13	360 00 01. 346	-1. 346	-1. 346	36000	00. 000	00. 000
67	Staveren (church)	278	97	1	0	1	45 51 04. 052	+0. 753	+1. 211	4551	04. 805	05. 263
		276	96	1	0	1	42 08 52. 250	-1. 247	+0. 402	4208	51. 003	52. 652
		273	95	2	0	2	48 08 53. 230	-3. 703	-1. 495	4808	49. 527	51. 735
		313	109	2	1	1	41 27 21. 104	+0. 783	-3. 041	4127	21. 887	18. 063
		322	112	2	0	2	41 18 52. 561	—	+3. 810	4118	52. 561	56. 371
		325	113	1	0	1	44 02 15. 596	+1. 127	-0. 941	4402	16. 723	14. 655
		328	114	1	0	1	49 40 05. 416	+2. 691	+0. 873	4940	08. 107	06. 289
		281	98	1	0	1	47 22 35. 387	—	-0. 416	4722	35. 387	34. 971
		8		11	1	10	359 59 59. 596	+0. 404	+0. 403	35959	60. 000	59. 999
68	Lemmer (reformed church)	331	115	2	0	2	85 46 22. 735	+0. 166	-1. 648	8546	22. 901	21. 117
		335	116	2	0	2	51 35 02. 411	+2. 200	+1. 915	5135	04. 611	04. 326
		284	99	2	0	2	73 39 20. 182	-1. 200	-0. 947	7339	18. 982	19. 205
		282	98	2	0	2	78 38 12. 843	-0. 044	-0. 668	7838	12. 799	12. 175
		330	114	2	0	2	70 21 03. 360	-2. 653	-0. 183	7021	00. 707	03. 177
		5		10	0	10	360 00 01. 531	-1. 531	-1. 531	36000	00. 000	00. 000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
69	Blokzijl (reformed church)	287	100	3	1	2	64 45 33.701	-3.535	-1.469	6445	30.166	32.232
		285	99	2	0	2	53 59 38.475	+1.089	+1.281	5359	39.564	39.756
		334	116	3	1	2	70 08 38.814	-1.200	-2.611	7008	37.614	36.203
		338	117	2	0	2	83 53 05.181	+2.078	+2.026	8353	07.259	07.207
		290	101	2	0	2	87 13 04.397	+1.000	+0.205	8713	05.397	04.602
		5		12	2	10	360 00 00.568	-0.568	-0.568	36000	00.000	00.000
70	Meppel (reformed church)	296	103	2	0	2	98 32 56.919	-1.624	+0.585	9832	55.295	57.504
		293	102	2	0	2	81 04 11.463	+1.195	-0.078	8104	12.658	11.385
		291	101	2	0	2	51 45 59.978	-2.247	-1.403	5145	57.731	58.575
		337	117	2	0	2	54 55 27.470	-1.863	-2.233	5455	25.607	25.237
		340	118	6	3	3	73 41 25.825	+2.884	+1.474	7341	28.709	27.299
		5		14	3	11	360 00 01.655	-1.655	-1.655	36000	00.000	00.000
71	Oosterend (reformed church)	317	110	2	0	2	69 44 39.186	-7.375	-1.352	6444	31.811	37.834
		311	108	3	2	1	54 04 06.022	+4.007	+1.868	5404	10.029	07.890
		309	107	2	0	2	63 04 17.501	-10.361	-3.834	6304	07.140	13.667
		3		7	2	5						
72	Robbezand (signal)	312	108	2	0	2	65 05 48.001	-3.300	-1.089	6505	44.701	46.912
		316	110	2	0	2	69 21 31.142	+1.057	-0.859	6921	32.199	30.283
		319	111	2	0	2	67 44 35.384	+3.200	+2.135	6744	38.584	37.519
		323	112	2	0	2	89 52 50.922	-0.248	-1.348	8952	50.674	49.574
		315	109	2	0	2	67 55 14.843	-2.001	+0.869	6755	12.842	15.712
		5		10	0	10	360 00 00.292	-0.292	-0.292	36000	00.000	00.000
73	Vlieland (beacon light)	344	119	2	0	2	62 29 59.600	+2.733	+1.362	6230	02.333	00.962
		320	111	2	0	2	47 11 38.271	-6.329	-3.933	4711	31.942	34.338
		318	110	3	1	2	40 53 52.873	+4.292	+0.187	4053	57.165	53.060
		3		7	1	6						
74	Harlingen (Western church)	346	120	1	0	1	51 37 34.677	-1.307	+0.706	5137	33.370	35.383
		349	121	2	0	2	50 58 15.238	+2.090	+3.346	5058	17.328	18.584
		353	122	4	2	2	51 05 39.717	-1.722	-1.055	5105	37.995	38.662
		326	113	2	0	2	52 25 09.023	-0.075	-0.552	5225	08.948	08.471
		324	112	1	0	1	48 48 18.870	-0.762	-3.470	4848	18.108	15.400
		321	111	1	0	1	65 03 48.522	+2.300	+0.971	6503	50.822	49.493
		343	119	1	0	1	40 01 14.574	-1.145	-0.568	4001	13.429	14.006
		7		12	2	10	359 59 60.621	-0.621	-0.622	35959	60.000	59.999

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
75	Sneek	352	122	3	1	2	70 48 33.445	-2.848	-0.576	7048	30.597	32.869
	(St. Martini	355	123	3	0	3	49 30 40.051	+0.137	-3.662	4930	40.188	36.389
	church)	358	124	2	0	2	45 36 07.492	———	-0.280	4536	07.492	07.212
		332	115	3	1	2	50 33 09.473	+4.035	+3.941	5033	13.508	13.414
		329	114	2	0	2	59 58 51.264	+1.127	+0.477	5958	52.391	51.741
		327	113	2	0	2	83 32 38.058	-2.234	+0.317	8332	35.824	38.375
		6		15	2	13	359 59 59.783	+0.217	+0.217	36000	00.000	00.000
76	Oldeholtpa	361	125	5	4	1	47 48 52.580	+6.014	+2.409	4748	58.594	54.989
	(reformed	364	126	6	1	5	41 31 14.143	+0.699	-2.507	4131	14.842	11.636
	church)	341	118	5	0	5	59 39 39.048	+1.860	+2.388	5939	40.908	41.436
		339	117	5	3	2	41 11 25.812	+2.104	+2.527	4111	27.916	28.339
		336	116	6	2	4	58 16 22.097	-3.324	-1.626	5816	18.773	20.471
		333	115	6	2	4	43 40 27.477	-2.630	-0.749	4340	24.847	26.728
		360	124	3	0	3	67 52 00.048	-5.928	-3.647	6751	54.120	56.401
		7		36	12	24	360 00 01.205	-1.205	-1.205	36000	00.000	00.000
77	Midsland	347	120	2	0	2	83 39 31.917	+2.200	+1.099	8339	34.117	33.016
	(church	345	119	2	0	2	77 28 47.554	-2.220	-1.426	7728	45.334	46.128
	tower)	2		4	0	4						
78	Ballum	368	127	3	2	1	44 01 51.040	+7.674	+2.970	4401	58.714	54.010
	(castle)	350	121	7	4	3	46 14 27.202	+1.982	+0.386	4614	29.184	27.588
		348	120	5	1	4	44 42 56.427	-2.200	-3.109	4442	54.227	53.318
		3		15	7	8						
79	Leeuwarden	356	123	2	0	2	87 36 21.057	+1.614	+2.216	8736	22.671	23.273
	(Oldehove)	354	122	2	0	2	58 05 48.707	+3.848	+0.908	5805	52.555	49.615
		351	121	2	0	2	82 47 15.351	-0.116	+0.225	8247	15.235	15.576
		367	127	5	2	3	59 24 00.645	-1.273	-0.478	5923	59.372	60.167
		371	128	3	0	3	72 06 32.043	-1.876	-0.674	7206	30.167	31.369
		5		14	2	12	359 59 57.803	+2.197	+2.197	36000	00.000	00.000
80	Dokkum	369	127	4	0	4	76 34 09.021	-5.942	-2.031	7634	03.079	06.990
	(tower)	485	—	3	2	1	90 42 07.124	*-2.147*	-0.299	9042	*04.977*	06.825
		486	129	2	0	2	41 35 39.856	———	-0.299	4135	39.856	39.557
		373	130	6	3	3	33 06 04.258	+6.000	+3.258	3306	10.258	07.516
		377	131	3	1	2	57 01 55.292	-3.486	-3.215	5701	51.806	52.077
		372	128	3	1	2	61 00 04.494	+5.530	+2.541	6100	10.024	07.035
		6		21	7	14	360 00 00.045	-0.045	-0.045	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
81	Drachten (reformed church)	362	125	4	1	3	53 55 24.745	-2.414	-0.954	5355	22.331	23.791
		359	124	4	1	3	66 31 56.513	+3.570	+1.572	6631	60.083	58.085
		357	123	4	3	1	42 52 59.382	-1.000	+2.199	4252	58.382	61.581
		370	128	6	4	2	46 53 27.163	-6.251	-4.463	4653	20.912	22.700
		376	131	4	2	2	83 33 14.515	+3.454	+0.945	8333	17.969	15.460
		380	132	4	3	1	66 12 57.246	+3.077	+1.137	6612	60.323	58.383
		6		26	14	12	359 59 59.564	+0.436	+0.436	36000	00.000	00.000
82	Ooster-wolde (reformed church)	365	126	2	0	2	100 51 25.802	-2.945	-0.781	10051	22.857	25.021
		363	125	3	1	2	78 15 42.347	-2.400	-0.254	7815	39.947	42.093
		379	132	3	1	2	81 54 17.447	-0.400	+2.132	8154	17.047	19.579
		382	133	2	0	2	55 01 03.414	+5.296	-0.836	5501	08.710	02.578
		385	134	4	3	1	43 57 30.487	+0.952	+0.242	4357	31.439	30.729
		5		14	5	9	359 59 59.497	+0.503	+0.503	36000	00.000	00.000
83	Beilen (reformed church)	366	126	4	3	1	37 37 22.662	+0.674	+1.718	3737	23.336	24.380
		387	134	4	2	2	75 31 35.708	+1.973	+1.087	7531	37.681	36.795
		388	135	5	3	2	84 20 01.699	-1.598	-0.154	8420	00.101	01.545
		392	136	3	2	1	30 14 27.108	———	-0.799	3014	27.108	26.309
		299	104	2	1	1	44 20 58.182	———	-0.309	4420	58.182	57.873
		297	103	2	1	1	41 16 40.644	+0.920	-0.458	4116	41.564	40.186
		342	118	3	1	2	46 38 55.800	-3.772	-2.889	4638	52.028	52.911
		7		23	13	10	359 59 61.803	-1.803	-1.804	35959	60.000	59.999
84	Coevorden (reformed church)	302	105	3	0	3	75 33 07.694	+0.500	+0.455	7533	08.194	08.149
		300	104	5	2	3	98 17 07.638	-2.416	-0.030	9817	05.222	07.608
		391	136	3	1	2	52 40 48.579	+0.710	+1.363	5240	49.289	49.942
		487	—	6	3	3	78 14 15.819	+3.665	-1.347	7814	19.484	14.472
		305	106	3	1	2	55 14 40.380	-2.569	-0.551	5514	37.811	39.829
		5		20	7	13	360 00 00.110	-0.110	-0.110	36000	00.000	00.000
85	Hornhuizen (reformed church)	397	138	5	3	2	59 02 04.459	+4.000	+2.143	5902	08.459	06.602
		374	130	4	3	1	111 05 01.186	-6.432	-1.659	11104	54.754	59.527
		489	129	3	2	1	53 11 07.362	———	-0.484	5311	07.362	06.878
		488	—	2	0	2	95 09 07.743	+1.477	-0.484	9509	09.220	07.259
		394	137	3	1	2	41 32 40.205	———	-0.471	4132	40.205	39.734
		5		17	9	8	360 00 00.955	-0.955	-0.955	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
86	Groningen (Martini tower)	383	133	3	2	1	47 50 24.148	+2.250	+1.762	4750	26.398	25.910
		381	132	6	3	3	31 52 46.094	-2.054	-2.652	3152	44.040	43.442
		378	131	6	5	1	39 24 52.397	———	+2.226	3924	52.397	54.623
		375	130	4	0	4	35 48 58.544	-2.168	-4.206	3548	56.376	54.338
		396	138	2	1	1	60 36 32.862	-0.001	-1.280	6036	32.861	31.582
		399	139	2	0	2	28 29 59.056	+0.001	+2.437	2829	59.057	61.493
		402	140	3	1	2	42 38 57.916	+3.357	+0.347	4238	61.273	58.263
		405	141	6	4	2	27 57 25.487	+2.866	+2.150	2757	28.353	27.637
		409	142	3	1	2	45 19 62.056	-2.811	+0.656	4519	59.245	62.712
		9		35	17	18	359 59 58.560	+1.440	+1.440	36000	00.000	00.000
87	Rolde (reformed church)	386	134	3	2	1	60 30 51.769	+0.001	+1.593	6030	51.770	53.362
		384	133	4	1	3	77 08 32.031	-5.602	+1.010	7708	26.429	33.041
		408	142	2	0	2	91 24 12.370	+3.780	-1.925	9124	16.150	10.445
		412	143	2	0	2	75 53 51.817	+4.437	+0.126	7553	56.254	51.943
		389	135	5	3	2	55 02 31.996	-2.599	-0.787	5502	29.397	31.209
		5		16	6	10	359 59 59.983	+0.017	+0.017	36000	00.000	00.000
88	Sleen (reformed church)	393	136	5	2	3	97 04 44.333	———	+0.142	9704	44.333	44.475
		390	135	4	3	1	40 37 25.441	+5.979	+2.719	4037	31.420	28.160
		411	143	5	3	2	53 43 56.668	+2.070	+0.519	5343	58.738	57.187
		490	—	4	0	4	64 35 31.531	. . . .	-2.113	6435	. . . .	29.418
		491	—	4	2	2	103 58 22.873	. . . .	-2.113	10358	. . . .	20.760
		5		22	10	12	360 00 00.846	-0.846	-0.846	36000	00.000	00.000
89	Uithuizermeden (reformed church)	400	139	4	3	1	86 23 50.103	+3.201	+0.954	8623	53.304	51.057
		398	138	3	0	3	60 21 26.314	-6.441	-3.311	6021	19.873	23.003
		395	137	4	2	2	88 03 07.755	-2.806	-1.120	8803	04.949	06.635
		414	144	3	1	2	77 42 53.053	-1.028	-1.129	7742	52.025	51.924
		415	145	4	2	2	47 28 45.498	+4.351	+1.882	4728	49.849	47.380
		5		18	8	10	359 59 62.723	-2.723	-2.724	35959	60.000	59.999
90	Holwierde (tower)	403	140	5	3	2	79 59 23.002	+3.401	+2.050	7959	26.403	25.052
		401	139	4	3	1	65 06 10.302	-1.954	-2.146	6506	08.348	08.156
		416	145	2	0	2	104 22 33.112	+3.700	-0.113	10422	36.812	32.999
		418	146	3	1	2	44 50 32.479	-4.078	-2.645	4450	28.401	29.834
		422	147	5	3	2	65 41 23.121	-3.085	+0.838	6541	20.036	23.959
		5		19	10	9	360 00 02.016	-2.016	-2.016	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
91	Midwolda	406	141	5	2	3	102 48 14.743	-0.673	-0.539	10248	14.070	14.204
	(reformed	404	140	2	0	2	57 21 39.019	-5.399	-1.045	5721	33.620	37.974
	church)	421	147	5	3	2	61 16 41.103	+1.676	-2.323	6116	42.779	38.780
		424	148	4	2	2	48 07 47.366	+6.060	+2.568	4807	53.426	49.934
		428	149	2	0	2	90 25 38.805	-2.700	+0.303	9025	36.105	39.108
		5		18	7	11	360 00 01.036	-1.036	-1.036	36000	00.000	00.000
92	Onstwedde	410	142	3	0	3	43 15 46.021	+0.392	+2.621	4315	46.413	48.642
	(reformed	407	141	3	1	2	49 14 22.426	-3.499	-2.924	4914	18.927	19.502
	church)	427	149	3	2	1	57 55 01.906	+3.070	+1.095	5755	04.976	03.001
		492	—	3	2	1	122 23 54.236	. . . .	-1.927	12223	. . . .	52.309
		493	—	2	1	1	36 48 45.873	. . . .	-1.927	3648	. . . .	43.946
		413	143	5	3	2	50 22 10.426	-3.680	+2.174	5022	06.746	12.600
		6		19	9	10	360 00 00.888	-0.888	-0.888	36000	00.000	00.000
93	Pilsum	431	150	4	2	2	95 45 11.111	-2.842	-1.473	9545	08.269	09.638
	(Pfarr-	419	146	4	2	2	79 00 45.636	+3.878	+1.337	7900	49.514	46.973
	kirche)	417	144/5	4	1	3	71 50 36.186	-11.819	-1.220	7150	24.367	34.966
		494	—	5	3	2	113 23 29.631	+ 8.219	-1.209	11323	37.850	28.422
		4		17	8	9	359 59 62.564	-2.564	-2.565	35959	60.000	59.999
94	Emden	430	150	3	0	3	47 25 46.286	-0.009	-2.353	4725	46.277	43.933
	(townhall)	433	151	4	2	2	46 42 35.453	-0.924	-1.861	4642	34.529	33.592
		437	152	4	1	3	75 40 36.186	-0.100	-0.651	7540	36.086	35.535
		425	148	4	2	2	81 00 23.667	-1.648	+1.009	8100	22.019	24.676
		423	147	4	3	1	53 01 57.002	+1.241	+1.313	5301	58.243	58.315
		420	146	4	3	1	56 08 44.657	-1.811	-0.707	5608	42.846	43.950
		6		23	11	12	360 00 03.251	-3.251	-3.250	36000	00.000	00.001
95	Leer	495	—	6	5	1	29 07 06.348	-5.792	-1.110	2907	00.556	05.238
	(great	429	149	7	4	3	31 39 18.207	+2.104	+1.062	3139	20.311	19.269
	church)	426	148	5	4	1	50 51 46.510	-0.760	+0.180	5051	45.750	46.690
		436	152	4	1	3	50 40 46.349	+0.488	+0.611	5040	46.837	46.960
		439	153	3	0	3	34 49 20.244	-0.099	-0.180	3449	20.145	20.064
		442	154	4	2	2	44 58 59.943	+3.472	+2.139	4459	03.415	02.082
		445	155	2	1	1	24 29 44.775	-1.870	-1.203	2429	42.905	43.572
		496	156	5	3	2	93 22 57.237	+2.844	-1.111	9322	60.081	56.126
		8		36	20	16	359 59 59.613	+0.387	+0.388	36000	00.000	00.001

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
96	Barssel (tower)	497	—	3	1	2	94 48 07.426	. . . .	+0.089	9448	. . . .	07.515
		499	156	3	1	2	44 00 59.629	-1.880	+0.089	4400	57.749	59.718
		447	155	2	0	2	122 42 18.227	———	-0.004	12242	18.227	18.223
		498	—	2	1	1	98 28 34.456	. . . .	+0.089	9828	. . . .	34.545
		4		10	3	7	359 59 59.738	+0.262	+0.263	36000	00.000	00.001
97	Hage (tower)	449	157	4	3	1	61 43 18.415	-0.554	-0.267	6143	17.861	18.148
		434	151	3	2	1	52 29 34.702	+0.847	+0.430	5229	35.549	35.132
		432	150	5	2	3	36 49 04.555	+1.705	+2.675	3649	06.260	07.230
		3		12	7	5						
98	Aurich (tower)	448	157	6	5	1	65 10 09.805	+1.695	-0.248	6510	11.500	09.557
		451	158	3	1	2	43 46 01.303	———	-0.706	4346	01.303	00.597
		455	159	6	2	4	68 38 27.114	———	-0.260	6838	27.114	26.854
		440	153	6	3	3	47 58 50.834	———	+1.145	4758	50.834	51.979
		438	152	6	5	1	53 38 40.268	-2.005	-1.587	5338	38.263	38.681
		435	151	5	3	2	80 47 51.404	-0.418	+0.928	8047	50.986	52.332
		6		32	19	13	360 00 00.728	-0.728	-0.728	36000	00.000	00.000
99	Strakholt (signal on the church)	441	153	6	5	1	97 11 49.808	———	-1.069	9711	49.808	48.739
		454	159	4	3	1	81 03 40.928	-0.138	-0.876	8103	40.790	40.052
		458	160	5	2	3	85 17 18.120	-0.809	-0.079	8517	17.311	18.041
		443	154	5	3	2	96 27 13.115	-1.024	+0.053	9627	12.091	13.168
		4		20	13	7	360 00 01.971	-1.971	-1.971	36000	00.000	00.000
100	Wester-stede (tower)	457	160	3	2	1	54 15 51.159	+3.182	+2.192	5415	54.341	53.351
		460	161	3	1	2	44 23 56.880	-1.254	-0.651	4423	55.626	56.229
		500	—	3	1	2	52 33 07.004	. . . .	-0.587	5233	. . . .	06.417
		501	—	3	1	2	52 14 27.955	. . . .	-0.587	5214	. . . .	27.368
		502	—	3	1	2	85 10 52.415	. . . .	-0.587	8510	. . . .	51.828
		446	155	4	3	1	32 47 59.580	———	-0.680	3247	59.580	58.900
		444	154	3	2	1	38 33 45.960	-0.314	-0.054	3833	45.646	45.906
		7		22	11	11	359 59 60.953	-0.953	-0.954	35959	60.000	59.999
101	Esens (small church tower)	503	—	3	1	2	50 35 52.340	. . . .	-0.133	5035	. . . .	52.207
		504	163	4	1	3	69 00 19.405	———	-0.132	6900	19.405	19.273
		452	158	7	4	3	90 31 40.509	-0.127	-0.131	9031	40.382	40.378
		450	157	4	0	4	53 06 31.966	-0.342	+1.310	5306	31.624	33.276
		505	—	3	1	2	96 45 34.999	. . . .	-0.133	9645	. . . . .	34.866
		5		21	7	14	359 59 59.219	+0.781	+0.781	36000	00.000	00.000

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	8+9=11	8+9=12	8+10=13
102	Jever (tower of the castle)	456	159	5	3	2	30 17 53.908	-0.700	+0.291	3017	53.208	54.199
		453	158	5	3	2	45 42 19.298	+0.167	+0.878	4542	19.465	20.176
		467	163	6	3	3	58 12 34.291	———	-0.406	5812	34.291	33.885
		468	—	6	3	3	109 59 04.896	———	-0.406	10959	*04.896*	04.490
		469	162	5	2	3	39 25 59.564	———	-0.406	3925	59.564	59.158
		461	161	5	4	1	35 55 18.309	+0.266	-0.470	3555	18.575	17.839
		459	160	4	2	2	40 26 50.160	-0.159	+0.093	4026	50.001	50.253
		7		36	20	16	360 00 00.426	-0.426	-0.426	36000	00.000	00.000
103	Varel (tower of the Luther-an church)	463	162	3	1	2	87 24 51.858	———	-0.056	8724	51.858	51.802
		464	—	3	0	3	50 05 53.067	. . . .	-0.056	5005	. . . .	53.011
		465	—	4	2	2	26 30 05.752	. . . .	-0.056	2630	. . . .	05.696
		466	—	3	0	3	96 18 22.318	. . . .	-0.056	9618	. . . .	22.262
		462	161	5	3	2	99 40 47.348	-0.254	-0.120	9940	47.094	47.228
		5		18	6	12	359 59 60.343	-0.343	-0.344	35959	60.000	59.999
104	Herentals (catholic church)	No observations										
105	Biesselt (mill)	No observations										
106	Borkum (beacon light)	No observations										

This is e.g. the case with angle 475 in the station Naarden (No. 41). As, however, Krayenhoff's sum of the adjusted spherical angles in that station must be 360^o, the adjusted angle 475 = $96^o 57' 53''.942$ and the correction $p'_{475} = -4''.211$ can easily be derived. I noted these amounts in italic numbers in columns 12 and 9. The adjusted angles can be found in the Table alphabétique des azimuths on the pages 182 - 202 of the second edition of Krayenhoff's book.

Table 9 gives also an easy survey of the 73 station equations in Krayenhoff's network. As e.g. in Amsterdam (station No. 40) the sum of the angles, reduced to horizon and centre is $360^o 00' 00''.149$, one finds:

$$p_{185} + p_{189} + p_{162} + p_{160} + p_{157} + p_{182} + 0''.149 = 0.$$

As already said the number of side equations is 54. Fifty one of them can easily be found for around each of the central points 12, 14, 17, 18, 22-25, 28, 29, 31-37, 40, 42, 43, 47-50, 54, 56, 59-61, 63, 66-70, 72, 74-76, 79, 81-83, 86, 87, 89-91, 94, 98, and 99 lies a number of triangles the base angles of which are measured or can be derived from other measured angles.

For each of these central points now holds that, after the adjustment, the sum of log sine left base angles must be alike to the sum of log sine right base angles. For Harlingen (station No. 74) e.g. one finds:

$$\log\sin(354+p_{354}) + \log\sin(327+p_{327}) + \log\sin(322+p_{322}) + \log\sin(319+p_{319}) + \log\sin(344+p_{344}) + \log\sin(347+p_{347}) + \log\sin(350+p_{350}) = \log\sin(352+p_{352}) + \log\sin(325+p_{325}) + \log\sin(323+p_{323}) + \log\sin(320+p_{320}) + \log\sin(345+p_{345}) + \log\sin(348+p_{348}) + \log\sin(351+p_{351}),$$

with e.g.

$$\log\sin(354+p_{354}) = \log\sin 354 + p_{354}\,\frac{M}{\rho}\cot 354.$$

In this formula M = 0.43429448 is the modulus of Briggs' system of logarithms. For $\rho = 206264''.806$, p is expressed in seconds. As $\frac{M}{\rho} = 0.0000021055$ and 354 = $58^o 05' 48''.707$ ($\cot 354 \simeq 0.62252$) one finds:

$$10^6 \log\sin(354+p_{354}) = 10^6 \log\sin 354 + 1.311\, p_{354}.$$

Worked out the condition runs as follows:

$$1.311\,p_{354} + 0.238\,p_{327} + 2.395\,p_{322} + 0.862\,p_{319} + 1.096\,p_{344} + 0.234\,p_{347} + 2.016\,p_{350} - 0.733\,p_{352} - 2.177\,p_{325} - 0.004\,p_{323} - 1.950\,p_{320} - 0.468\,p_{345} - 2.127\,p_{348} - 0.266\,p_{351} - 32.127 = 0.$$

It will be clear that multiplication by 10^6 saves the writing of a great number of ciphers zero.

A 52nd side equation can be borrowed from a chain of triangles around the former Zuiderzee (see Fig. 12). Starting e. g. from Harderwijk-Kampen=HK one finds for Urk-Kampen (UK):

$$\sin UK = \frac{\sin HK \sin (239+p_{239})}{\sin (\alpha + p_{\alpha}}$$

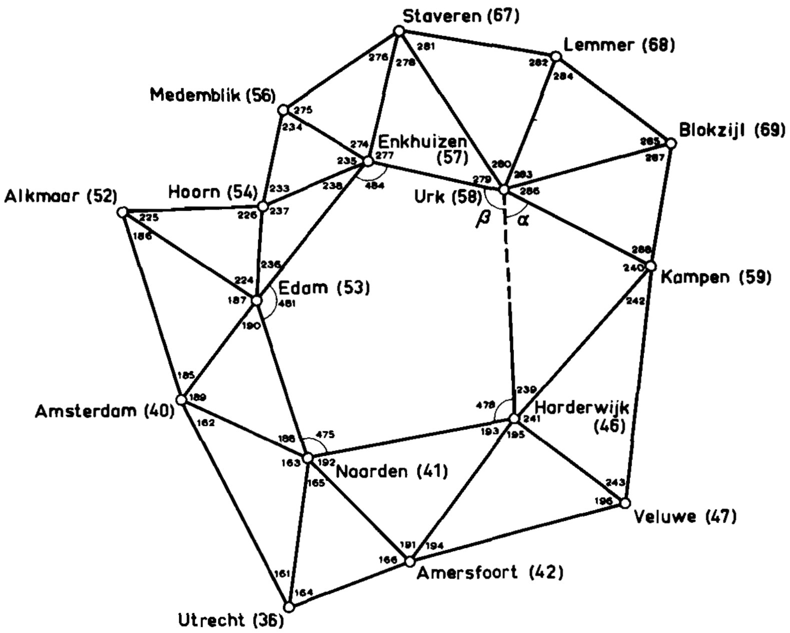
and, in the same way:

$$\sin UB = \frac{\sin UK \sin (288+p_{288})}{\sin (287+p_{287})}$$

whence:

$$\sin UB = \frac{\sin HK \sin (239+p_{239}) \sin (288+p_{288})}{\sin (\alpha + p_{\alpha}) \sin (287+p_{287})}$$

in which UB (in arc measure) is the length of the side Urk-Blokzijl.



$$\alpha + p_{\alpha} = 59°49'27''.348 - p_{239} - p_{240}$$

$$\beta + p_{\beta} = 105°28'58''.430 + p_{239} + p_{240} - p_{286} - p_{283} - p_{280} - p_{279}$$

Fig. 12.

Proceeding in the same way one finds the sides Urk-Lemmer, Urk-Staveren, Staveren-Enkhuizen, ..., Naarden-Amersfoort, Amersfoort-Harderwijk, Harderwijk-Veluwe, and, finally, Harderwijk-Kampen:

$$\sin HK = \frac{\sin HK \sin (239+p_{239}) \sin(288+p_{288}) \ldots\ldots\ldots \sin (194+p_{194}) \sin (243+p_{243})}{\sin (\alpha + p_{\alpha}) \sin (287+p_{287}) \ldots\ldots\ldots \sin (196+p_{196}) \sin (242+p_{242})}$$

from which follows (in a logarithmic form) a similar condition as found in the preceding 51 side equations. It will be clear that the amount $\alpha + p_\alpha$ in the condition must be derived from triangle Urk-Kampen-Harderwijk $(E = 1''.887)$:

$$(\alpha + p_\alpha) = 180^\circ 00' 01''887 - (239 + p_{239} + 240 + p_{240}) = 59^\circ 49' 27''.348 - p_{239} - p_{240}.$$

An analogous amount $\beta + p_\beta$, necessary for the determination of the still missing 53rd and 54th side equation in the network is mentioned in Fig. 12. It is an angle of the Zuiderzee pentagon the other four angles of which are also known.

If one starts from a provisional length of an arbitrary side of this pentagon - I used the length of the chord Harderwijk-Urk = 34896.065 m borrowed from Krayenhoff's tableau définitif (triangle 84) - then the lengths of all the other sides of the pentagon can be computed by a repeated application of the sine rule in the spherical triangles of Fig. 12. The computation is an extension of the computations necessary for the determination of the 52nd side equation just mentioned. As the spherical excess of all triangles is known the computation can rather easily be done by the application of Legendre's theorem. To the length of Krayenhoff's chord Harderwijk-Urk = 34896.065 m the small amount of 0.043 m must then be added in order to find the spherical side of the pentagon.

The results, computed with a Brunsviga table calculating machine and Peters' eight place trigonometric tables and verified by a computer computation, are shown in table 10. The spherical length of the side Urk-Enkhuizen e.g. is $20851.9043 + 0.1064\, p_{239} - 0.0240\, p_{240} - \ldots - 0.0477\, p_{287} + 0.0342\, p_{288}$, etc.

I continued the computation via the sides Harderwijk-Amersfoort, Harderwijk-Veluwe, and Harderwijk-Kampen to the side Harderwijk-Urk from which it started. As the two lengths must be equal to each other the difference found between them must be zero. The computation described is an alternative determination of the 52nd side equation mentioned before. The equation is mentioned in table 13 and it runs:

$$-1.4897 - 0.2498\, p_{161} + 0.2164\, p_{162} + \ldots - 0.0798\, p_{287} + 0.0572\, p_{288} = 0.$$

It shows that, if one computes the spherical sides of the triangles in Fig. 12 with Krayenhoff's uncorrected angles, the amount found for Harderwijk-Urk is 1.49 m smaller than that from which one started. In my opinion it is a small amount if the primitive goniometer used for the angular measurement and the perimeter of the pentagon (140 km) is taken into consideration.

In a pentagon with 5 known sides and 5 known angles are three redundant data. One of them, relating to the sum of the angles of the pentagon, was already worked up in the 149th polygon condition.

Table 10

angles i	Harderwijk-Urk	Urk-Enkhuizen	Enkhuizen-Edam	Edam-Naarden	Naarden-Harderwijk
	34896.108	20851.9043	27028.6651	25348.7895	31529.5874
161					-0.2258
162					+0.1955
164					+0.0833
166					-0.0649
185				-0.0913	-0.1135
186				+0.1511	+0.1879
188				-0.1141	-0.1419
189				+0.0243	
190					+0.1108
191					+0.0261
193					-0.1624
225				-0.1685	-0.2095
226				+0.0132	+0.0164
233			-0.0933	-0.0875	-0.1089
234			+0.0471	+0.0442	+0.0550
236			-0.1913	-0.1794	-0.2232
237			-0.0688		
238				+0.2322	+0.2889
239		+0.1064	+0.1380	+0.1294	+0.1609
240		-0.0240	-0.0311	-0.0292	-0.0363
275			-0.0580	-0.0544	-0.0677
276			+0.1448	+0.1358	+0.1689
277		-0.0005	-0.0007	-0.0006	-0.0008
278		+0.0981			
279			+0.1336	+0.1253	+0.1559
281		-0.0930	-0.1206	-0.1131	-0.1407
282		+0.0203	+0.0263	+0.0247	+0.0307
284		-0.0296	-0.0384	-0.0360	-0.0448
285		+0.0735	+0.0952	+0.0893	+0.1111
287		-0.0477	-0.0618	-0.0579	-0.0721
288		+0.0342	+0.0443	+0.0415	+0.0517

The two remaining side equations are similar to those of a closed polygon in a flat plane: the sum of the projections of the sides on the x- and y-axis of an arbitrary coordinate system must be zero. As the determination of the two conditions in a flat plane is easiest, I reduced the lengths of the sides of the pentagon on the conformal sphere to lengths in the plane of projection of the R.D. and the angles between the great circles on the sphere to angles between chords in the plane of projection. This could easily be done as Krayenhoff's stations Naarden, Edam, Enkhuizen, and Urk are, apart from small alterations, the same as those computed in the R.D.-coordinate system. Harderwijk is not an identical point (see section 4, page 17) but provisional coordinates of Krayenhoff's station could be computed. With those of the other 4 points they are mentioned in table 11. The amounts Δl_P (in mm per 100 m) in the last column are the corrections to a distance of 100 m on the sphere in the respective points in order to find the corresponding distance in the plane of projection.

Table 11

Angular points P Zuiderzee pentagon	prov. coordinates (km)		Δl_P (mm/100m)
	X'_P	Y'_P	
Naarden	-15.346	+15.661	-8.915
Edam	-23.099	+39.794	-7.911
Enkhuizen	- 6.395	+61.043	-6.898
Urk	+13.944	+56.448	-7.135
Harderwijk	+15.620	+21.592	-8.774

They can be computed with the formula:

$$\Delta l_P = -9.210 + \frac{X'^2_P + Y'^2_P}{1629.38} \quad \ldots\ldots\ldots\ldots \quad (8)$$

X'_P and Y'_P in this formula are expressed in km [49].

The correction from the chord PQ on the sphere to the chord PQ in the plane of projection is:

$$\Delta l_{PQ} = (\Delta l_P + \Delta l_Q) : 2 \quad \ldots\ldots\ldots\ldots \quad (9)$$

(Δl_{PQ} in mm per 100 m) and that from the arc on the sphere to the chord on the sphere

$$C_{PQ} = -l^3_{PQ} : 9777.2 \ldots\ldots\ldots\ldots (10)$$

(l_{PQ} in km, C_{PQ} in cm) [50].

As for the side Naarden-Harderwijk $l_{PQ} = -2.7885$ m and $C_{PQ} = -0.0321$ m, the length of the side Naarden-Harderwijk in the plane of projection is:

$$(31529.5874 - 2.7885 - 0.0321) - 0.2258\ p_{161} + 0.1955\ p_{162} + \dots\dots\dots\dots$$

$$= 31526.7668 - 0.2258\ p_{161} + 0.1955\ p_{162} + \dots\dots\dots\dots\dots$$

It will be clear that the computation of the reductions of the terms in the upper row in table 10 will do: the influence of the reduction on $-0.2258\ p_{161} + 0.1955\ p_{162}$ etc. is so small that it may be neglected. The principal term, the amount 31526.7668 m, is also mentioned in Fig. 14 and, in the same way, that of the other sides of the pentagon. The corrections to these amounts must also be borrowed from table 10.

As in the conformal stereographic projection circles on the sphere are circles in the plane of projection and great circles through Amersfoort (C) are projected as straight lines, the small angle ϵ between arc and chord in the plane of projection for a side PQ is the half of the spherical excess E of the triangle PQC (see Fig. 13).

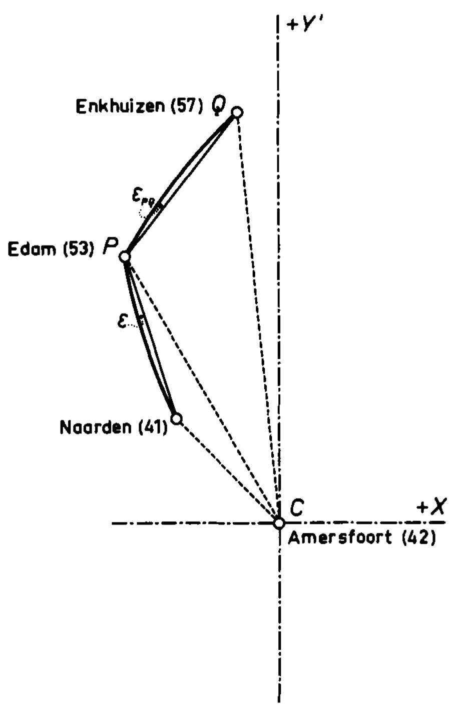


Fig. 13

If O is the area of this spherical triangle then:

$$E : 4\pi = O : 4\pi R^2, \text{ whence:}$$

$$E_{rad.} = O : R^2 \quad \text{and:}$$

$$\epsilon''_{PQ} = (\rho : 2R^2)O$$

As the spherical area is approximately equal to the area in the plane of projection and the latter is:

$$(X'_Q\ Y'_P - X'_P\ Y'_Q) : 2, \text{ one finds:}$$

$$\epsilon''_{PQ} = (\rho : 4R^2)(X'_Q Y'_P - X'_P Y'_Q)\ .$$

For R, X', and Y' in km one finds:

$$\epsilon''_{PQ} = 0.0012658\ (X'_Q Y'_P - X'_P Y'_Q) \quad \text{(11)}$$

The formula gives, with its sign, ϵ in a clockwise direction from the arc PQ to the chord.

For P = Edam and Q = Enkhuizen e.g. one finds $\epsilon_{PQ} = +1''.463$. As for P = Edam and Q = Naarden, $\epsilon = -0''.315$ and the measured spherical angle 481 at Edam between Enkhuizen and Naarden is $124^{\circ}01'02''.685$, that between the chords is: $124^{\circ}01'02''.685 + (-0''.315 - 1''.463) = 124^{\circ}01'00''.907$. With the other reduced angles the amount is mentioned in column 2 of table 12.

From the angles of the flat pentagon now follow in an arbitrary coordinate system xy the gridbearings ψ of the sides.

If the positive x-axis is chosen along the side Harderwijk-Urk (see Fig. 14) the gridbearings are the amounts in column 3 of table 12. The two amounts for Harderwijk-Urk are of course the same: their difference is the 149th polygon condition already mentioned on page 57.

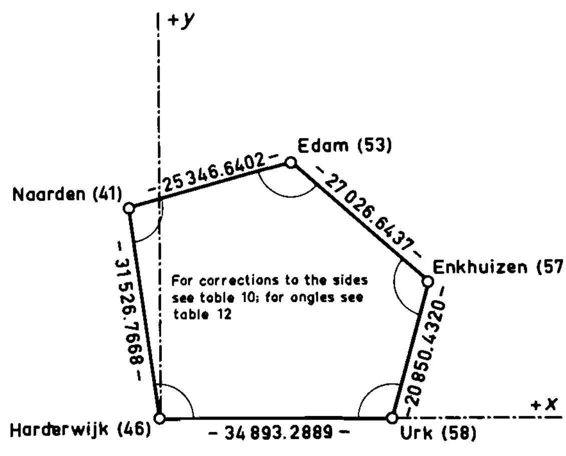


Fig. 14.

Table 12

angular point Zuiderzee pentagon	angles in plane of projection	Gridbearings ψ in coordinate system xy Fig. 14
1	2	3
Harderwijk		
		$90^{\circ}00'00''.000$
Urk	$105^{\circ}28'56''.160$ $+p_{239}+p_{240}-p_{279}$ $-p_{280}-p_{283}-p_{286}$	
		$15^{\circ}28'56''.160+p_{239}$ $+p_{240}-p_{279}-p_{280}-p_{283}$ $-p_{286}$
Enkhuizen	$115^{\circ}26'22''.152$ $+p_{484}$	
		$310^{\circ}55'18''.312+p_{239}$ $+p_{240}-p_{279}-p_{280}-p_{283}$ $-p_{286}+p_{484}$
Edam	$124^{\circ}01'00''.907$ $+p_{481}$	
		$254^{\circ}56'19''.219+p_{239}$ $+p_{240}-p_{279}-p_{280}-p_{283}$ $-p_{286}+p_{484}+p_{481}$
Naarden	$96^{\circ}57'58''.567$ $+p_{475}$	
		$171^{\circ}54'17''.786+p_{239}+$ $p_{240}-p_{279}-p_{280}-p_{283}-$ $p_{286}+p_{484}+p_{481}+p_{475}$
Harderwijk	$98^{\circ}05'34''.508$ $+p_{478}$	
		$89^{\circ}59'52''.294+p_{239}+$ $p_{240}-p_{279}-p_{280}-p_{283}-$ $p_{286}+p_{484}+p_{481}+p_{475}$ $+p_{478}$
Urk		

A combination, finally, of the lengths l of the sides of the pentagon (Fig. 14) with their reductions (table 10) and the gridbearings ψ in table 12 gives, as $\Sigma l \sin\psi = 0$ and $\Sigma l \cos\psi = 0$, the 53rd and 54th side equation in the network. With the 52nd already mentioned they can be found in table 13.

The 53rd e.g. runs as follows:

$$+1.1735-0.0318\, p_{161} + \ldots . -0.1832\, p_{481} - 0.0974\, p_{484} = 0.$$

Table 13

angles i	Side equation No. 52	53	54	angles i	Side equation No. 52	53	54
	-1.4897	+1.1735	-0.6834	238	+0.3197	-0.1836	-0.3463
161	-0.2498	-0.0318	+0.2235	239	+0.2764	-0.1781	0.0000
162	+0.2164	+0.0275	-0.1936	240	+0.0984	+0.0402	0.0000
164	+0.0922	+0.0117	-0.0825	242	-0.2425		
166	-0.0718	-0.0091	+0.0643	243	+0.1038		
185	-0.1256	+0.0721	+0.1361	275	-0.0749	+0.0868	+0.0431
186	+0.2080	-0.1194	-0.2253	276	+0.1869	-0.2167	-0.1077
188	-0.1571	+0.0902	+0.1702	277	-0.0009	+0.0009	0.0000
189		-0.0235	-0.0063	278		+0.0262	+0.0946
190	+0.1226	+0.0156	-0.1097	279	+0.1725	-0.2000	-0.0993
191		+0.0037	-0.0259	281	-0.1557	+0.1557	0.0000
192	+0.1123			282	+0.0340	-0.0340	0.0000
193	-0.1797	-0.0229	+0.1607	284	-0.0496	+0.0496	0.0000
194	+0.2122			285	+0.1229	-0.1229	0.0000
196	-0.1276			287	-0.0798	+0.0798	0.0000
225	-0.2319	+0.1332	+0.2512	288	+0.0572	-0.0572	0.0000
226	+0.0182	-0.0104	-0.0197	475		-0.1513	-0.1907
233	-0.1205	+0.1397	+0.0694	478		0.0000	-0.1692
234	+0.0608	-0.0705	-0.0350	481		-0.1832	-0.0720
236	-0.2470	+0.2864	+0.1423	484		-0.0974	+0.0270
237		+0.0520	-0.0451				

14. Analysis of the closing errors in the angles around the central points

It is obvious that Krayenhoff noticed that in all central points of his network the sum of the angles measured there had to be 360°. In tableau I of his Précis Historique he mentions even the amounts found for the "tours d'horizon" and their differences with 360°. For Amsterdam (station No. 40) e.g. it is 0".149, for Naarden (station No. 41) 3".501. They may be found in section 13 (table 9).

This is so much the more remarkable because Krayenhoff's predecessor Delambre, from whom he borrowed so much, took no account of these station conditions in the 8 cases in which this had to be done. He only paid attention to the closing errors in the triangles. At Vouzon, e.g. (about 30 km south of Orléans) the closing error in the tour d'horizon of 5 measured angles is 1".76. After the adjustment of the network, however, it increases to 9".49. [51]

The closing errors in Krayenhoff's tours d'horizon are very small. In order to investigate their reliability I computed the standard deviation $m\sqrt{n}$ in the sum of n angles around the several central points, assuming for the standard error m in an angle the amounts $m = \frac{2''.4}{\sqrt{2.28}} = \pm\ 1''.6$ for the years 1802-1807 and $m = \frac{4''.9}{\sqrt{2.12}} = \pm\ 3''.4$ for the years 1810 and 1811 found in section 8.

Table 14

Central points		m	n	$m\sqrt{n}$	Closing error	Central points		m	n	$m\sqrt{n}$	Closing error
No.	Name	''		''	''	No.	Name	''		''	''
1	2	3	4	5	6	1	2	3	4	5	6
55	Schagen	1.6	6	3.9	-1.855	75	Sneek	1.6	6	3.9	-0.217
47	Veluwe	1.6	7	4.2	-2.030	43	Imbosch	1.6	5	3.6	-0.193
48	Zutphen	1.6	6	3.9	+2.160	24	Hilvarenbeek	1.6	5	3.6	-0.171
90	Holwierde	3.4	5	7.6	+2.016	63	Uelsen	1.6	5	3.6	+0.138
79	Leeuwarden*	-	5	5.5	-2.197	40	Amsterdam	1.6	6	3.9	+0.149
59	Kampen	1.6	6	3.9	-2.269	25	Helmond	1.6	6	3.9	+0.127
35	Nieuwkoop	1.6	5	3.6	+2.361	84	Coevorden*	-	5	5.5	+0.110
89	Uithuizermeden	3.4	5	7.6	+2.723	46	Harderwijk	1.6	5	3.6	-0.084
14	Hulst	1.6	6	3.9	-2.309	28	Rotterdam	1.6	6	3.9	-0.060
93	Pilsum	3.4	4	6.8	+2.564	80	Dokkum	3.4	6	8.3	+0.045
18	Hoogstraten	1.6	6	3.9	+2.648	31	Gouda	3.4	6	8.3	+0.037
94	Emden	3.4	6	8.3	+3.251	87	Rolde	3.4	5	7.6	-0.017
41	Naarden	1.6	5	3.6	+3.501	22	Willemstad	1.6	6	3.9	-0.017

*3 angles in 1807 and 2 in 1810 (see table 3)

The results may be found in table 14. Next to the 13 largest closing errors in column 6 in the left part of the table and the 13 smallest in the right part one finds in column 5 the amounts $m''\sqrt{n}$, the standard deviation which can be expected in the sum of the n angles. For Naarden (station No. 41) one finds in the left part $m\sqrt{n} = 3''.6$ and for the closing error $+3''.501$. In the right part the amounts for Amsterdam (station No. 40) are $3''.9$ and $+0''.149$, respectively.

For the computation of column 5 for the stations Leeuwarden and Coevorden I took into account that 3 angles were measured with the accurate instrument and 2 with the less accurate one. Therefore the amount in column 5 is:

$$\sqrt{3(5.87:2.28)+2(24.33:2.12)} = 5''.5.$$

As can be seen from the table the closing error is, even for all the 26 cases, smaller than the standard error in the sum of n angles that can be expected in connection with the accuracy of the angular measurement. For the stations with the smallest closing errors the disharmony is of course the most remarkable. At Dokkum (station No. 80) e.g. the closing error is $0''.045$ whereas a standard deviation $m\sqrt{n} = 8''.3$ in this closing error could be expected. The station Gouda shows the same great disharmony. Here, however, is some doubt: it might be possible that the amount $3''.4$ in column 3 must be $1''.6$. In that case the angles at Gouda should have been measured with the "accurate" instrument.

Anyhow it is clear that Krayenhoff's closing errors in column 6 are much too low. There can be only one conclusion: in retaining or rejecting measured series Krayenhoff will have been guided by the wish to make the "errors" as small as possible. This endeavour, though often applied in those days, was rightly condemned by Gauss and Cohen Stuart. I am convinced, however, that Krayenhoff was honest in this matter, and did not try to flatter intentionally his observations. There are even indications that he left it to Van Swinden to decide which series should be used for the computation of the network and which should be rejected. These indications can be borrowed from Krayenhoff's letter to Van Swinden, dated June 3rd, 1803 [52] . It was found in the archives of the Netherlands Geodetic Commission by Mr. Van der Schraaf, assistant secretary of that commission. He gives an excerpt of the text of this letter in his paper [28] , already mentioned before (page 74). In my opinion it is an important letter which may throw some light on the responsibility for the arbitrariness in the choice of the series which should be used for the computation. I therefore give a translation of the passage in question with the quintessence of it underlined:
"I take the liberty, persuaded as I am by your permission so kindly given, to "submit these operations which have been faithfully registered without withholding "the smallest detail or arbitrary arrangement, respectfully to your judgment as "a University professor in order to give an equal sharp statement as given about "the observations of the French astronomers and therefore to choose from all the "observations those which, with rejection of the others, should be used for the "composition of the triangles which will only depend on your decision ..... ".
Unfortunately I know no answer to this letter. Baron Krayenhoff at Amersfoort could not give me any information either concerning correspondence between his ancestor and professor Van Swinden.

Though Krayenhoff's letter dates from the beginning of his measurements and was written 12 years before the publication of his Précis Historique, it remains possible, however, that Van Swinden influenced Krayenhoff's decisions in this matter. In a report, dated May 13th, 1813, signed by Van Swinden, Florijn, and Vrolik, but composed by Van Swinden [53] , he does not say anything more concerning this question than: "Monsieur Krayenhoff a examiné ses observations d'après cette règle" (sum angles around a central point is 360°) "et il a trouvé que les déviations sont excessivement petites".

15. Analysis of the closing errors in the triangles

In order to investigate the reliability of the closing errors in the triangles I arranged the greater part of the observations from table 9 in the 160 triangles 2-161 of table 15. Their number is the same as that in tableaux II and III of the Précis Historique. I have not copied No. 1. It is the last triangle, Duinkerken-Mont Cassel-Watten, of Delambre's network. With the exception of the numbers 20, 21, 29, 41, 53, 79, 84, 129, 137, 144, 145, and 156 (12 in total) all the angles of these triangles were measured. They give rise to the 160 - 12 = 148 conditions already mentioned before.

The closing errors range between $0''.000$ in triangle Oldenzaal-Uelsen-Bentheim (91) and $7''.657$ in triangle Oosterland-Kijkduin-Oosterend (107). According to Krayenhoff the large deviation in the latter triangle was caused by an error in the eccentricity of the signal Kijkduin during the measurement of the angle 309 at Oosterend: by a gust of wind the signal was blown down and, without Krayenhoff's knowledge, "erected anew in a place that differed more than a foot from the original one" [54] .

Table 15

No. triangle	Stations: Name	Stations: No.	No. angle	Observed spherical angles reduced to horizon and centre ° ′ ″	Corrections (sec. of arc) p′	Corrections (sec. of arc) p	Adj. spherical angles Précis Hist. ° ′	Adj. spherical angles Précis Hist. ″	Adj. spherical angles Least sq. ″	Opposite sides (chords) (metres) Précis Hist.	Opposite sides (chords) (metres) Least squares	Differences (cm) v	Differences (cm) v′
1	2	3	4	5	6	7	5+6 8	9	5+7 10	11	12	12–11 =13	14
2	Duinkerken	1	1	51 07 03.361	———	−0.464	51 07	03.361	02.897	21415.27	21416.16	89	− 3
	Mont Cassel	2	2	35 21 23.420	−0.697	−0.464	35 21	22.723	22.956	15919.29	15919.99	70	+ 2
	Hondschoote	3	3	93 31 35.472	−0.696	−0.464	93 31	34.776	35.008	27458.58	27459.76	+118	0
				180 00 02.253	−1.393	−1.392	180 00	00.860	00.861				
3	Duinkerken	1	4	43 51 34.238	———	+0.657	43 51	34.238	34.895	20070.83	20071.85	+102	+16
	Hondschoote	3	5	102 48 10.174	+0.985	+0.657	102 48	11.159	10.831	28246.59	28247.93	+134	+13
	Nieuwpoort	4	6	33 20 14.407	+0.984	+0.656	33 20	15.391	15.063	15919.29	15919.99	70	+ 2
				179 59 58.819	+1.969	+1.970	180 00	00.788	00.789				
4	Hondschoote	3	7	38 12 04.083	+0.203	+0.203	38 12	04.286	04.286	13234.92	13235.59	67	+10
	Nieuwpoort	4	8	72 06 23.004	+0.202	+0.203	72 06	23.206	23.207	20365.86	20366.89	+103	+15
	Diksmuide	5	9	69 41 32.944	+0.203	+0.203	69 41	33.147	33.147	20070.83	20071.85	+102	+16
				180 00 00.031	+0.608	+0.609	180 00	00.639	00.640				
5	Nieuwpoort	4	10	96 31 19.610	+0.384	+0.385	96 31	19.994	19.995	22125.02	22126.13	+111	+16
	Diksmuide	5	11	47 00 49.849	+0.384	+0.384	47 00	50.233	50.233	16290.31	16291.14	83	+13
	Oostende	6	12	36 27 49.930	+0.384	+0.384	36 27	50.314	50.314	13234.92	13235.59	67	+10
				179 59 59.389	+1.152	+1.153	180 00	00.541	00.542				
6	Diksmuide	5	13	42 07 50.732	+0.010	+0.011	42 07	50.742	50.743	21402.67	21403.74	+107	+15
	Oostende	6	14	93 57 52.236	+0.011	+0.011	93 57	52.247	52.247	31828.73	31830.31	+158	+21
	Brugge	7	15	43 54 18.195	+0.010	+0.011	43 54	18.205	18.206	22125.02	22126.13	+111	+16
				180 00 01.163	+0.031	+0.033	180 00	01.194	01.196				
7	Diksmuide	5	16	59 38 07.335	−0.248	−0.247	59 38	07.087	07.088	27465.58	27466.96	+138	+20
	Brugge	7	17	31 12 30.031	−0.248	−0.247	31 12	29.783	29.784	16493.82	16494.64		+11
	Hooglede	8	18	89 09 24.522	−0.247	−0.247	89 09	24.275	24.275	31828.73	31830.31	+158	+21
				180 00 01.888	−0.743	−0.741	180 00	01.145	01.147				
8	Brugge	7	19	38 38 56.617	+0.056	+0.056	38 38	56.673	56.674	17373.71	17374.58	87	+12
	Hooglede	8	20	60 28 54.848	+0.056	+0.056	60 28	54.904	54.904	24207.27	24208.49	+122	+18
	Tielt	9	21	80 52 09.417	+0.056	+0.057	80 52	09.473	09.473	27465.58	27466.96	+138	+20
				180 00 00.882	+0.168	+0.169	180 00	01.050	01.051				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12−11 = 13	14
9	Brugge	7	22	46 46 43.116	−0.462	−0.308	46 46	42.654	42.808	28630.25	28631.74	+149	+26
	Tielt	9	23	95 11 15.090	−0.463	−0.308	95 11	14.627	14.782	39128.01	39130.02	+201	+33
	Gent	10	24	38 02 04.464	———	−0.307	38 02	04.464	04.157	24207.27	24208.49	+122	+18
				180 00 02.670	−0.925	−0.923	180 00	01.745	01.747				
10	Brugge	7	25	50 54 24.760	+0.684	+0.229	50 54	25.444	24.989	31280.97	31282.54	+157	+22
	Gent	10	26	25 13 03.257	———	+0.228	25 13	03.257	03.485	17171.79	17172.73	+ 94	+20
	Aardenburg	11	27	103 52 32.617	———	+0.229	103 52	32.617	32.846	39128.01	39130.02	+201	+33
				180 00 00.634	+0.684	+0.686	180 00	01.318	01.320				
11	Gent	10	28	43 56 12.180	———	−0.060	43 56	12.180	12.120	21911.14	21912.24	+110	+16
	Aardenburg	11	29	38 11 30.503	+1.192	+1.030	38 11	31.695	31.533	19524.97	19525.93	+ 96	+12
	Assenede	12	30	97 52 17.551	−0.355	−0.131	97 52	17.196	17.420	31280.97	31282.54	+157	+22
				180 00 00.234	+0.837	+0.839	180 00	01.071	01.073				
12	Aardenburg	11	31	78 29 46.076	———	−0.282	78 29	46.076	45.794	31738.84	31740.39	+155	+19
	Middelburg	13	32	42 34 11.773	———	+0.328	42 34	11.773	12.101	21911.14	21912.24	+110	+16
	Assenede	12	33	58 56 04.360	−0.703	−0.747	58 56	03.657	03.613	27744.08	27745.42	+134	+15
				180 00 02.209	−0.703	−0.701	180 00	01.506	01.508				
13	Middelburg	13	34	33 52 06.765	−0.344	+0.028	33 52	06.421	06.793	21798.10	21799.26	+116	+22
	Hulst	14	35	54 14 07.427	+2.027	+1.362	54 14	09.454	08.789	31738.84	31740.39	+155	+19
	Assenede	12	36	91 53 46.218	−0.345	−0.049	91 53	45.873	46.169	39093.31	39095.30	+199	+31
				180 00 00.410	+1.338	+1.341	180 00	01.748	01.751				
14	Hulst	14	37	32 11 52.788	+0.010	−0.090	32 11	52.798	52.698	19524.97	19525.93	+ 96	+12
	Gent	10	38	36 30 14.930	———	+0.578	36 30	14.930	15.508	21798.10	21799.26	+116	+22
	Assenede	12	39	111 17 53.274	———	−0.476	111 17	53.274	52.798	34140.26	34141.99	+173	+26
				180 00 00.992	+0.010	+0.012	180 00	01.002	01.004				
15	Hulst	14	40	116 57 17.281	———	+0.086	116 57	17.281	17.367	50721.39	50723.94	+255	+37
	Antwerpen	15	41	36 52 05.744	−0.670	−0.631	36 52	05.074	05.113	34140.26	34141.99	+173	+26
	Gent	10	42	26 10 39.576	———	−0.122	26 10	39.576	39.454	25103.00	25104.23	+123	+15
				180 00 02.601	−0.670	−0.667	180 00	01.931	01.934				
16	Hulst	14	43	38 05 46.579	+0.272	+0.835	38 05	46.851	47.414	26714.21	26715.71	+150	+35
	Middelburg	13	44	77 21 50.324	———	+0.286	77 21	50.324	50.610	42249.37	42251.62	+225	+43
	Zierikzee	16	45	64 32 24.164	+1.237	+0.392	64 32	25.401	24.556	39093.31	39095.30	+199	+31
				180 00 01.067	+1.509	+1.513	180 00	02.576	02.580				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	9	5+7 10	11	12	12–11 =13	14
17	Hulst	14	46	47 29 05.400	—	+0.502	47 29	05.400	05.902	31143.07	31144.81	+174	+40
	Zierikzee	16	47	43 01 13.763	-1.409	-1.635	43 01	12.354	12.128	28825.86	28827.37	+151	+27
	Bergen op Zoom	17	48	89 29 44.515	—	-0.272	89 29	44.515	44.243	42249.37	42251.62	+225	+43
				180 00 03.678	-1.409	-1.405	180 00	02.269	02.273				
18	Hulst	14	49	71 01 48.216	—	-0.387	71 01	48.216	47.829	31474.31	31475.87	+156	+21
	Bergen op Zoom	17	50	48 57 37.261	—	-0.368	48 57	37.261	36.893	25103.00	25104.23	+123	+15
	Antwerpen	15	51	60 00 37.149	-0.896	-0.139	00 00	36.253	37.010	28825.85	28827.37	+152	+28
				180 00 02.626	-0.896	-0.894	180 00	01.730	01.732				
19	Bergen op Zoom	17	52	58 08 57.352	+0.560	+1.159	58 08	57.912	58.511	32182.15	32183.79	+164	+26
	Antwerpen	15	53	65 40 32.739	+0.821	-0.072	65 40	33.560	32.667	34523.71	34525.34	+163	+15
	Hoogstraten	18	54	56 10 30.861	—	+0.298	56 10	30.861	31.159	31474.31	31475.87	+156	+21
				180 00 00.952	+1.381	+1.385	180 00	02.333	02.337				
20	Antwerpen	15	55	48 05 13.808	+1.000	+0.582	48 05	14.808	14.390	25706.84	25708.04	+120	+ 9
	Hoogstraten	18	56	63 13 31.656	—	-0.483	63 13	31.656	31.173	30840.90	30842.37	+147	+14
	Herentals	104	—	—	—	—	68 41	15.403	16.307	32182.15	32183.79	+164	+26
				—	—	—	180 00	01.867	01.870				
21	Hoogstraten	18	57	52 06 35.704	—	-0.649	52 06	35.704	35.055	33995.62	33996.90	+128	-18
	Lommel	19	58	36 38 22.186	-1.623	-0.598	36 38	20.563	21.588	25706.84	25708.04	+120	+ 9
	Herentals	104	—	—	—	—	91 15	05.941	05.569	43066.39	43068.12	+173	-12
				—	—	—	180 00	02.208	02.212				
22	Zierikzee	16	59	51 16 31.236	+0.750	-0.138	51 16	31.986	31.098	30181.71	30183.25	+154	+24
	Willemstad	22	60	58 09 07.407	—	-0.216	58 09	07.407	07.191	32862.24	32864.00	+176	+35
	Brielle	21	61	70 34 24.678	-1.707	-0.599	70 34	22.971	24.079	36483.89	36485.93	+204	+47
				180 00 03.321	-0.957	-0.953	180 00	02.364	02.368				
23	Willemstad	22	62	57 32 10.481	—	+0.029	57 32	10.481	10.510	31143.07	31144.81	+174	+40
	Zierikzee	16	63	41 11 24.955	+1.542	+0.637	41 11	26.497	25.592	24308.43	24309.66	+123	+18
	Bergen op Zoom	17	64	81 16 25.064	-0.151	+0.728	81 16	24.913	25.792	36483.89	36485.93	+204	+47
				180 00 00.500	+1.391	+1.394	180 00	01.891	01.894				
24	Willemstad	22	65	89 20 33.430	—	-0.090	89 20	33.430	33.340	35376.12	35377.82	+170	+18
	Bergen op Zoom	17	66	47 15 25.087	—	-0.418	47 15	25.087	24.669	25982.08	25983.27	+119	+ 7
	Breda	23	67	43 24 04.399	-1.319	-0.809	43 24	03.080	03.590	24308.43	24309.66	+123	+18
				180 00 02.916	-1.319	-1.317	180.00	01.597	01.599				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12−11 =13	14
25	Bergen op Zoom	17	68	34 51 50.312	—	-0.420	34 51	50.312	49.892	20955.84	20956.78	+ 94	+ 4
	Breda	23	69	70 20 38.816	—	-0.061	70 20	38.816	38.755	34523.71	34525.34	+163	+15
	Hoogstraten	18	70	74 47 34.387	-1.750	-1.267	74 47	32.637	33.120	35376.12	35377.82	+170	+18
				180 00 03.515	-1.750	-1.748	180 00	01.765	01.767				
26	Breda	23	71	67 56 48.937	+1.378	+0.642	67 56	50.315	49.579	27695.68	27696.79	+111	- 8
	Hoogstraten	18	72	67 31 20.833	—	+0.077	67 31	20.833	20.910	27611.73	27612.88	+115	- 4
	Hilvarenbeek	24	73	44 31 50.208	—	+0.661	44 31	50.208	50.869	20955.84	20956.78	+ 94	+ 4
				179 59 59.978	+1.378	+1.380	180 00	01.356	01.358				
27	Hoogstraten	18	74	46 10 29.207	-0.898	-0.623	46 10	28.309	28.584	31142.93	31144.24	+131	- 3
	Hilvarenbeek	24	75	93 54 55.522	-0.531	-0.708	93 54	54.991	54.814	43066.37	43068.12	+175	-10
	Lommel	19	76	39 54 38.875	—	-0.095	39 54	38.875	38.780	27695.68	27696.79	+111	- 8
				180 00 03.604	-1.429	-1.426	180 00	02.175	02.178				
28	Hilvarenbeek	24	77	65 27 16.041	—	-0.210	65 27	16.041	15.831	36602.65	36604.24	+159	+ 2
	Lommel	19	78	63 50 11.437	-0.600	+0.068	63 50	10.837	11.505	36116.03	36117.68	+165	+10
	Helmond	25	79	50 42 35.777	-0.068	-0.523	50 42	35.709	35.254	31142.93	31144.24	+131	- 3
				180 00 03.255	-0.668	-0.665	180 00	02.587	02.590				
29	Helmond	25	80	57 21 25.713	—	-0.846	57 21	25.713	24.867	30931.08	30932.36	+128	- 5
	Nederweert	20	81	85 10 07.255	-1.500	-0.512	85 10	05.755	06.743	36602.65	36604.24	+159	+ 2
	Lommel	19	—	—	—	—	37 28	30.273	30.134	22349.01	22349.95	+ 94	- 2
				—	—	—	180 00	01.741	01.744				
30	Helmond	25	82	101 29 46.907	-1.344	-0.196	101 29	45.563	46.711	39049.08	39050.78	+170	+ 2
	Nederweert	20	83	44 23 21.748	+1.315	+0.546	44 23	23.063	22.294	27875.36	27876.50	+114	- 6
	Vierlingsbeek	26	84	34 06 52.917	—	-0.376	34 06	52.917	52.541	22349.01	22349.95	+ 94	- 2
				180 00 01.572	-0.029	-0.026	180 00	01.543	01.546				
31	Brielle	21	85	57 30 31.318	+5.242	+2.630	57 30	36.560	33.948	21233.15	21234.10	+ 95	+ 4
	Rotterdam	28	86	60 06 09.816	—	+1.226	60 06	09.816	11.042	21823.07	21824.28	+121	+27
	Den Haag	27	87	62 23 14.662	—	+1.387	62 23	14.662	16.049	22305.92	22307.16	+124	+28
				179 59 55.796	+5.242	+5.243	180 00	01.038	01.039				
32	Brielle	21	88	56 21 12.969	—	-0.728	56 21	12.969	12.241	25738.28	25739.54	+126	+15
	Rotterdam	28	89	77 28 19.415	-0.829	-1.090	77 28	18.586	18.325	30181.71	30183.25	+154	+24
	Willemstad	22	90	46 10 29.862	—	+0.991	46 10	29.862	30.853	22305.92	22307.16	+124	+28
				180 00 02.246	-0.829	-0.827	180 00	01.417	01.419				

Table 15 (continued)

1	2	3	4	5	6	7	5+6		5+7	11	12	12−11	14
							8	9	10			=13	
33	Rotterdam	28	91	52 36 40.963	−0.028	+0.575	52 36	40.935	41.538	20493.21	20494.26	+105	+17
	Dordrecht	29	92	86 16 35.683	−0.029	+0.440	86 16	35.654	36.123	25738.28	25739.54	+126	+15
	Willemstad	22	93	41 06 44.271	+0.017	−1.054	41 06	44.288	43.217	16959.66	16960.37	+ 71	− 2
				180 00 00.917	−0.040	−0.039	180 00	00.877	00.878				
34	Willemstad	22	94	67 40 54.532	———	+0.358	67 40	54.532	54.890	26279.99	26281.27	+128	+15
	Dordrecht	29	95	66 08 56.624	+1.000	+0.005	66 08	57.624	56.629	25982.08	25983.27	+119	+ 7
	Breda	23	96	46 10 10.812	−1.723	−1.084	46 10	09.089	09.728	20493.21	20494.26	+105	+17
				180 00 01.968	−0.723	−0.721	180 00	01.245	01.247				
35	Dordrecht	29	97	77 00 26.407	———	+0.312	77 00	26.407	26.719	30047.13	30048.63	+150	+21
	Breda	23	98	44 32 14.595	+0.389	+0.326	44 32	14.984	14.921	21628.01	21629.07	+106	+13
	Gorinchem	32	99	58 27 20.846	−0.837	−1.084	58 27	20.009	19.762	26279.99	26281.27	+128	+15
				180 00 01.848	−0.448	−0.446	180 00	01.400	01.402				
36	Breda	23	100	46 10 45.883	+1.906	+1.530	46 10	47.789	47.413	27985.58	27986.82	+124	+ 4
	Gorinchem	32	101	83 02 43.733	+0.800	+0.251	83 02	44.533	43.984	38501.81	38503.58	+177	+11
	's-Hertogen-	33	102	50 46 30.588	−0.800	+0.128	50 46	29.788	30.716	30047.12	30048.63	+151	+22
	bosch			180 00 00.204	+1.906	+1.909	180 00	02.110	02.113				
37	Breda	23	103	41 25 15.627	+0.300	+0.386	41 25	15.927	16.013	25503.39	25504.59	+120	+10
	Hilvarenbeek	24	104	92 49 53.184	−0.928	+0.029	92 49	52.256	53.213	38501.81	38503.58	+177	+11
	's-Hertogen-	33	105	45 44 52.895	+0.700	−0.340	45 44	53.595	52.555	27611.73	27612.88	+115	− 4
	bosch			180 00 01.706	+0.072	+0.075	180 00	01.778	01.781				
38	Hilvarenbeek	24	106	63 16 04.874	+1.630	+0.400	63 16	06.504	05.274	33558.32	33559.72	+140	− 4
	's-Hertogenb.	33	107	73 59 10.573	−0.787	−0.090	73 59	09.786	10.483	36116.03	36117.68	+165	+10
	Helmond	25	108	42 44 45.790	———	+0.536	42 44	45.790	46.326	25503.38	25504.59	+121	+11
				180 00 01.237	+0.843	+0.846	180 00	02.080	02.083				
39	's-Hertogenb.	33	109	59 02 50.019	−0.662	−0.717	59 02	49.357	49.302	31858.33	31859.64	+131	− 6
	Helmond	25	110	56 21 03.842	+1.285	+1.118	56 21	05.127	04.960	30924.48	30925.73	+125	− 8
	Grave	34	111	64 36 07.766	———	+0.225	64 36	07.766	07.991	33558.32	33559.72	+140	− 4
				180 00 01.627	+0.623	+0.626	180 00	02.250	02.253				
40	Helmond	25	112	51 20 22.098	———	−0.217	51 20	22.098	21.881	26123.38	26124.43	+105	− 7
	Grave	34	113	56 25 55.298	−0.400	−0.293	56 25	54.898	55.005	27875.36	27876.50	+114	− 6
	Vierlingsbeek	26	114	72 13 43.931	+0.826	+0.939	72 13	44.757	44.870	31858.33	31859.64	+131	− 6
				180 00 01.327	+0.426	+0.429	180 00	01.753	01.756				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12–11 =13	14
41	Vierlingsbeek	26	115	23 57 11.703	-0.880	-1.145	23 57	10.823	10.558	11217.44	11217.85	+ 41	- 7
	Grave	34	116	47 02 19.299	-0.745	-0.469	47 02	18.554	18.830	20219.99	20220.83	+ 84	- 3
	Biesselt	105	—	—	—	—	109 00	31.165	31.155	26123.38	26124.43	+105	- 7
				—	—	—	180 00	00.542	00.543				
42	Rotterdam	28	117	36 09 43.225	+0.306	+0.334	36 09	43.531	43.559	15402.90	15403.55	+ 65	- 1
	Den Haag	27	118	89 24 20.957	+2.143	+1.066	89 24	23.100	22.023	26102.08	26103.16	+108	- 4
	Leiden	30	119	54 25 55.896	-1.700	-0.650	54 25	54.196	55.246	21233.15	21234.10	+ 95	+ 4
				180 00 00.078	+0.749	+0.750	180 00	00.827	00.828				
43	Rotterdam	28	120	56 16 44.929	—	-1.920	56 16	44.929	43.009	22062.39	22063.14	+ 75	-20
	Leiden	30	121	43 58 05.908	+1.600	+2.884	43 58	07.508	08.792	18415.53	18416.39	+ 86	+ 7
	Gouda	31	122	79 45 10.672	-2.098	-1.461	79 45	08.574	09.211	26102.08	26103.16	+108	- 4
				180 00 01.509	-0.498	-0.497	180 00	01.011	01.012				
44	Gouda	31	123	48 22 15.785	+1.806	+1.112	48 22	17.591	16.897	16959.66	16960.37	+ 71	- 2
	Rotterdam	28	124	77 22 21.592	+0.611	+0.935	77 22	22.203	22.527	22140.69	22141.71	+102	+ 7
	Dordrecht	29	125	54 15 22.676	-1.700	-1.328	54 15	20.976	21.348	18415.53	18416.39	+ 86	+ 7
				180 00 00.053	+0.717	+0.719	180 00	00.770	00.772				
45	Dordrecht	29	126	76 18 37.923	+1.416	+1.259	76 18	39.339	39.182	27043.15	27044.42	+127	+11
	Gouda	31	127	50 59 27.052	-1.044	-0.636	50 59	26.008	26.416	21628.01	21629.07	+106	+13
	Gorinchem	32	128	52 41 57.129	-1.300	-1.549	52 41	55.829	55.580	22140.69	22141.71	+102	+ 7
				180 00 02.104	-0.928	-0.926	180 00	01.176	01.178				
46	Gouda	31	129	59 43 08.897	+0.650	+1.472	59 43	09.547	10.369	19730.88	19731.66	+ 78	- 7
	Leiden	30	130	45 21 05.907	-2.580	-0.828	45 21	03.327	05.079	16254.73	16255.47	+ 74	+ 4
	Nieuwkoop	35	131	74 55 45.056	+2.853	+0.280	74 55	47.909	45.336	22062.39	22063.14	+ 75	-20
				179 59 59.860	+0.923	+0.924	180 00	00.783	00.784				
47	Gouda	31	132	55 23 26.199	+0.649	+0.299	55 23	26.848	26.498	24320.31	24321.44	+113	+ 8
	Nieuwkoop	35	133	91 14 12.929	-2.500	-1.982	91 14	10.429	10.947	29542.33	29543.74	+141	+14
	Utrecht	36	134	33 22 20.827	+2.895	+2.729	33 22	23.722	23.556	16254.72	16255.47	+ 75	+ 5
				179 59 59.955	+1.044	+1.046	180 00	00.999	01.001				
48	Gouda	31	135	65 46 31.432	—	-0.821	65 46	31.432	30.611	30797.10	30798.46	+136	+ 4
	Gorinchem	32	136	61 01 09.789	+2.150	+2.685	61 01	11.939	12.474	29542.33	29543.74	+141	+14
	Utrecht	36	137	53 12 20.865	-2.394	-2.106	53 12	18.471	18.759	27043.15	27044.42	+127	+11
				180 00 02.086	-0.244	-0.242	180 00	01.842	01.844				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12-11 =13	14
49	Gorinchem	32	138	51 15 01.187	-1.369	-0.353	51 14	59.818	60.834	33841.41	33842.92	+151	+ 5
	Utrecht	36	139	83 32 16.923	—	-1.712	83 32	16.923	15.211	43117.29	43119.01	+172	-13
	Rhenen	37	140	45 12 44.843	+1.034	+1.734	45 12	45.877	46.577	30797.09	30798.46	+137	+ 5
				180 00 02.953	-0.335	-0.331	180 00	02.618	02.622				
50	's-Hertogenb.	33	141	86 06 43.267	+1.336	+1.123	86 06	44.603	44.390	43117.29	43119.01	+172	-13
	Gorinchem	32	142	53 31 47.872	—	-0.508	53 31	47.872	47.364	34753.43	34754.76	+133	-16
	Rhenen	37	143	40 21 30.338	-0.360	+0.365	40 21	29.978	30.703	27985.58	27986.82	+124	+ 4
				180 00 01.477	+0.976	+0.980	180 00	02.453	02.457				
51	's-Hertogenb.	33	144	44 19 53.092	-0.221	-0.538	44 19	52.871	52.554	25031.71	25032.64	+ 93	-15
	Rhenen	37	145	59 41 27.650	-1.170	-0.614	59 41	26.480	27.036	30924.48	30925.73	+125	- 8
	Grave	34	146	75 58 41.229	+1.318	+1.082	75 58	42.547	42.311	34753.42	34754.76	+134	-15
				180 00 01.971	-0.073	-0.070	180 00	01.898	01.901				
52	Grave	34	147	69 36 06.705	-0.330	-0.738	69 36	06.375	05.967	23833.27	23834.12	+ 85	-17
	Rhenen	37	148	30 31 27.217	—	+0.087	30 31	27.217	27.304	12914.81	12915.30	+ 49	- 7
	Nijmegen	38	149	79 52 27.174	—	+0.322	79 52	27.174	27.496	25031.71	25032.64	+ 93	-15
				180 00 01.096	-0.330	-0.329	180 00	00.766	00.767				
53	Grave	34	150	46 20 50.424	-0.564	-0.528	46 20	49.860	49.896	9623.82	9624.19	+ 37	- 4
	Nijmegen	38	151	57 29 42.299	+0.564	+0.235	57 29	42.863	42.534	11217.44	11217.85	+ 41	- 7
	Biesselt	105	—	—	—	—	76 09	27.542	27.835	12914.81	12915.30	+ 49	- 7
				—	—	—	180 00	00.265	00.265				
54	Nieuwkoop	35	152	67 22 13.986	-1.000	-1.452	67 22	12.986	12.534	26939.81	26940.92	+111	- 5
	Leiden	30	153	70 05 48.204	—	+0.920	70 05	48.204	49.124	27443.52	27444.73	+121	+ 3
	Haarlem	39	154	42 31 59.378	+0.696	+0.229	42 31	60.074	59.607	19730.88	19731.66	+ 78	- 7
				180 00 01.568	-0.304	-0.303	180 00	01.264	01.265				
55	Nieuwkoop	35	155	36 32 08.209	+0.716	+2.599	36 32	08.925	10.808	16789.28	16790.26	+ 98	+26
	Haarlem	39	156	66 46 54.724	+0.500	+0.146	66 46	55.224	54.870	25917.88	25919.05	+117	+ 6
	Amsterdam	40	157	76 40 54.984	+1.938	+0.410	76 40	56.922	55.394	27443.52	27444.73	+121	+ 3
				179 59 57.917	+3.154	+3.155	180 00	01.071	01.072				
56	Utrecht	36	158	46 51 34.789	+1.380	+0.954	46 51	36.169	35.743	25917.87	25919.05	+118	+ 7
	Nieuwkoop	35	159	89 55 42.181	-2.430	-1.806	89 55	39.751	40.375	35519.27	35520.94	+167	+14
	Amsterdam	40	160	43 12 46.973	-1.300	-1.495	43 12	45.673	45.478	24320.31	24321.44	+113	+ 8
				180 00 03.943	-2.350	-2.347	180 00	01.593	01.596				

Table 15 (continued)

1	2	3	4	5	6	7	8 (5+6)	9 (5+6)	5+7 10	11	12	12-11 =13	14
57	Utrecht	36	161	34 06 09.301	-1.211	-0.048	34 06	08.090	09.253	20925.08	20926.22	+114	+24
	Amsterdam	40	162	38 01 09.588	+0.342	+0.172	38 01	09.930	09.760	22987.37	22988.39	+102	+ 3
	Naarden	41	163	107 52 42.537	+0.600	-0.391	107 52	43.137	42.146	35519.27	35520.94	+167	+14
				180 00 01.426	-0.269	-0.267	180 00	01.157	01.159				
58	Utrecht	36	164	61 23 56.660	—	+0.731	61 23	56.660	57.391	21926.44	21927.50	+106	+12
	Naarden	41	165	51 36 31.081	-1.300	-1.081	51 36	29.781	30.000	19574.09	19575.02	+ 93	+ 9
	Amersfoort	42	166	66 59 32.774	+1.784	+0.835	66 59	34.558	33.609	22987.37	22988.39	+102	+ 3
				180 00 00.515	+0.484	+0.485	180 00	00.999	01.000				
59	Rhenen	37	167	34 59 33.730	+1.838	+2.244	34 59	35.568	35.974	19574.09	19575.02	+ 93	+ 9
	Utrecht	36	168	47 31 22.466	-2.501	-2.380	47 31	19.965	20.086	25173.83	25174.96	+113	+ 5
	Amersfoort	42	169	97 29 06.032	-0.330	-0.855	97 29	05.702	05.177	33841.41	33842.92	+151	+ 5
				180 00 02.228	-0.993	-0.991	180 00	01.235	01.237				
60	Rhenen	37	170	61 42 57.299	-0.296	-2.210	61 42	57.003	55.089	33374.16	33375.06	+ 90	-54
	Amersfoort	42	171	76 39 40.413	-0.778	-1.291	76 39	39.635	39.122	36876.58	36877.73	+115	-44
	Veluwe	47	172	41 37 26.029	-0.600	+1.830	41 37	25.429	27.859	25173.83	25174.96	+113	+ 5
				180 00 03.741	-1.674	-1.671	180 00	02.067	02.070				
61	Rhenen	37	173	40 06 15.236	—	+1.841	40 06	15.236	17.077	23900.60	23901.61	+101	- 2
	Veluwe	47	174	56 13 18.524	-0.843	-1.306	56 13	17.681	17.218	30839.28	30840.22	+ 94	-39
	Imbosch	43	175	83 40 28.485	+0.450	-0.925	83 40	28.935	27.560	36876.58	36877.73	+115	-44
				180 00 02.245	-0.393	-0.390	180 00	01.852	01.855				
62	Nijmegen	38	176	82 34 23.739	+0.324	+1.196	82 34	24.063	24.935	30839.28	30840.22	+ 94	-39
	Rhenen	37	177	47 24 02.641	—	-2.401	47 24	02.641	00.240	22892.99	22893.43	+ 44	-54
	Imbosch	43	178	50 01 34.664	—	+1.531	50 01	34.664	36.195	23833.27	23834.12	+ 85	-17
				180 00 01.044	+0.324	+0.326	180 00	01.368	01.370				
63	Nijmegen	38	179	48 58 05.286	-2.300	-1.166	48 58	02.986	04.120	20948.25	20948.86	+ 61	-29
	Imbosch	43	180	75 30 30.884	—	+0.335	75 30	30.884	31.219	26886.92	26887.58	+ 66	-50
	Hettenheuvel	44	181	55 31 23.184	+4.120	+2.652	55 31	27.304	25.836	22892.99	22893.43	+ 44	-54
				179 59 59.354	+1.820	+1.821	180 00	01.174	01.175				
64	Amsterdam	40	182	69 52 42.395	-1.503	+0.938	69 52	40.892	43.333	28882.63	28884.84	+221	+97
	Haarlem	39	183	77 02 28.593	+3.558	+2.159	77 02	32.151	30.752	29977.07	29978.99	+192	+63
	Alkmaar	52	184	33 04 48.800	-0.648	-1.689	33 04	48.152	47.111	16789.28	16790.26	+ 98	+26
				179 59 59.788	+1.407	+1.408	180 00	01.195	01.196				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12-11 =13	14
65	Amsterdam	40	185	53 24 03.079	—	+0.056	53 24	03.079	03.135	24089.83	24091.34	+151	+47
	Alkmaar	52	186	39 07 37.587	+1.753	+0.457	39 07	39.340	38.044	18935.42	18936.48	+106	+25
	Edam	53	187	87 28 20.879	-2.146	-0.904	87 28	18.733	19.975	29977.07	29978.99	+192	+63
				180 00 01.545	-0.393	-0.391	180 00	01.152	01.154				
66	Naarden	41	188	47 07 13.651	-2.482	-1.970	47 07	11.169	11.681	18935.42	18936.48	+106	+25
	Amsterdam	40	189	78 48 23.130	+0.374	-0.230	78 48	23.504	22.900	25349.06	25350.42	+136	+27
	Edam	53	190	54 04 25.662	+0.648	+0.741	54 04	26.310	26.403	20925.08	20926.22	+114	+24
				180 00 02.443	-1.460	-1.459	180 00	00.983	00.984				
67	Amersfoort	42	191	80 17 59.800	—	+0.668	80 17	59.800	60.468	31530.45	31531.73	+128	- 8
	Naarden	41	192	56 25 38.079	+3.892	+1.640	56 25	41.971	39.719	26652.02	26652.90	+ 88	-27
	Harderwijk	46	193	43 16 21.185	-1.500	+0.086	43 16	19.685	21.271	21926.44	21927.50	+106	+12
				179 59 59.064	+2.392	+2.394	180 00	01.456	01.458				
68	Amersfoort	42	194	38 33 41.746	-1.441	-0.122	38 33	40.305	41.624	20811.16	20811.89	+ 73	-16
	Harderwijk	46	195	88 28 12.125	+4.314	+1.319	88 28	16.439	13.444	33374.16	33375.06	+ 90	-54
	Veluwe	47	196	52 58 04.658	—	+1.678	52 58	04.658	06.336	26652.02	26652.90	+ 88	-27
				179 59 58.529	+2.873	+2.875	180 00	01.402	01.404				
69	Imbosch	43	197	73 10 53.214	-0.257	-1.434	73 10	52.957	51.780	25382.91	25383.84	+ 93	-16
	Veluwe	47	198	42 29 06.914	+2.700	+2.230	42 29	09.614	09.144	17909.95	17910.58	+ 63	-14
	Zutphen	48	199	64 20 02.273	-3.808	-2.160	64 19	58.465	60.113	23900.60	23901.61	+101	- 2
				180 00 02.401	-1.365	-1.364	180 00	01.036	01.037				
70	Hettenheuvel	44	200	45 38 33.689	-2.460	-1.807	45 38	31.229	31.882	17909.95	17910.58	+ 63	-14
	Imbosch	43	201	77 36 32.560	—	+0.688	77 36	32.560	33.248	24465.89	24466.72	+ 83	-22
	Zutphen	48	202	56 44 53.203	+3.934	+2.595	56 44	57.137	55.798	20948.25	20948.86	+ 61	-29
				179 59 59.452	+1.474	+1.476	180 00	00.926	00.928				
71	Hettenheuvel	44	203	69 01 23.529	+1.700	+0.048	69 01	25.229	23.577	30914.94	30915.68	+ 74	-59
	Zutphen	48	204	63 20 07.051	-1.200	-1.199	63 20	05.851	05.852	29587.82	29588.62	+ 80	-47
	Groenlo	49	205	47 38 32.910	-2.281	-0.628	47 38	30.629	32.282	24465.89	24466.72	+ 83	-22
				180 00 03.490	-1.781	-1.779	180 00	01.709	01.711				
72	Bocholt	45	206	71 06 29.541	+2.081	+1.410	71 06	31.622	30.951	29587.82	29588.62	+ 80	-47
	Hettenheuvel	44	207	46 35 39.391	—	-0.260	46 35	39.391	39.131	22719.49	22720.10	+ 61	-37
	Groenlo	49	208	62 17 50.492	—	+0.933	62 17	50.492	51.425	27687.62	27688.47	+ 85	-34
				179 59 59.424	+2.081	+2.083	180 00	01.505	01.507				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	9	5+7 10	11	12	12−11 =13	14
73	Bocholt	45	209	44 43 11.632	-0.180	-1.052	44 43	11.452	10.580	26914.95	26915.38	+43	-73
	Groenlo	49	210	98 50 31.745	-0.160	-0.249	98 50	31.585	31.496	37796.46	37797.23	+77	-86
	Ahaus	51	211	36 26 18.268	+0.222	+1.186	36 26	18.490	19.454	22719.49	22720.10	+61	-37
				180 00 01.645	-0.118	-0.115	180 00	01.527	01.530				
74	Groenlo	49	212	55 27 04.992	+2.102	+0.798	55 27	07.094	05.790	25880.60	25881.08	+48	-63
	Zutphen	48	213	44 51 14.201	-0.626	-0.808	44 51	13.575	13.393	22161.78	22162.25	+47	-48
	Harikerberg	50	214	79 41 42.538	-1.780	-0.292	79 41	40.758	42.246	30914.94	30915.68	+74	-59
				180 00 01.731	-0.304	-0.302	180 00	01.427	01.429				
75	Groenlo	49	215	95 45 59.600	+0.600	-0.593	95 45	60.200	59.007	36543.42	36543.98	+56	-101
	Harikerberg	50	216	47 07 15.812	+0.150	+0.371	47 07	15.962	16.183	26914.95	26915.38	+43	-73
	Ahaus	51	217	37 06 46.771	-1.433	-0.459	37 06	45.338	46.312	22161.78	22162.25	+47	-48
				180 00 02.183	-0.683	-0.681	180 00	01.500	01.502				
76	Ahaus	51	218	49 16 03.562	-1.295	-1.081	49 16	02.267	02.481	27853.83	27854.29	+46	-74
	Harikerberg	50	219	46 55 41.986	+0.600	+0.501	46 55	42.586	42.487	26851.81	26852.22	+41	-74
	Oldenzaal	61	220	83 48 17.027	—	-0.113	83 48	17.027	16.914	36543.42	36543.98	+56	-101
				180 00 02.575	-0.695	-0.693	180 00	01.880	01.882				
77	Ahaus	51	221	33 34 42.066	+1.320	+0.482	33 34	43.386	42.548	15623.94	15624.05	+11	-56
	Oldenzaal	61	222	74 30 58.834	—	-0.425	74 30	58.834	58.409	27223.64	27223.98	+34	-83
	Bentheim	62	223	71 54 20.278	-1.476	-0.211	71 54	18.802	20.067	26851.81	26852.22	+41	-74
				180 00 01.178	-0.156	-0.154	180 00	01.022	01.024				
78	Edam	53	224	60 01 39.912	-2.553	-0.502	60 01	37.359	39.410	20988.38	20989.85	+147	+57
	Alkmaar	52	225	36 06 37.800	+2.719	+0.837	36 06	40.519	38.637	14279.56	14280.02	+ 46	-15
	Hoorn	54	226	83 51 42.875	—	-0.168	83 51	42.875	42.707	24089.83	24091.34	+151	+47
				180 00 00.587	+0.166	+0.167	180 00	00.753	00.754				
79	Alkmaar	52	479	32 45 24.013	+1.034	-0.070	32 45	25.047	23.943	9623.07	—	—	—
	Schagen	55	483	66 25 55.084	-2.067	+1.266	66 25	53.017	56.350	16301.54	—	—	—
	Petten	—	—	—	—	—	80 48	42.328	40.099	17557.94	17558.29	+35	-40
				—	—	—	180 00	00.392	00.392				
80	Hoorn	54	227	45 08 18.486	-2.793	-0.569	45 08	15.693	17.917	17557.94	17558.29	+ 35	-40
	Alkmaar	52	228	76 56 19.088	+2.734	-0.888	76 56	21.822	18.200	24129.08	24130.57	+149	+45
	Schagen	55	229	57 55 25.418	-2.025	-0.626	57 55	23.393	24.792	20988.38	20989.85	+147	+57
				180 00 02.992	-2.084	-2.083	180 00	00.908	00.909				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12−11 =13	14
81	Hoorn	54	230	58 45 27.585	—	+1.230	58 45	27.585	28.815	20786.47	20787.82	+135	+46
	Schagen	55	231	38 16 46.153	-0.060	-1.972	38 16	46.093	44.181	15061.34	15062.10	+ 76	+11
	Medemblik	56	232	82 57 48.191	-1.083	-0.400	82 57	47.108	47.791	24129.08	24130.57	+149	+45
				180 00 01.929	-1.143	-1.142	180 00	00.786	00.787				
82	Hoorn	54	233	54 32 14.155	+0.963	-1.355	54 32	15.118	12.800	14932.21	14932.73	+52	-12
	Medemblik	56	234	70 13 24.339	+0.003	+0.172	70 13	24.342	24.511	17251.77	17252.52	+75	+ 1
	Enkhuizen	57	235	55 14 22.578	-1.503	+0.647	55 14	21.075	23.225	15061.34	15062.10	+76	+11
				180 00 01.072	-0.537	-0.536	180 00	00.535	00.536				
83	Edam	53	236	34 24 30.351	-1.403	-0.246	34 24	28.948	30.105	17251.78	17252.52	+ 74	0
	Hoorn	54	237	117 42 17.226	+1.503	+0.536	117 42	18.729	17.762	27029.50	27030.50	+100	-16
	Enkhuizen	57	238	27 53 13.893	-1.019	-1.208	27 53	12.874	12.685	14279.56	14280.02	+ 46	-15
				180 00 01.470	-0.919	-0.918	180 00	00.551	00.552				
84	Harderwijk	46	239	43 31 47.020	+4.340	+3.433	43 31	51.360	50.453	24702.33	24703.15	+ 82	-24
	Kampen	59	240	76 38 47.519	—	+1.900	76 38	47.519	49.419	24896.06	34897.44	+138	-12
	Urk	58	—	—	—	—	59 49	23.005	22.015	31005.09	31006.16	+107	-26
				—	—	—	180 00	01.884	01.887				
85	Harderwijk	46	241	86 38 05.072	-2.469	-2.926	86 38	02.603	02.146	36313.15	36314.38	+123	-33
	Kampen	59	242	34 53 52.053	+0.290	+0.432	34 53	52.343	52.485	20811.16	20811.89	+ 73	-16
	Veluwe	47	243	58 28 06.683	—	+0.317	58 28	06.683	07.000	31005.09	31006.16	+107	-26
				180 00 03.808	-2.179	-2.177	180 00	01.629	01.631				
86	Veluwe	47	244	48 13 56.330	+0.773	-1.393	48 13	57.103	54.937	34703.12	34703.58	+ 46	-103
	Kampen	59	245	80 27 56.965	—	-0.580	80 27	56.965	56.385	45885.43	45886.46	+103	- 94
	Lemelerberg	60	246	51 18 09.445	-0.371	+2.379	51 18	09.074	11.824	36313.15	36314.38	+123	- 33
				180 00 02.740	+0.402	+0.406	180 00	03.142	03.146				
87	Veluwe	47	247	59 59 58.832	—	-1.327	59 59	58.832	57.505	39812.56	39813.33	+ 77	-94
	Lemelerberg	60	248	33 30 52.529	-1.598	+0.354	33 30	50.931	52.883	25382.91	25383.84	+ 93	-16
	Zutphen	48	249	86 29 12.047	+0.740	+0.119	86 29	12.787	12.166	45885.43	45886.46	+103	-94
				180 00 03.408	-0.858	-0.854	180 00	02.550	02.554				
88	Zutphen	48	250	44 14 33.385	-1.200	-0.707	44 14	32.185	32.678	27902.21	27902.80	+59	-61
	Lemelerberg	60	251	40 19 37.920	-2.069	-2.215	40 19	35.851	35.705	25880.61	25881.08	+47	-64
	Harikerberg	50	252	95 25 52.981	+0.800	+0.456	95 25	53.781	53.437	39812.56	39813.33	+77	-94
				180 00 04.286	-2.469	-2.466	180 00	01.817	01.820				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12-11 =13	14
89	Harikerberg	50	253	90 49 25.721	+1.192	-0.075	90 49	26.913	25.646	39707.92	39708.54	+62	-109
	Lemelerberg	60	254	44 32 21.135	———	+0.178	44 32	21.135	21.313	27853.84	27854.29	+45	-75
	Oldenzaal	61	255	44 38 14.838	-0.922	+0.170	44 38	13.916	15.008	27902.21	27902.80	+59	-61
				180 00 01.694	+0.270	+0.273	180 00	01.964	01.967				
90	Oldenzaal	61	256	52 56 09.399	-0.545	-0.446	52 56	08.854	08.953	31711.92	31712.43	+51	-85
	Lemelerberg	60	257	34 43 02.479	———	-0.588	34 43	02.479	01.891	22633.72	22633.98	+26	-71
	Uelsen	63	258	92 20 50.480	———	+0.492	92 20	50.480	50.972	39707.92	39708.54	+62	-109
				180 00 02.358	-0.545	-0.542	180 00	01.813	01.816				
91	Oldenzaal	61	259	104 06 20.671	+0.698	+0.045	104 06	21.369	20.716	30475.56	30475.83	+27	-104
	Uelsen	63	260	29 48 57.592	-0.699	-0.818	29 48	56.893	56.774	15623.93	15624.05	+12	-55
	Bentheim	62	261	46 04 42.605	———	+0.773	46 04	42.605	43.378	22633.72	22633.98	+26	-71
				180 00 00.868	-0.001	0.000	180 00	00.867	00.868				
92	Bentheim	62	262	48 15 55.697	+0.630	+0.200	48 15	56.327	55.897	27815.48	27815.56	+ 8	-112
	Uelsen	63	263	76 53 21.043	+0.328	-0.298	76 53	21.371	20.745	36302.66	36302.83	+17	-139
	Kirch Hesepe	64	264	54 50 45.189	-0.800	+0.259	54 50	44.389	45.448	30475.56	30475.83	+27	-104
				180 00 01.929	+0.158	+0.161	180 00	02.087	02.090				
93	Schagen	55	265	57 06 16.779	+3.076	+1.333	57 06	19.855	18.112	19547.08	19548.07	+ 99	+15
	Kijkduin	65	266	68 19 09.784	-4.001	-1.992	68 19	05.783	07.792	21632.42	21633.71	+129	+36
	Oosterland	66	267	54 34 35.391	-0.158	-0.423	54 34	35.233	34.968	18970.16	18971.21	+105	+23
				180 00 01.954	-1.083	-1.082	180 00	00.871	00.872				
94	Schagen	55	268	52 13 23.621	+4.263	+0.589	52 13	27.884	24.210	18685.26	18686.09	+ 83	+ 3
	Oosterland	66	269	61 33 33.953	-4.928	-2.237	61 33	29.025	31.716	20786.47	20787.82	+135	+46
	Medemblik	56	270	66 13 04.686	-0.696	+0.288	66 13	03.990	04.974	21632.42	21633.71	+129	+36
				180 00 02.260	-1.361	-1.360	180 00	00.899	00.900				
95	Medemblik	56	271	74 28 04.814	———	+0.183	74 28	04.814	04.997	24169.54	24170.36	+82	-22
	Oosterland	66	272	57 23 02.881	+3.740	+1.349	57 23	06.621	04.230	21129.95	21130.50	+55	-36
	Staveren	67	273	48 08 53.230	-3.703	-1.494	48 08	49.527	51.736	18685.24	18686.09	+85	+ 5
				180 00 00.925	+0.037	+0.038	180 00	00.962	00.963				
96	Enkhuizen	57	274	71 43 32.073	-2.093	-1.721	71 43	29.980	30.352	21129.95	21130.50	+55	-36
	Medemblik	56	275	66 07 38.243	+1.503	-0.517	66 07	39.746	37.726	20348.61	20349.04	+43	-45
	Staveren	67	276	42 08 52.250	-1.247	+0.402	42 08	51.003	52.652	14932.21	14932.73	+52	-12
				180 00 02.566	-1.837	-1.836	180 00	00.729	00.730				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	9	5+7 10	11	12	12-11 =13	14
97	Enkhuizen	57	277	89 42 24.939	+0.128	+0.894	89 42	25.067	25.833	29061.34	29062.14	+80	-45
	Staveren	67	278	45 51 04.052	+0.753	+1.211	45 51	04.805	05.263	20852.78	20853.39	+61	-29
	Urk	58	279	44 26 31.201	——	-1.223	44 26	31.201	29.978	20348.61	20349.04	+43	-45
				180 00 00.192	+0.881	+0.882	180 00	01.073	01.704				
98	Urk	58	280	53 59 14.365	-1.255	-0.213	53 59	13.110	14.152	23977.22	23977.97	+75	-28
	Staveren	67	281	47 22 35.387	——	-0.416	47 22	35.387	34.971	21811.42	21811.99	+57	-37
	Lemmer	68	282	78 38 12.843	-0.044	-0.668	78 38	12.799	12.175	29061.34	29062.14	+80	-45
				180 00 02.595	-1.299	-1.297	180 00	01.296	01.298				
99	Urk	58	283	52 21 02.584	——	-0.414	52 21	02.584	02.170	21347.81	21348.34	+53	-39
	Lemmer	68	284	73 39 20.182	-1.200	-0.976	73 39	18.982	19.206	25872.68	25873.37	+69	-42
	Blokzijl	69	285	53 59 38.475	+1.089	+1.280	53 59	39.564	39.755	21811.40	21811.99	+59	-35
				180 00 01.241	-0.111	-0.110	180 00	01.130	01.131				
100	Urk	58	286	43 54 46.072	+0.800	+0.129	43 54	46.872	46.201	18941.23	18941.70	+47	-34
	Blokzijl	69	287	64 45 33.701	-3.535	-1.469	64 45	30.166	32.232	24702.33	24703.15	+82	-24
	Kampen	59	288	71 19 42.104	+1.979	+0.585	71 19	44.083	42.689	25872.68	25873.37	+69	-42
				180 00 01.877	-0.756	-0.755	180 00	01.121	01.122				
101	Kampen	59	289	41 00 58.629	-1.000	-1.049	41 00	57.629	57.580	15825.22	15825.55	+33	-35
	Blokzijl	69	290	87 13 04.397	+1.000	+0.206	87 13	05.397	04.603	24085.49	24086.01	+52	-52
	Meppel	70	291	51 45 59.978	-2.247	-1.403	51 45	57.731	58.575	18941.23	18941.70	+47	-34
				180 00 03.004	-2.247	-2.246	180 00	00.757	00.758				
102	Kampen	59	292	55 38 40.461	+1.000	+0.980	55 38	41.461	41.441	29000.81	29001.25	+44	-81
	Meppel	70	293	81 04 11.463	+1.195	-0.078	81 04	12.658	11.385	34703.12	34703.58	+46	-103
	Lemelerberg	60	294	43 17 10.477	-2.851	-1.556	43 17	07.626	08.921	24085.49	24086.01	+52	-52
				180 00 02.401	-0.656	-0.654	180 00	01.745	01.747				
103	Lemelberg	60	295	40 10 24.576	+0.621	-0.207	40 10	25.197	24.369	28358.59	28359.09	+50	-72
	Meppel	70	296	98 32 56.919	-1.624	+0.585	98 32	55.295	57.504	43471.30	43472.20	+90	-97
	Beilen	83	297	41 16 40.644	+0.920	-0.458	41 16	41.564	40.186	29000.81	29001.25	+44	-81
				180 00 02.139	-0.083	-0.080	180 00	02.056	02.059				
104	Lemelerberg	60	298	37 21 55.759	+2.885	+0.811	37 21	58.644	56.570	26661.23	26661.48	+25	-90
	Beilen	83	299	44 20 58.182	——	-0.309	44 20	58.182	57.873	30708.27	30708.90	+63	-69
	Coevorden	84	300	98 17 07.638	-2.416	-0.030	98 17	05.222	07.608	43471.30	43472.20	+90	-97
				180 00 01.579	+0.469	+0.472	180 00	02.048	02.051				

Table 15 (continued)

1	2	3	4	5	6	7	8	5+6 9	5+7 10	11	12	12−11 =13	14
105	Lemelerberg	60	301	34 46 26.178	+2.885	+0.347	34 46	29.063	26.525	18677.56	18677.54	− 2	−82
	Coevorden	84	302	75 33 07.694	+0.500	+0.456	75 33	08.194	08.150	31711.92	31712.43	+51	−85
	Uelsen	63	303	69 40 26.484	−2.337	+0.247	69 40	24.147	26.731	30708.27	30708.90	+63	−69
				180 00 00.356	+1.048	+1.050	180 00	01.404	01.406				
106	Uelsen	63	304	91 16 24.539	+2.570	+0.239	91 16	27.109	24.778	33847.52	33847.40	−12	−158
	Coevorden	84	305	55 14 40.380	−2.569	−0.551	55 14	37.811	39.829	27815.48	27815.56	+ 8	−112
	Kirch Hesepe	64	306	33 28 55.993	+0.400	+0.715	33 28	56.393	56.708	18677.56	18677.54	− 2	−82
				180 00 00.912	+0.401	+0.403	180 00	01.313	01.315				
107	Oosterland	66	307	55 01 17.974	−1.127	−2.045	55 01	16.847	15.929	17964.44	17965.02	+58	−19
	Kijkduin	65	308	61 54 32.966	+3.830	−1.778	61 54	36.796	31.188	19342.31	19342.71	+40	−43
	Oosterend	71	309	63 04 17.501	−10.361	−3.834	63 04	07.140	13.667	19547.08	19548.07	+99	+15
				180 00 08.441	−7.658	−7.657	180 00	00.783	00.784				
108	Oosterland	66	310	60 50 06.007	———	−0.071	60 50	06.007	05.936	18621.67	18621.95	+28	−52
	Oosterend	71	311	54 04 06.022	+4.007	+1.868	54 04	10.029	07.890	17267.71	17267.85	+14	−60
	Robbezand	72	312	65 05 48.001	−3.300	−1.089	65 05	44.701	46.912	19342.31	19342.71	+40	−43
				180 00 00.030	+0.707	+0.708	180 00	00.737	00.738				
109	Staveren	67	313	41 27 21.104	+0.783	−3.041	41 27	21.887	18.063	17267.72	17267.85	+13	−61
	Oosterland	66	314	70 37 25.140	+1.127	+2.082	70 37	26.267	27.222	24605.12	24605.86	+74	−32
	Robbezand	72	315	67 55 14.843	−2.001	+0.869	67 55	12.842	15.712	24169.54	24170.36	+82	−22
				180 00 01.087	−0.091	−0.090	180 00	00.996	00.997				
110	Robbezand	72	316	69 21 31.142	+1.057	−0.859	69 21	32.199	30.283	26616.01	26616.94	+93	−21
	Oosterend	71	317	69 44 39.186	−7.375	−1.352	69 44	31.811	37.834	26682.48	26683.78	+130	+15
	Vlieland	73	318	40 53 52.873	+4.292	+0.187	40 53	57.165	53.060	18621.67	18621.95	+28	−52
				180 00 03.201	−2.026	−2.024	180 00	01.175	01.177				
111	Robbezand	72	319	67 44 35.384	+3.200	+2.135	67 44	38.584	37.519	27233.36	27234.72	+136	+19
	Vlieland	73	320	47 11 38.271	−6.329	−3.933	47 11	31.942	34.338	21587.64	21589.01	+137	+44
	Harlingen	74	321	65 03 48.522	+2.300	+0.971	65 03	50.822	49.493	26682.48	26683.78	+130	+15
				180 00 02.177	−0.829	−0.827	180 00	01.348	01.350				
112	Staveren	67	322	41 18 52.561	———	+3.810	41 18	52.561	56.371	21587.65	21589.01	+136	+43
	Robbezand	72	323	89 52 50.922	−0.248	−1.348	89 52	50.674	49.574	32699.00	32700.35	+135	− 6
	Harlingen	74	324	48 48 18.870	−0.762	−3.470	48 48	18.108	15.400	24605.12	24605.86	+ 74	−32
				180 00 02.353	−1.010	−1.008	180 00	01.343	01.345				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12-11 =13	14
113	Staveren	67	325	44 02 15.596	+1.127	-0.941	44 02	16.723	14.655	22875.28	22875.95	+57	-41
	Harlingen	74	326	52 25 09.023	-0.075	-0.552	52 25	08.948	08.471	26079.13	26080.12	+99	-13
	Sneek	75	327	83 32 38.058	-2.234	+0.317	83 32	35.824	38.375	32699.00	32700.35	+135	- 6
				180 00 02.677	-1.182	-1.176	180 00	01.495	01.501				
114	Staveren	67	328	49 40 05.416	+2.691	+0.873	49 40	08.107	06.289	21109.90	21110.44	+54	-37
	Sneek	75	329	59 58 51.264	+1.127	+0.477	59 58	52.391	51.741	23977.22	23977.97	+75	-28
	Lemmer	68	330	70 21 03.360	-2.653	-0.183	70 21	00.707	03.177	26079.13	26080.12	+99	-13
				180 00 00.040	+1.165	+1.167	180 00	01.205	01.207				
115	Lemmer	68	331	85 46 22.735	+0.166	-1.618	85 46	22.901	21.117	30486.61	30487.10	+49	-82
	Sneek	75	332	50 33 09.473	+4.035	+3.940	50 33	13.508	13.413	23606.58	23606.98	+40	-62
	Oldeholtpa	76	333	43 40 27.477	-2.630	-0.749	43 40	24.847	26.728	21109.88	21110.44	+56	-35
				179 59 59.685	+1.571	+1.573	180 00	01.256	01.258				
116	Blokzijl	69	334	70 08 38.814	-1.200	-2.611	70 08	37.614	36.203	23606.58	23606.98	+40	-62
	Lemmer	68	335	51 35 02.411	+2.200	+1.915	51 35	04.611	04.326	19665.53	19665.88	+35	-50
	Oldeholtpa	76	336	58 16 22.097	-3.324	-1.626	58 16	18.773	20.471	21347.81	21348.34	+53	-39
				180 00 03.322	-2.324	-2.322	180 00	00.998	01.000				
117	Meppel	70	337	54 55 27.470	-1.863	-2.233	54 55	25.607	25.237	19665.53	19665.88	+35	-50
	Blokzijl	69	338	83 53 05.181	+2.078	+2.026	83 53	07.259	07.207	23892.88	23893.34	+46	-57
	Oldeholtpa	76	339	41 11 25.812	+2.104	+2.527	41 11	27.916	28.339	15825.22	15825.55	+33	-35
				179 59 58.463	+2.319	+2.320	180 00	00.782	00.783				
118	Meppel	70	340	73 41 25.825	+2.884	+1.474	73 41	28.709	27.299	31536.29	31536.70	+41	-95
	Oldeholtpa	76	341	59 39 39.048	+1.860	+2.388	59 39	40.908	41.436	28358.61	28359.09	+48	-74
	Beilen	83	342	46 38 55.800	-3.772	-2.889	46 38	52.028	52.911	23892.88	23893.34	+46	-57
				180 00 00.673	+0.972	+0.973	180 00	01.645	01.646				
119	Harlingen	74	343	40 01 14.574	-1.145	-0.567	40 01	13.429	14.007	17939.31	17940.25	+ 94	+17
	Vlieland	73	344	62 29 59.600	+2.733	+1.362	62 30	02.333	00.962	24744.91	24746.03	+112	+ 6
	Midsland	77	345	77 28 47.554	-2.220	-1.426	77 28	45.334	46.128	27233.36	27234.72	+136	+19
				180 00 01.728	-0.632	-0.631	180 00	01.096	01.097				
120	Harlingen	74	346	51 37 34.677	-1.307	+0.706	51 37	33.370	35.383	27572.33	27573.93	+160	+41
	Midsland	77	347	83 39 31.917	+2.200	+1.099	83 39	34.117	33.016	34954.86	34956.58	+172	+22
	Ballum	78	348	44 42 56.427	-2.200	-3.109	44 42	54.227	53.318	24744.91	24746.03	+112	+ 6
				180 00 03.021	-1.307	-1.304	180 00	01.714	01.717				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12–11 =13	14
121	Harlingen	74	349	50 58 15.238	+2.090	+3.346	50 58	17.328	18.584	27370.59	27372.08	+149	+31
	Ballum	78	350	46 14 27.202	+1.982	+0.386	46 14	29.184	27.588	25447.82	25448.89	+107	- 2
	Leeuwarden	79	351	82 47 15.351	-0.116	+0.226	82 47	15.235	15.577	34954.85	34956.58	+173	+23
				179 59 57.791	+3.956	+3.958	180 00	01.747	01.749				
122	Sneek	75	352	70 48 33.445	-2.848	-0.575	70 48	30.597	32.870	25447.82	25448.89	+107	- 2
	Harlingen	74	353	51 05 39.717	-1.722	-1.055	51 05	37.995	38.662	20968.18	20969.04	+ 86	- 4
	Leeuwarden	79	354	58 05 48.707	+3.848	+0.908	58 05	52.555	49.615	22875.28	22875.95	+ 67	-31
				180 00 01.869	-0.722	-0.722	180 00	01.147	01.147				
123	Sneek	75	355	49 30 40.051	+0.137	-3.662	49 30	40.188	36.389	23434.13	23434.35	+22	-79
	Leeuwarden	79	356	87 36 21.057	+1.614	+2.216	87 36	22.671	23.273	30785.95	30786.72	+77	-55
	Drachten	81	357	42 52 59.382	-1.000	+2.199	42 52	58.382	61.581	20968.17	20969.04	+87	- 3
				180 00 00.490	+0.751	+0.753	180 00	00.241	01.243				
124	Sneek	75	358	45 36 07.492	———	-0.280	45 36	07.492	07.212	23746.63	23747.09	+46	-56
	Drachten	81	359	66 31 56.513	+3.570	+1.572	66 31	60.083	58.085	30486.61	30487.10	+49	-82
	Oldeholtpa	76	360	67 52 00.048	-5.928	-3.647	67 51	54.120	56.401	30785.95	30786.72	+77	-55
				180 00 04.053	-2.358	-2.355	180 00	01.695	01.698				
125	Oldeholtpa	76	361	47 48 52.580	+6.014	+2.409	47 48	58.594	54.989	17972.02	17972.04	+ 2	-75
	Drachten	81	362	53 55 24.745	-2.414	-0.954	53 55	22.331	23.791	19602.60	19603.03	+43	-41
	Oosterwolde	82	363	78 15 42.347	-2.400	-0.254	78 15	39.947	42.093	23746.63	23747.09	+46	-56
				179 59 59.672	+1.200	+1.201	180 00	00.872	00.873				
126	Oldeholtpa	76	364	41 31 14.143	+0.699	-2.507	41 31	14.842	11.636	21286.10	21286.03	- 7	-99
	Oosterwolde	82	365	100 51 25.802	-2.945	-0.781	100 51	22.857	25.021	31536.29	31536.70	+41	-95
	Beilen	83	366	37 37 22.662	+0.674	+1.718	37 37	23.336	24.380	19602.60	19603.03	+43	-41
				180 00 02.607	-1.572	-1.570	180 00	01.035	01.037				
127	Leeuwarden	79	367	59 24 00.645	-1.273	-0.478	59 23	59.372	60.167	24221.58	24222.82	+124	+20
	Ballum	78	368	44 01 51.040	+7.674	+2.970	44 01	58.714	54.010	19559.60	19560.10	+ 50	-34
	Dokkum	80	369	76 34 09.021	-5.942	-2.031	76 34	03.079	06.990	27370.58	27372.08	+150	+32
				180 00 00.706	+0.459	+0.461	180 00	01.165	01.167				
128	Drachten	81	370	46 53 27.163	-6.251	-4.463	46 53	20.912	22.700	19559.60	19560.10	+50	-34
	Leeuwarden	79	371	72 06 32.043	-1.876	-0.674	72 06	30.167	31.369	25497.07	25497.57	+50	-60
	Dokkum	80	372	61 00 04.494	+5.530	+2.541	61 00	10.024	07.035	23434.13	23434.35	+22	-79
				180 00 03.700	-2.597	-2.596	180 00	01.103	01.104				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	9	5+7 10	11	12	12–11 =13	14
129	Dokkum	80	486	41 35 39.856	—	-0.299	41 35	39.856	39.557	16671.46	—	—	—
	Hornhuizen	85	489	53 11 07.362	—	-0.484	53 11	07.362	06.878	20105.06	—	—	—
	Schiermonnik-oog	—	—	—	—	—	85 13	13.632	14.411	25025.88	25025.88	0	-108
				—	—	—	180 00	00.850	00.846				
130	Dokkum	80	373	33 06 04.258	+6.000	+3.258	33 06	10.258	07.516	23356.46	23356.32	-14	-114
	Hornhuizen	85	374	111 05 01.186	-6.432	-1.659	111 04	54.754	59.527	39903.84	39904.03	+19	-153
	Groningen	86	375	35 48 58.544	-2.168	-4.206	35 48	56.376	54.338	25025.88	25025.88	0	-108
				180 00 03.988	-2.600	-2.607	180 00	01.388	01.381				
131	Drachten	81	376	83 33 14.515	+3.454	+0.945	83 33	17.969	15.460	39903.84	39904.03	+19	-153
	Dokkum	80	377	57 01 55.292	-3.486	-3.215	57 01	51.806	52.077	33690.82	33691.06	+24	-121
	Groningen	86	378	39 24 52.397	—	+2.227	39 24	52.397	54.624	25497.07	25497.57	+50	- 60
				180 00 02.204	-0.032	-0.043	180 00	02.172	02.161				
132	Oosterwolde	82	379	81 54 17.447	-0.400	+2.132	81 54	17.047	19.579	33690.85	33691.06	+21	-124
	Drachten	81	380	66 12 57.246	+3.077	+1.136	66 12	60.323	58.382	31140.02	31140.04	+ 2	-132
	Groningen	86	381	31 52 46.094	-2.054	-2.652	31 52	44.040	43.442	17972.04	17972.04	0	- 77
				180 00 00.787	+0.623	+0.616	180 00	01.410	01.403				
133	Oosterwolde	82	382	55 01 03.414	+5.296	-0.836	55 01	08.710	02.578	26170.72	26169.99	-73	-186
	Groningen	86	383	47 50 24.148	+2.250	+1.762	47 50	26.398	25.910	23677.32	23677.12	-20	-122
	Rolde	87	384	77 08 32.031	-5.602	+1.010	77 08	26.429	33.041	31140.02	31140.04	+ 2	-132
				179 59 59.593	+1.944	+1.936	180 00	01.537	01.529				
134	Oosterwolde	82	385	43 57 30.487	+0.952	+0.242	43 57	31.439	30.729	16974.02	16973.82	-20	- 93
	Rolde	87	386	60 30 51.769	+0.001	+1.593	60 30	51.770	53.362	21286.10	21286.03	- 7	- 99
	Beilen	83	387	75 31 35.708	+1.973	+1.087	75 31	37.681	36.795	23677.32	23677.12	-20	-122
				179 59 57.964	+2.926	+2.922	180 00	00.890	00.886				
135	Beilen	83	388	84 20 01.699	-1.598	-0.154	84 20	00.101	01.545	25942.02	25942.16	+14	- 98
	Rolde	87	389	55 02 31.996	-2.599	-0.787	55 02	29.397	31.209	21365.61	21365.86	+25	- 67
	Sleen	88	390	40 37 25.441	+5.979	+2.719	40 37	31.420	28.160	16974.04	16973.82	-22	- 95
				179 59 59.136	+1.782	+1.778	180 00	00.918	00.914				
136	Coevorden	84	391	52 40 48.579	+0.710	+1.363	52 40	49.289	49.942	21365.61	21365.86	+25	- 67
	Beilen	83	392	30 14 27.108	—	-0.799	30 14	27.108	26.309	13530.68	13530.72	+ 4	- 54
	Sleen	88	393	97 04 44.333	—	+0.142	97 04	44.333	44.475	26661.23	26661.48	+25	- 90
				180 00 00.020	+0.710	+0.706	180 00	00.730	00.726				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12–11 =13	14
137	Hornhuizen	85	394	41 32 40.205	—	-0.471	41 32	40.205	39.734	20152.67	20152.34	-33	-120
	Uithuizermeden	89	395	88 03 07.755	-2.806	-1.120	88 03	04.949	06.635	30369.42	30369.02	-40	-171
	Borkum	106	—	—	—	—	50 24	16.046	14.825	23415.06	23414.63	-43	-144
				—	—	—	180 00	01.200	01.194				
138	Groningen	86	396	60 36 32.862	-0.001	-1.280	60 36	32.861	31.582	23415.06	23414.63	-43	-144
	Hornhuizen	85	397	59 02 04.459	+4.000	+2.143	59 02	08.459	06.602	23044.09	23043.62	-47	-146
	Uithuizermeden	89	398	60 21 26.314	-6.441	-3.311	60 21	19.873	23.003	23356.46	23356.32	-14	-114
				180 00 03.635	-2.442	-2.448	180 00	01.193	01.187				
139	Groningen	86	399	28 29 59.056	+0.001	+2.437	28 29	59.057	61.493	12122.20	12122.24	+ 4	- 48
	Uithuizermeden	89	400	86 23 50.103	+3.201	+0.954	86 23	53.304	51.057	25355.05	25354.53	-52	-161
	Holwierde	90	401	65 06 10.302	-1.954	-2.146	65 06	08.348	08.156	23044.09	23043.62	-47	-146
				179 59 59.461	+1.248	+1.245	180 00	00.709	00.706				
140	Groningen	86	402	42 38 57.916	+3.357	+0.347	42 38	61.273	58.263	20400.46	20399.43	-103	-191
	Holwierde	90	403	79 59 23.002	+3.401	+2.050	79 59	26.403	25.052	29652.11	29651.06	-105	-233
	Midwolda	91	404	57 21 39.019	-5.399	-1.044	57 21	33.620	37.975	25355.05	25354.53	- 52	-161
				179 59 59.937	+1.359	+1.353	180 00	01.296	01.290				
141	Groningen	86	405	27 57 25.487	+2.866	+2.150	27 57	28.353	27.637	18353.46	18352.64	- 80	-159
	Midwolda	91	406	102 48 14.743	-0.673	-0.539	102 48	14.070	14.204	38174.70	38173.23	-147	-311
	Onstwedde	92	407	49 14 22.426	-3.499	-2.924	49 14	18.927	19.502	29652.11	29651.06	-105	-233
				180 00 02.656	-1.306	-1.313	180 00	01.350	01.343				
142	Rolde	87	408	91 24 12.370	+3.780	-1.925	91 24	16.150	10.445	38174.70	38173.23	-147	-311
	Groningen	86	409	45 20 02.056	-2.811	+0.656	45 19	59.245	62.712	27158.18	27157.56	- 62	-179
	Onstwedde	92	410	43 15 46.021	+0.392	+2.621	43 15	46.413	48.642	26170.72	26169.99	- 73	-186
				180 00 00.447	+1.361	+1.352	180 00	01.808	01.799				
143	Sleen	88	411	53 43 56.668	+2.070	+0.519	53 43	58.738	57.187	27158.22	27157.56	-66	-183
	Rolde	87	412	75 53 51.817	+4.437	+0.126	75 53	56.254	51.943	32668.88	32668.10	-78	-218
	Onstwedde	92	413	50 22 10.426	-3.680	+2.174	50 22	06.746	12.600	25942.02	25942.16	+14	- 98
				179 59 58.911	+2.827	+2.819	180 00	01.738	01.730				
144	Uithuizermeden	89	414	77 42 53.053	-1.028	-1.129	77 42	52.025	51.924	28502.87	28501.79	-108	-231
	Pilsum	93	—	—	—	—	43 41	50.457	54.782	20152.66	20152.34	- 32	-119
	Borkum	106	—	—	—	—	58 35	18.766	14.535	24895.79	24894.54	-125	-232
	See also No. 145			—	—	—	180 00	01.248	01.241				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12−11 =13	14
145	Uithuizermeden	89	415	47 28 45.498	+4.351	+1.882	47 28	49.849	47.380	18942.59	18941.33	-126	-207
	Holwierde	90	416	104 22 33.112	+3.700	-0.113	104 22	36.812	32.999	24895.79	24894.54	-125	-232
	Pilsum	93	—	—	—	—	28 08	33.910	40.184	12122.20	12122.24	+ 4	- 48
	See also No. 144			—	—	—	180 00	00.571	00.563				
146	Holwierde	90	418	44 50 32.479	-4.078	-2.645	44 50	28.401	29.834	16084.14	16083.30	- 84	-153
	Pilsum	93	419	79 00 45.636	+3.878	+1.337	79 00	49.514	46.973	22391.69	22390.31	-138	-234
	Emden	94	420	56 08 44.657	-1.811	-0.707	56 08	42.846	43.950	18942.56	18941.33	-123	-204
				180 00 02.772	-2.011	-2.015	180 00	00.761	00.757				
147	Midwolda	91	421	61 16 41.103	+1.676	-2.323	61 16	42.779	38.780	22391.69	22390.31	-138	-234
	Holwierde	90	422	65 41 23.121	-3.085	+0.838	65 41	20.036	23.959	23268.94	23267.96	- 98	-198
	Emden	94	423	53 01 57.002	+1.241	+1.313	53 01	58.243	58.315	20400.46	20399.43	-103	-191
				180 00 01.226	-0.168	-0.172	180 00	01.058	01.054				
148	Midwolda	91	424	48 07 47.366	+6.060	+2.568	48 07	53.426	49.934	22340.23	22338.87	-136	-232
	Emden	94	425	81 00 23.667	-1.648	+1.009	81 00	22.019	24.676	29631.01	29629.72	-129	-256
	Leer	95	426	50 51 46.510	-0.760	+0.180	50 51	45.750	46.690	23268.94	23267.96	- 98	-198
				179 59 57.543	+3.652	+3.757	180 00	01.195	01.300				
149	Onstwedde	92	427	57 55 01.906	+3.070	+1.095	57 55	04.976	03.001	29631.01	29629.72	-129	-256
	Midwolda	91	428	90 25 38.805	-2.700	+0.302	90 25	36.105	39.107	34970.61	34969.30	-131	-281
	Leer	95	429	31 39 18.207	+2.104	+1.062	31 39	20.311	19.269	18353.46	18352.64	- 82	-161
				179 59 58.918	+2.474	+2.459	180 00	01.392	01.377				
150	Emden	94	430	47 25 46.286	-0.009	-2.353	47 25	46.277	43.933	19765.55	19764.18	-137	-222
	Pilsum	93	431	95 45 11.111	-2.842	-1.473	95 45	08.269	09.638	26703.96	26702.40	-156	-271
	Hage	97	432	36 49 04.555	+1.705	+2.675	36 49	06.260	07.230	16084.14	16083.30	- 84	-153
				180 00 01.952	-1.146	-1.151	180 00	00.806	00.801				
151	Emden	94	433	46 42 35.453	-0.924	-1.861	46 42	34.529	33.592	19690.90	19689.64	-126	-211
	Hage	97	434	52 29 34.702	+0.847	+0,430	52 29	35.549	35.132	21459.93	21458.62	-131	-223
	Aurich	98	435	80 47 51.404	-0.418	+0.928	80 47	50.986	52.332	26703.96	26702.40	-156	-271
				180 00 01.559	-0.495	-0.503	180 00	01.064	01.056				
152	Leer	95	436	50 40 46.349	+0.488	+0.611	50 40	46.837	46.960	21459.94	21458.62	-132	-224
	Emden	94	437	75 40 36.186	-0.100	-0.651	75 40	36.086	35.535	26877.54	26875.85	-169	-285
	Aurich	98	438	53 38 40.268	-2.005	-1.587	53 38	38.263	38.681	22340.23	22338.87	-136	-232
				180 00 02.803	-1.617	-1.627	180 00	01.186	01.176				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	5+6 9	5+7 10	11	12	12–11 =13	14
153	Leer	95	439	34 49 20.244	-0.099	-0.180	34 49	20.145	20.064	15469.82	15468.83	- 99	-166
	Aurich	98	440	47 58 50.834	———	+1.145	47 58	50.834	51.979	20126.42	20125.25	-117	-204
	Strakholt	99	441	97 11 49.808	———	-1.069	97 11	49.808	48.739	26877.54	26875.85	-169	-285
				180 00 00.886	-0.099	-0.104	180 00	00.787	00.782				
154	Leer	95	442	44 58 59.943	+3.472	+2.139	44 59	03.415	02.082	22823.75	22822.21	-154	-252
	Strakholt	99	443	96 27 13.115	-1.024	+0.053	96 27	12.091	13.168	32081.97	32080.02	-195	-333
	Westerstede	100	444	38 33 45.960	-0.314	-0.054	38 33	45.646	45.906	20126.42	20125.25	-117	-204
				179 59 59.018	+2.134	+2.138	180 00	01.152	01.156				
155	Leer	95	445	24 29 44.775	-1.870	-1.203	24 29	42.905	43.572	15807.85	15807.00	- 85	-153
	Westerstede	100	446	32 47 59.580	———	-0.680	32 47	59.580	58.900	20653.30	20651.92	-138	-227
	Barssel	96	447	122 42 18.227	———	-0.004	122 42	18.227	18.223	32081.97	32080.02	-195	-333
				180 00 02.582	-1.870	-1.887	180 00	00.712	00.695				
156	Leer	95	496	93 22 57.237	+2.844	-1.111	93 22	60.081	56.126	30459.03	———	——	——
	Barssel	96	499	44 00 59.629	-1.880	+0.089	44 00	57.749	59.718	21201.68	———	——	——
	Aschendorf	—	—	———	———	———	42 36	03.282	05.263	20653.30	20651.92	-138	-227
				———	———	———	180 00	01.112	01.107				
157	Aurich	98	448	65 10 09.805	+1.695	-0.248	65 10	11.500	09.557	22344.50	22342.83	-167	-263
	Hage	97	449	61 43 18.415	-0.554	-0.267	61 43	17.861	18.148	21682.19	21680.69	-150	-243
	Esens	101	450	53 06 31.966	-0.342	+1.310	53 06	31.624	33.276	19690.90	19689.64	-126	-211
				180 00 00.186	+0.799	+0.795	180 00	00.985	00.981				
158	Aurich	98	451	43 46 01.303	———	-0.706	43 46	01.303	00.597	20954.23	20952.64	-159	-249
	Esens	101	452	90 31 40.509	-0.127	-0.131	90 31	40.382	40.378	30291.36	30289.16	-220	-350
	Jever	102	453	45 42 19.298	+0.167	+0.877	45 42	19.465	20.175	21682.19	21680.69	-150	-243
				180 00 01.110	+0.040	+0.040	180 00	01.150	01.150				
159	Strakholt	99	454	81 03 40.928	-0.138	-0.876	81 03	40.790	40.052	30291.37	30289.16	-220	-350
	Aurich	98	455	68 38 27.114	———	-0.260	68 38	27.114	26.854	28557.65	28555.57	-208	-331
	Jever	102	456	30 17 53.908	-0.700	+0.291	30 17	53.208	54.199	15469.82	15468.83	- 99	-166
				180 00 01.950	-0.838	-0.845	180 00	01.112	01.105				
160	Westerstede	100	457	54 15 51.159	+3.182	+2.192	54 15	54.341	53.351	28557.65	28555.57	-208	-331
	Strakholt	99	458	85 17 18.120	-0.809	-0.079	85 17	17.311	18.041	35062.45	35060.04	-241	-392
	Jever	102	459	40 26 50.160	-0.159	+0.093	40 26	50.001	50.253	22823.76	22822.21	-155	-253
				179 59 59.439	+2.214	+2.206	180 00	01.653	01.645				

Table 15 (continued)

1	2	3	4	5	6	7	5+6 8	9	5+7 10	11	12	12−11 =13	14
161	Westerstede	100	460	44 23 56.880	-1.254	-0.651	44 23	55.626	56.229	24885.63	24884.00	-163	-270
	Jever	102	461	35 55 18.309	+0.266	-0.470	35 55	18.575	17.839	20867.51	20865.97	-154	-244
	Varel	103	462	99 40 47.348	-0.254	-0.120	99 40	47.094	47.228	35062.45	35060.04	-241	-392
				180 00 02.537	-1.242	-1.241	180 00	01.295	01.296				
162	Jever	102	469	39 25 59.564	—	-0.406	39 25	59.564	59.158	19752.66	—	—	—
	Varel	103	463	87 24 51.858	—	-0.056	87 24	51.858	51.802	31066.21	—	—	—
	Stolham	—	—	—	—	—	53 09	09.827	10.283	24885.63	24884.00	-163	-270
				—	—	—	180 00	01.249	01.243				
163	Esens	101	504	69 00 19.405	—	-0.133	69 00	19.405	19.272	24565.26	—	—	—
	Jever	102	467	58 12 34.291	—	-0.406	58 12	34.291	33.885	22364.68	—	—	—
	Wangeroge	—	—	—	—	—	52 47	07.417	07.951	20954.23	20952.64	-159	-249
				—	—	—	180 00	01.113	01.108				

Conformably to table 14 I mentioned in the left part of table 16 the 13 largest and in the right part the 13 smallest closing errors (column 4) and the standard deviations in the sum of the three angles (column 3) which could be expected in the several triangles (column 1) because of the accuracy of the angular measurement. If all three angles are measured with the same instrument this standard deviation is of course $m''\sqrt{3}$ with $m = 1''.6$ or $m = 3''.4$. In some exceptional cases (triangles 121, 103 and 33) column 3 must be computed from $(m_A^2 + m_B^2 + m_C^2)^{\frac{1}{2}}$.

Table 16

Triangle	m "	m √3 "	closing error "	Triangle	m "	m √3 "	closing error "
1	2	3	4	1	2	3	4
107	1.6	2.8	+7.657	99	1.6	2.8	+0.110
31	1.6	2.8	-5.243	153	3.4	5.9	+0.104
121*	—	4.1	-3.958	109	1.6	2.8	+0.090
148	3.4	5.9	-3.757	103*	—	4.1	+0.080
55	1.6	2.8	-3.155	37	1.6	2.8	-0.075
68	1.6	2.8	-2.875	51	1.6	2.8	+0.070
143	3.4	5.9	-2.819	131	3.4	5.9	+0.043
130	3.4	5.9	+2.607	158	3.4	5.9	-0.040
128	3.4	5.9	+2.596	33**	—	4.1	+0.039
88	1.6	2.8	+2.466	95	1.6	2.8	-0.038
149	3.4	5.9	-2.459	30	1.6	2.8	+0.026
138	3.4	5.9	+2.448	14	1.6	2.8	-0.012
67	1.6	2.8	-2.394	91	1.6	2.8	0.000

* 2 angles in 1807, 1 in 1810; (see table 3)

** 2 angles in 1802, 1 in 1810

One sees that for 24 out of 26 triangles and especially in the right part of the table the closing error is much smaller than the standard deviation in the sum of the three angles that may be expected. As in section 14 here, too, is only one conclusion: From the several series of measured angles Krayenhoff must have chosen those, which gave a small closing error.

It does not appear from the Précis Historique whether Krayenhoff saw the relation between the angles of the Zuiderzee pentagon, expressed by the 149th polygon condition:

$$p_{240} + p_{239} + p_{478} + p_{475} + p_{481} + p_{484} - p_{279} - p_{280} - p_{283} - p_{286} - 7''.706 = 0$$

on page 57 .

If one substitutes, however, in this equation the amounts p found by Krayenhoff (they can be borrowed from table 9, column 9) then one finds + $0\overset{''}{.}097 = 0$.
The very small difference of about $0\overset{''}{.}1$ shows in my opinion that Krayenhoff must have seen the condition the angles had to comply with.

16. Analysis of the closing errors in the side equations

A survey of the reliability of the closing errors in the side (sine) equations, finally, is given in table 17. The 13 largest closing errors (left part, column 5) are mentioned there next to the standard deviations M in the closing errors (column 4) which can be expected on the ground of the accuracy of the angular measurement. In the same way one finds in the right part of the table the 13 smallest closing errors and the corresponding amounts M.

Table 17

Central Points No.	Name	M^2	M	Closing error	Central Points No.	Name	M^2	M	Closing error
1	2	3	4	5	1	2	3	4	5
86	Groningen	354.9	18.8	-37.488	98	Aurich	461.5	21.5	+4.985
72	Robbezand	79.2	8.9	+37.369	70	Meppel	245.8	15.7	+4.967
81	Drachten	322.8	18.0	+37.166	14	Hulst	146.1	12.1	+4.522
79	Leeuwarden	248.6	15.8	-37.127	60	Lemelerberg	172.3	13.1	-3.406
74	Harlingen	148.0	12.2	-32.127	36	Utrecht*	123.4	11.1	-2.952
43	Imbosch	78.2	8.8	-32.006	23	Breda	76.6	8.8	-2.086
31	Gouda*	184.0	13.6	+31.540	67	Staveren	52.3	7.2	-1.920
82	Oosterwolde	425.6	20.6	-30.927	33	's-Hertogenbosch	77.0	8.8	+1.457
76	Oldeholtpa	206.6	14.4	+30.414	32	Gorinchem	75.7	8.7	+1.345
59	Kampen	91.4	9.6	-29.078	87	Rolde	364.5	19.1	-0.670
89	Uithuizermeden	1787.	42.3	+27.150	22	Willemstad*	80.4	9.0	-0.476
94	Emden	338.6	18.4	+24.512	63	Uelsen	132.9	11.5	-0.407
90	Holwierde	892.8	29.9	-24.024	24	Hilvarenbeek	81.9	9.0	+0.089

*Assumed that at Dordrecht, Gouda and Leiden Krayenhoff measured with the less accurate instrument

The amounts in column 5 are the known terms of the side equations around the central points mentioned in column 2. For Harlingen (station No. 74) e.g. this term is -32.127. It can be found from the equation:

$$1.311\ p_{354} + 0.238\ p_{327} + 2.395\ p_{322} + 0.862\ p_{319} + 1.096\ p_{344} + 0.234\ p_{347} +$$
$$+ 2.016\ p_{350} - 0.733\ p_{352} - 2.177\ p_{325} - 0.004\ p_{323} - 1.950\ p_{320} - 0.468\ p_{345} -$$
$$- 2.127\ p_{348} - 0.266\ p_{351} - 32.127 = 0,$$

derived in section 13 (page 76).

The square M^2 of the standard deviation in the sum of the amounts:

$$1.311\ p_{354} + 0.238\ p_{327} + \ldots\ldots - 0.266\ p_{351} \text{ is:}$$
$$M^2 = 1.311^2 m^2_{354} + 0.238^2 m^2_{327} + \ldots\ldots\ldots + 0.266^2 m^2_{351} \text{ with}$$
$$m^2_{354} = m^2_{327} = m^2_{322} = m^2_{319} = m^2_{344} = m^2_{347} = m^2_{352} = m^2_{325} = m^2_{323} =$$
$$= m^2_{320} = m^2_{345} = m^2_{351} = 2.61 \text{ and } m^2_{350} = m^2_{348} = 11.5.$$

One finds $M^2 = 148.0$ or $M = \pm\ 12.2$. The closing error is rather large, about 2.6 M. The closing error in the side equation around Robbezand (station No. 72) is very bad. It is more than 4M. It might be possible that this bad result must be attributed to lateral refraction: for all the ten angles concerned with the equation the signal on the sand-bank in the middle of the Dutch shallows was one of the sighting points. As already remarked, Krayenhoff was convinced of the existence of lateral refraction [55] . Gauss too was of that opinion. Bessel, however, considered lateral refraction the scapegoat for bad observers [56] .

I dare not say whether the strongly heated sand and heather fields of the Veluwe will also have furthered lateral refraction. If so it might be an explanation for the great disharmony between the amounts in column 4 and 5 for the station Imbosch.

For the other side equations round the stations in the left part of the table the disharmony is not too great. For the numbers 89, 94 and 90 the results are even very good and for the other ones the closing error is smaller than 2.5 M.

In the right half of the table all closing errors are very much smaller than the amounts M. It is obvious to assume that for:

$$\simeq 5 < |\text{closing error}| < \simeq 24$$

the harmony between the closing errors and the computed M's will in general be better which pleads both for the observations used by Krayenhoff and the amounts m computed in table 6 (section 8).

For the side equation round Amsterdam (station No. 40) e.g. M = 11.0 and the (absolute) amount of the closing error is 22.309. For those round Bergen op Zoom

(No. 17, M = 8.4), Rotterdam (No. 28, M = 11.2), Grave (No. 34, M = 11.0), Amersfoort (No. 42, M = 9.3), and Oldenzaal (No. 61, M = 11.3) the absolute amounts of the closing errors are 9.229, 19.575, 7.680, 20.334, and 11.317, respectively.

17. Consideration on the rejection of series measured in the triangulation

As already remarked in sections 14 and 15 the closing errors round the central points and those in the triangles of the network are much too low in connection with the standard deviation in the angular measurement. Apparently Krayenhoff was inclined - may be advised by van Swinden - to reject those series which made several closing errors too large in his opinion. For Cohen Stuart this rejectable method was the reason to condemn Krayenhoff's triangulation. He was of the opinion that in principle all the 1514 series in which the 505 angles of table 4 (see section 7) were measured, had to be used and that the 389 series mentioned in column 5 of that table were wrongly rejected. "In order not to fall myself into arbitrariness" he remarks on page 29 of his book "I rejected only those series (11 in total) where an apparent error could be proved" [57] .

In retaining all the other 378 series Cohen Stuart went much too far in my opinion. He took into too little account that the dynamic Krayenhoff on all hours of the day tried to get results from his measurements, even under less favourable weather conditions. His observation registers in which he noted faithfully all his observations prove this. It is clear that of the great number of measurements or - better - of attempts for measurements, many had to be rejected because too heavy heat shimmer, too strong wind and arising fog, rain or darkness made the results of these series unreliable. It won't do to retain the series as Cohen Stuart did.

For the station Enkhuizen (No. 57; see table 5 in section 7), I already gave some examples. Some other ones are given in table 18. The first part of the table relates to all the observations in Amsterdam (station No. 40), the second part to all the observations at Rhenen (station No. 37) and the third part to the 6 series of angle 87 measured at the station Den Haag (No. 27). The fourth part finally, relates to the 36 series with 16 repetitions each in which the 7 angles at Jever (station No. 102) were measured with the less accurate instrument. As one sees from the asterisks in column 2 only 16 of the 36 series at this station were retained. The weather conditions during the measurements at Jever were translated from the Dutch text in the copy at Topografische Dienst at Delft.

Table 18

Number		Eccentric angle	Weather conditions *)
angle	series		
185	2	53°24'29".000	Very faint objects; inconvenient heat shimmer; very uncomfortable position
185	7*	33".333	Slight fog; objects rather visible; towards the end of the series it begins to rain
185	11*	30".000	Twilight (june 18th, 8 p.m.) makes the objects less visible, especially the weighhouse steeple at Alkmaar
189	3	78°49'09".166	Very faint objects; now and then heat shimmer
189	10	09".772	Tower of Edam very clearly visible, that of Naarden less clearly
189	12*	11".136	A kind of haze makes the objects less visible
162	4*	38°01'29".423	Slight fog; Utrecht very faint and only now and then lit up. Also Naarden
162	13	(31".50)	Very faint objects; series stopped because of rain
162	14*	29".583	Slight fog, inconvenient heat shimmer; objects, however, visible
160	5	43°13'16".000	Slight fog
160	15*	13".958	Notwithstanding the hazy atmosphere the objects rather visible
160	17*	13".958	Objects clearly visible; inconvenient heat shimmer
157	6*	76°41'25".000	Slight fog; Haarlem rather visible
157	16*	21".250	Inconvenient heat shimmer; the tower of the abbey at Nieuwkoop very faint
157	18*	23".571	Very faint objects; haze forces to break off the observation
157	19*	23".750	Objects very clearly visible; very good observation
182	1*	69°52'47".000	Very faint objects
182	8*	49".750	Rather visible objects; good observation
182	9*	50".250	As the preceding observation

*) The series marked with an asterisk were retained.

173	9	$40^{\circ}06'40''.750$	Veluwe very faint; heavy heat shimmer
173	20*	$30''.000$	Very good observation
170	7*	$61^{\circ}43'26''.666$	Heavy heat shimmer; Veluwe hardly visible; doubtful observation
170	8*	$25''.909$	Very good observation
167	6	$34^{\circ}59'41''.136$	Heat shimmer; Amersfoort hardly visible; doubtful observation
167	19*	$37''.500$	Very good observation
140	5	$45^{\circ}13'07''.500$	Gorinchem bad; very inconvenient heat shimmer; very doubtful observation
140	16*	$01''.590$	Very good observation
140	17*	$00''.000$	Excellent observation
143	4*	$40^{\circ}21'45''.000$	Good observation
143	18*	$45''.000$	Very good observation
145	3*	$59^{\circ}41'50''.625$	Very good observation
145	11*	$45''.000$	Very good observation
145	14*	$40''.000$	Good but objects lit up only by intervals; doubtful observation
148	2*	$30^{\circ}31'50''.417$	Very good observation
148	10*	$45''.208$	Very good observation
148	13*	$40''.500$	Nijmegen faint; doubtful observation
177	1*	$47^{\circ}24'13''.636$	Very faint objects
177	12*	$13''.333$	Horizon hazy; still good observation, Imbosch faint
177	15*	$15''.000$	Imbosch very faint; heavy heat shimmer; still good observation

87	2*	$62^{\circ}23'40''.909$	Very strong wind; good visibility
87	3*	$43''.250$	Less strong wind; rather good observation
87	4*	$36''.500$	Very inconvenient strong wind; doubtful observation
87	5*	$42''.000$	Very inconvenient strong wind; doubtful observation
87	7	$45''.500$	Very good visibility; very good observation
87	9	$44''.250$	Excellent objects; excellent observation

456	3	30°17'58".125	In the beginning rather good visibility; at the end faint
456	10*	65".000	Very clear objects
456	20*	66".250	Clear objects
456	28	63".750	Because of twilight it is very difficult to point at Strakholt
456	35	63".750	Visible objects
453	2*	45°42'43".750	Visible objects
453	9	40".000	Visible objects
453	19	39".375	Visible objects
453	34	43".750	The objects now and then lit up
453	36*	41".875	Excellent objects
467	1	58°12'26".250	Visible objects
467	8*	45".000	Rather visible objects
467	17	41".250	Visible objects
467	18*	41".250	Visible objects
467	25	36".250	Visible objects; Esens at the end faint
467	33*	43".125	Clearly visible objects
468	4	109°59'25".000	Very clearly visible objects
468	14*	37".500	Faint objects; can be distinguished, however
468	15*	45".000	Rather visible objects; Wangeroge partially lit up
468	21	30".000	Excellent objects
468	24*	38".750	Visible objects; very strong wind
468	30	33".750	Excellent objects
469	5	39°26'18".750	Very clearly visible objects
469	13*	15".937	Stolham very faint; Varel clearly visible
469	16	25".625	Stolham very faint; Varel clearly visible
469	22*	13".125	Good observation
469	29*	15".000	Excellent observation
461	6	35°55'35".000	Westerstede faint; rather good observation
461	12*	33".750	Westerstede lit up and faint; Varel very clearly visible
461	23	45".000	Good observation
461	27	38".125	Clearly visible objects
461	31	37".500	Excellent objects
459	7*	40°27'03".750	The objects very faint
459	11	04".375	Visible objects
459	26	06".250	Clearly visible objects
459	32*	01".875	Clearly visible objects

In my opinion it is absolutely justified that at Enkhuizen (see table 5) Krayenhoff did not use the series 6 (Edam very faint, doubtful observation) and 7 (inconvenient heat shimmer, doubtful observation) in his computation. Because of the weather conditions it seems also justified to reject series 2 of the measurements in Amsterdam and the series 9, 6 and 5 at Rhenen. It seems even unjustifiable to maintain series 13 in Amsterdam. For after only 10 repetitions the series of the angle 162 between Utrecht and Naarden was stopped because of the rain. I should be able to supply these examples with many others.

On the other hand Krayenhoff is also inconsequent in his retaining or rejecting series and his method is in many cases incomprehensible if one does not see it against the background of getting good closing errors in the station equations and the triangles. In table 5 e. g. he retained series 12 notwithstanding the weather description "very strong wind, objects rather visible, doubtful observation". I am convinced that he would have rejected it if it would have influenced the said errors in an unfavourable way.

In table 18 one can ask oneself why series 10 in Amsterdam was rejected and series 12 was retained and why, notwithstanding the unfavourable weather conditions, the series 16 and 18 were retained. Series 18 was even broken off after 14 repetitions because of fog. For the same reason it is incomprehensible that at Rhenen the series 7 and 13 were retained. Here too the examples can be extended with many others.

A clear example of arbitrariness I give for the station Den Haag. It concerns the eccentric angle 87 between Brielle and Rotterdam. The series 2, 3, 4 and 5 are retained. The series 7 and 9, however, measured under ideal weather conditions on April 28th, 1810 at $8^h.30$ and $9^h.30$ a. m., respectively, are rejected. It is impossible to trace the reasons for the rejection. However, it cannot have been his intention to make the closing error in triangle 31 look better than it is, for according to table 9 (column 9) the sum of the measured angles 85, 86 and 87, reduced to horizon and centre, must be corrected with $+5''.242$ in order to find a closure. This amount would have been smaller if the series 7 and 9 should have been retained and the doubtful observation in series 4 would have been rejected. That he did not do that might, at any rate for this case, plead for the care with which Krayenhoff selected the series to be used for the computation of his network.

In my opinion the most serious and inadmissible arbitrariness occurs in the choice of the series in the northern part of the network. The station Jever (No. 102) is an example. It seems correct (see the 4th part of table 18) that for the determination of the eccentric angles 456 and 453 the series 10, 20, 2 and 36 were retained and that 3, 28 and 34 were rejected. But why were the series 35, 9 and 19 rejected, measured under the same weather conditions as those of series 2 ? For the determination of angle 467 the weather condition in series 1 was apparently better than in series 8. Nevertheless 1 was rejected and 8 retained and with 8 the series 18 and 35. Apparently Krayenhoff wished to use here those observations which gave the highest amount for the angle.

From the six series in which angle 468 was measured the same preference is perceptible: the series with the "very good objects" (No. 4) and "excellent objects" (Nos.21 and 30) were rejected and series 14 (faint objects) was retained. For the determination of angle 469 it is incomprehensible that series 5 was rejected and 13 was retained but if 13 had to be used why should 16 then be rejected?

Only one series (No. 12) was used for the determination of angle 461. It is the worst of the five measured series. It is incomprehensible that 12 (Westerstede faint) was retained and the good and even excellent observations in the series 23, 27 and 31 were rejected. Apparently Krayenhoff found it necessary to use here the series with the lowest amount of the angle. The same can be said of the measurement of angle 459: the two lowest amounts in the series 7 and 32 were used notwithstanding "the objects (were) very faint" during the measurement of series 7. As one sees the weather conditions during the measurement of the series 11 and 26 were much better; nevertheless these series were rejected.

It must be said that it is here a question of a serious arbitrariness, apparently only in order to find small closing errors. In table 19 I give a survey of the series which should have been used with less arbitrariness (column 2). The means of these series are in column 4. Columns 5 and 6 give the reductions to horizon and centre. I borrowed the amounts from Krayenhoff's computations. The reduced angles and those according to the Précis Historique are in the columns 7 and 8. As one sees the closing error in the station Jever is $-7''.246$ instead of $+0''.426$ found by Krayenhoff. The first amount is much better in harmony with the accuracy of the angular measurement than the latter.

Table 19

Angles	Series Retained	Rej.	Measured mean	Reduction to hor.	Reduction to centre	Reduced angles	Angles acc. to P. H.	Diff.
1	2	3	4	5	6	4+5+6=7	8	7-8=9
456	10*,20*,35	3, 28	30°17'65".000	+0".258	-11".975	30°17'53".283	30°17'53".908	-0".625
453	2*, 9, 19, 36*	34	45°42'41".250	+0".255	-23".770	45°42'17".735	45°42'19".298	-1".563
467	1,8*,17,18*, 25,33*	—	58°12'38".854	+0".184	- 9".018	58°12'30".020	58°12'34".291	-4".271
468	4,14*,15*, 21,24*,30	—	109°59'35".000	+0".880	-36".400	109°58'59".480	109°58'64".896	-5".416
469	5,13*,22*, 29*	16	39°26'15".703	+0".115	-15".238	39°25'60".580	39°25'59".564	+1".016
461	6,12*,23, 27,31	—	35°55'37".875	+0".121	-15".562	35°55'22".434	35°55'18".309	+4".125
459	7*,11,26, 32*	—	40°27'04".062	+0".337	-15".177	40°26'49".222	40°26'50".160	-0".938
	32	4	360°01'57".744	+2".150	-127".140	359°59'52".754	360°00'00".426	-7".672

18. Krayenhoff's computation of his triangulation network and his efforts to make it a closing mathematical figure

As an introduction to the computation of his triangulation network I already explained in some preceding sections how Krayenhoff reduced the measured space angles to the horizon (section 10) and to centre (section 11) and how the spherical angles of the several triangles, reduced to horizon and centre, were reduced to plane angles between the chords on the sphere (section 12). The further computation of the network can then be distinguished into:

a) a provisional computation of the lengths of the sides of the network;

b) a final computation of the lengths of the sides.

Both computations are carried out in the volume folio mentioned in section 4 under d . The provisional lengths may be found in tableau II of the Précis Historique, the final lengths in tableau III.

As Krayenhoff measured no base line in his network he had to start from the length of the chord Duinkerken-Mont Cassel = 27458.585 m of Delambre's triangulation [58] .

For the computation of the provisional lengths of the sides Krayenhoff only took into account that in any triangle the sum of the measured angles reduced to horizon, centre and chords had to be 180°. In this phase he paid no attention to the other conditions. The condition was of course never quite satisfied. In contradistinction to Delambre who corrected each of the angles with a third of the closing error, Krayenhoff gave corrections which were dependent on the accuracy of the several angles. If, in his opinion, the angles of a triangle had the same weight, each of the angles was corrected with the same amount (e.g. in triangle 8). In other cases one of the angles remained unaltered; the closing error was then distributed over the two other angles (e.g. in triangle 9). In still other cases two angles remained unaltered and the "worst" angle obtained the whole of the closing error (e.g. in triangle 10).

After this very individual adjustment the provisional lengths of the sides (chords) were computed with the use of a 7 place logarithmic trigonometric table. Up to and including the sides of triangle Aardenburg-Brugge-Gent (10) there was only one way to do this as the first part of the triangulation network is a chain. After triangle 10, however, the results of the computation are dependent on the chosen route. Krayenhoff computed a great number of sides of his network in several chains of triangles marked in green, red, blue and yellow on a map belonging to the first edition of his Précis Historique. A small part of this map is represented in Fig. 15. The first

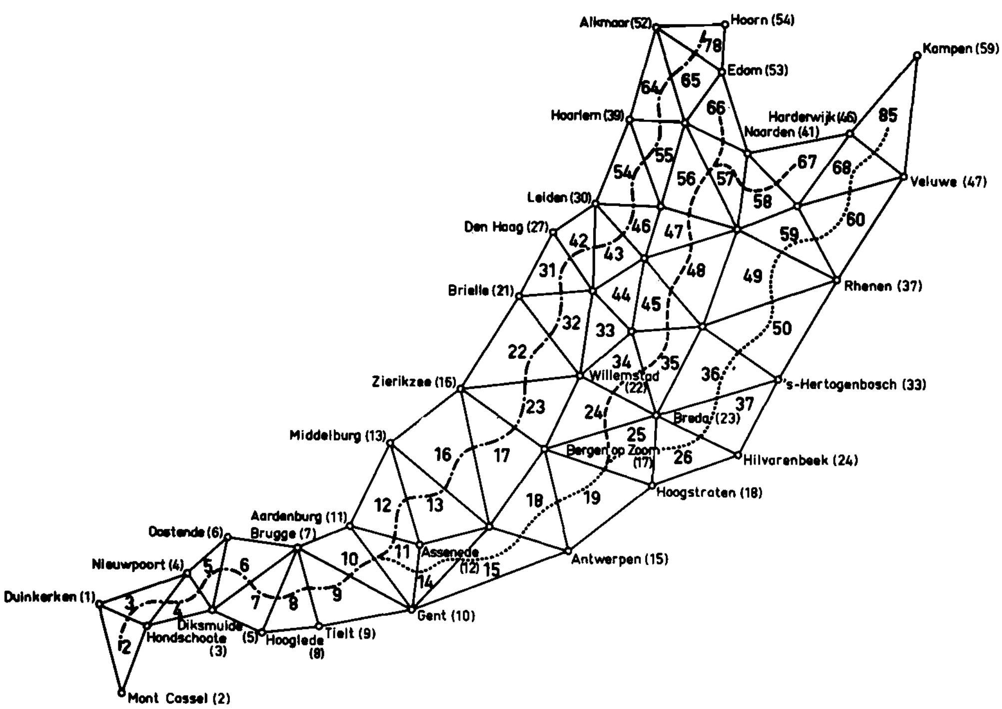


Fig. 15

(red) chain, marked with a dash-dot-dash line, follows the western part of the network. The second (blue) chain, marked with a dotted line, begins at the side Gent-Aardenburg of triangle 11 and goes in a northeastern direction. A third (green) chain, marked with a dashed line, starts from the side Bergen op Zoom-Hoogstraten in the blue chain and takes its way through the triangles 25, 24, 34, 35, 45, 48, 47, 56, 57, 58, 67 (,56, 57,66, respectively).

An example of the computation of the triangles 34-37 is given in table 20. The logarithms of the sines of the adjusted angles and those of the sides have been left out.

Table 20

No. triangle	Stations		Angles between chords		Opposite sides (metres)
	No.	Name	"measured"	adjusted	
1	2	3	4	5	6
34	22	Willemstad	$67^\circ 40' 54''.094$	$67^\circ 40' 54''.094$	26280.054
	29	Dordrecht	$66^\circ 08' 56''.186$	$66^\circ 08' 55''.467$	25982.048
	23	Breda	$46^\circ 10' 10''.439$	$46^\circ 10' 10''.439$	20493.428
			$180^\circ 00' 00''.719$	$180^\circ 00' 00''.000$	
35	29	Dordrecht	$77^\circ 00' 25''.834$	$77^\circ 00' 25''.694$	30047.141
	23	Breda	$44^\circ 32' 14''.181$	$44^\circ 32' 14''.042$	21627.959
	32	Gorinchem.	$58^\circ 27' 20''.434$	$58^\circ 27' 20''.264$	26280.054
			$180^\circ 00' 00''.449$	$180^\circ 00' 00''.000$	
36	23	Breda	$46^\circ 10' 45''.292$	$46^\circ 10' 45''.935$	27985.413
	32	Gorinchem	$83^\circ 02' 42''.796$	$83^\circ 02' 43''.439$	38501.810
	33	's-Hertogenbosch	$50^\circ 46' 29''.983$	$50^\circ 46' 30''.626$	30047.296
			$179^\circ 59' 58''.071$	$180^\circ 00' 00''.000$	
37	23	Breda	$41^\circ 25' 15''.184$	$41^\circ 25' 15''.219$	25503.359
	24	Hilvarenbeek	$92^\circ 49' 52''.231$	$92^\circ 49' 52''.267$	38501.810
	33	's-Hertogenbosch	$45^\circ 44' 52''.479$	$45^\circ 44' 52''.514$	27611.651
			$179^\circ 59' 59''.894$	$180^\circ 00' 00''.000$	

The differences in length between the common sides in the several chains are mentioned in foot notes on the concerning pages of the Précis Historique.

The common sides Assenede-Hulst (triangles 13 and 14) e.g. and Hulst-Bergen op Zoom (triangles 17 and 18) give differences of 0.14 m and 0.12 m, respectively, in the computation of the red and the blue chain. As the two sides, however, lie close to the side Aardenburg-Gent from which the computation in the two chains was started, these small differences could be expected. Bigger differences can be expected e.g. for the sides Gouda-Nieuwkoop (triangles 46 and 47) and Nieuwkoop-Amsterdam (triangles 55 and 56) in the red and the green chain as they lie rather far from the side Bergen op Zoom-Hoogstraten from which the computation in the latter chain started. The differences for these sides appear to be 0.65 m and 0.82 m, respectively. As can be seen from table 20 the difference of the computation of Breda-Gorinchem in the triangles 35 and 36 is 0.16 m. For the sides Gorinchem-Utrecht (triangles 48 and 49), Amersfoort-Utrecht (triangles 58 and 59) and Amersfoort-Harderwijk (triangles 67 and 68) they are 0.09 m, 0.01 m, and 0.88 m, respectively.

On page 30 of the Précis Historique Krayenhoff gives a table in which the differences larger than 1 m are mentioned. Table 21 gives the sides with differences larger than 0.90 m.

Table 21

Triangle		Common Side	Length (metres)		Diff. (m)
a	b		a	b	
69	70	Imbosch-Zutphen	17909.51	17910.44	0.93
94	95	Medemblik-Oosterland	18684.56	18685.47	0.91
100	84	Urk-Kampen	24702.72	24701.62	1.10
108	109	Oosterland-Robbezand	17266.49	17267.47	0.98
121	127	Leeuwarden-Ballum	27370.20	27373.63	3.43
131	132	Groningen-Drachten	33692.45	33691.27	1.18
139	140	Groningen-Holwierde	25354.37	25355.55	1.18
142	143	Rolde-Onstwedde	27158.26	27156.94	1.32
151	152	Emden-Aurich	21460.17	21458.94	1.23
158	159	Jever-Aurich	30291.85	30290.04	1.81

It appears that the greatest difference by far occurs in the common side Leeuwarden-Ballum of the triangles 121 and 127 (see Fig. 16). It is 3.43 m. It is peculiar that just in the apex Ballum (castle; station No. 78) of these triangles and also in the angular point Ballum of triangle 120, Krayenhoff made some alterations in the angles 350, 368 and 348 measured there. As in this phase of the computation, however, he did not alter the base angles of the triangles 121 and 127 at Harlingen, Leeuwarden and Dokkum, the large difference cannot be caused by this alteration.

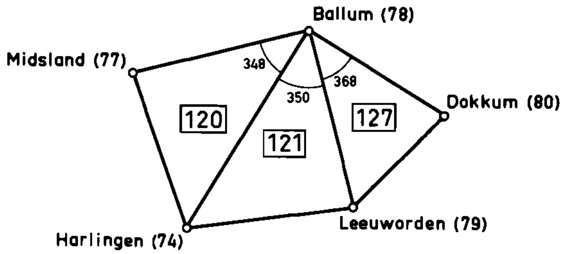


Fig. 16

In my opinion it must be imputed to too long a distance Leeuwarden-Dokkum = 19560.910 m in this phase of the computation and a less favourable form of triangle 127: the length Leeuwarden-Ballum is about 1.4 times that of Leeuwarden-Dokkum. Of course the alterations influence the closing errors of the three triangles in Fig. 16. I have shown that in table 22.

Table 22

Tri-angle	Angle	Spherical angle	Reduct. to chords	Angle between chords	Angle pages 106-107 P. H.	Diff.	Closing error P. H.	Correct closing error
1	2	3	4	3+4=5	6	5-6=7	8	7+8=9
120	348	$44^\circ 42' 56''.427$	$-0''.471$	$44^\circ 42' 55''.956$	$44^\circ 42' 52''.459$	$+3''.497$	$-2''.204$	$+1''.293$
121	350	$46^\circ 14' 27''.202$	$-0''.484$	$46^\circ 14' 26''.718$	$46^\circ 14' 32''.115$	$-5''.397$	$+1''.432$	$-3''.965$
127	368	$44^\circ 01' 51''.040$	$-0''.349$	$44^\circ 01' 50''.691$	$44^\circ 01' 48''.347$	$+2''.344$	$-2''.842$	$-0''.498$

According to tableau I the spherical angles in column 2 have the values mentioned in column 3 (see also table 9, station No. 78). The reductions to the chords (I have not verified them) are the amounts in column 4. The "angles des cordes" on the pages 106 and 107 of the Précis Historique are therefore the amounts in column 5 instead of those in column 6. The difference for angle 350 amounts to $-5''.397$ (column 7). As, according to the Précis Historique, the closing error in the (plane) triangle 121 is $+1''.432$ (column 8), it ought to be $-3''.965$ (column 9). I don't know why Krayenhoff made the alterations: in any case not in order to obtain better closing errors: those in column 9 of the table are a little better than those of column 8.

Mentioning the results as given in tableau II, the computation of a triangulation before Krayenhoff's time would have been finished. For - I remarked it already - even Delambre did not use the station equations and the side (sine) equations in his network. As far as I know Krayenhoff was the first who was not satisfied with these results. In his Précis Historique (pages 30-33) he writes on the differences in table 20:

"Pour faire disparaître ces différences, grandes ou petites, j'ai dû entreprendre
"un second calcul dont voici les principes.
"J'ai fait à chaque tour d'horizon l'addition des logarithmes des sinus des angles
"inverses ou opposés des triangles dont il est composé, ayant leur sommet au
"centre de la station; les sommes de ces deux séries de logarithmes doivent
"nécessairement être égales et ce qui diffère doit être considéré comme erreur.
"Par exemple le tour d'horizon à Breda (23e station) est composé de sept triangles.
"Les séries des logarithmes des sinus des angles inverses pris du premier calcul
" [59] sont comme il suit (see table 23):

Table 23

Stations		Number		Left (right) base angles between chords (tableau II)		Corrected log sine (tableau III)
No.	Name	tri-angle	an-gle	angle	log sine	
1	2	3	4	5	6	7
29	Dordrecht	35	97	77°00'25".694	9.9887364	9.9887365
32	Gorinchem	36	101	83°02'43".439	9.9967928	9.9967928
33	's-Hertogenbosch	37	105	45°44'52".514	9.8550808	9.8550821
24	Hilvarenbeek	26	73	44°31'50".193	9.8458978	9.8458970
18	Hoogstraten	25	70	74°47'31".972	9.9845187	9.9845187
17	Bergen op Zoom	24	66	47°15'24".676	9.8659349	9.8659349
22	Willemstad	34	94	67°40'54".094	9.9661833	9.9661833
					9.5031447	9.5031453
32	Gorinchem	35	99	58°27'20".264	9.9305595	9.9305586
33	's-Hertogenbosch	36	102	50°46'30".626	9.8891171	9.8891146
24	Hilvarenbeek	37	104	92°49'52".267	9.9994696	9.9994697
18	Hoogstraten	26	72	67°31'20".354	9.9656854	9.9656854
17	Bergen op Zoom	25	68	34°51'49".824	9.7571136	9.7571136
22	Willemstad	24	65	89°20'31".983	9.9999713	9.9999714
29	Dordrecht	34	95	66°08'55".467	9.9612304	9.9612320
					9.5031469	9.5031453

"J'ai traité de même plusieurs tours d'horizon à la fois qui étaient en rapports "plus ou moins éloignés entr'eux et après avoir reconnu les différences des "logarithmes des sinus des angles inverses de tous les triangles qui les composent, "j'ai de nouveau consulté mes régistres pour examiner s'il y avait moyen de faire "disparaître ces irrégularités, en substituant aux observations sur lesquelles "j'avais basé mon premier calcul d'autres qui fussent plus d'accord entr'elles et "qui remplissent les conditions désirées, ce qui m'a souvent réussi. Après cette "substitution il me devait rester encore des petites corrections à faire pour que "la somme des angles sphériques à chaque tour d'horizon fut exactement de 360° "et celle des triangles de 180° après avoir diminué chaque angle de sa portion "dans l'excès sphérique.
"C'est après cette méthode, à la vérité longue et pénible, mais la seule qui put "atteindre un but satisfaisant que le calcul définitif de la triangulation a été "exécuté, tel qu'il est présenté dans le tableau No. III. On y verra que les "valeurs des cotés communs, déduites de deux séries de triangles, sont égales "entr'elles à moins d'un centimètre. J'aurais même pu parvenir à un plus grand "degré de précision en me servant de tables de logarithmes à plus de sept "décimals mais cela m'a paru absolument inutile".

From this ample quotation - the underlining is mine - it appears how Krayenhoff adjusted his network as a "thinking observer" (the expression is of Van der Plaats [60]). He had not yet the disposal of the strict scientific method of the least squares, and even if he had known it he could not have used it in practice as it requires the solution of 276 normal equations. It testifies to his great perseverance and his devotion to his work that he followed the way described, "longue et pénible" indeed.

First making up a great number of sine conditions from which that round the station Breda in table 22 is an example. Then the comparison of all the angles in these equations with the results of the observation registers. For, dependent on the accuracy which Krayenhoff attached to the several observations, he had to decide - see the underlined quotation - which angles and to which amounts they had to be altered in order that they should satisfy the several conditions. Then the determination of the small corrections to the angles in the several stations and triangles, necessary to satisfy the station and triangle conditions and, finally, a second computation of the sides of the network with the adjusted angles.

The results of this computation can be found in tableau III of the Précis Historique . As an example I give in table 24 the computation of the same triangles 34-37 as given in table 20 for tableau II.

Table 24

No. triangle	Stations No.	Stations Name	Adjusted angles between chords	log sine	log opposite sides	opposite sides (metres)
1	2	3	4	5	6	7
34	22	Willemstad	67°40'54".095	9.9661833	4.4196252	26279.991
	29	Dordrecht	66°08'57".188	9.9612320	4.4146739	25982.077
	23	Breda	46°10'08".717	9.8581681	4.3116100	20493.214
			180°00'00".000			
35	29	Dordrecht	77°00'25".845	9.9887365	4.4778031	30047.134
	23	Breda	44°32'14".579	9.8459500	4.3350166	21628.010
	32	Gorinchem	58°27'19".576	9.9305586	4.4196252	26279.991
			180°00'00".000			
36	23	Breda	46°10'47".204	9.8582460	4.4469343	27985.577
	32	Gorinchem	83°02'43".606	9.9967928	4.5854811	38501.810
	33	's-Hertogenbosch	50°46'29".190	9.8891146	4.4778029	30047.121
			180°00'00".000			
37	23	Breda	41°25'15".493	9.8205865	4.4065979	25503.388
	24	Hilvarenbeek	92°49'51".320	9.9994697	4.5854811	38501.810
	33	's-Hertogenbosch	45°44'53".187	9.8550821	4.4410935	27611.726
			180°00'00".000			

The log sines of the left and right base angles between the chords in column 5 of table 24 agree with the amounts in column 7 of table 23 where the adjustment of the sine conditions took place. The angles in column 4 of table 24 are of course computed from the log sines in column 5.

Because of the alteration of the angles of the triangles the lengths of the sides in tableau III (table 24) don't agree with those in tableau II (table 20). For every triangle Krayenhoff mentions in foot notes the differences of the two computed sides with the analogous amounts from the computation in tableau II. Those for the sides Dordrecht-Breda and Breda-Willemstad in triangle 34 (see tables 20 and 24) are -0.06 m and -0.21 m. For the triangles 35, 36 and 37 these amounts are -0.01 m and +0.05 m, +0.16 m and -0.17 m, and +0.03 m and 0.00 m, respectively. These very small amounts rise in triangle 67 to +1.03 m for the side Amersfoort-Harderwijk and to +1.06 m for the side Naarden-Harderwijk.

As Krayenhoff started his adjustment in the southern part of the network it is obvious to suppose that, also because of the use of a less accurate repetition circle, there will be the tendency that in the northern part the corrections to the angles will be greater in order to make the network a closing mathematical figure. The correctness of this supposition can be seen from the amounts p' in column 9 of table 9. If one leaves $p' = -10''.361$ at the station Oosterend (No. 71) out of consideration (as already said this correction is also influenced by the re-erection of the signal Kijkduin), then the correction $p' = -11''.819$ to angle 417, measured at the station Pilsum (No. 93) between the sighting points Holwierde and Borkum is very large, even the largest in the network.

But several other amounts p' are also large, especially if one takes into consideration that they are given in thousandths of a second. Those larger than $5''$ are mentioned in table 25. From the columns 1 and 2 can be seen that almost all corrections relate to angles measured in the northern part of the network.

Table 25

Station No.	Station Name	Triangle	Angle	Corr. p'	Station No.	Station Name	Triangle	Angle	Corr. p'
1	2	3	4	5	1	2	3	4	5
21	Brielle	31	85	$+5''.242$	81	Drachten	128	370	$-6''.251$
71	Oosterend	110	317	$-7''.375$	85	Hornhuizen	130	374	$-6''.432$
73	Vlieland	111	320	$-6''.329$	82	Oosterwolde	133	382	$+5''.296$
76	Oldeholtpa	124	360	$-5''.928$	87	Rolde	133	384	$-5''.602$
76	"	125	361	$+6''.014$	88	Sleen	135	390	$+5''.979$
78	Ballum	127	368	$+7''.674$	89	Uithuizermeden	138	398	$-6''.441$
80	Dokkum	127	369	$-5''.942$	91	Midwolda	140	404	$-5''.399$
80	"	128	372	$+5''.530$	91	"	148	424	$+6''.060$
80	"	130	373	$+6''.000$	93	Pilsum	144/5	417	$-11''.819$

It seems logical to assume that in this part there will also be the tendency that the differences between the lengths of the sides in tableau II and tableau III will be the greatest. This is confirmed indeed by Krayenhoff's computation. For the side Drachten-Groningen in triangle 131 this difference is -1.63 m and for the side Dokkum-Groningen -1.62 m. In triangle 143 the difference between the two computations is + 1.80 m for the side Sleen-Onstwedde and +1.27 m for the side Rolde-Onstwedde. They are the largest in the network, only - and to a great extent - surpassed by the differences -2.58 m in the side Dokkum-Ballum and

-3.05 m in the side Leeuwarden-Ballum of triangle 127. As the latter side, computed in triangle 121 of tableau III (see also Fig. 16), differs "but" +0.39 m from the computation in tableau II, the bad harmony between both results will probably have to be imputed to too long a distance Dokkum-Leeuwarden = 19560.910 m found in tableau II and in the rather bad shape of triangle 127. I discussed this matter already in the text belonging to Fig. 16 (see page 122). In tableau III the length Leeuwarden-Dokkum is 19559.600 m (difference -1.31 m).

The large differences in length found from the computation in two chains in tableau II (see table 20) and the big corrections p' to the angles in the stations in column 2 of table 25 make it likely that in the part of Krayenhoff's network bordered approximately by the angular points Ballum, Dokkum, Groningen, Oosterwolde, Oldeholtpa, Sneek and Harlingen and formed by the triangles 127, 128, 131, 132, 125, 124, 123, 122 and 121 the accuracy is the worst. It is just this part of the triangulation - I mentioned it already in my Introduction (section 1) - that was criticized by Gauss and, later on, by Verdam. The big corrections p', given e.g. to the angles Drachten ($p'_{370} = -6''.251$), Leeuwarden ($p'_{371} = -1''.876$) and Dokkum ($p'_{372} = +5''.530$) of triangle 128, however, need not be representative for the accuracy of the measurements. It might be possible that with much smaller corrections to the observations a closure of the network can be obtained. Gauss demonstrated it already in his criticism: for the part of the network just mentioned one finds in an adjustment according to the least squares of the 27 measured angles $[pp] = 97.88$ whereas Krayenhoff's $[p'p']$ for the same part of the triangulation amounts to $[p'p'] = 341.42$ [61].

I remarked already that in Krayenhoff's time an adjustment of the network according to the least squares would have been impossible even if one had known the method. But Krayenhoff was the first who adjusted a whole triangulation according to a method found by himself. He did not only see all the conditions in the several triangles and the conditions the angles around the central points had to comply with but also the greater part (51) of the 54 side (sine) conditions. If one substitutes e.g. in the side equation round Harlingen in section 13 (see page 76) Krayenhoff's amounts p', one finds +0.376 = 0. As the equation was multiplied by 10^6 the sum of log sine left base angles round Harlingen differs but 0.00000038 from the sum of log sine right base angles. As Krayenhoff used a seven place logarithmic trigonometric table for his computations the amount may be neglected. After the computation of the lengths of the sides in the several chains the differences in the common sides appear seldom to be more than 1 or 2 cm. The biggest difference, 4 cm, can be found in the side Rolde-Onstwedde of the triangles 142 and 143. They seem to prove - I return to this subject in section 22 - that Krayenhoff succeeded in making an almost closing mathematical figure of his network.

In my opinion he was the first who used sine conditions in the adjustment of a triangulation, though not in the form Gauss used them later on. It seems that the geometrical theorem underlying these conditions which holds for the flat plane as well as for the sphere, was published for the first time (in 1803) by Carnot [62] in his Géométrie de position but that Krayenhoff found it anew. At any rate he was the first who applied it in a geodetic problem. The very simple proof of the theorem may be found on page 31 of the second edition of the Précis Historique. The proof is not included in the first edition.

It is not known whether professor Van Swinden [3] and (or) Jacob de Gelder [45], Huguenin [63] and (or) other collaborators in the triangulation [64] advised or cooperated in a design for the adjustment. The three mentioned above were at any rate good mathematicians.

Nothing in the text and the tables of his book shows that Krayenhoff noticed the 52nd, 53rd and 54th side equation round the former Zuiderzee and mentioned in table 13. They differ considerably - especially the 53rd and 54th - from the normal shape. It cannot be taken him amiss: even the great Gauss made in 1834 [65] a similar mistake in the adjustment of 63 measured angles in 21 triangles of the triangulation of Oldenburg. They surround (see Fig. 17) an "empty" heptagon. From

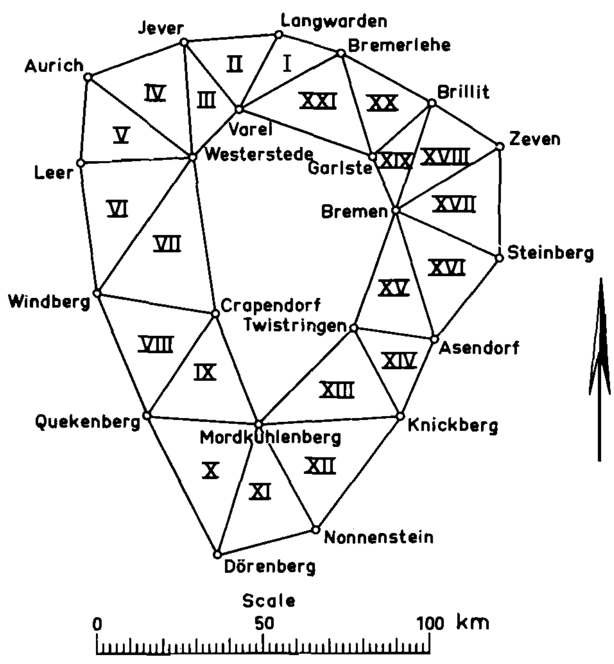


Fig. 17

the "adjusted" angles of the triangles follow the angles of the inner heptagon. Their sum is $900^{\circ}00'32''.714$. As the spherical excess of the heptagon is $17''.814$, the error in the sum of the inner angles is $14''.900$. In the adjustment of the 21 triangles which proceeded according to the measurement of the angles Gauss apparently forgot to take into account the polygon condition, similar to Krayenhoff's condition for the Zuiderzee pentagon [66] and also the three special conditions similar to those mentioned in the numbers 52, 53 and 54 of Krayenhoff's network and represented in Fig. 14 and table 13.

On page 87 (year 1891) of his paper [20] Van der Plaats says that Krayenhoff must have noticed one of the two conditions 53 and 54. This opinion seems hardly credible. Why should he have seen one of them and overlooked the other which is completely of the same structure ? Moreover, Van der Plaats' explanation given there is incorrect. Therefore Krayenhoff's ignorance of the three equations 52-54 cannot have influenced his choice of the series of angles which occur in these equations. I already said (see section 13 page 78 and table 13) that a closing error of -1.49 m for the not-adjusted angles in condition 52 is very small in my opinion. Those for the conditions 53 and 54 and for all p''s = 0 (see table 13) are also very small: +1.17 m and -0.68 m respectively. For Krayenhoff's amounts p' which can be borrowed from table 9 (column 9) these closing errors can even be reduced to -0.64 m, +0.80 m and +0.38 m, respectively.

In order to get a reliable impression of the accuracy of Krayenhoff's triangulation, an adjustment of the whole network according to the method of the least squares will have to be done and, after that, a comparison of the lengths of its sides with the sides between the identical points of the R. D.-triangulation network.

19. Adjustment of the spherical angles of the triangulation network according to the method of the least squares

In order to determine the corrections $p_1, p_2, \ldots\ldots p_{504}, p_{505}$ to the 505 spherical angles measured,in such a way that $[pp]$ = minimum, one must determine the 276 normal equations from the 276 condition equations mentioned in section 13.

Their general form is:

$$[\Phi a]\ K_a + [\Phi b]\ K_b + \ldots\ldots\ [\Phi\theta]K_\theta + [\Phi\zeta]K_\zeta + W_\Phi = 0$$

$$(\Phi = a, \ldots\ldots, \zeta).$$

The first equation therefore runs as follows:

$$[aa]\,K_a + [ab]K_b + \ldots\ldots + [a\theta]\,K_\theta + [a\zeta]\,K_\zeta + W_a = 0,$$

the 276th:

$$[\zeta a]\,K_a + [\zeta b]\,K_b + \ldots\ldots + [\zeta\theta]K_\theta + [\zeta\zeta]K_\zeta + W_\zeta = 0.$$

The computation of the coefficients $[aa]$, ..., $[\zeta\zeta]$ and the solution of the 276 correlates K_Φ (Φ= a, ..., ζ) was done with the I. B. M. 360/65-computer of the Delft University of Technology. The corrections p_i (i = 1, 2, ..., 504, 505) can then be computed from:

$$p_i = a_iK_a + b_iK_b + \ldots\ldots + \theta_iK_\theta + \zeta_iK_\zeta .$$

From these amounts follows:

$$[pp] = -[KW]$$

which gives an insight into the internal accuracy of the triangulation. These computations were also carried out by the computer. The result is given in the tables 9 (column 10) and 15 (column 7). The amounts p_i are rounded off to a thousandth of a second. The adjusted angles $\alpha_i + p_i$ may be found in column 13 of table 9 and column 10 of table 15. The large corrections p' which Krayenhoff gave (and had to give because of his arbitrary adjustment) to the angles of triangle 128 - I discussed them on page 127 of section 18 - are now reduced to much more reasonable amounts p. As $[pp] = -[KW] = 870.1$, $m_\alpha^2 = \frac{870.1}{276} = 3.152$, the internal accuracy of the triangulation network can be characterized by the standard deviation $m_\alpha = \pm\ 1''.775$ in the measured angle. It deviates but little from the mean of the amounts $\sqrt{\frac{2''.4}{2.25}} = 1''.6$ (southern part of the network, accurate instrument) and $\frac{4''.9}{\sqrt{2.12}} = 3''.4$ (northern part of the network, less accurate instrument), found at the end of section 8. The largest negative p (p_{370}) is $-4''.463$, the largest positive one, (p_{332}), is $+3''.941$.

In the histogram of Fig. 18 the several p's have been arranged in a surveyable manner. The class interval is $0''.4$. The number of p's between $-0''.2$ and $+0''.2$ e. g. is 76, that between $+1''.4$ and $+1''.8$ is 13, etc. On the same scale and for the same class interval the figure also gives the number of p's which must be expected according to Gauss' law of probabilities (standard deviation $m = \pm\ 1''.775$) if the distribution of errors would have been a normal one (between $-0''.2$ and $+0''.2$, forty five, between $+1''.4$ and $+1''.8$, thirty one, etc.). As can be seen the numbers in the corresponding classes don't match very well. According to the adjustment of the network the number of the small p's is much too high and accordingly that of the larger p's too low. Here too one can see that Krayenhoff's results of his

Frequency distribution for 505 corrections p ($m = \pm 1\overset{''}{.}775$)

p'' between	Krayenh.	Normal	p'' between	Krayenh.	Normal
-5.8 -5.4	-	-	+0.2 +0.6	67	44
-5.4 -5.0	-	1	+0.6 +1.0	40	41
-5.0 -4.6	-	1	+1.0 +1.4	34	36
-4.6 -4.2	2	2	+1.4 +1.8	13	31
-4.2 -3.8	2	4	+1.8 +2.2	18	24
-3.8 -3.4	3	6	+2.2 +2.6	13	18
-3.4 -3.0	4	9	+2.6 +3.0	10	13
-3.0 -2.6	6	13	+3.0 +3.4	2	9
-2.6 -2.2	9	18	+3.4 +3.8	1	6
-2.2 -1.8	19	24	+3.8 +4.2	2	4
-1.8 -1.4	21	31	+4.2 +4.6	-	2
-1.4 -1.0	35	36	+4.6 +5.0	-	1
-1.0 -0.6	45	41	+5.0 +5.4	-	1
-0.6 -0.2	83	44	+5.4 +5.8	-	-
-0.2 +0.2	76	45		505	505

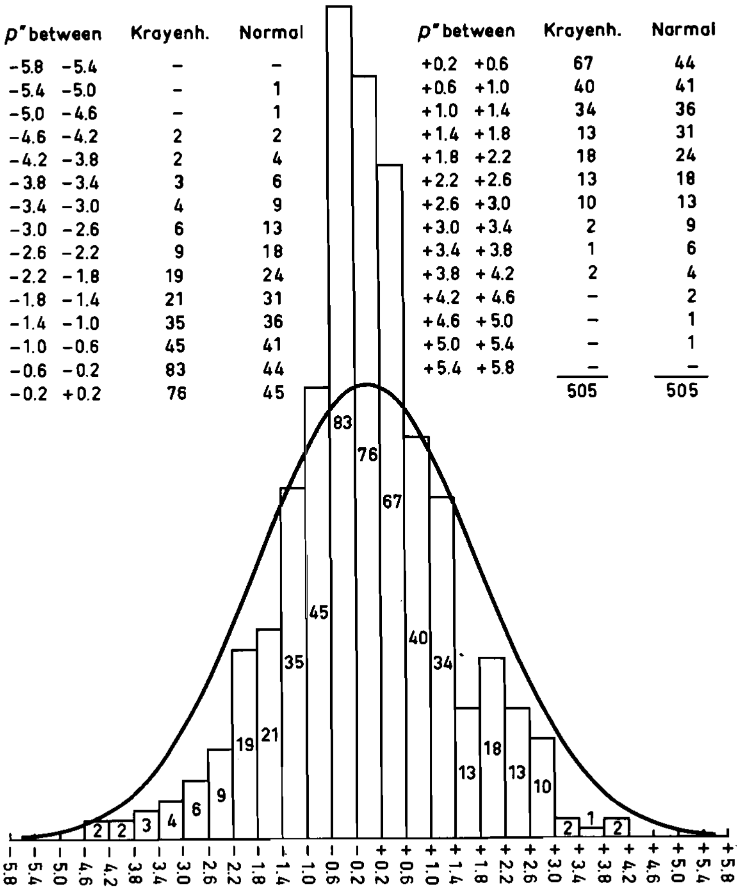


Fig. 18

angular measurements were influenced by his endeavour to get small closing errors in the station equations and in the angle equations for the several triangles.

20. Provisional adaptation of the adjusted network to the points Rhenen and Gorinchem of the R. D.-triangulation

In order to compute the final lengths of the sides of the triangulation network one could start, as Krayenhoff did, from the side Duinkerken-Mont Cassel, the only length that he had at his disposal. It is better, however, to use a more reliable length, not situated in the utmost southern part of the network but in the centre. For a provisional computation of the coordinates of the angular points of Krayenhoff's network in the R. D.-coordinate system I started from the base-line Rhenen (No. 37)-Gorinchem (No. 32). The coordinates of its terminal points, both of them determined by the R.D. in 1895, are:

$$X'_{37} = +\,12163.650 \quad , \quad Y'_{37} = -\,22038.969$$
$$X'_{32} = -\,28577.386 \quad , \quad Y'_{32} = -\,36148.952$$

If one assumes that the spires of both towers remained unaltered between Krayenhoff's time and 1895 - I cannot prove the contrary - one then finds from these coordinates a length of the chord on the conformal sphere of 43118.84 m. Starting from the side Duinkerken-Mont Cassel Krayenhoff found 43117.29 m for the same length in tableau III of his Précis Historique (page 125). The relative difference is 1.55 : 43118.84 = 0.000036. From this almost ideal very long baseline the provisional coordinates of the angular points of Krayenhoff's network can be computed in the following way.

In Fig. 19 I called Gorinchem the left base point L and Rhenen the right base point R of triangle 49.

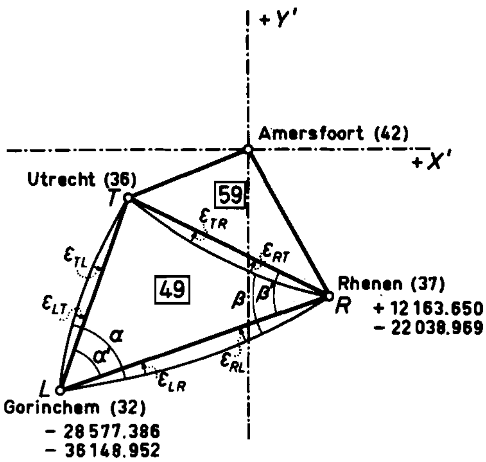


Fig. 19

Utrecht is the apex T that must be computed from the coordinates of L and R and the adjusted spherical angles 138 = α and 140 = β between the arcs of the circles in triangle 49.

If the angles between arc and chord in R and L are ϵ_{RT}, ϵ_{RL}, ϵ_{LR} and ϵ_{LT}, respectively (from arc to chord to the right is positive), the angles β' and α' between the chords can be computed from:

$$\left.\begin{aligned} \beta' &= \beta + (\epsilon_{RT} - \epsilon_{RL}) \\ \alpha' &= \alpha + (\epsilon_{LR} - \epsilon_{LT}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

From the gridbearing

$$\overline{RL} = \text{arc tan}\ \frac{X'_L - X'_R}{Y'_L - Y'_R} \quad \text{of RL}$$

and the angle β' follows the gridbearing of RT:

$$\overline{RT} = \overline{RL} + \beta'$$

In the same way:

$$\overline{LT} = \overline{LR} - \alpha'$$

From intersection then the coordinates of T follow.

For ϵ_{RL} (in seconds of arc) one has (see formula 11 in section 13):

$$\epsilon''_{RL} = 0.0012658\ (X'_L Y'_R - X'_R Y'_L)$$

and, in the same way:

$$\left.\begin{aligned} \epsilon''_{RT} &= 0.0012658\ (X'_T Y'_R - X'_R Y'_T) \\ \epsilon''_{LR} &= 0.0012658\ (X'_R Y'_L - X'_L Y'_R) = -\epsilon''_{RL} \\ \epsilon''_{LT} &= 0.0012658\ (X'_T Y'_L - X'_L Y'_T) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (13)$$

(X' and Y' in km).

For the computation of ϵ''_{RT} and ϵ''_{LT} in (13) the unknown coordinates of T are necessary. As, however, ϵ''_{RT} and ϵ''_{LT} are very small, (very) provisional coordinates are sufficient. They can be found from an intersection for which $\alpha' \simeq \alpha$ and $\beta' \simeq \beta$.

From the formula for α' and β' in (12) and the analogous formula $\gamma' = \gamma + {}$ $+ (\epsilon_{TL} - \epsilon_{TR})$ for the apex angle T of triangle RLT follows:

$$(\epsilon''_{RT} - \epsilon''_{RL}) + (\epsilon''_{LR} - \epsilon''_{LT}) + (\epsilon''_{TL} - \epsilon''_{TR}) = -E''$$

in which E is the spherical excess of the triangle.

As $\epsilon_{LR} = -\epsilon_{RL}$, $\epsilon_{TL} = -\epsilon_{LT}$ and $\epsilon_{RT} = -\epsilon_{TR}$, one finds:

$$\epsilon''_{RL} + \epsilon''_{LT} + \epsilon''_{TR} = \tfrac{1}{2} E'' \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

The formula gives the opportunity of checking the computed ϵ's as the spherical excess of the triangle was already known.

As an example I give underneath the computation of the coordinates X_{36}, Y_{36} of the apex Utrecht of triangle 49.

As $\alpha = 138 = 51^\circ 15' 00''.834$, $\beta = 140 = 45^\circ 12' 46''.577$, $\epsilon_{RL} = +1''.354$ and $\overline{RL} = \text{arc tan}\ \dfrac{X'_L - X'_R}{Y'_L - Y'_R} = 250^\circ 53' 50''.350$, one finds for the (very)provisional coordinates of T:

$$X_{T'} = -18223.0 \quad , \quad Y_{T'} = -7146.0$$

and from these coordinates and those of R and L:

$$\epsilon_{LT} = +0''.575 \text{ and } \epsilon_{RT} = +0''.618 \text{ (see formula 13).}$$

The check of formula (14) gives:

$$\epsilon''_{RL} + \epsilon''_{LT} + \epsilon''_{TR} = +1''.311 = \tfrac{1}{2}E'' .$$

Therefore (formula 12):

$$\alpha' = \alpha + (\epsilon''_{LR} - \epsilon''_{LT}) = 51^{\circ}14'58''.905$$

$$\beta' = \beta + (\epsilon''_{RT} - \epsilon''_{RL}) = 45^{\circ}12'45''.841$$

The intersection of T (Utrecht), now with the correct base angles α' and β' gives the required coordinates:

$$X_{36} = -18222.796 \quad , \quad Y_{36} = -7146.191.$$

In a similar way the coordinates $X_{42} = -0.587$, $Y_{42} = -0.575$ of the apex Amersfoort of triangle 59 can be computed from the base angles and the coordinates of the base points Utrecht (L) and Rhenen (R) of that triangle.

As the spires of Utrecht and Amersfoort in 1895 are assumed to be identical with those in Krayenhoff's time, the coordinates in the R. D.-system:

$$X'_{36} = -18222.582, \quad Y'_{36} = -7145.440$$

$$X'_{42} = 0.000, \quad Y'_{42} = 0.000$$

already give some impression of the accuracy of Krayenhoff's measurements.

Proceeding in the same way all the angular points of Krayenhoff's triangulation network were computed in the XY-coordinate system. The computation, done with the computer, was checked by computing the coordinates of 54 points of the network in a second way. Those of Nieuwkoop (No. 35) e.g. in the first computation were determined by intersection from Gouda and Utrecht (triangle 47) and those of Amsterdam (No. 40) from Haarlem and Nieuwkoop (triangle 55). If in a second computation Nieuwkoop is determined from Utrecht and Amsterdam by intersection in triangle 56, one must find the same coordinates, apart of course from small rounding-off errors. The second computation is a check on the condition equations, the solution of the normal equations and the determination of the adjusted angles of the network.

21. Final adaptation of Krayenhoff's adjusted triangulation to 65 identical points of the R. D.-network

The coordinates XY of all the 106 angular points of Krayenhoff's triangulation, computed in the preceeding section, are mentioned in the columns 3 and 4 of table 26. The sequence of the stations is the same as in table 9. In columns 5 and 6 one finds behind the stations Gorinchem (No. 32) and Rhenen (No. 37) the (same) coordinates from which the computation in columns 3 and 4 was started. The other coordinates in columns 5 and 6 - in total 65 pairs - are those of the points which are identical or are supposed to be identical with the towers of Krayenhoff's network. With the exception of Oosterwolde (No. 82) and Midwolda (No. 91) all these points were determined in the first order triangulation network between 1885 and 1907.

In his publication "Topografische Kaart en Rijksdriehoeksmeting " [67] Heuvelink also gives a list of identical points. It is almost the same as mine. In 1920, however, Heuvelink had not yet the disposal of the coordinates $X'Y'$ of Gent (No. 10), Antwerpen (No. 15), Kirch Hesepe (No. 64), Oosterwolde (No. 82), Midwolda (No. 91), Leer (No. 95) and Herentals (No. 104).

The R. D.-observation pillar in the isle of Vlieland is identical with the beacon-light No. 73 in Krayenhoff's time [68] . The lighthouse in the isle of Borkum, already determined in R. D.-coordinates in 1888, is not identical with the former beacon-light No. 106, though it was built almost in the same place.

In my opinion Heuvelink mentions wrongly as identical points Harikerberg (No. 50, $X' = +78741.774$, $Y' = +9584.122$) and Lemelerberg (No. 60, $X' = +69322.687$, $Y' = +35847.558$) though the stones which marked Krayenhoff's triangulation points could not be found again in 1894 and 1889 respectively.

Calling, as Heuvelink did, Dordrecht (No. 29, $X' = -50150.388$, $Y' = -37683.375$), Nieuwkoop (No. 35, $X' = -41573.752$, $Y' = -350.257$) and Edam (No. 53, $X' = -23098.677$, $Y' = +39794.418$) identical points, is a mistake. The municipal land surveying department at Dordrecht informed me that it appeared from deformation measurements that points on the platform of the tower are moving away from each other and that since very long the tower is sagging to the north-northwest. It is true that provisions could be made in order to stop the deformation but the sagging of the tower still proceeds. With regard to the tower of Edam the municipality wrote to me [69] that between 1803 and 1899 the tower sagged to the northeast. The municipality of Nieuwkoop was as kind as to inform me in detail on the sagging to the southwest of the tower of the former abbey [70].

Table 26

Stations i		System Krayenhoff		System R. D.		System XY adapted to system X'Y'		Differ-ences (cm)	
No.	Name	X_i	Y_i	X'_i	Y'_i	X''_i	Y''_i	v_i	w_i
1	2	3	4	5	6	7	8	9	10
1	Duinkerken	-211162. 91	-120207. 05			-211164. 95	-120204. 23		
2	Mont Cassel	-204324. 09	-146809. 50			-204326. 48	-146806. 88		
3	Hondschoote	-196668. 81	-126802. 18			-196670. 89	-126799. 59		
4	Nieuwpoort	-184511. 53	-110825. 57			-184513. 33	-110823. 09		
5	Diksmuide	-176949. 20	-121691. 01			-176951. 12	-121688. 68		
6	Oostende	-172281. 58	-100059. 02			-172283. 17	-100056. 68		
7	Brugge	-151094. 57	-103116. 63			-151096. 13	-103114. 60		
8	Hooglede	-161870. 93	-128385. 42			-161872. 89	-128383. 33		
9	Tielt	-144602. 84	-126441. 49			-144604. 71	-126439. 64		
10	Gent	-116435. 66	-121288. 16	-116438. 10	-121287. 08	-116437. 34	-121286. 70	+76	+38
11	Aardenburg	-135313. 46	- 96340. 74	-135315. 21	- 96338. 52	-135314. 85	- 96338. 91	+36	-40
12	Assenede	-114116. 58	-101899. 20	-114118. 36	-101897. 84	-114117. 97	-101897. 70	+39	+15
13	Middelburg	-123063. 27	- 71444. 56	-123064. 60	- 71443. 32	-123064. 26	- 71442. 81	+34	+51
14	Hulst	- 93008. 85	- 96449. 34	- 93009. 69	- 96448. 54	- 93010. 08	- 96448. 12	-39	+42
15	Antwerpen	- 68921. 95	-103524. 08	- 68922. 96	-103523. 10	- 68923. 19	-103523. 23	-23	-13
16	Zierikzee	-101897. 15	- 55143. 18	-101897. 97	- 55142. 05	-101897. 82	- 55141. 67	+15	+38
17	B. O. Zoom	- 76334. 53	- 72933. 95	- 76335. 44	- 72933. 16	- 76335. 36	- 72932. 88	+ 8	+28
18	Hoogstraten	- 43543. 01	- 83733. 63	- 43543. 86	- 83733. 53	- 43543. 87	- 83733. 07	- 1	+46
19	Lommel	- 5073. 99	-103095. 01	- 5074. 79	-103095. 49	- 5074. 97	-103095. 08	-18	+41
20	Nederweert	+ 25174. 22	- 96630. 15	+ 25173. 46	- 96630. 71	+ 25173. 45	- 96630. 63	- 1	+ 9
21	Brielle	- 84301. 49	- 27387. 66	- 84301. 96	- 27386. 16	- 84301. 69	- 27386. 30	+27	-13
22	Willemstad	- 65636. 50	- 51105. 90	- 65636. 93	- 51105. 36	- 65636. 98	- 51104. 90	- 5	+46
23	Breda	- 42438. 73	- 62807. 03	- 42438. 97	- 62806. 85	- 42439. 28	- 62806. 41	-31	+44
24	Hilv. beek	- 17429. 39	- 74507. 96	- 17429. 86	- 74508. 43	- 17430. 01	- 74507. 74	-15	+69
25	Helmond	+ 18678. 59	- 75246. 04	+ 18678. 37	- 75246. 75	+ 18678. 10	- 75246. 34	-27	+42
26	Vierl. beek	+ 43200. 11	- 61990. 79	+ 43199. 65	- 61991. 94	+ 43199. 90	- 61991. 40	+25	+54
27	Den Haag	- 74077. 14	- 8107. 89	- 74077. 22	- 8106. 07	- 74077. 03	- 8106. 60	+19	-53
28	Rotterdam	- 62066. 03	- 25616. 79	- 62066. 00	- 25615. 64	- 62066. 12	- 25615. 74	-12	-10
29	Dordrecht	- 50150. 21	- 37684. 40			- 50150. 43	- 37683. 57		
30	Leiden	- 61286. 22	+ 472. 87	- 61286. 17	+ 473. 98	- 61285. 93	+ 474. 01	+24	+ 3
31	Gouda	- 46450. 88	- 15855. 63	- 46450. 83	- 15854. 87	- 46450. 78	- 15854. 77	+ 5	+11
32	Gorinchem	- 28577. 39	- 36148. 95	- 28577. 39	- 36148. 95	- 28577. 50	- 36148. 42	-11	+53
33	's-Bosch	- 5494. 42	- 51970. 21	- 5494. 58	- 51970. 34	- 5494. 68	- 51970. 07	-10	+27
34	Grave	+ 24356. 87	- 43898. 76			+ 24356. 85	- 43899. 02		
35	Nieuwkoop	- 41573. 31	- 350. 61			- 41572. 96	- 349. 76		
36	Utrecht	- 18222. 80	- 7146. 19	- 18222. 58	- 7145. 44	- 18222. 45	- 7145. 70	+13	-26
37	Rhenen	+ 12163. 65	- 22038. 97	+ 12163. 65	- 22038. 97	+ 12163. 90	- 22038. 97	+25	00
38	Nijmegen	+ 32735. 26	- 34071. 23	+ 32735. 42	- 34071. 93	+ 32735. 42	- 34071. 57	00	+36
39	Haarlem	- 51065. 98	+ 25397. 80	- 51065. 62	+ 25399. 10	- 51065. 30	+ 25398. 89	+33	-21
40	Amsterdam	- 34299. 80	+ 24524. 64	- 34299. 28	+ 24525. 50	- 34299. 06	+ 24525. 49	+22	- 1
41	Naarden	- 15346. 30	+ 15659. 36	- 15346. 01	+ 15660. 58	- 15345. 61	+ 15659. 90	+40	-67
42	Amersfoort	- 0. 59	- 0. 58	0. 00	0. 00	- 0. 07	- 0. 32	- 7	-32
43	Imbosch	+ 41641. 56	- 12983. 32			+ 41642. 05	- 12983. 71		
44	Hettenheuvel	+ 58285. 14	- 25702. 52			+ 58285. 51	- 25703. 20		
45	Bocholt	+ 84518. 60	- 34555. 07			+ 84518. 95	- 34556. 16		

Table 26 (continued)

1	2	3	4	5	6	7	8	9	10
46	Harderwijk	+ 15619. 22	+ 21592. 60			+ 15620. 11	+ 21592. 72		
47	Veluwe	+ 32148. 66	+ 8950. 05	+ 32149. 33	+ 8950. 75	+ 32149. 44	+ 8949. 89	+11	-86
48	Zutphen	+ 55316. 02	- 1418. 45	+ 55316. 85	- 1418. 50	+ 55316. 73	- 1418. 99	-12	-49
49	Groenlo	+ 84421. 44	- 11836. 27	+ 84422. 95	- 11836. 40	+ 84422. 11	- 11837. 27	-83	-87
50	Harikerberg	+ 78740. 00	+ 9584. 13			+ 78740. 97	+ 9583. 29		
51	Ahaus	+110997. 99	- 7585. 10			+110998. 83	- 7586. 46		
52	Alkmaar	- 43134. 02	+ 53170. 06	- 43132. 50	+ 53171. 31	- 43132. 90	+ 53171. 14	-41	-17
53	Edam	- 23100. 44	+ 39792. 40			- 23099. 44	+ 39793. 15		
54	Hoorn	- 22163. 72	+ 54040. 54			- 22162. 51	+ 54041. 33		
55	Schagen	- 39878. 90	+ 70422. 87			- 39877. 52	+ 70423. 98		
56	Medemblik	- 19155. 91	+ 68798. 18	- 19154. 01	+ 68799. 85	- 19154. 48	+ 68798. 99	-47	-86
57	Enkhuizen	- 6396. 96	+ 61041. 59	- 6395. 43	+ 61042. 26	- 6395. 59	+ 61042. 18	-16	- 8
58	Urk	+ 13942. 35	+ 56446. 81	+ 13944. 28	+ 56447. 59	+ 13943. 73	+ 56447. 09	-55	-49
59	Kampen	+ 35867. 95	+ 45070. 46	+ 35869. 42	+ 45070. 45	+ 35869. 26	+ 45070. 38	-16	- 7
60	Lemelerberg	+ 69321. 08	+ 35847. 41			+ 69322. 39	+ 35846. 81		
61	Oldenzaal	+105090. 63	+ 18608. 64	+105091. 42	+ 18608. 61	+105091. 82	+ 18607. 46	+41	-115
62	Bentheim	+120696. 12	+ 17852. 54	+120696. 78	+ 17852. 64	+120697. 37	+ 17851. 13	+59	-151
63	Uelsen	+100642. 96	+ 40800. 75			+100644. 46	+ 40799. 72		
64	K. Hesepe	+125193. 75	+ 53875. 92	+125194. 07	+ 53875. 60	+125195. 53	+ 53874. 59	+146	-101
65	Kijkduin	- 44739. 49	+ 88759. 95			- 44737. 87	+ 88761. 20		
66	Oosterland	- 25331. 18	+ 86433. 30	- 25328. 84	+ 86434. 74	- 25329. 52	+ 86434. 26	-68	-48
67	Staveren	- 1811. 99	+ 80866. 03			- 1810. 32	+ 80866. 63		
68	Lemmer	+ 21815. 10	+ 76786. 92	+ 21816. 73	+ 76787. 67	+ 21816. 80	+ 76787. 16	+ 7	-51
69	Blokzijl	+ 38749. 85	+ 63790. 33	+ 38751. 73	+ 63790. 81	+ 38751. 44	+ 63790. 28	-30	-53
70	Meppel	+ 54254. 91	+ 60626. 16	+ 54256. 50	+ 60626. 04	+ 54256. 51	+ 60625. 88	+ 1	-16
71	Oosterend	- 34454. 18	+103488. 67	- 34451. 61	+103490. 80	- 34452. 30	+103489. 83	-69	-98
72	Robbezand	- 16004. 54	+100964. 95			- 16002. 63	+100965. 84		
73	Vlieland	- 21939. 77	+126979. 93	- 21935. 61	+126980. 44	- 21937. 52	+126981. 00	-191	+55
74	Harlingen	+ 1656. 47	+113380. 71	+ 1659. 31	+113381. 11	+ 1658. 63	+113381. 39	-68	+28
75	Sneek	+ 18202. 68	+ 97584. 99	+ 18204. 88	+ 97585. 41	+ 18204. 67	+ 97585. 37	-21	- 4
76	Oldeholtpa	+ 44711. 71	+ 82529. 67	+ 44713. 93	+ 82529. 12	+ 44713. 59	+ 82529. 61	-34	+49
77	Midsland	- 6816. 40	+136631. 04			- 6813. 94	+136631. 93		
78	Ballum	+ 19975. 88	+143152. 85			+ 19978. 54	+143153. 38		
79	Leeuwarden	+ 26891. 77	+116668. 52	+ 26894. 38	+116668. 84	+ 26894. 07	+116668. 85	-31	+ 1
80	Dokkum	+ 40666. 35	+130556. 02			+ 40668. 90	+130556. 21		
81	Drachten	+ 47794. 81	+106075. 11	+ 47796. 98	+106074. 53	+ 47797. 04	+106075. 10	+ 6	+57
82	Oosterwolde	+ 60823. 03	+ 93695. 73	+ 60825. 19	+ 93695. 13	+ 60825. 13	+ 93695. 48	- 6	+35
83	Beilen	+ 76025. 80	+ 78797. 67	+ 76027. 53	+ 78796. 64	+ 76027. 74	+ 78797. 15	+21	+51
84	Coevorden	+ 91499. 07	+ 57086. 41	+ 91500. 34	+ 57085. 68	+ 91500. 76	+ 57085. 58	+42	-10
85	Hornhuizen	+ 64678. 78	+137608. 53	+ 64681. 42	+137608. 70	+ 64681. 53	+137608. 40	+12	-30
86	Groningen	+ 78881. 15	+119065. 31	+ 78883. 13	+119065. 17	+ 78883. 69	+119064. 90	+56	-26
87	Rolde	+ 84499. 16	+ 93505. 04	+ 84501. 39	+ 93503. 96	+ 84501. 36	+ 93504. 45	- 3	+49
88	Sleen	+ 95501. 12	+ 70011. 49	+ 95502. 75	+ 70010. 74	+ 95503. 01	+ 70010. 65	+27	- 9
89	Uith. meden	+ 87945. 02	+140252. 78	+ 87947. 55	+140252. 60	+ 87947. 89	+140252. 32	+34	-28
90	Holwierde	+ 98769. 38	+134793. 54			+ 98772. 23	+134792. 91		
91	Midwolda	+108449. 59	+116835. 62	+108451. 41	+116834. 15	+108452. 21	+116834. 78	+80	+63
92	Onstwedde	+111159. 07	+ 98683. 17	+111160. 76	+ 98681. 52	+111161. 44	+ 98682. 21	+69	+69
93	Pilsum	+111232. 25	+149059. 59	+111236. 22	+149058. 28	+111235. 34	+149058. 83	-88	+55
94	Emden	+121106. 75	+136362. 15			+121109. 70	+136361. 20		
95	Leer	+137723. 72	+121428. 60	+137725. 86	+121426. 10	+137726. 52	+121427. 36	+66	+126

Table 26 (continued)

1	2	3	4	5	6	7	8	9	10
96	Barssel	+157483.11	+115414.06			+157485.90	+115412.51		
97	Hage	+125540.99	+162697.47			+125544.34	+162696.56		
98	Aurich	+138955.10	+148279.84			+138958.29	+148278.68		
99	Strakholt	+149962.61	+137408.10			+149965.69	+137406.74		
100	Westerstede	+169529.04	+125652.99			+169532.03	+125651.30		
101	Esens	+147160.78	+168352.02			+147164.29	+168350.82		
102	Jever	+166631.75	+160600.32			+166635.23	+160598.81		
103	Varel	+182849.51	+141719.49			+182852.78	+141717.67		
104	Herentals	- 38561.50	-108953.69	- 38562.32	-108954.04	- 38562.70	-108953.30	-38	+74
105	Biesselt	+ 35556.31	- 43271.88			+ 35556.34	- 43272.30		
106	Borkum	+ 84989.78	+160189.07			+ 84992.93	+160188.73		

From Krayenhoff's measurements and the R. D.-coordinates $X'Y'$ I had already predicted these changes. It remains possible, however, that there are still other R. D.-points which are assumed to be identical with Krayenhoff's stations but have in fact changed a little. It is very difficult to discover such alterations. They occur especially in the western part of The Netherlands where several towers are built on weak peat ground.

Whether the R. D.-station Veluwe is identical indeed with Krayenhoff's "Observatoire" (No. 47) (see my considerations on page 17) is also subject to some light doubt.

It will be clear that by the great number of identical points in both networks the (provisional) choice of the base Rhenen-Gorinchem was rather arbitrary. In order to adapt Krayenhoff's adjusted network as well as possible to the R. D.-triangulation a similarity transformation was applied on all the 65 identical points. The result of the transformation, the coordinates $X''Y''$, is mentioned in the columns 7 and 8 of table 26. The columns 9 and 10 give for the identical points the remaining differences v and w (in cm) in X- and Y-direction. In Fig. 20 these differences are represented as vectors. The smallest (in Nederweert, No. 20) is 9 cm, the largest (in Vlieland, No. 73) is 199 cm.

From the directions and the lengths of the rather long vectors in Oldenzaal (No. 61), Bentheim (No. 62) and Kirch Hesepe (No. 64) can be seen that the distances Oldenzaal-Bentheim and Bentheim-Kirch Hesepe of the network are hardly influenced by the coordinate differences in columns 9 and 10 of table 26. The mutual position of the three points is almost the same as in the R. D. for the spherical R. D.-angle $94^{\circ}20'39''.3$ in Bentheim between Oldenzaal and Kirch Hesepe also corresponds exactly with the sum of the two adjusted angles 261 and 262.

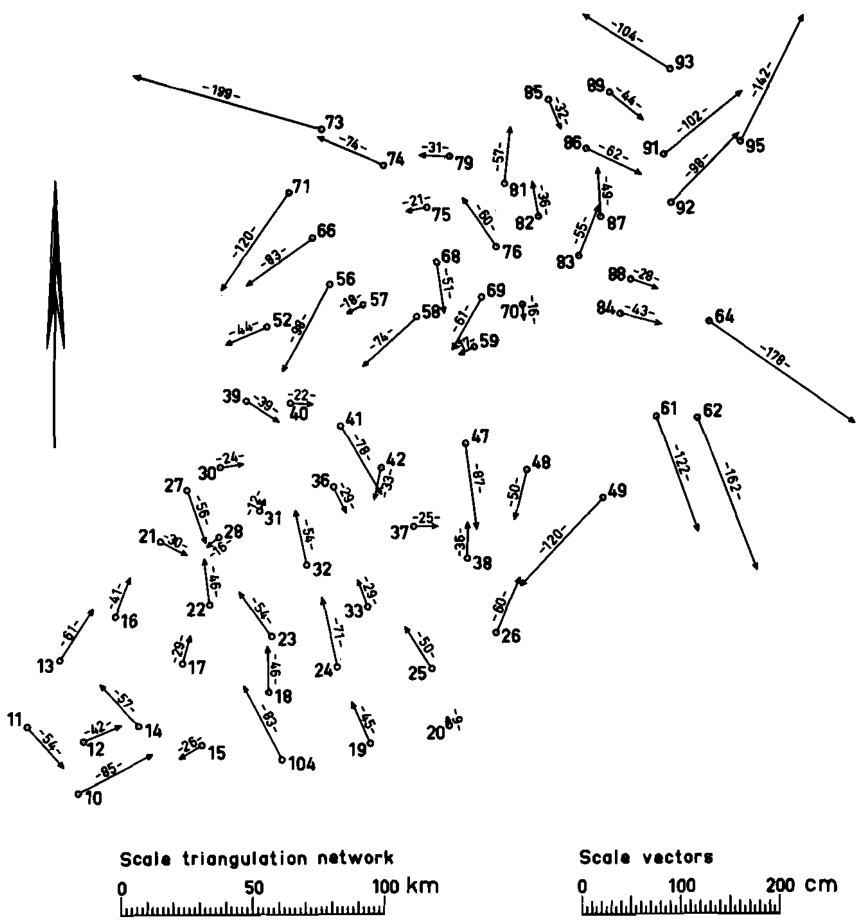


Fig. 20

The position of Coevorden with respect to Kirch Hesepe and Bentheim is also very good. The sum $88^{\circ}19'42''.2$ of the two adjusted spherical angles 264 and 306 is but $0''.4$ greater than the amount $88^{\circ}19'41''.8$ found from the R. D.-coordinates of the three points.

From the coordinates $X''Y''$ in columns 7 and 8 in table 26 finally follow the 263 distances between the projections of the angular points of Krayenhoff's network and from these distances the final lengths of the chords k on the conformal sphere. Of course the formulae (8) and (9) in section 13, necessary for this computation, must now be used with the opposite sign. All the lengths k can be found in column 12 of table 15. The differences $v = k - k'$ (k' are the chords in the Précis Historique) are given in column 13. As one sees from $v = +1.18$ m in triangle 2 the side Duinkerken-Mont Cassel is 1.18 m longer than the amount from which Krayenhoff started his computation. If in the triangulation chain between Assenede and Duinkerken no important errors are made in the carrying-forward of the "ideal" baseline then one might conclude that Krayenhoff's baseline has a relative error of about + 1.18 m : 27459 m = + 0.000043. It does not seem quite impossible if we take into account that it was derived from the measurement of a baseline near Melun, about 30 km south of Paris and about 280 km from Duinkerken.

As the amounts v relate to the ideal baseline and the amounts k' to the lengths computed with Krayenhoff's baseline, I reduced the v's to amounts v' = v-0.000043 k'. They are mentioned in column 14 of table 15. v' for Duinkerken-Mont Cassel is of course zero. Apart from small errors in Krayenhoff's computation of the side lengths the other ones are only caused by the different adjustments of the angles in the network. The very large negative amounts v (v') in the northern part of the triangulation will be analysed in section 22.

22. Comparison between the side lengths in tableau III of the Précis Historique and those found from the adjustment according to the method of the least squares .

As I already remarked before, a great number of tests at random in Krayenhoff's adjusted network give the impression that the triangulation is about a closing mathematical figure. The closure of the angles round the station Breda (station No. 23, table 9, columns 11 and 12), the closure of the sine equations round that station and the closure of the angles in the triangles round Breda is an arbitrary example.

In his system of adjustment of the triangulation, however, Krayenhoff was dependent on a chain of triangles in which a side length was computed. A length of 26279.991 m for the chord Dordrecht-Breda e.g. in his Précis Historique (see triangle 34 in table 24) does not implicate that a same amount will be found along an arbitrary other route. Moreover the network cannot be a closing mathematical figure because Krayenhoff overlooked the three conditions round the Zuiderzee.

In order to show that in reality the closure of the network is but a seeming closure, I compared the lengths of a great number (297) of sides in column 12 of table 15 with those found in the Précis Historique (column 11). They are the rays in those 51 central points of the network which are surrounded by numbered triangles. If one takes as an example the central point Amsterdam (No. 40), surrounded by the triangles 57, 56, 55, 64, 65 and 66, one sees that even all the sides in these triangles have positive amounts v. For the 6 rays to the surrounding points in this station I copied these differences in column 4 of table 27. With the lengths of the rays k in column 5 one finds for the relative differences the amounts in column 6. They have a mean of about +0.000054 or, in other words, Krayenhoff's too small distances in Amsterdam must be multiplied by about 1.000054 in order to find the side lengths in column 12 of table 15 which match as well as possible the side lengths in the R. D.-system.

Table 27

Station	Rays to	Tri-angle	Diff. v (cm) table 15	Distan-ces k(km) table 15	Relative differ-ences
1	2	3	4	5	6
Amsterdam	Naarden	57	+114	20.925	+0.000054
(No. 40)	Utrecht	56	+167	35.519	+ 47
	Nieuwkoop	55	+117	25.918	+ 45
	Haarlem	64	+ 98	16.789	+ 58
	Alkmaar	65	+192	29.977	+ 64
	Edam	66	+106	18.935	+ 56
					+0.000054
Aurich	Esens	157	-150	21.681	-0.000069
(No. 98)	Jever	158	-220	30.289	- 73
	Strakholt	159	- 99	15.469	- 64
	Leer	153	-169	26.876	- 63
	Emden	152	-132	21.459	- 62
	Hage	151	-126	19.690	- 64
					-0.000066

In the second example Aurich (station No. 98) in table 26 all the differences in column 4 are negative. Krayenhoff's side lengths are about 6.6 cm per km too long. They must be multiplied by about 0.999934 in order to find the lengths of the sides belonging to an "ideal" baseline and an adjustment of the angles according to the least squares.

The amounts 1.000054 in Amsterdam and 0.999934 at Aurich are once again mentioned in table 28 together with the 49 other scale factors which were computed in an analogous way. For a good survey of the local situation they are also mentioned on the map in Fig. 21. It appears that the factor 1.000050 in Assenede (No. 12) agrees very well yet with that of the baseline Duinkerken-

Table 28

Station No.	Station Name	Scale factor	Station No.	Station Name	Scale factor	Station No.	Station Name	Scale factor
1	2	3	1	2	3	1	2	3
12	Assenede	1.000050	40	Amsterdam	1.000054	70	Meppel	1.000019
14	Hulst	1.000052	42	Amersfoort	1.000040	72	Robbezand	1.000033
17	Bergen op Zoom	1.000051	43	Imbosch	1.000030	74	Harlingen	1.000046
18	Hoogstraten	1.000045	47	Veluwe	1.000033	75	Sneek	1.000026
22	Willemstad	1.000051	48	Zutphen	1.000028	76	Oldeholtpa	1.000013
23	Breda	1.000047	49	Groenlo	1.000023	79	Leeuwarden	1.000030
24	Hilvarenbeek	1.000043	50	Harikerberg	1.000018	81	Drachten	1.000013
25	Helmond	1.000043	54	Hoorn	1.000051	82	Oosterwolde	1.000002
28	Rotterdam	1.000047	56	Medemblik	1.000044	83	Beilen	1.000008
29	Dordrecht	1.000047	59	Kampen	1.000027	86	Groningen	0.999985
31	Gouda	1.000045	60	Lemelerberg	1.000018	87	Rolde	0.999986
32	Gorinchem	1.000046	61	Oldenzaal	1.000013	89	Uith. meden	0.999980
33	's-Hertogenbosch	1.000043	63	Uelsen	1.000008	90	Holwierde	0.999961
34	Grave	1.000039	66	Oosterland	1.000037	91	Midwolda	0.999957
35	Nieuwkoop	1.000044	67	Staveren	1.000031	94	Emden	0.999944
36	Utrecht	1.000046	68	Lemmer	1.000025	98	Aurich	0.999934
37	Rhenen	1.000038	69	Blokzijl	1.000023	99	Strakholt	0.999935

Mont-Cassel (1.000043), found at the end of section 21. The stations along the west side of the network, the red chain in Fig. 15, as far as about Hoorn (No. 54) and Medemblik (No. 56) also have about the same scale factors as Assenede. More to the east and the north, however, the picture changes. At Gorinchem (No. 32), 's-Hertogenbosch (No. 33) and Utrecht (No. 36) where the factor is about 1.000045 there is still a good agreement with that at Bergen op Zoom (No. 17), Hoogstraten (No. 18), Breda (No. 23) and Hilvarenbeek (No. 24). At Rhenen (No. 37), however, it falls to 1.000038 and at Veluwe (No. 47) to 1.000033. At Kampen (No. 59), Lemmer (No. 68), Blokzijl (No. 69) and Sneek (No. 75) it is about 1.000025 and east of Kampen at Lemelerberg (No. 60) and Uelsen (No. 63) 1.000018 and 1.000008 respectively. At Oosterwolde (No. 82) and Beilen (No. 83) the scale of Krayenhoff's network is about the same as that of the R.D. At Groningen (No. 86) and Rolde (No. 87) the factor is about 0.999985. Still more to the northeast it falls rapidly to 0.999934 at Aurich (No. 98), about the end of the triangulation.

Fig. 21

Though the several amounts cannot lay claim to a high accuracy - the relative differences for the different rays at the same station sometimes differ too much - it is unmistakable that a constant scale in Krayenhoff's network is out of the question. As, however, the scale factor gradually changes from about 1. 000050 in the south to about 0. 999935 in the north of the triangulation the not-constant scale cannot be found from local checks. Notwithstanding the great trouble to make his network a closing mathematical figure, Krayenhoff did not succeed in his attempt. He could not even succeed for the plain reason that his time was not yet ripe for such an operation.

23. Comparison of the angles and sides (chords) of the adjusted network with the results of the R. D.

A reliable judgment of the accuracy of Krayenhoff's triangulation can only be obtained - Van der Plaats remarked it already - if its angles and sides are compared with those of another triangulation of uncontested higher order. A survey of this comparison is given in table 29. In this table I mentioned the 57 triangles of which the angular points (column 2) of Krayenhoff's triangulation coincide or are assumed to coincide with those of the R. D. Column 4 gives the

Table 29

No. triangle	Station	Spherical angles No.	Spherical angles Adj. least sq. ° ' "	Spherical angles acc. R. D. ° ' "	Diff. seconds	Opposite sides (chords) From adj.	Opposite sides (chords) R. D.	Diff. cm
1	2	3	4	5	5-4=6	7	8	8-7=9
11	Gent	28	43 56 12.120	43 56 10.580	-1.540	21912.24	21912.35	+ 11
	Aardenburg	29	38 11 31.533	38 11 31.756	+0.223	19525.93	19526.21	+ 28
	Assenede	30	97 52 17.420	97 52 18.737	+1.317	31382.54	31382.91	+ 37
			180 00 01.073	180 00 01.073	0			
12	Aardenburg	31	78 29 45.794	78 29 47.726	+1.932	31740.39	31740.02	- 37
	Middelburg	32	42 34 12.101	42 34 15.547	+3.446	21912.24	21912.35	+ 11
	Assenede	33	58 55 63.613	58 55 58.235	-5.378	27745.42	27744.62	- 80
			180 00 01.508	180 00 01.508	0			
13	Middelburg	34	33 52 06.793	33 52 09.189	+2.396	21799.26	21799.94	+ 68
	Hulst	35	54 14 08.789	54 14 01.592	-7.197	31740.39	31740.02	- 37
	Assenede	36	91 53 46.169	91 53 50.970	+4.801	39095.30	39095.80	+ 50
			180 00 01.751	180 00 01.751	0			
14	Hulst	37	32 11 52.698	32 11 51.841	-0.857	19525.93	19526.21	+ 28
	Gent	38	36 30 15.508	36 30 17.105	+1.597	21799.26	21799.94	+ 68
	Assenede	39	111 17 52.798	111 17 52.058	-0.740	34141.99	34142.75	+ 76
			180 00 01.004	180 00 01.004	0			
15	Hulst	40	116 57 17.367	116 57 26.699	+9.332	50723.94	50725.06	+112
	Antwerpen	41	36 52 05.113	36 52 01.586	-3.527	34141.99	34142.75	+ 76
	Gent	42	26 10 39.454	26 10 33.649	-5.805	25104.23	25103.92	- 31
			180 00 01.934	180 00 01.934	0			
16	Hulst	43	38 05 47.414	38 05 47.709	+0.295	26715.71	26715.95	+ 24
	Middelburg	44	77 21 50.610	77 21 47.710	-2.900	42251.62	42251.78	+ 16
	Zierikzee	45	64 32 24.556	64 32 27.161	+2.605	39095.30	39095.80	+ 50
			180 00 02.580	180 00 02.580	0			
17	Hulst	46	47 29 05.902	47 29 05.043	-0.859	31144.81	31144.80	- 1
	Zierikzee	47	43 01 12.128	43 01 10.425	-1.703	28827.37	28827.22	- 15
	B. O. Zoom	48	89 29 44.243	89 29 46.805	+2.562	42251.62	42251.78	- 16
			180 00 02.273	180 00 02.273	0			
18	Hulst	49	71 01 47.829	71 01 47.115	-0.714	31475.87	31475.55	- 32
	B. O. Zoom	50	48 57 36.893	48 57 36.138	-0.755	25104.23	25103.92	- 31
	Antwerpen	51	60 00 37.010	60 00 38.479	+1.469	28827.37	28827.22	- 15
			180 00 01.732	180 00 01.732	0			
19	B. O. Zoom	52	58 08 58.511	58 08 55.067	-3.444	32183.79	32183.25	- 54
	Antwerpen	53	65 40 32.667	65 40 37.377	+4.710	34525.34	34525.48	+ 14
	Hoogstraten	54	56 10 31.159	56 10 29.893	-1.266	31475.87	31475.55	- 32
			180 00 02.337	180 00 02.337	0			
20	Antwerpen	55	48 05 14.390	48 05 17.786	+3.396	25708.04	25708.39	+ 35
	Hoogstraten	56	63 13 31.173	63 13 35.781	+4.608	30842.37	30842.68	+ 31
	Herentals	—	68 41 16.307	68 41 08.303	-8.004	32183.79	32183.25	- 54
			180 00 01.870	180 00 01.870	0			

Table 29 (continued)

1	2	3	4	5	6	7	8	9
21	Hoogstraten	57	52 06 35.055	52 06 33.149	-1.906	33996.90	33996.76	- 14
	Lommel	58	36 38 21.588	36 38 23.154	+1.566	25708.04	25708.39	+ 35
	Herentals	—	91 15 05.569	91 15 05.909	+0.340	43068.12	43068.25	+ 13
			180 00 02.212	180 00 02.212	0			
22	Zierikzee	59	51 16 31.098	51 16 34.062	+2.964	30183.25	30183.92	+ 67
	Willemstad	60	58 09 07.191	58 09 07.430	+0.239	32864.00	32864.36	+ 36
	Brielle	61	70 34 24.079	70 34 20.876	-3.203	36485.93	36486.12	+ 19
			180 00 02.368	180 00 02.368	0			
23	Willemstad	62	57 32 10.510	57 32 09.469	-1.041	31144.81	31144.80	- 1
	Zierikzee	63	41 11 25.592	41 11 24.225	-1.367	24309.66	24309.55	- 11
	B. O. Zoom	64	81 16 25.792	81 16 28.200	+2.408	36485.93	36486.12	+ 19
			180 00 01.894	180 00 01.894	0			
24	Willemstad	65	89 20 33.340	89 20 36.100	+2.760	35377.82	35378.16	+ 34
	B. O. Zoom	66	47 15 24.669	47 15 24.573	-0.096	25983.27	25983.50	+ 23
	Breda	67	43 24 03.590	43 24 00.926	-2.664	24309.66	24309.55	- 11
			180 00 01.599	180 00 01.599	0			
25	B. O. Zoom	68	34 51 49.892	34 51 49.217	-0.675	20956.78	20956.82	+ 4
	Breda	69	70 20 38.755	70 20 37.294	-1.461	34525.34	34525.48	+ 14
	Hoogstraten	70	74 47 33.120	74 47 35.256	+2.136	35377.82	35378.16	+ 34
			180 00 01.767	180 00 01.767	0			
26	Breda	71	67 56 49.579	67 56 50.334	+0.755	27696.79	27696.85	+ 6
	Hoogstraten	72	67 31 20.910	67 31 19.918	-0.992	27612.88	27612.84	- 4
	Hilv. beek	73	44 31 50.869	44 31 51.106	+0.237	20956.78	20956.82	+ 4
			180 00 01.358	180 00 01.358	0			
27	Hoogstraten	74	46 10 28.584	46 10 26.003	-2.581	31144.24	31143.99	- 25
	Hilv. beek	75	93 54 54.814	93 54 57.704	+2.890	43068.12	43068.25	+ 13
	Lommel	76	39 54 38.780	39 54 38.471	-0.309	27696.79	27696.85	+ 6
			180 00 02.178	180 00 02.178	0			
28	Hilv. beek	77	65 27 15.831	65 27 16.545	+0.714	36604.24	36604.30	+ 6
	Lommel	78	63 50 11.505	63 50 12.790	+1.285	36117.68	36117.78	+ 10
	Helmond	79	50 42 35.254	50 42 33.255	-1.999	31144.24	31143.99	- 25
			180 00 02.590	180 00 02.590	0			
29	Helmond	80	57 21 24.867	57 21 23.869	-0.998	30932.36	30932.26	- 10
	Nederweert	81	85 10 06.743	85 10 10.498	+3.755	36604.24	36604.30	+ 6
	Lommel	—	37 28 30.134	37 28 27.377	-2.757	22349.95	22349.56	- 39
			180 00 01.744	180 00 01.744	0			
30	Helmond	82	101 29 46.711	101 29 49.102	+2.391	39050.78	39050.26	- 52
	Nederweert	83	44 23 22.294	44 23 20.794	-1.500	27876.50	27875.99	- 51
	Vierl. beek	84	34 06 52.541	34 06 51.650	-0.891	22349.95	22349.56	- 39
			180 00 01.546	180 00 01.546	0			
31	Brielle	85	57 30 33.948	57 30 35.607	+1.659	21234.10	21234.63	+ 53
	Rotterdam -	86	60 06 11.042	60 06 10.253	-0.789	21824.28	21824.66	+ 38
	den Haag	87	62 23 16.049	62 23 15.179	-0.870	22307.16	22307.55	+ 39
			180 00 01.039	180 00 01.039	0			

Table 29 (continued)

1	2	3	4	5	6	7	8	9
32	Brielle	88	56 21 12.241	56 21 12.459	+0.218	25739.54	25740.10	+ 56
	Rotterdam	89	77 28 18.325	77 28 18.951	+0.626	30183.25	30183.92	+ 67
	Willemstad	90	46 10 30.853	46 10 30.009	-0.844	22307.16	22307.55	+ 39
			180 00 01.419	180 00 01.419	0			
36	Breda	100	46 10 47.413	46 10 47.138	-0.275	27986.82	27986.66	- 16
	Gorinchem	101	83 02 43.984	83 02 44.607	+0.623	38503.58	38503.43	- 15
	's Bosch	102	50 46 30.716	50 46 30.368	-0.348	30048.63	30048.45	- 18
			180 00 02.113	180 00 02.113	0			
37	Breda	103	41 25 16.013	41 25 19.458	+3.445	25504.59	25504.94	+ 35
	Hilv. beek	104	92 49 53.213	92 49 48.957	-4.256	38503.58	38503.43	- 15
	's Bosch	105	45 44 52.555	45 44 53.366	+0.811	27612.88	27612.84	- 4
			180 00 01.781	180 00 01.781	0			
38	Hilv. beek	106	63 16 05.274	63 16 05.688	+0.414	33559.72	33559.94	+ 22
	's Bosch	107	73 59 10.483	73 59 08.520	-1.963	36117.68	36117.78	+ 10
	Helmond	108	42 44 46.326	42 44 47.875	+1.549	25504.59	25504.94	+ 35
			180 00 02.083	180 00 02.083	0			
42	Rotterdam	117	36 09 43.559	36 09 40.965	-2.594	15403.55	15403.19	- 36
	den Haag	118	89 24 22.023	89 24 16.005	-6.018	26103.16	26103.01	- 15
	Leiden	119	54 25 55.246	54 25 63.858	+8.612	21234.10	21234.63	+ 53
			180 00 00.828	180 00 00.828	0			
43	Rotterdam	120	56 16 43.009	56 16 46.691	+3.682	22063.14	22063.31	+ 17
	Leiden	121	43 58 08.792	43 58 06.822	-1.970	18416.39	18416.14	- 25
	Gouda	122	79 45 09.211	79 45 07.499	-1.712	26103.16	26103.01	- 15
			180 00 01.012	180 00 01.012	0			
48	Gouda	135	65 46 30.611	65 46 34.385	+3.774	30798.46	30799.12	+ 66
	Gorinchem	136	61 01 12.474	61 01 07.966	-4.508	29543.74	29543.76	+ 2
	Utrecht	137	53 12 18.759	53 12 19.493	+0.734	27044.42	27044.85	+ 43
			180 00 01.844	180 00 01.844	0			
49	Gorinchem	138	51 15 00.834	51 15 01.177	+0.343	33842.92	33842.93	+ 1
	Utrecht	139	83 32 15.211	83 32 10.185	-5.026	43119.01	43118.84	- 17
	Rhenen	140	45 12 46.577	45 12 51.260	+4.683	30798.46	30799.12	+ 66
			180 00 02.622	180 00 02.622	0			
50	's Bosch	141	86 06 44.390	86 06 43.353	-1.037	43119.01	43118.84	- 17
	Gorinchem	142	53 31 47.364	53 31 48.782	+1.418	34754.76	34754.82	+ 6
	Rhenen	143	40 21 30.703	40 21 30.322	-0.381	27986.82	27986.66	- 16
			180 00 02.457	180 00 02.457	0			
57	Utrecht	161	34 06 09.253	34 06 07.527	-1.726	20926.22	20925.77	- 45
	Amsterdam	162	38 01 09.760	38 01 13.861	+4.101	22988.39	22988.77	+ 38
	Naarden	163	107 52 42.146	107 52 39.771	-2.375	35520.94	35520.76	- 18
			180 00 01.159	180 00 01.159	0			
58	Utrecht	164	61 23 57.391	61 23 60.416	+3.025	21927.50	21928.08	+ 58
	Naarden	165	51 36 30.000	51 36 27.927	-2.073	19575.02	19575.23	+ 21
	Amersfoort	166	66 59 33.609	66 59 32.657	-0.952	22988.39	22988.77	+ 38
			180 00 01.000	180 00 01.000	0			

Table 29 (continued)

1	2	3	4	5	6	7	8	9
59	Rhenen	167	34 59 35.974	34 59 37.781	+1.807	19575.02	19575.23	+ 21
	Utrecht	168	47 31 20.086	47 31 21.605	+1.519	25174.96	25175.08	+ 12
	Amersfoort	169	97 29 05.177	97 29 01.851	-3.326	33842.92	33842.93	+ 1
			180 00 01.237	180 00 01.237	0			
60	Rhenen	170	61 42 55.089	61 42 49.628	-5.461	33375.06	33375.03	- 3
	Amersfoort	171	76 39 39.122	76 39 46.186	+7.064	36877.73	36878.53	+ 80
	Veluwe	172	41 37 27.859	41 37 26.256	-1.603	25174.96	25175.08	+ 12
			180 00 02.070	180 00 02.070	0			
64	Amsterdam	182	69 52 43.333	69 52 45.398	+2.065	28884.84	28885.00	+ 16
	Haarlem	183	77 02 30.752	77 02 27.988	-2.764	29978.99	29978.96	- 3
	Alkmaar	184	33 04 47.111	33 04 47.810	+0.699	16790.26	16790.38	+ 12
			180 00 01.196	180 00 01.196	0			
99	Urk	283	52 21 02.170	52 21 06.858	+4.688	21348.34	21348.61	+ 27
	Lemmer	284	73 39 19.206	73 39 16.007	-3.199	25873.37	25873.13	- 24
	Blokzijl	285	53 59 39.755	53 59 38.266	-1.489	21811.99	21811.78	- 21
			180 00 01.131	180 00 01.131	0			
100	Urk	286	43 54 46.201	43 54 51.750	+5.549	18941.70	18942.18	+ 48
	Blokzijl	287	64 45 32.232	64 45 30.635	-1.597	24703.15	24703.00	- 15
	Kampen	288	71 19 42.689	71 19 38.737	-3.952	25873.37	25873.13	- 24
			180 00 01.122	180 00 01.122	0			
101	Kampen	289	41 00 57.580	41 00 55.324	-2.256	15825.55	15825.33	- 22
	Blokzijl	290	87 12 64.603	87 12 59.753	-4.850	24086.01	24085.95	- 6
	Meppel	291	51 45 58.575	51 45 65.681	+7.106	18941.70	18942.18	+ 48
			180 00 00.758	180 00 00.758	0			
115	Lemmer	331	85 46 21.117	85 46 28.490	+7.373	30487.10	30487.47	+ 37
	Sneek	332	50 33 13.413	50 33 12.686	-0.727	23606.98	23607.13	+ 15
	Oldeholtpa	333	43 40 26.728	43 40 20.082	-6.646	21110.44	21109.93	- 51
			180 00 01.258	180 00 01.258	0			
116	Blokzijl	334	70 08 36.203	70 08 42.223	+6.020	23606.98	23607.13	+ 15
	Lemmer	335	51 34 64.326	51 34 52.618	-11.708	19665.88	19664.92	- 96
	Oldeholtpa	336	58 16 20.471	58 16 26.159	+5.688	21348.34	21348.61	+ 27
			180 00 01.000	180 00 01.000	0			
117	Meppel	337	54 55 25.237	54 55 20.171	-5.066	19665.88	19664.92	- 96
	Blokzijl	338	83 53 07.207	83 53 09.124	+1.917	23893.34	23892.61	- 73
	Oldeholtpa	339	41 11 28.339	41 11 31.488	+3.149	15825.55	15825.33	- 22
			180 00 00.783	180 00 00.783	0			
118	Meppel	340	73 41 27.299	73 41 29.575	+2.276	31536.70	31536.16	- 54
	Oldeholtpa	341	59 39 41.436	59 39 41.410	-0.026	28359.09	28358.51	- 58
	Beilen	342	46 38 52.911	46 38 50.661	-2.250	23893.34	23892.61	- 73
			180 00 01.646	180 00 01.646	0			
122	Sneek	352	70 48 32.870	70 48 33.081	+0.211	25448.89	25448.56	- 33
	Harlingen	353	51 05 38.662	51 05 42.060	+3.398	20969.04	20969.03	- 1
	Leeuwarden	354	58 05 49.615	58 05 46.006	-3.609	22875.95	22875.39	- 56
			180 00 01.147	180 00 01.147	0			

Table 29 (continued)

1	2	3	4	5	6	7	8	9
123	Sneek	355	49 30 36.389	49 30 38.740	+2.351	23434.35	23434.28	- 7
	Leeuwarden	356	87 36 23.273	87 36 18.514	-4.759	30786.72	30786.29	- 43
	Drachten	357	42 53 01.581	42 53 03.989	+2.408	20969.04	20969.03	- 1
			180 00 01.243	180 00 01.243	0			
124	Sneek	358	45 36 07.212	45 36 06.509	-0.703	23747.09	23746.96	- 13
	Drachten	359	66 31 58.085	66 31 64.808	+6.723	30487.10	30487.47	+ 37
	Oldeholtpa	360	67 51 56.401	67 51 50.381	-6.020	30786.72	30786.29	- 43
			180 00 01.698	180 00 01.698	0			
125	Oldeholtpa	361	47 48 54.989	47 48 55.402	+0.413	17972.04	17971.97	- 7
	Drachten	362	53 55 23.791	53 55 23.196	-0.595	19603.03	19602.89	- 14
	Oosterwolde	363	78 15 42.093	78 15 42.275	+0.182	23747.09	23746.96	- 13
			180 00 00.873	180 00 00.873	0			
126	Oldeholtpa	364	41 31 11.636	41 31 15.077	+3.441	21286.03	21285.95	- 8
	Oosterwolde	365	100 51 25.021	100 51 19.201	-5.820	31536.70	31536.16	- 54
	Beilen	366	37 37 24.380	37 37 26.759	+2.379	19603.03	19602.89	- 14
			180 00 01.037	180 00 01.037	0			
132	Oosterwolde	379	81 54 19.579	81 54 16.653	-2.926	33691.06	33690.92	- 14
	Drachten	380	66 12 58.382	66 12 61.521	+3.139	31140.04	31140.18	+ 14
	Groningen	381	31 52 43.442	31 52 43.229	-0.213	17972.04	17971.97	- 7
			180 00 01.403	180 00 01.403	0			
133	Oosterwolde	382	55 01 02.578	55 01 09.489	+6.911	26169.99	26170.86	+ 87
	Groningen	383	47 50 25.910	47 50 23.513	-2.397	23677.12	23677.09	- 3
	Rolde	384	77 08 33.041	77 08 28.527	-4.514	31140.04	31140.18	+ 14
			180 00 01.529	180 00 01.529	0			
134	Oosterwolde	385	43 57 30.729	43 57 32.382	+1.653	16973.82	16973.96	+ 14
	Rolde	386	60 30 53.362	60 30 52.121	-1.241	21286.03	21285.95	- 8
	Beilen	387	75 31 36.795	75 31 36.383	-0.412	23677.12	23677.09	- 3
			180 00 00.886	180 00 00.886	0			
135	Beilen	388	84 19 61.545	84 19 53.967	-7.578	25942.16	25941.50	- 66
	Rolde	389	55 02 31.209	55 02 33.530	+2.321	21365.86	21365.56	- 30
	Sleen	390	40 37 28.160	40 37 33.417	+5.257	16973.82	16973.96	+ 14
			180 00 00.914	180 00 00.914	0			
136	Coevorden	391	52 40 49.942	52 40 53.567	+3.625	21365.86	21365.56	- 30
	Beilen	392	30 14 26.309	30 14 29.959	+3.650	13530.72	13530.76	+ 4
	Sleen	393	97 04 44.475	97 04 37.200	-7.275	26661.48	26660.86	- 62
			180 00 00.726	180 00 00.726	0			
138	Groningen	396	60 36 31.582	60 36 30.015	-1.567	23414.63	23414.40	- 23
	Hornhuizen	397	59 02 06.602	59 02 09.906	+3.304	23043.62	23043.72	+ 10
	Uith. meden	398	60 21 23.003	60 21 21.266	-1.737	23356.32	23356.07	- 25
			180 00 01.187	180 00 01.187	0			
141	Groningen	405	27 57 27.637	27 57 26.070	-1.567	18352.64	18352.72	+ 8
	Midwolda	406	102 48 14.204	102 48 21.627	+7.423	38173.23	38173.64	+ 41
	Onstwedde	407	49 14 19.502	49 14 13.646	-5.856	29651.06	29650.89	- 17
			180 00 01.343	180 00 01.343	0			

Table 29 (continued)

1	2	3	4	5	6	7	8	9
142	Rolde	408	91 24 10.445	91 24 14.126	+3.681	38173.23	38173.64	+ 41
	Groningen	409	45 19 62.712	45 19 54.713	-7.999	27157.56	27156.82	- 74
	Onstwedde	410	43 15 48.642	43 15 52.960	+4.318	26169.99	26170.86	+ 87
			180 00 01.799	180 00 01.799	0			
143	Sleen	411	53 43 57.187	53 43 57.043	-0.144	27157.56	27156.82	- 74
	Rolde	412	75 53 51.943	75 53 51.695	-0.248	32668.10	32667.21	- 89
	Onstwedde	413	50 22 12.600	50 22 12.992	+0.392	25942.16	25941.50	- 66
			180 00 01.730	180 00 01.730	0			
149	Onstwedde	427	57 55 03.001	57 55 06.780	+3.779	29629.72	29629.76	+ 4
	Midwolda	428	90 25 39.107	90 25 33.502	-5.605	34969.30	34968.94	- 36
	Leer	429	31 39 19.269	31 39 21.095	+1.826	18352.64	18352.72	+ 8
			180 00 01.377	180 00 01.377				
Number Triangles			Sides					
52 - 62			Rhenen - Nijmegen			23834.12	23834.52	+ 40
54			Leiden - Haarlem			26940.92	26941.11	+ 19
69 - 87			Veluwe - Zutphen			25383.84	25384.20	+ 36
71 - 74			Zutphen - Groenlo			30915.68	30916.22	+ 54
77 - 91			Oldenzaal - Bentheim			15624.05	15623.84	- 21
82 - 96			Medemblik - Enkhuizen			14932.73	14932.86	+ 13
85 - 86			Veluwe - Kampen			36314.38	36313.62	- 76
92			Bentheim - Kirch Hesepe			36302.83	36302.22	- 61
94 - 95			Oosterland - Medemblik			18686.09	18685.66	- 43
97			Enkhuizen - Urk			20853.39	20853.68	+ 29
106			Coevorden - Kirch Hesepe			33847.40	33846.28	-112
107 - 108			Oosterland - Oosterend			19342.71	19343.14	+ 43
110			Oosterend - Vlieland			26616.94	26616.16	- 78
111 - 119			Vlieland - Harlingen			27234.72	27233.52	-120
144 - 145			Uithuizermeden - Pilsum			24894.54	24895.39	+ 85

adjusted spherical angles according to the method of the least squares, column 5 those computed from the R. D. -coordinates in columns 5 and 6 of table 26. In an analogous way column 7 gives the opposite sides (chords) according to the adjustment (see also column 12 in table 15). The R. D. -sides are in column 8. Columns 6 and 9 finally give the differences in the angles and side lengths respectively. The side lengths in the two systems of a number of identical sides in not-identical triangles are mentioned at the end of the table.

In his publication [67] Heuvelink makes a similar comparison between the R. D.-results and the amounts published by Krayenhoff in tableau III of his Précis Historique. Because of Krayenhoff's arbitrary adjustment of his network, however, this comparison is not a standard for the determination of the accuracy of the triangulation. Moreover the triangles 33 on page 9 of the publication [67], 34, 35 (page 10), 44, 45, 46, 54, 55 (page 11), 47, 56 (page 12), 86, 87, 88, 89, 74, 102, 103, 104 (page 13), 65 and 66 (page 16) don't belong in that table for the reason already mentioned in section 21.

A graphical survey of the 171 v's in column 6 of table 29 is given in the histogram of Fig. 22. It gives a picture of the external accuracy of the triangulation and it agrees fairly well with a normal distribution with a standard deviation m = $\pm$ 3".57 as is sketched in the figure. As could be expected the external accuracy is worse than the inner accuracy: 29 v's are even greater than 5". The greatest v's occur

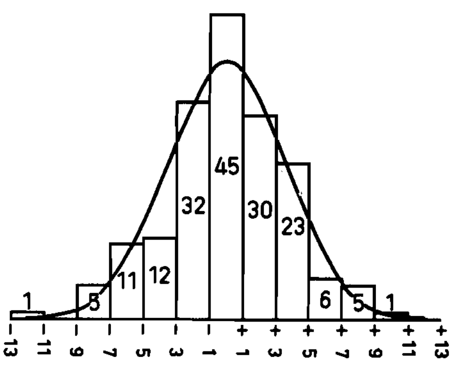


Fig. 22

in triangle 116 (about + 6".0, - 11".7 and + 5".7, respectively) though the lengths of the sides of the triangle are about 20 km. It might be possible - I already remarked it before - that similar changes as described for Edam, Dordrecht and Nieuwkoop influenced the results of the computations. If, e. g., I had not known the non-identity of the R. D. point Edam and Krayenhoff's station No. 53, the v's for the angles 185, 186, and 187 in triangle 65 would have been -3".933, +13".993 and -10".060 respectively and those for the angles 188, 189 and 190 of triangle 66, +15".680, -5".685 and -9".994 respectively. They would have considerably spoilt the external accuracy of the triangulation.

In my opinion the amounts v in column 9 are small and very often even considerably smaller than Baeyer's demand for distances between far distant points. The worst relative error in the side length of a triangle is 1 to 20,000 for the side Blokzijl-Oldeholtpa in the triangles 116 and 117.

It will be clear that the accuracy of the lengths between far distant points in the network, necessary for the determination of the shape of the earth, will be still much better. As an example I computed the R. D.-distances from the astronomical stations Amsterdam (No. 40) to Gent (No. 10) in the south and Leer (No. 95) in the north. The first chord is 167356.02 m, the second one 197436.23 m. As the corrections from chord to arc are +4.79 m and +7.87 m respectively, the lengths on the sphere are 167360.81 and 197444.10 m respectively. From the coordinates in columns 7 and 8 of table 26 one finds in the same way 167360.20 m and 197445.11 m. The differences are +0.61 m and -1.01 m respectively, the relative differences +0.0000036 and -0.0000051. They are about a factor 10 better than Baeyer demanded.

In the utmost northern part of the network where reliable data for a correct comparison are missing, I made a superficial comparison between Krayenhoff's adjusted results and the adjusted angles and side lengths of Gauss' triangulation in Oldenburg. From preliminary computations it appeared that the angular points Jever, Westerstede, Aurich and Leer of the triangles IV and V of the Oldenburg-triangulation (see Fig. 17 in section 18) are identical with Krayenhoff's stations Nos. 102, 100, 98 and 95. The triangles are once again represented in Fig. 23.

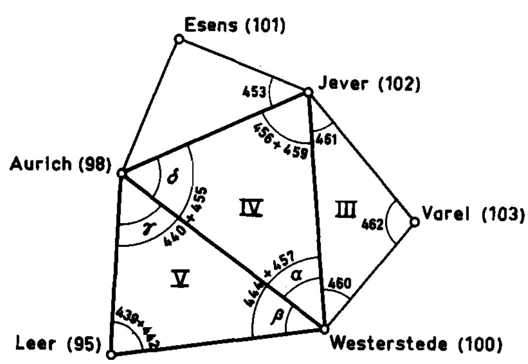


Fig. 23

The adjusted angles Leer in V and Jever in IV are therefore comparable with the sum of Krayenhoff's adjusted angles 439 and 442 (at Leer) and 456 and 459 (at Jever), respectively. Though Krayenhoff has not measured the angles α and β at Westerstede and γ and δ at Aurich, I could compute them from the coordinates X"Y" in table 26. As these coordinates are rounded-off at cm, α, β, γ, and δ are rounded-off at tenths of a second in column 4 of table 30. $\alpha + \beta$ is of course alike to 444+457, $\gamma + \delta$ to 440+455. In column 5 are the results of the angles according to the adjusted Oldenburg-triangulation. I borrowed them from [65] page 25. The agreement is excellent as may be seen from the differences in column 6. For the triangles IV and V the largest difference is +4".5 in γ.

Table 30

No. tri-angle	Station	Spherical angles		Sph. angles Oldenb. tr.	Diff. v. seconds	Opposite sides (chords)		Diff. v. m.
		No.	Kr. least sq.			Kr. least sq.	Oldenb.	
1	2	3	4	5	5-4=6	7	8	8-7=9
IV	Jever	456+459	70°44'44".452	70°44'46".559	+2.107	38029.68	38030.78	+1.10
	Westerstede	α	48°45'22".5	48°45'22".089	-0.4	30289.16	30289.88	+0.72
	Aurich	δ	60°29'55".6	60°29'53".893	-1.7	35060.04	35060.78	+0.74
			180°00'02".541	180°00'02".541	0			
V	Aurich	γ	56°07'23".3	56°07'27".758	+4.5	32080.02	32081.49	+1.47
	Westerstede	β	44°04'16".7	44°04'15".059	-1.6	26875.85	26876.47	+0.62
	Leer	439+442	79°48'22".146	79°48'19".331	-2.815	38029.68	38030.78	+1.10
			180°00'02".147	180°00'02".148	+0.001			
III	Westerstede	460	44°23'56".229	44°23'50".443	-5.786	24884.00	24883.67	-0.33
	Jever	461	35°55'17".839	35°55'28".795	+10.956	20865.97	20867.85	+1.88
	Varel	462	99°40'47".228	99°40'42".059	-5.169	35060.04	35060.78	+0.74
			180°00'01".296	180°00'01".297	+0.001			

The deviations in triangle III are larger. They even amount to +10".956 for the angle at Jever between Varel and Westerstede. It might be possible that it must be ascribed to Krayenhoff's arbitrary choice of series 12 (35°55'18".309) at Jever for his further computations which I also used for the rigorous adjustment of the triangulation. If, for the computation of angle 461, he would have used the mean 35°55'22".434 of the four series mentioned in table 19, the amount +10".956 in column 6 of table 30 would probably have been less. It is also possible, however, that it is caused by an alteration in the spire of the Lutheran church at

Varel between Krayenhoff's measurements in 1811 and those for the Oldenburg-triangulation in 1831. For in the years 1827, 1828 (and 1833) extensive repairs of the tower and its skylight might have altered the position of the spire [71] . This possible alteration,however, is independent of Gauss' own measurement (in 1825) of the angle at Jever between Varel and Esens. According to his letter to Schumacher [72] it amounts to $152^{\circ}22'36''.585$: "Der Winkel zwischen Varel und Esens ist nach meiner Messung auf das Zentrum reduziert $152^{\circ}22'36''.585$ mit Vorbehalt einer kleinen Reduktion wegen des Umstandes dass mein Heliotrop in Varel nicht genau im Zentrum des Turms stand welche Reduktion ich noch nicht berechnet habe die aber nur einen Bruch einer Sekunde betragen kann während Krayenhoff $35^{\circ}55'18''.309 + 40^{\circ}26'50''.160 + 30^{\circ}17'53''.908 + 45^{\circ}42'19''.298 = 152^{\circ}22'21''.675$ (findet) oder nach seiner eignen Ausgleichung $152^{\circ}22'21''.243$ ".

The amounts mentioned in the above quotation are of course Krayenhoff's angles 461 + (456+459) + 453 (see Fig. 23). In the adjustment of the angles according to the least squares this sum is $152^{\circ}22'22''.466$, a large difference indeed with Gauss' measurement ($14''.119$ smaller). I cannot find a plausible explanation for it. A substitution of the adjusted amount $152^{\circ}22'22''.466$ by the sum of the angles 461, 456, 459 and 453 in column 7 of table 20 ($152^{\circ}22'22''.674$) gives no solution: both the amounts differ too little from each other. The result $70^{\circ}45'27''.036$ of Gauss' measurement of the angle Westerstede-Jever-Aurich is about $42''$ greater than the mean of the amounts $70^{\circ}44'44''.452$ and $70^{\circ}44'46''.559$ of the adjusted Krayenhoff- and Oldenburg triangulations respectively (see table 30) and almost $45''$ greater than the sum of the angles 456 and 459 in column 7 of table 19. It is certain, however, that the explanation cannot be found from computations from the table on the pages 206 and 207 of Gaede's paper [9] . The differences found from these computations are much greater.

The lengths of the sides (chords) of the triangles IV, V and III of the Oldenburg-triangulation are mentioned in column 8 of table 30. I borrowed them from the arcs [73] on page 25 of [65] and the reductions c from arc to chord:

$$c_{cm} = - l_{km}^{3} : 9777.2 \text{ (see formula 10 in section 13).}$$

The length of the chord Jever-Varel e.g. is: $24883.69 - (24.884^3 : 9777.2)\text{cm} = 24883.67$.

The chords in Krayenhoff's adjusted system are in column 7 of the table. I borrowed them from column 12 of table 15 (section 15). If one leaves the dubious triangle III out of consideration, the v's in column 9 are all positive which might be ascribed to a difference in scale between the two triangulations of about 0. 000029 (3 cm per km).

24. Final consideration of the geodetic part of the triangulation

The excellent side lengths in the adjusted triangulation network - it must be said once again - are obtained by the introduction of a fictitious "ideal" baseline. As there was no baseline available - it seems logical that the one near Melun could not be used as it lies about 280 km south of Duinkerken and the distances Duinkerken-Amsterdam and Amsterdam-Jever are about 230 km and 250 km respectively - this "baseline" was found from a similarity transformation by which the figure of the network with its 106 angular points and 505 adjusted angles was adapted as well as possible to 65 identical points of the R. D. -triangulation network.

Another baseline would have given worse results. If one can agree with this ideal baseline - there is hardly any alternative and the measurement of a baseline was not excluded by Baeyer - and if one wishes to accentuate Baeyer's requirements for a relative accuracy of 0. 00005 for long distances in the network, then Cohen Stuart wrongly rejected Krayenhoff's triangulation as Van der Plaats anticipated already.

Concerning Baeyer's requirement that the closing error in the angles of a triangle should only exceptionally surpass $3''$, it must be said that this requirement relates to triangles in a triangulation chain [74] . According to Cohen Stuart - and I agree with him - this requirement is not necessary for triangles in a triangulation network, the angles of which can be verified in other ways.

According to the Précis Historique the number of triangles with closing errors greater than $3''$ is only 5 (the numbers 31, 55, 107, 121 and 148) though I also agree with Cohen Stuart that this number was kept low by a choice from the measured series. Especially in the northern part of the network (see the considerations for the station Jever in tables 18 and 19) this arbitrariness went much too far. Already before, however, I explained that I don't agree with Cohen Stuart's thesis that in principle all the measured series had to be used for the computation. The result would have been unnecessarily worse. If one admits that there is a talk of a choice of the series, one must also admit that this choice, based on the reliability of the observations in the series, was a good one. The corrections p to all the 505 angles are very low, much lower than appears from Krayenhoff's primitive adjustment ($m = \pm 1''.775$, see Fig. 18). The two largest are $p_{370} = -4''.463$ and $p_{375} = -4''.206$ in the triangles 128 and 130. With an intentionally influenced choice of the series they could not have been so good. For the consequences of such a choice on the corrections to the other observations could not possibly be predicted the more - I repeat it once again - because Krayenhoff saw "only" 273 of the 276 conditions the angles of the network had to comply with.

It seems to me that it is justified that Krayenhoff's triangulation was highly praised at the time of its completion (1811). If it had (could have) been adjusted according to the least squares and provided with the baseline already mentioned before, it would even have satisfied the requirements of more than 50 years later (1864).

Cohen Stuart, however, rejected the measurement. After all we must be grateful for this rejection because in an indirect way this rejection was the motive for the measurement of the R. D.-triangulation network. Measured with the utmost care according to scientific methods and adjusted according to the least squares it satisfies high requirements of accuracy still in our days.

It will be clear that the judgment on the triangulation given above, relates to the geodetic part. On the astronomical part Cohen Stuart only writes a few sentences on page 32 of his booklet. In the next chapter this part of the triangulation will also be submitted to an extensive investigation.

II - ASTRONOMICAL PART OF THE TRIANGULATION

25. Introduction

In order to give Krayenhoff's adjusted network its correct place on the ellipsoidical earth it is necessary that at least of one angular point the geographical latitude φ and the longitude λ are known and the azimuth of at least one side. As, however, Delambre had already determined the coordinates of the common point Duinkerken of both triangulations ($\varphi = 51^\circ 02' 08''.73$, $\lambda = + 0^\circ 02' 23''.000$ with respect to Cassini's meridian of Paris) [75] and the astronomical azimuth $205^\circ 12' 29''.65$ of the side Watten-Duinkerken (counted from the south in a clockwise direction [76]) all the data for the computation of the coordinates of all the angular points of the network and the azimuths of all the sides were already available. Krayenhoff must be praised that, notwithstanding these data, he thought it necessary to check his computations by measuring the latitudes φ of his stations Amsterdam (No. 40) and Jever (No. 102) and the azimuths A of the sides Amsterdam-Utrecht and Jever-Varel. The determinations of latitudes on the Naval Observatory in Den Haag (The Hague) and on the cathedral at Utrecht, already executed between 1801 and 1803 are left out of consideration: they are not used for the computation of the network. The determinations of latitudes in Amsterdam and at Jever will be discussed in the sections 26 and 27 respectively. They were executed with the repetition circle (diameter about 38 cm) already mentioned in section 5 and probably pictured next to Krayenhoff's portrait in Fig. 1. The determinations of azimuths will be discussed in the sections 28 and 29.

It is obvious that the coordinates φ_i and λ_i of the angular points (i = 1, 2, ..., 105, 106) and the azimuths A of the sides of the network are dependent on the reference ellipsoid on which they are computed. For the computation of his own triangulation Delambre used the ellipsoid of which the radius of the equator was 6375737 metres and the flattening 1 : 334. As the results of this triangulation were not yet known when Krayenhoff computed his own network, he had to use the same data. From these data and those mentioned above follows, according to Précis Historique page 34, an azimuth of $25^\circ 19' 42''.433$ for the side Duinkerken-Watten and, from the adjusted angle $42^\circ 06' 09''.730$ of triangle 1 at Duinkerken, an azimuth of $343^\circ 13' 32''.703$ for the side Duinkerken-Mont Cassel. From this azimuth and the coordinates of Duinkerken the computation of the angular points 2, 3, ..., 105, 106 of the network and the azimuths of the sides was started. The computation with Delambre's formulae in his Méthodes analytiques can be found in the volume folio mentioned under f at the end of section 4. The results φ and λ (λ with respect to the meridian of Paris) are mentioned in the alphabetic order of the stations on the pages 149-154 of the Précis Historique. The azimuths are shown on the pages 155-174. The coordinates of Amsterdam e.g. are $\varphi = 52^{\circ}22'30''.188$, $\lambda = +2^{\circ}32'54''.360$ and the azimuth Amsterdam-Utrecht $A = 332^{\circ}41'20''.350$, those of Jever $\varphi = 53^{\circ}34'23''.433$, $\lambda = +5^{\circ}34'10''.416$ and the azimuth Jever-Varel $A = 321^{\circ}20'33''.733$.

Later on, after the appearance of the first edition of the Précis Historique, Krayenhoff computed the φ's, λ's and A's a second time, now according to the results of Delambre's triangulation on the ellipsoid with a radius of the equator of 6356356.1 m and a flattening of 0.003229489 or about 1 : 309.65 [77] . For this computation he started from the coordinates of Amsterdam $\varphi = 52^{\circ}22'30''.13$, found from his own determination (see section 26), $\lambda = 0$ (the meridian of Amsterdam) and from the azimuth Amsterdam-Utrecht $A = 332^{\circ}41'19''.940$ also found from his own observations (see section 28). The results of the computations of the coordinates φ and λ may be found on the pages 177-181, of the azimuths on the pages 182-202 of the second edition of the Précis Historique. φ and λ of Duinkerken, e.g., are $51^{\circ}02'09''.65$ and $-2^{\circ}30'28''.35$ respectively, φ and λ of Jever $53^{\circ}34'22''.71$ and $+3^{\circ}01'12''.22$. The azimuths Duinkerken-Mont Cassel and Jever-Varel are $343^{\circ}13'33''.569$ and $321^{\circ}20'30''.411$, respectively.

I don't know where Krayenhoff computed these φ's, λ's and A's. The computation cannot be found in the volume folio f already mentioned before.

26. Determination of the latitude in Amsterdam (station No. 40)

For the determination of latitudes in Amsterdam and at Jever Krayenhoff used a repetition circle with a diameter of 14 (Paris) inches (about 38 cm). It was made at his own expense by Lenoir in Paris. "It was an excellent instrument that could be handled in an easy way without the risk of getting disadjusted and above all I was very pleased with the accuracy of both the levels" [78] . In section 6 I already explained how, with a vertical limb, zenith distances could be measured with it. I don't know whether the instrument still exists and, if so, where it is. Baron Krayenhoff at Amersfoort could give me no information concerning this question.

For the determination of latitudes Krayenhoff applied the measurement of circum meridian zenith distances of the pole star (α Ursae minoris). The chronometer used for the time registration was made by the clockmaker Knebel in Amsterdam. "La marche m'en était parfaitement connue, au moyen des observations correspondantes du soleil, répétées autant de fois que les circonstances le permirent" [79] . As I shall prove in section 27 it was adjusted to keep mean solar time.

If one knows the approximate longitude of the stations where the latitude must be determined (Krayenhoff used for Amsterdam $\lambda = +2^\circ 32' 53'' = +10^m 12^s$ east of Paris), then it is possible to compute in advance from the known right ascension α of the star at which moment (Krayenhoff expressed it in mean solar time) on a fixed day Polaris will pass the meridian of the Western tower in Amsterdam, either in upper or in lower transit. In the example of table 31 this moment is $12^h 18^m 43^s$ ($24^h 18^m 43^s$) mean solar time (see column 7). It refers to the observation of Polaris on October 3rd, 1810 [80]. At that moment the star passed the meridian in upper transit. As the timekeeper was $15^m 54^s$ slow at that moment, the chronometer time of transit was $12^h 02^m 49^s$ (column 7).

Table 31

No. obs.	Reading time keeper	Reading limb	Hour angle t	Corrections –	Corrections +	Remarks
1	2	3	4	5	6	7
						Approx. geogr. coordinates:
1	$11^h 30^m 51^s$	$0^\circ 00' 00''$	$-31^m 58^s$	$61''.975$	$0''.013$	$\lambda = -2^\circ 32' 53''$
2	32 26	71 51 05	-30 23	55.996	0.011	$\varphi = +52^\circ 22' 30''$
3	34 24		-28 25	48.989	0.008	
4	35 39		-27 10	44.780	0.007	Decl. $\delta = +88^\circ 17' 53''.485$
5	38 53		-23 56	34.764	0.004	Barometer 763 mm
						Thermometer + $11^\circ.5$
18	12 01 26		-01 23	00.116	0.000	Time transit $12^h 18^m 43^s$
19	03 17		+00 28	00.013	0.000	Corr. time keeper – $15^m 54^s$
20	04 34	718 19 40	+01 45	00.186	0.000	Trans. on time keeper $12^h 02^m 49^s$
21	06 36		+03 47	00.869	0.000	
22	07 55		+05 06	01.579	0.000	40 observations $1436^\circ 43' 30''$
						Sum reductions – $935''.37$
36	12 31 36		+28 47	50.262	0.009	Arc $35^\circ 55' 05''.250$
37	33 56		+31 07	58.463	0.011	Reduction – $23''.384$
38	35 58		+33 09	66.639	0.015	Reduced arc $35^\circ 54' 41''.866$
39	37 00		+34 11	70.851	0.017	Refraction + $41''.089$
40	38 18	$1436^\circ 43' 30''$	+35 29	76.331	0.020	co declination $1^\circ 42' 06''.515$
		$1436^\circ 43' 30''$		$935''.510$	$0''.141$	co latitude $37^\circ 37' 29''.470$
						Latitude $52^\circ 22' 30''.530$
						Very clear sky; very good observation.

About half an hour before transit Krayenhoff began his observations in a series (in this case series 8) and he continued them till about half an hour after transit according to the method of the measurement of vertical angles already described before. In that time about 40 observations on the star were made. For every pointing when the star passed through the intersection point of the cross wires the timekeeper was read by the calling out method and the chronometer reading noted down in column 2. The first reading with, as usual in Krayenhoff's measurements, zero on the limb, is $11^h30^m51^s$ in table 31. As the time of transit on the timekeeper is $12^h02^m49^s$, the hour angle t in column 4 is -31^m58^s. It is expressed in mean solar time. In the table I only give the time observations 1-5, 18-22 and 36-40 and the accessory hour angles $\underline{t}$ for the 40 pointings at the star. After the reading $12^h02^m49^s$ on the timekeeper, $\underline{t}$ alters of course from negative into positive. The star is then at the west side of the meridian.

As the measured zenith distance of the star is not constant, it has no sense to give the reading on the limb for all the 40 measurements. In order to have some check, however, on the regularity of a series, Krayenhoff used to note the sum of the first 2, 10, 20, 30, 40 measured angles. The latter amount in column 3 is $1436^\circ 43' 30''$.

In order to find the zenith distance of the star when it passes the meridian, all circum meridian zenith distances must be reduced to meridian zenith distance. How this was done is not described in the Précis Historique and here too one must consult Delambre's Méthodes analytiques in order to find how Krayenhoff computed these reductions. One can find the derivation of the formula used on the pages 47-52 of the book. Underneath I give an other derivation. A similar one can be found e.g. in R. Roelofs: "Astronomy applied to landsurveying" (Amsterdam 1950, page 143).

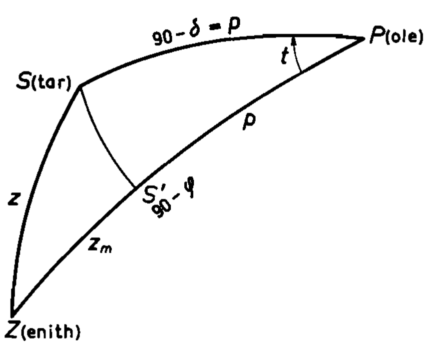


Fig. 24

In Fig. 24, P is the celestial North pole, Z the zenith and S the star that passed the meridian in S' between P and Z in upper transit. On the moment of its observation its hour angle is t. If one calls δ the declination of the star, z its zenith distance, z_m its meridian zenith distance and φ the latitude of the station then:

$$PS = 90^{\circ} - \delta = p = PS', \quad SZ = z, \quad S'Z = z_m \text{ and } PZ = p + z_m = 90^{\circ} - \varphi .$$

In triangle PSZ holds:

$$\cos z = \sin\delta \sin\varphi + \cos\delta \cos\varphi \cos t .$$

If Δz is the correction given to z in order to find z_m then:

$$z + \Delta z = z_m \quad \text{or} \quad z = z_m - \Delta z, \text{ so that:}$$

$$\cos z = \cos (z_m - \Delta z) = \cos z_m + \Delta z \sin z_m - \frac{(\Delta z)^2}{2} \cos z_m$$

and, as $z_m = (90^{\circ} - \varphi) - (90^{\circ} - \delta) = \delta - \varphi$:

$$\cos z = \cos (\delta - \varphi) + \Delta z \sin (\delta - \varphi) - \frac{(\Delta z)^2}{2} \cos(\delta - \varphi) = \cos\delta \cos\varphi + \sin\delta \sin\varphi +$$

$$+ \Delta z \sin (\delta - \varphi) - \frac{(\Delta z)^2}{2} \cos (\delta - \varphi) = \sin\delta \sin\varphi + \cos\delta \cos\varphi \cos t$$

whence:

$$\Delta z \sin (\delta - \varphi) - \frac{(\Delta z)^2}{2} \cos (\delta - \varphi) = -\cos\delta \cos\varphi (1 - \cos t) =$$

$$= - 2 \cos\delta \cos\varphi \sin^2 \tfrac{1}{2} t.$$

If in the first instance the small amount with $(\Delta z)^2$ is neglected, then a good provisional amount for Δz is:

$$\Delta z = \frac{- 2 \cos\delta \cos\varphi}{\sin(\delta - \varphi)} \sin^2 \tfrac{1}{2} t \quad \text{whence:}$$

$$\frac{(\Delta z)^2}{2} \cos (\delta - \varphi) = \frac{2 \cos^2\delta \cos^2\varphi}{\sin(\delta - \varphi)} \cot (\delta - \varphi) \sin^4 \tfrac{1}{2} t$$

A better approximation for Δz is therefore:

$$\Delta z_{rad} = \frac{-2 \cos\delta \cos\varphi}{\sin(\delta - \varphi)} \sin^2 \tfrac{1}{2} t + 2 \left(\frac{\cos\delta \cos\varphi}{\sin(\delta - \varphi)}\right)^2 \cot (\delta - \varphi) \sin^4 \tfrac{1}{2} t, \text{ or:}$$

$$\Delta z'' = - 2 \rho'' \frac{\cos\delta \cos\varphi}{\sin (\delta - \varphi)} \sin^2 \tfrac{1}{2} t + 2 \rho'' \left(\frac{\cos\delta \cos\varphi}{\sin (\delta - \varphi)}\right)^2 \cot (\delta - \varphi) \sin^4 \tfrac{1}{2} t \quad \ldots\ldots\ldots\ldots (15)$$

from which Δz can be computed if δ and t are known and an approximate value of φ .

It will be clear - Delambre mentions it in the example on page 156 of his Méthodes analytiques - that, in order to find the correct hour angles t, the chronometer must be adjusted to keep sidereal time. Krayenhoff's timekeeper, however, kept, as already said, mean solar time. If he made no mistake in the computation of the moment of transit ($12^h18^m43^s$ mean solar time) - I could not verify that - the hour angles in column 4 had to be multiplied by 366.2422 : 365.2422 = 1.002738. The great hour angle in the 40th observation is then $+35^m35^s = +8^{\circ}53'45''$ instead of the amount $+35^m29^s = +8^{\circ}52'15''$ used by Krayenhoff. The small difference is hardly of any practical influence on the final result, though it exceeds the influence of the second correction term in a considerable manner (some tenths of a second of arc). A consideration on the influence of an error dt in an hour angle t will be given in section 27 (page 166).

Krayenhoff's computation of his final result can be found in column 7 of table 31. The sum of the measured angles $1436^{\circ}43'30''$ augmented with the sum of the reductions $-935''.510$ and $+0''.141$ in columns 5 and 6 is the 40-multiple of the "measured" zenith distance $35^{\circ}54'41''.866$ of the star. If this distance is augmented with the refraction $+41''.089$ and the amount $p = 90^{\circ} - \delta = 1^{\circ}42'06''.515$ one finds $90^{\circ} - \varphi = 37^{\circ}37'29''.470$ and $\varphi = 52^{\circ}22'30''.530$. The readings of the barometer 28 inches 2.4 lines = 338.4 lines = 338.4 x 2.256 mm = 763 mm and the thermometer $+11^{\circ}.5$ were of course necessary for the computation of the refraction.

At lower transit (see Fig. 25) the star S passes the meridian in S'. Its hour angle, at that moment 180°, is $180^{\circ} + t$ when it is in S. In an analogous way as already described one can derive the reductions Δz which must be given to the measured zenith distances z = ZS of the star in order to find the meridian zenith distances z_m = S'Z. The formula runs as follows:

$$\Delta z'' = + 2\rho''\frac{\cos\delta\cos\varphi}{\sin(\delta+\varphi)}\sin^2\tfrac{1}{2}t - 2\rho''\left(\frac{\cos\delta\cos\varphi}{\sin(\delta+\varphi)}\right)^2\cot(\delta+\varphi)\sin^4\tfrac{1}{2}t \qquad (16)$$

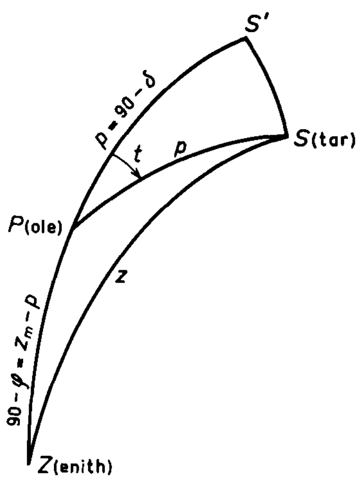


Fig. 25

In Amsterdam Krayenhoff measured 24 latitudes: 11 between September 20th and December 16th, 1810 (upper transit of Polaris) and 13 between April 23rd and May 25th, 1811 (lower transit). For his computation of the latitude φ of the station he used all observations; not a single series was rejected. A survey of the series is given in table 32 with the dates of the measurements (column 2) and the number of repetitions (column 3). Though both means are about alike, the measurements in 1810 are much better than those of 1811. Those in 1810 give a standard deviation in the determination of a latitude $m = \pm 1''.089$, those in 1811 $m = \pm 3''.302$. The standard deviation in the mean $52^\circ 22' 30''.187$ of the 11 series in 1810 is $M = \pm 0''.328$, that in the mean $52^\circ 22' 30''.315$ of the 13 series in 1811 is $M = \pm 0''.957$. I cannot explain this large difference. Unfavourable weather conditions during the measurements cannot have been the cause: only on May 11th (series 5) Krayenhoff calls them "good". On all the other observation days they were "very good". In 1810, however, they were "difficult" for the series 1, 2, 4 and 10 and even "very difficult" for the series 5 and 6. For the final result of the latitude φ of his eccentric station on the tower Krayenhoff used $\varphi = 52^\circ 22' 30''.251$,

Table 32

Series	Date 1810	No. rep.	φ	v''	Series	Date 1811	No. rep.	φ	v''
1	2	3	4	5	1	2	3	4	5
1	Sept. 20	14	$52^\circ 22' 28''.968$	+1.219	1	April 23	26	$52^\circ 22' 26''.156$	+4.159
2	" 24	18	29.322	+0.865	2	" 24	26	27.154	+3.161
3	" 25	22	30.069	+0.118	3	" 25	28	35.380	-5.065
4	" 27	30	30.295	-0.108	4	" 26	34	33.233	-2.918
5	" 30	30	29.354	+0.833	5	May 11	30	30.791	-0.476
6	Oct. 1	22	29.387	+0.800	6	" 12	32	30.410	-0.095
7	" 2	42	32.030	-1.843	7	" 13	36	31.536	-1.221
8	" 3	40	30.530	-0.343	8	" 14	36	34.956	-4.641
9	" 4	30	32.322	-2.135	9	" 19	36	25.115	+5.200
10	" 26	42	29.963	+0.224	10	" 20	36	27.835	+2.480
11	Dec. 16	34	29.817	+0.370	11	" 23	40	27.766	+2.549
			$52^\circ 22' 30''.187$	0.000	12	" 24	32	29.280	+1.035
					13	" 25	36	34.484	-4.169
								$52^\circ 22' 30''.315$	-0.001

$$m^2 = \frac{[vv]}{10} = 1.1853; \quad m = \pm 1''.089$$

$$M^2 = \frac{1.1853}{11} = 0.1078; \quad M = \pm 0''.328$$

$$m^2 = \frac{[vv]}{12} = 11.905; \quad m = \pm 3''.302$$

$$M^2 = \frac{11.905}{13} = 0.9158; \quad M = \pm 0''.957$$

the mean of the results of 1810 and 1811. It has a standard deviation M = ± 0".506. As the measurements were executed outside the centre on the north side of the (balustrade of the) first gallery of the tower (about midway between the points E and F in Fig. 9 of section 11), a correction -0".122 was computed because of the reduction to centre [81] .

The latitude φ of the station Amsterdam is therefore $52°22'30".129$. It is but $1".828$ smaller than the amount $52°22'31".957$ found with De Groot's formula [82] from the R.D.-coordinates X' = -34299.277, Y' = +24525.501 of the spire S. In my opinion it is a very good result. The sensitivity of the level A on the lower telescope of the repetition circle used must therefore have been much better than the amount of 25" per 2 mm found for the instrument pictured in Figures 3 and 4.

27. Determination of the latitude at Jever (station No. 102)

The latitude of the station Jever was determined in an analogous way to that of Amsterdam by measuring circum meridian zenith distances of the pole star in lower transit. The measurements were carried out between August 28th and September 7th, 1811 in an eccentric station 8.787 m north of the centre "in a drawing room of the castle". The results of the seven series measured are mentioned in the left part of table 33 (column 4). The standard deviation in a measured φ is m = ± 0".845, that in the mean $53°34'23".713$ of the 7 series M = ± 0".319.

Table 33

Series	Date 1811	No. rep.	φ	v"	Urs. maj.	Date 1811	No. rep.	φ	v"
1	2	3	4	5	1	2	3	4	5
1	Aug. 28	28	53°34'23".172	+0.541	α	Aug. 26	30	53°34'24".933	-1.188
2	" 31	36	22.503	+1.210					
3	Sept. 3	36	23.609	+0.104	γ	" 26	36	23.099	+0.646
4	" 4	36	24.817	-1.104	γ	" 27	36	21.347	+2.398
5	" 5	36	24.197	-0.484					
6	" 6	36	23.133	+0.580	ϵ	" 25	36	25.062	-1.317
7	" 7	36	24.563	-0.850	ϵ	" 26	36	23.655	+0.090
			53°34'23".713	-0.003	ϵ	" 27	36	24.373	-0.628
								53°34'23".745	+0.001

$$m^2 = \frac{[vv]}{6} = 0.7133;\ m = \pm\ 0".845$$

$$M^2 = \frac{0.7133}{7} = 0.1019;\ M = \pm\ 0".319$$

$$m^2 = \frac{[vv]}{5} = 1.9432;\ m = \pm\ 1".394$$

$$M^2 = \frac{1.9432}{6} = 0.3239;\ M = \pm\ 0".569$$

In the same eccentric station Krayenhoff also measured the circum meridian zenith distances of the stars α, γ and ϵ (in lower transit) of the constellation Ursae Majoris (Great Bear). As can be seen from columns 1 and 2 of the right part of the table there was one measured series on the α-star (Dubhe), two on the γ-star and three on the ϵ-star. In the same night (September 26th) even three series could be measured on α, γ and ϵ, respectively. As the stars pass the meridian of a station in the sequence of their right ascensions (R. A.), one can make a programme - and apparently Krayenhoff did so - how the measurements for that night had to be arranged. As Krayenhoff does not mention these R. A. in his computation registers, Dr. Van Herk of the Leiden Observatory was as kind as to give me both R. A. and δ (declination) of the three stars for the year 1811_0. He borrowed them from the Connaissance des Temps of that year. One can find them in columns 2 and 3 of table 34.

Table 34

Urs. maj.	Connaissance des Temps		According to Roelofs		Acc. to Krayenhoff
	R. A. 1811_0	$\delta\,1811_0$	R. A. 1811_0	$\delta\,1811_0$	δ Aug. 26, 1811
1	2	3	4	5	6
α	$10^h51^m57^s.5$	$62^\circ46'06''$	$10^h51^m54^s.74$	$62^\circ45'55''.3$	$62^\circ45'59''.048$
γ	11 43 49.8	54 44 43	11 43 51.67	54 44 44.3	54 44 41.620
ϵ	12 45 40.2	56 59 16	12 45 42.83	56 59 14.9	56 59 21.296

Prof. Roelofs, former professor of geodesy and photogrammetry at the Delft University of Technology, computed for me the coordinates 1811_0 from the amounts 1968_0. They are mentioned in the columns 4 and 5. The amounts for δ used by Krayenhoff on August 26th are given in column 6. A part of the differences between the comparable columns 2 and 4 and 3 and 5 respectively will be caused by the inaccurate determinations of right ascensions and declinations in the beginning of the 19th century, another part perhaps by the conversion of R. A. and δ 1968_0 into the corresponding amounts 1811_0.

For the arrangement of the programme mentioned above the right ascensions of the three stars are once again given in table 35, both according to the Connaissance des Temps (column 2) and to Roelofs (column 4).

As one sees from columns 3 and 5 the differences in transit between α and γ, γ and ϵ, and α and ϵ are about 51^m54^s, 61^m51^s and 113^m45^s respectively. They are expressed in sidereal time. Assuming that the right ascensions 1811_0

Table 35

Urs. Maj.	Right ascension Conn. des Temps 1811_0	Diff.	Right ascension acc. to Roelofs 1811_0	Diff.	Transit acc. to Krayenhoff (mean solar time)	Differences sol. time	Differences sid. time
1	2	3	4	5	6	7	8
α	$10^h51^m57^s.5$		$10^h51^m54^s.74$		$12^h33^m00^s.7$		
		$51^m52^s.3$		$51^m56^s.93$		$51^m45^s.1$	$51^m53^s.6$
γ	11 43 49.8		11 43 51.67		13 24 45.8		
		61 50.4		61 51.16		61 40.9	61 51.0
ϵ	12 45 40.2		12 45 42.83		14 26 26.7		
		$113^m42^s.7$		$113^m48^s.09$		$113^m26^s.0$	$113^m44^s.6$

in the table are about the same as those used by Krayenhoff on August 26th, 1811 then the measurements to α must have been ended shortly before those to γ began. The difference of almost 62 (sidereal) minutes between transit of γ and ϵ allows to continue the measurements of the zenith distances of γ till about 25 minutes after transit and to begin those of ϵ about 25 minutes before transit. Then there is about 12 minutes to switch over from one programme to the other one, that it to say from a zenith distance of about $(180^\circ - \varphi) - \delta_\gamma \simeq 71^\circ 41'$ to a zenith distance of about $(180^\circ - \varphi) - \delta_\epsilon \simeq 69^\circ 26'$. The meridian zenith distance of Dubhe is about $63^\circ 40'$.

These theoretical considerations are confirmed by the observation register and by the computation of the hour angles of the stars in the volume e mentioned at the end of section 4. For the α-star Krayenhoff says there that its transit is at $12^h33^m00^s.7 + 3^m29^s.7 = 12^h36^m30^s$. For the γ- and ϵ-star it is $13^h24^m45^s.8 + 3^m29^s.2 = 13^h28^m15^s$ and $14^h26^m26^s.7 + 3^m28^s.3 = 14^h29^m55^s$ respectively. It will be clear that the amounts of about 3^m29^s are the corrections which must be given to the true moments of transit in order to find the chronometer moments. The "true" moments are repeated in column 6 of the table. The differences in column 7 differ considerably from those in columns 3 and 5. If, however, they are multiplied by 1.002738, that is to say if one passes from mean solar time to sidereal time , they are about alike. The consideration is a proof that Krayenhoff's time keeper kept mean solar time indeed as already indicated in the heads of columns 6 and 7. Apparently the moments of transit in mean solar time of the three stars are computed in a correct manner. He only omitted - and I assume he was fully aware of that - to convert the small hour angles into sidereal time.

As the declination δ of e.g. the α-star of the Great Bear is much smaller than that of Polaris, it will be clear that the terms:

$$\frac{\cos\varphi\ \cos\delta}{\sin(\varphi+\delta)} \text{ and } \left(\frac{\cos\varphi\cos\delta}{\sin(\varphi+\delta)}\right)^2 \cot(\varphi+\delta)$$

in (16) will be much greater than the corresponding terms in the computation of the meridian zenith distance of the pole star.

For Dubhe they are 0.303212 and 0.045517 respectively. As the first pointing at that star was made at an hour angle $t=-21^m48^s$ mean solar time ($\frac{1}{2}\,t=-10^m54^s=-2^o43'30''$) one finds for the two correction terms in (16) $+\,282''.723$ (Krayenhoff $+283''.24$) and $+\,0''.096$ (Krayenhoff $+\,0''.07$) respectively. The influence of a small error dt in t on the computation of

$$\Delta z_1'' = +2\rho''\frac{\cos\varphi\ \cos\delta}{\sin(\varphi+\delta)}\ \sin^2\tfrac{1}{2}t$$

can be found by differentiating this formula with respect to t. The result is:

$$d\,\Delta z_1'' = \frac{\cos\varphi\ \cos\delta}{\sin(\varphi+\delta)}\ \sin t\ dt''$$

In the example of Dubhe already given before $t=-21^m48^s=-5^o27'00''$ and $\frac{\cos\varphi\ \cos\delta}{\sin(\varphi+\delta)} = 0.303212$ so that:

$$d\,\Delta z_1'' = -\ 0.02880\ dt''.$$

The hour angle $t=-\ 21^m48^s$ in this example, however, was expressed in mean solar time. Its equivalent in sidereal time is $-21^m51^s.58$. dt is therefore $-3^s.58=-53''.7$ so that $d\,\Delta z_1''=+\,1''.547$, not an insignificant amount if a computation in thousandths of a second is taken into consideration.

A small error $d\varphi$ in the assumed latitude $\varphi = 53^o34'23''$ on the computation of:

$$\Delta z_1(\text{rad}) = 2\cos\delta\ \sin^2\tfrac{1}{2}t\ \frac{\cos\varphi}{\sin(\varphi+\delta)}$$

can be expressed by:

$$d\,\Delta z_1'' = -\,\frac{2\cos\varphi\ \cos\delta}{\sin(\varphi+\delta)}\sin^2\tfrac{1}{2}t\ \{\tan\varphi + \cot(\varphi+\delta)\}\ d\varphi''$$

For:

$$\frac{\cos\varphi\cos\delta}{\sin(\varphi+\delta)} = 0.303212,\ t=-5^o27'00'',\ \text{and}\ \delta_\alpha \simeq +\,62^o45'59''$$

one finds:

$$d\,\Delta z_1'' = -\ 0.0012\ d\varphi''$$

This amount can be neglected.

The latitudes found from the six series measured on the three stars of Ursae Majoris are mentioned in column 4 of the right part of table 33. The standard deviation in the determination is $m=\pm\,1''.394$; $M=\pm\,0''.569$. Like in table 32 here, too, the means in the left and right part of the table are almost alike. For the final φ of his eccentric station Krayenhoff used the mean of the two means $\varphi = 53^{\circ}34'23''.729 \pm 0''.326$. As the reduction to centre is $-0''.284$, the latitude of the centre of the station Jever is $\varphi = 53^{\circ}34'23''.445$. From the R. D.-coordinates $X'' = +\ 166635.23$, $Y'' = +160598.81$ in columns 7 and 8 of table 26 in section 21, computed from Krayenhoff's adjusted triangulation network, one finds with De Groot's formula $\varphi = 53^{\circ}34'23''.929$, a difference of about $0''.5$ with the measurement. It is an excellent result in my opinion.

28. Determination of astronomical azimuths. General considerations and results of the measurement of the azimuth Amsterdam-Utrecht

"I tried", says Krayenhoff on page 13 of his Précis Historique, "to do azimuth observations of the sun with the repetition circle with which I determined latitudes in Amsterdam and at Jever. The instrument was perfectly suited for such a determination as it could be set up in any arbitrary plane through the station and the sun's movement could be followed in an easy way. But, either by the lack of experience of the two observers in this kind of observations or by the refraction which is very variable in Holland, I never obtained satisfactory results with it. I deleted them even from my observation registers though I had already made a considerable number of observations. I was therefore obliged to change my working method. I replaced the repetition circle by a transit instrument with a telescope of 1. 03 m length mounted on an axis of 0. 772 m, excellently centred and provided with a good level".

In eccentric stations at the east, south and west side of the gallery of the towers in Amsterdam and at Jever the instrument was set up in the vertical plane through the station and the spire where to the azimuth had to be determined. "In order to be sure", says Krayenhoff on page 36 of the Précis Historique, "that the telescope moved in the vertical plane, the level was minutely centred. Just before the sun's transit I made once again sure of the correct position of the telescope and, after having placed the opaque glass in front of it, it was turned (in a vertical direction) towards the sun in order to registrate the moments on which the vertical cross-wire touched the right and the left side of the sun. By a second observer these moments were read on the chronometer in seconds or half-seconds. They were immediately noted down in the registers". Apparently Krayenhoff used here the method of time observation by calling out.

How Krayenhoff computed his azimuths can only be seen from the formulae (1) up to and including (4) on page 42 of his book . He gives them without any com-

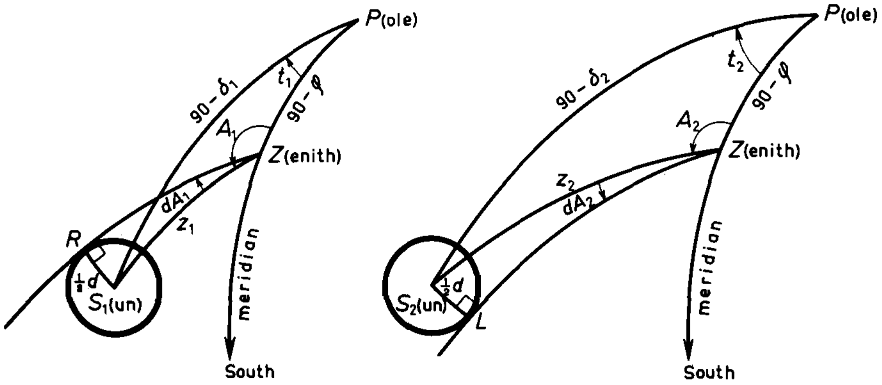


Fig. 26a Fig. 26b

ment and there are even mistakes in them, fortunately, however, not in their application in the computation register.

A derivation of the correct formulae can be found from Figures 26a and 26b. In Fig. a P is the (north) pole, Z the zenith and S_1 the sun when the hour angle t_1 of its centre, expressed in mean solar time, corresponds with the time between the sun's transit through the meridian and the moment its right side R touches the vertical cross-wire. As both moments are known, t_1 is known. If δ_1 is the sun's declination and φ the latitude of the station then $S_1P = 90^o - \delta_1$ and $ZP = 90^o - \varphi$ are known. z_1 is the sun's zenith distance and A_1 its azimuth. Conformably to Krayenhoff's computations it is counted from the north in an anti-clockwise direction. From Napier's analogies [83] one finds in the spherical triangle PS_1Z:

$$\tan \tfrac{1}{2} (A_1 - S_1) = \frac{\sin \frac{1}{2} \{ (90^o - \delta_1) - (90^o - \varphi) \}}{\sin \frac{1}{2} \{ (90^o - \delta_1) + (90^o - \varphi) \}} \cot \tfrac{1}{2} t_1$$

or:

$$\tan \tfrac{1}{2} (A_1 - S_1) = \frac{\sin \frac{1}{2} (\varphi - \delta_1)}{\cos \frac{1}{2} (\varphi + \delta_1)} \cot \tfrac{1}{2} t_1 \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

and

$$\tan \tfrac{1}{2} (A_1 + S_1) = \frac{\cos \frac{1}{2} \{ (90^o - \delta_1) - (90^o - \varphi) \}}{\cos \frac{1}{2} \{ (90^o - \delta_1) + (90^o - \varphi) \}} \cot \tfrac{1}{2} t_1$$

or:

$$\tan \tfrac{1}{2} (A_1 + S_1) = \frac{\cos \frac{1}{2} (\varphi - \delta_1)}{\sin \frac{1}{2} (\varphi + \delta_1)} \cot \tfrac{1}{2} t_1 \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

From (17) and (18) Krayenhoff computed with logarithms $\frac{1}{2} (A_1 - S_1) = p$ and $\frac{1}{2} (A_1 + S_1) = q$ so that $A_1 = p + q$. It is Krayenhoff's formula (3). (1) and (2) agree with (17) and (18) respectively.

The angle PZR, the demanded azimuth, can be found by diminishing A_1 with the amount dA_1. It can be computed from the right angled triangle S_1RZ as $\frac{1}{2}d$, the radius of the sun's disc on the day of the observation, is known and z_1 can be computed in triangle S_1PZ.

According to Delambre's second and fourth formula one finds in the latter triangle:

$$\frac{\sin\frac{1}{2}(A_1 - S_1)}{\cos\frac{1}{2}t_1} = \frac{\sin\frac{1}{2}\{(90^o - \delta_1) - (90^o - \varphi)\}}{\sin\frac{1}{2}z_1}$$

or:

$$\sin\tfrac{1}{2}z_1 = \frac{\cos\frac{1}{2}t_1 \sin\frac{1}{2}(\varphi - \delta_1)}{\sin\frac{1}{2}(A_1 - S_1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

and:

$$\frac{\cos\frac{1}{2}(A_1 - S_1)}{\sin\frac{1}{2}t_1} = \frac{\sin\frac{1}{2}\{(90^o - \varphi) + (90^o - \delta_1)\}}{\sin\frac{1}{2}z_1}$$

or:

$$\sin\tfrac{1}{2}z_1 = \frac{\sin\frac{1}{2}t_1 \cos\frac{1}{2}(\varphi + \delta_1)}{\cos\frac{1}{2}(A_1 - S_1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

As $\frac{1}{2}(A_1 - S_1)$ was already found in (17), z_1 can be computed from (19) or (and) (20). (19) agrees with the first part of Krayenhoff's formula (4). According to (20) the second part, however, is wrong. As he made no use of this check, he did not find his error.

From Fig. 26a now follows:

$$\sin dA_1 = \sin\tfrac{1}{2}d : \sin(z_1 - r_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

and therefore:

$$A = A_1 - dA_1 \ .$$

In (21) $\frac{1}{2}$d is the sun's radius. It ranges from about 16'18" on January 1st to about 15'45" on July 2nd. r_1 in the formula is the refraction; z_1 must be reduced with this amount as the sun's zenith distance belonging to the moment of the observation is somewhat smaller than the computed distance z_1.

The second contact of the vertical cross-wire with the sun's disc - now at its left side L - is shown in Fig. 26b. The hour angle of the sun's centre S_2 is now t_2, a little larger than t_1. The azimuth A_2 can be computed with (17) and (18), z_2 with (19) and (or) (20) and dA_2 with (21).

$A = A_2 + dA_2$ is the azimuth to the terrestrial object (the spire). Apart from observation errors it must be alike to $A = A_1 - dA_1$. As an example I give Krayenhoff's computation of the azimuth Amsterdam-Nieuwkoop, measured as series 51 on April 26th, 1811 (see table 36).

Table 36

Con-tact	Chron. time	Chron. corr.	Hour angle t		Declination δ
			in time	in arc	
1	2	3	4	5	6
1(R)	$12^h38^m21^s.5$	$+2^m11^s.067$	$0^h40^m32^s.567$	$10^\circ08'08''.505$	$13^\circ18'52''.40$
2(L)	$12^h40^m34^s.0$	$+2^m11^s.081$	$0^h42^m45^s.081$	$10^\circ41'16''.215$	$13^\circ18'54''.20$

The ridiculous "accuracy" of the hour angles in column 4 and 5 is of course for Krayenhoff's responsibility.

For $\varphi = 52^\circ22'30''$ one finds with (17) and (18):

$$p = \tfrac{1}{2}(A_1 - S_1) = 77^\circ26'09''.435$$
$$q = \tfrac{1}{2}(A_1 + S_1) = 87^\circ04'42''.555 \qquad \text{and therefore:}$$
$$p + q = A_1 \qquad = 164^\circ30'51''.990$$

From (19) follows $z_1 = 39^\circ53'45''.484$.

As $r_1 = 47''.330$ one finds from (21):

$$dA_1 = 24'48''.888 \qquad \text{so that:}$$
$$A = A_1 - dA_1 = 164^\circ06'03''.102.$$

In an analogous way Krayenhoff computes for the second contact with the sun's disc:

$$A = A_2 + dA_2 = 164^\circ06'09''.509.$$

It differs $6''.4$ from the first computation. The mean, used in the further computations is $164^\circ06'06''.305$ or, counted from the south in a clockwise direction, $A = 15^\circ53'53''.695$.

As according to Krayenhoff the reduction to centre is $-8''.136$ the azimuth from the station Amsterdam to Nieuwkoop is $15^\circ53'45''.559$. In order to find the azimuth to Utrecht it must be diminished with the adjusted spherical angle 160. As (see table 9, columns 11-12) Krayenhoff found $43^\circ12'45''.673$ for it, the result is $332^\circ40'59''.886$.

On page 36 of the Précis Historique Krayenhoff gives a sketch of the eccentric stations A, H and K on the first gallery of the Western tower where he did his observations. It gives the impression that A, where, among others, the azimuth to Nieuwkoop was measured, lies midway between B and H of Fig. 9. According to this sketch K lies midway between H and E and H midway between B and F. The sketch in Calcul des Observations astronomiques, however, deviates from that in the Précis Historique. As is explicitly said in the text, A, H and K lie on the balustrade. In Fig. 27 I marked the three points in accordance with the latter description.

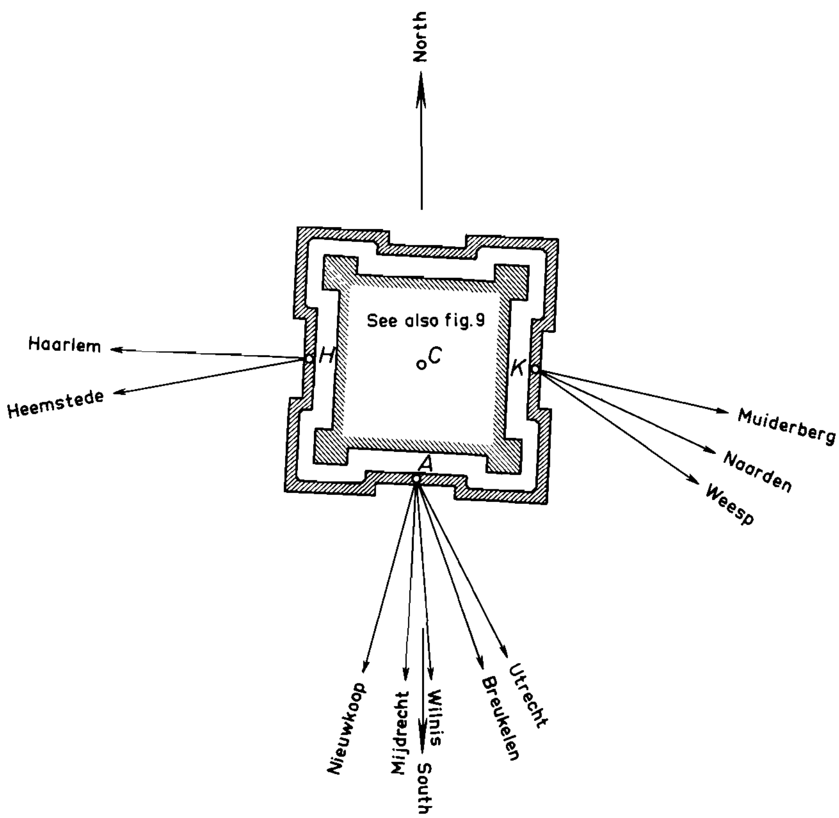


Fig. 27

In K Krayenhoff measured the azimuths to Muiderberg, Naarden and Weesp, in A to Utrecht, Breukelen, Wilnis, Mijdrecht and Nieuwkoop and in H those to Heemstede and Haarlem. As Muiderberg, Weesp, Breukelen, Wilnis, Mijdrecht and Heemstede are no angular points of Krayenhoff's triangulation network and all azimuths had to be reduced to the azimuth Amsterdam-Utrecht, in K the horizontal angles between Naarden and Muiderberg and between Naarden and Weesp had to be measured with the repetition circle and to be reduced to centre. In an analogous way in A the horizontal angles were measured between Utrecht and Breukelen, Wilnis and Mijdrecht, respectively, and in H the angle between Haarlem and Heemstede. I don't know how Krayenhoff got knowledge of the approximate distances to the several sighting points, necessary for the computation of the reduction to centre. One can find the concerning data in the observation registers octavo X and XI. In column 1 of table 39 are the amounts (reduced to centre) which the ray to Utrecht must turn to the right in order to coincide with that to the other 9 towers.

It will be clear that from the provisional azimuth of the side Amsterdam-Utrecht found from the computation of the triangulation and the angles mentioned above provisional azimuths to Muiderberg, etc. could be found. They enabled Krayenhoff to make a programme, that's to say to compute in advance at which chronometer time of a certain day the sun's transit through the vertical of Muiderberg, Naarden, etc. could be expected.

Table 37

Station Amsterdam, April 26th, 1811					
Station	Series	Transit sun through vertical of	Chronometer times First contact R h m s	 Second contact L h m s	Remarks
1	2	3	4	5	6
K	44	Muiderberg	8 03 26.0	8 06 10.5	
	45	Naarden	8 33 06.5	8 35 47.5	Chron. at noon
	46	Weesp	9 12 24.0	9 14 57.5	Apr. 25: $11^h 57^m 58^s.901$
A	47	Utrecht	10 43 28.0	10 45 45.5	" 26: $11^h 57^m 49^s.193$
	48	Breukelen	11 04 59.0	11 07 14.0	Chronometer rate
	49	Wilnis	11 45 47.0	11 47 59.5	in 24 hours:
	50	Mijdrecht	12 05 20.0	12 07 31.0	$0^s.893$ fast on
	51	Nieuwkoop	12 38 21.5	12 40 34.0	mean time
H	52	Heemstede	16 25 48.0	16 28 34.5	
	53	Haarlem	17 27 09.5	17 29 57.5	

Table 37 (see also Précis Historique, page 37), gives a survey of the programme measured on April 26th, 1811. The preparations for the measurement in K of the azimuth to Muiderberg will have been started at about half past seven. After the sun's transit through the vertical of Weesp at about $9^h 15^m$ the instrument could be carried to A where the measurement of the azimuth to Utrecht could be expected at about $10^h 43^m$. That to Nieuwkoop could be observed at the same station at about $12^h 40^m$ and the observation in H of the azimuth to Heemstede at about $16^h 25^m$. The programme ended that day after the observation of the sun's transit through Haarlem at about $17^h 30^m$.

Column 6 (Remarks) gives the reading of the chronometer at noon on April 25th and 26th respectively and the chronometer rate per 24 hours. Only with these data and those in columns 4 and 5 a correct computation of the hour angle is not possible because nowhere in his registers Krayenhoff mentions the equation of time e, the difference between the hour angles of the true sun and the mean sun at apparent noon of the observation days. It is also not immediately clear whether "Chronometer at noon" means "at apparent noon" or "at mean noon". The latter question can easily be solved. As on April 26th e (apparent time minus mean time) is + $2^m12^s.6$ the (fictitious) mean sun will pass the meridian of Amsterdam $2^m12^s.6$ later than the true sun and the vertical plane through Amsterdam and Nieuwkoop about the same amount later. As, according to column 2 of table 36, the first contact of the (true) sun with the vertical cross-wire through Nieuwkoop was at $12^h38^m21^s.5$ chronometer time, that with the mean sun would be at about $12^h40^m34^s.1$ chronometer time. If one supposes that at local mean noon of April 26th the chronometer reading is $11^h57^m49^s.2$ (see column 6 of table 37) the hour angle of the (mean) sun would be about $12^h40^m34^s.1 - 11^h57^m49^s.2 = 42^m44^s.9$ which is in contradiction with the amount $0^h40^m32^s.567$ in column 4 of table 36. "Noon" in table 37 means therefore local apparent noon.

In order to check the computation of the chronometer rate $-0^s.893$ per 24 hours mentioned in table 37 and that of some hour angles t, Dr. Van Herk of Leiden Observatory was as kind as to send me a copy of the pages 44 and 45 of the Connaissance des Temps of 1811 in which for April the amounts e can be found in the column "Temps moyen au midi vrai". For my use of course they had to be reduced from the meridian of Paris to that of Amsterdam. For April 22nd up to and including April 26th, I mentioned them in column 3 of table 38. From these amounts and the chronometer readings at local apparent noon in column 2 - I borrowed them from Krayenhoff's observation registers - the chronometer readings at local mean noon could be computed (column 4). From these amounts follow the chronometer corrections (column 5) and the rate of the chronometer in 24 hours (column 6). As one sees the amounts are negative: the chronometer gains. Between the two succesive transits of the mean sun on April 26th and April 27th the rate is $-0^s.471$. Apparently Krayenhoff used the amount $-0^s.893$ of the previous days. The difference is hardly of any practical influence. Between transit of the true sun on April 26th at a chronometer time $11^h57^m49^s.193$ and the first contact in the vertical of Nieuwkoop at $12^h38^m21^s.5$ clock time are $40^m32^s.307$ clock time or $40^m32^s.282$ mean time. In this time interval e increases from $+2^m12^s.6$ to $+2^m12^s.887$ so that the hour angle t is $t_1 = 40^m32^s.282 + 2^m12^s.887 - 2^m12^s.6 = 40^m32^s.569$. According to table 36 Krayenhoff finds $40^m32^s.567$. For Haarlem (first contact at $17^h27^m09^s.5$ (see table 37) the computation is:

$$t_1 = \left\{ (17^h27^m09^s.5 - 11^h57^m49^s.193) - 0^s204 \right\} + 2^m14^s.933 - 2^m12^s.6 = \\ = 5^h29^m22^s.436.$$

Table 38

Date 1811	Chronometer at local apparent noon	Equation of time (apparent-mean)	Chronometer at local mean noon	Chronometer correction	Rate per 24 hours
1	2	3	4=2+3	5	6
April 22	$11^h58^m31^s.527$	$+1^m26^s.8$	$11^h59^m58^s.327$	$+1^s.673$	
					$-0^s.788$
23	$58^m20^s.115$	$+1^m39^s.0$	$11^h59^m59^s.115$	$+0^s.885$	
					$-0^s.893$
24	$58^m09^s.308$	$+1^m50^s.7$	$12^h00^m00^s.008$	$-0^s.008$	
					$-0^s.893$
25	$57^m58^s.901$	$+2^m02^s.0$	$12^h00^m00^s.901$	$-0^s.901$	
					$-0^s.892$
26	$57^m49^s.193$	$+2^m12^s.6$	$12^h00^m01^s.793$	$-1^s.793$	
					$-0^s.471$
27	$57^m39^s.464$	$+2^m22^s.8$	$12^h00^m02^s.264$	$-1^s.264$	

With a chronometer rate $-0^s.471$ per 24 hours (table 38 column 6), which is somewhat better indeed, $t_1 = 5^h29^m22^s.532$ according to my computation. Krayenhoff finds $5^h29^m22^s.517$. The differences, $-0^s.081$ and $+0^s.015$ respectively, are of course of no importance because the uncertainty in the reading of the moment the vertical cross-wire touches the sun's limb is more than 0.5 seconds. The accuracy of the computation is not at all adapted to that of the observation. In the examples mentioned above I followed this bad working method only for a check on Krayenhoff's computations.

In Amsterdam Krayenhoff measured 89 azimuths. Only the first 53 of them were computed. The computation of 3 azimuths to Utrecht, 2 to Breukelen, 4 to Wilnis, 4 to Mijdrecht, 3 to Nieuwkoop, 1 to Heemstede, 2 to Haarlem, 5 to Muiderberg, 6 to Naarden and 6 to Weesp, all of them measured between April 27th and May 23rd, 1811, were omitted. The results of the computation, all of them reduced to centre, are given in table 39 in sequence of the stations A, H and K where they were measured. Column 2 gives the sequence of the 53 measurements.

Originally it was Krayenhoff's intention to use only those series which deviated no more than 20" (!) from the azimuth Amsterdam-Utrecht that could be computed from the azimuth Duinkerken-Mont Cassel and the adjusted angles of the

Table 39

Azimuth from Amsterdam to	Series	Date 1811	Azimuth	v'' +	v'' −	Computation stand. deviation
1	2	3	4	5	6	7
Utrecht	1*	April 1	$332°41'12''.000$	19.549		
	7*	2	$41'14''.830$	16.719		[vv] = 1984.3
	12*	19	$41'16''.703$	14.846		
	17	20	$41'43''.198$		11.649	m^2 = 396.9
$0°00'00''.000$	26	23	$41'60''.970$		29.421	
	32	24	$41'41''.593$		10.044	m = ± 19''.9
	39^o	25	$40'12''.052$	———	———	
	47^o	26	$40'34''.561$	———	———	
	6		$332°41'31''.549$	51.114	51.114	
Breukelen	18	April 20	$340°23'66''.910$		39.532	[vv] = 3168.3
	33*	24	$23'40''.543$		13.165	m^2 = 1056.1
$7°42'05''.848$	40	25	$22'56''.354$	31.024		
	48	26	$23'05''.704$	21.674		m = ± 32''.5
	4		$340°23'27''.378$	52.698	52.697	
Wilnis	19	April 20	$355°47'71''.181$		30.531	[vv] = 1325.5
	27*	23	$47'23''.115$	17.535		
	34*	24	$47'35''.019$	5.631		m^2 = 441.8
$23°06'13''.590$	41*	25	$47'33''.287$	7.363		
	49^o	26	$46'34''.658$	———	———	m = ± 21''.0
	4		$355°47'40''.650$	30.529	30.531	
Mijdrecht	2	April 1	$3°19'18''.787$	25.964		[vv] = 3762.4
	8*	2	$19'58''.090$		13.339	
	20	20	$19'84''.746$		39.995	m^2 = 627.1
	28*	23	$19'53''.239$		8.488	
$30°38'25''.597$	35*	24	$19'42''.608$	2.143		m = ± 25''.0
	42*	25	$19'46''.138$		1.387	
	50	26	$19'09''.648$	35.103		
	7		$3°19'44''.751$	63.210	63.209	
Nieuwkoop	3*	April 1	$15°53'49''.302$	31.232		
	13	19	$54'47''.401$		26.867	[vv] = 3921.0
	21	20	$54'41''.694$		21.160	
	29*	23	$54'22''.223$		1.689	m^2 = 653.5

Table 39 (continued)

1	2	3	4	5	6	7
43°12′45″.673	36*	April 24	54′14″.392	6.142		
	43	25	54′43″.164		22.630	m = ± 25″.6
	51	26	53′45″.559	34.975		
	7		15°54′20″.534	72.349	72.346	
Heemstede	4°	April 1	80°09′49″.085	———	———	[vv] = 837.6
	9*	2	10′30″.483		0.056	
107°29′24″.184	22*	20	10′52″.723		22.296	m^2 = 279.2
	37	24	10′25″.982	4.445		
	52	26	10′12″.519	17.908		m = ± 16″.7
	4		80°10′30″.427	22.353	22.352	
Haarlem	5	April 1	92°34′30″.320	2.343		[vv] = 44.1
	10	3	34′29″.598	3.065		
119°53′42″.595	38	24	34′38″.071		5.408	m^2 = 22.1
	53°	26	34′05″.070	———	———	m = ± 4″.7
	3		92°34′32″.663	5.408	5.408	
Muiderberg	14*	April 20	287°46′82″.975		17.197	[vv] = 811.0
	23	23	46′47″.772	18.006		
310°05′58″.980	30	24	46′56″.415	9.363		m^2 = 270.3
	44*	26	46′75″.948		10.170	m = ± 16″.4
	4		287°47′05″.778	27.369	27.367	
Naarden	6*	April 2	294°39′61″.725	7.033		[vv] = 767.8
	15*	20	39′78″.983		10.225	
321°58′50″.070	24	23	39′48″.829	19.929		m^2 = 191.9
	31*	24	39′83″.312		14.554	
	45*	26	39′70″.942		2.184	m = ± 13″.9
	5		294°40′08″.758	26.962	26.963	
Weesp	11*	April 19	304°36′92″.915		13.632	[vv] = 1181.7
	16*	20	36′99″.497		20.214	
331°55′59″.390	25	23	36′59″.675	19.608		m^2 = 393.9
	46*	26	36′65″.044	14.239		m = ± 19″.8
	4		304°37′19″.283	33.847	33.846	

Table 40

Series	Azimuth Amsterdam-Utrecht	v″ +	v″ −	Series	Azimuth Amsterdam-Utrecht	v″ +	v″ −
1	2	3	4	1	2	3	4
1	332°41′12″.000	7.940		9	332°41′06″.299	13.641	
7	41′14″.830	5.110		22	41′28″.539		8.599
12	41′16″.703	3.237		37	41′01″.798	18.142	
17	41′43″.198		23.258	52	40′48″.335	31.605	
26	41′60″.970		41.030				
32	41′41″.593		21.653	5	40′47″.725	32.215	
				10	40′47″.003	32.937	
18	41′61″.062		41.122	38	40′55″.476	24.464	
33	41′34″.695		14.755				
40	40′50″.506	29.434		14	41′23″.995		4.055
48	40′59″.856	20.084		23	40′48″.792	31.148	
				30	40′57″.435	22.505	
19	41′57″.591		37.651	44	41′16″.968	2.972	
27	41′09″.525	10.415					
34	41′21″.429		1.489	6	41′11″ 655	8.285	
41	41′19″.697	0.243		15	41′28″.913		8.973
				24	40′58″.759	21.181	
2	40′53″.190	26.750		31	41′33″.242		13.302
8	41′32″.493		12.553	45	41′20″.872		0.932
20	41′59″.149		39.209				
28	41′27″.642		7.702	11	41′33″.525		13.585
35	41′17″.011	2.929		16	41′40″.107		20.167
42	41′20″.541		0.601	25	41′00″.285	19.655	
50	40′44″.051	35.889		46	41′05″ 654	14.286	
				48	332°41′19″.940	451.432	451.445
3	41′03″.629	16.311					
13	41′61″.728		41.788				
21	41′56″.021		36.081				
29	41′36″.550		16.610				
36	41′28″.719		8.779				
43	41′57″.491		37.551				
51	40′59″.886	20.054					

$[vv] = 24514.2$

$m^2 = 521.6$

$m = \pm 22''.8$

$M^2 = 521.6 : 48 = 10.87$

$M = \pm 3''.3$

triangulation network. According to the Table alphabétique des azimuths on page 155 of the Précis Historique this azimuth is $332^\circ 41' 20''.350$. Of the 8 series directly measured to Utrecht only the numbers 1, 7 and 12 fulfil this condition. Also series 33 for the azimuth to Breukelen could be used because $(340^\circ 23' 40''.543 - 7^\circ 42' 05''.848) - 332^\circ 41' 20''.350 = 14''.335$ is less than $20''$. In column 2 of table 39 I marked these series with an asterisk. In total there are only 26 of these series. Later on Krayenhoff receded from this intention and finally he rejected only 5 series. I marked them with an $^\circ$. For the direct azimuth to Utrecht it concerns the series 39 and 47. On page 42 of the Précis Historique Krayenhoff says that these two series and the numbers 49 to Wilnis, 4 to Heemstede and 53 to Haarlem were rejected "because of their too big deviations which do suppose some dis-adjustment in the setting up of the telescope". It is clear that by this method Krayenhoff introduced anew an element of arbitrariness which does not exclude an intentional influence upon the final result. It must be said, however, that the rejected series are very bad indeed. The other series must be called bad as can be seen from the standard deviations m in the determination of an azimuth computed in column 7 of table 39. m's of $19''.9$ for an azimuth to Utrecht, $32''.5$ to Breukelen and $25''.6$ to Nieuwkoop, e.g. show that this part of the triangulation is very bad indeed. In sections 30 and 31 I shall treat the influences which will have caused these bad results.

The computation of the final azimuth Amsterdam-Utrecht is given in table 40. The sequence of the numbers of the series in column 1 is the same as that in column 2 of table 39. The azimuths in column 2 for the series 1-32 are of course the same as those in column 4 of table 39. Those for the series 18-48 in column 4 of table 39 must be diminished by $7^\circ 42' 05''.848$ in order to find those in column 2 of table 40, etc. The mean azimuth is $332^\circ 41' 19''.940$. The standard deviation in any azimuth is $\pm 22''.8$, that in the mean of the 48 measurements $\pm 3''.3$. It is almost incomprehensible that the azimuth differs but $0''.5$ from the amount $332^\circ 41' 19''.452$ which can be computed from the R.D.-coordinates $X' Y'$ of the two stations [84] . In the Table alphabétique des azimuths on page 155 of the Précis Historique Krayenhoff finds $332^\circ 41' 20''.350$.

29. Measurement and computation of the azimuth Jever-Varel

The determination of the azimuth Jever-Varel was done in the same way as that of the side Amsterdam-Utrecht, already amply discussed in the preceding section. In the eccentric stations E, S and W of the castle at Jever, Krayenhoff measured 85 azimuths to the surrounding points indicated in Fig. 28. The measurements were done between August 13th and September 6th, 1811. On page 44 of the Précis

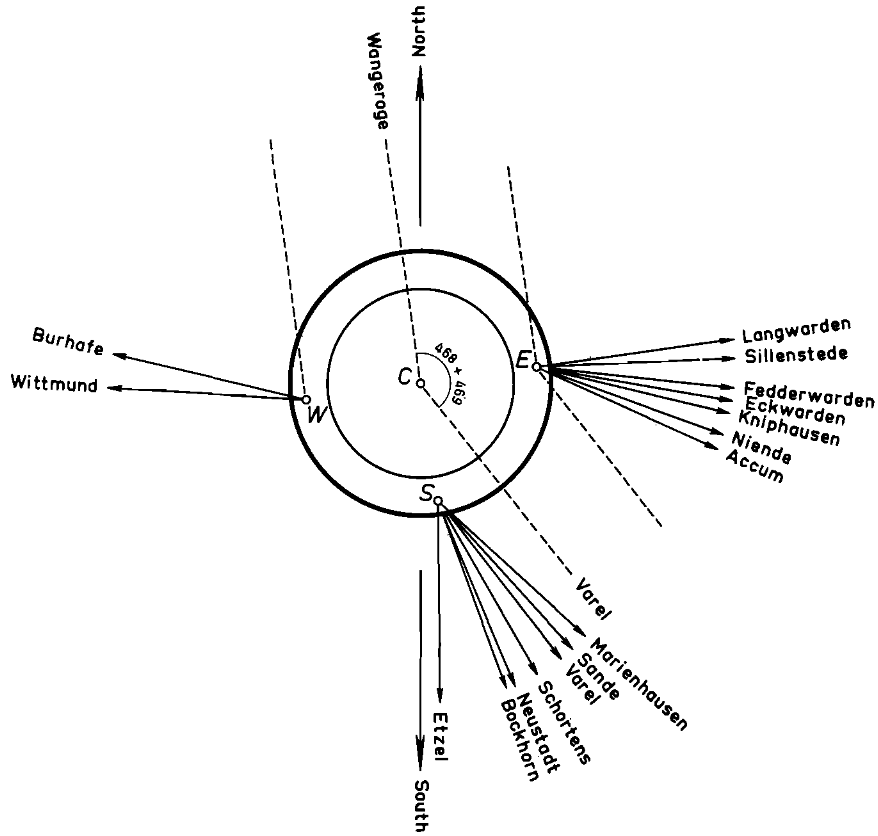


Fig. 28

Historique Krayenhoff says that he computed the first 62 of them and that he rejected 16 observations, "because of their too great irregularity that made me suppose an error in the observation".

In contradistinction to what he did in Amsterdam the results of the computation of the rejected observations are not mentioned in his registers. A survey of the measured azimuths, reduced to centre, can be found in table 41. It is arranged in the same way as table 39.

It must be said that Krayenhoff's statement of 62 computed azimuths does not agree with his registers for apparently he computed the azimuth $339^\circ 34' 29''.089$ to Bockhorn, measured as series 70 on September 5th. If one assumes that the series 71 and 72 measured on the same day to Etzel and Wittmund respectively were also computed then one must conclude that 72 azimuths were computed and that only the last 13, measured on September 6th, remained uncomputed. Apparently 46 computed azimuths were retained and therefore 26 rejected.

Table 41

Azimuth from Jever to	Series	Date 1811	Azimuth	v'' +	v'' −	Computation stand. deviation
1	2	3	4	5	6	7
Varel	6	Aug. 13	rejected			[vv] = 281.9
	11	19	$321^{\circ}20'61''.458$		1.350	
	37	24	20 52 . 758	7.350		m^2 = 940
$0^{\circ}00'00''.000$	49	25	20 73 . 308		13.200	
	56	Sept. 1	rejected			m = $\pm 9''.7$
	67	5	20 52 . 908	7.200		
	80	6	not computed			
	4		$321^{\circ}21'00''.108$	14.550	14.550	
Schortens	12	Aug. 19	rejected			The only series retained gives a difference 36''.4 in the computation of the azimuth from the right and the left side of the sun's disc
	38	24	$329^{\circ}34'59''.641$	——	——	
	50	25	rejected			
$8^{\circ}14'11''.193$	57	Sept. 1	rejected			
	68	5	rejected			
	81	6	not computed			
	1		$329^{\circ}34'59''.641$	——	——	
Neustadt	13	Aug. 19	$336^{\circ}53'58''.885$	33.277		[vv] = 1620.7
	39	24	54 52 . 938		20.776	
	51	25	54 37 . 088		4.926	m^2 = 540.2
$15^{\circ}33'47''.555$	59	Sept. 4	rejected			
	69	5	54 39 . 738		7.576	m = $\pm 23''.2$
	82	6	not computed			
	4		$336^{\circ}54'32''.162$	33.277	33.278	
Bockhorn	40	Aug. 24	rejected			
	52	25	rejected			
	60	Sept. 4	rejected			
$18^{\circ}13'30''.154$	70	5	$339^{\circ}34'29''.089$	——	——	
	83	6	not computed			
	1		$339^{\circ}34'29''.089$	——	——	

Table 41 (continued)

1	2	3	4	5	6	7
Etzel	41	Aug. 24	rejected			The azimuth to Etzel is about 359°21′, almost in the meridian
	53	25	rejected			
38°00′38″.750	61	Sept. 4	rejected			
	71	5	rejected			
	84	6	not computed			
Wittmund	14	Aug. 19	rejected			
	72	Sept. 5	rejected			
132°00′26″.201	85	6	not computed			
Burhafe 141°49′34″.393	15	Aug. 19	rejected			
Langwarden	20	Aug. 21	262°21′32″.184		15.750	[vv] = 496.1
301°00′40″.000	28	24	21 00 .684	15.750		m^2 = 496.1
	2		262°21′16″.434	15.750	15.750	m = ± 22″.3
Sillenstede	16	Aug. 20	267°40′22″.644	4.872		[vv] = 815.2
	21	21	40 20 .944	6.572		
	26	22	40 39 .994		12.478	m^2 = 203.8
306°19′59″.011	29	24	40 44 .205		16.689	
	42	25	40 09 .794	17.722		m = ± 14″.3
	54	27	rejected			
	5		267°40′27″.516	29.166	29.167	
Fedderwarden	1	Aug. 13	276°10′21″.700	1.614		[vv] = 44.0
	17	20	10 20 .769	2.545		
	22	21	10 29 .019		5.705	m^2 = 14.7
314°50′02″.826	30	24	10 21 .769	1.545		
	55	27	rejected			m = ± 3″.8
	73	Sept. 6	not computed			
	4		276°10′23″.314	5.704	5.705	

Table 41 (continued)

1	2	3	4	5	6	7
Eckwarden	18	Aug. 20	$279^{\circ}46'18''.052$	2.410		[vv] = 395.5
	23	21	46 32 .852		12.390	
	31	24	46 28 .652		8.190	m^2 = 98.9
$318^{\circ}25'02''.181$	43	25	46 09 .952	10.510		
	62	Sept. 5	46 12 .802	7.660		m = $\pm 9''.9$
	74	6	not computed			
	5		$279^{\circ}46'20''.462$	20.580	20.580	
Kniphausen	19	Aug. 20	rejected			[vv] = 322.4
	24	21	$283^{\circ}28'05''.446$		11.217	
	32	24	27 56 .796		2.567	m^2 = 161.2
$322^{\circ}06'55''.076$	44	25	27 40 .446	13.783		
	75	Sept. 6	not computed			m = $\pm 12''.7$
	3		$283^{\circ}27'54''.229$	13.783	13.784	
Niende	2	Aug. 13	$289^{\circ}48'24''.808$		15.267	[vv] = 392.0
	7	19	47 58 .308	11.233		
	25	21	48 11 .158		1.617	m^2 = 78.4
	27	22	rejected			
$328^{\circ}27'55''.535$	33	24	48 06 .008	3.533		m = $\pm 8''.9$
	45	25	48 11 .258		1.717	
	63	Sept. 5	48 05 .708	3.833		
	76	6	not computed			
	6		$289^{\circ}48'09''.541$	18.599	18.601	
Accum	3	Aug. 13	$294^{\circ}02'27''.087$		6.387	[vv] = 2134.6
	8	19	01 41 .587	39.113		
	34	24	02 40 .837		20.137	m^2 = 711.5
$332^{\circ}41'36''.432$	46	25	02 33 .287		12.587	
	64	Sept. 5	rejected			m = $\pm 26''.7$
	77	6	not computed			
	4		$294^{\circ}02'20''.700$	39.113	39.111	

Table 41 (continued)

1	2	3	4	5	6	7
Marienhausen	4	Aug. 13	312°04′55″.019		31.313	[vv] = 1592.9
	9	19	04 02 .169	21.537		
	35	24	04 11 .719	11.987		m^2 = 531.0
	47	25	04 25 .919		2.213	
350°43′11″.849	58	Sept. 4	rejected			m = ± 23″.0
	65	5	rejected			
	78	6	not computed			
	4		312°04′23″.706	33.524	33.526	
Sande	5	Aug. 13	317°03′46″.063		1.617	[vv] = 727.8
	10	19	04 02 .663		18.217	
	36	24	03 24 .613	19.833		m^2 = 363.9
355°42′56″.455	48	25	rejected			
	66	Sept. 5	rejected			m = ± 19″.1
	79	6	not computed			
	3		317°03′44″.446	19.833	19.834	

As table 41 shows, four observations to Schortens (the series 12, 50, 57 and 68) were rejected. Series 81 was not computed. Only one series (38) was retained and in this series the difference between the computation from the right and the left side of the sun's disc is 36″.4.

To Etzel none of the 5 azimuths was retained and all azimuths to Wittmund and Burhafe were rejected or not computed. It will be clear that, just like in Amsterdam, here too the results of the determination of the azimuths are very bad. The standard deviations in column 7 are about the same as those in table 39.

In order to reduce the 46 retained azimuths to 46 values for the azimuth Jever-Varel they must be diminished by the horizontal angles mentioned in column 1. They are derived from Krayenhoff's measurements in his eccentric stations. I give these angles, reduced to centre, in table 42. In part I Varel is the left sighting point and Schortens, Neustadt, etc. the right one. In part II Varel is right and Sande, Marienhausen, etc. left. In III and IV where the angles between Wangeroge and Fedderwarden, Eckwarden, Niende, Accum, Burhafe and Wittmund are given, Wangeroge is left and right sighting point respectively.

Table 42

I	Varel-Schortens	$8^{\circ}14'11''.193$	II	Sande -Varel	$4^{\circ}17'03''.545$
	" -Neustadt	$15^{\circ}33'47''.555$		Marienhausen - "	$9^{\circ}16'48''.151$
	" -Bockhorn	$18^{\circ}13'30''.154$		Kniphausen - "	$37^{\circ}53'04''.924$
	" -Etzel	$38^{\circ}00'38''.750$		Sillenstede - "	$53^{\circ}40'00''.989$
				Langwarden - "	$58^{\circ}59'20''.000$
III	Wangeroge-Fedderwarden	$104^{\circ}15'07''.286$			
	" -Eckwarden	$107^{\circ}50'06''.641$	IV	Burhafe -Wangeroge	$68^{\circ}45'21''.147$
	" -Niende	$117^{\circ}52'59''.995$		Wittmund- "	$78^{\circ}34'29''.339$
	" -Accum	$122^{\circ}06'40''.892$			

As Wangeroge (see Fig. 2 and the dotted line in Fig. 28) is a sighting-point in Krayenhoff's first order triangulation network, the angle between Wangeroge and Varel was not measured again because it is the sum of the angles 468 and 469. In his adjustment Krayenhoff found $149^{\circ}25'04''.460$ for it (see table 9).

I treated this less interesting part of the measurements rather fully because Krayenhoff made several mistakes in his reduction of the measured azimuths to the azimuth Jever-Varel. A serious mistake in his sloppy computations of this part of his work is that, instead of the reduction $-318^{\circ}25'02''.181$ for the 5 azimuths to Eckwarden, he used $-318^{\circ}26'02''.181$. Four similar mistakes of $1'$ in the reduction of the azimuths in the series 4 and 47 to Marienhausen, in series 39 to Neustadt and in series 45 to Niende and some small errors make that the mean azimuth in table 43 is $321^{\circ}20'45''.325$ instead of $321^{\circ}20'34''.905$ mentioned in his computation on page 45 of the Précis Historique. The standard deviation in a determination of an azimuth is $25''.2$, that one in the mean of the 46 determinations $3''.7$. The amounts are almost the same as those found in Amsterdam. Starting from the azimuth Duinkerken-Mont Cassel and with the "adjusted" angles of his triangulation network Krayenhoff found $321^{\circ}20'33''.733$. Later on he computed the azimuth again, now starting from his measured azimuth Amsterdam-Utrecht = $332^{\circ}41'19''.940$ (see table 40). According to page 192 of the Précis Historique he now found $321^{\circ}20'30''.411$. From the coordinates $X''Y''$ of Jever and Varel in table 26, I find $321^{\circ}20'37''.69$. It differs $7''.64$ from the azimuth in table 43. Because of the large standard deviation in the latter determination and the arbitrary adjustment of the not quite closing network the difference is explicable.

Table 43

Series	Azimuth Jever-Varel	v"		Series	Azimuth Jever-Varel	v"	
		+	−			+	−
1	2	3	4	1	2	3	4
11	321°20'61".458		16.133				
37	20 52 .758		7.433	24	321°20'70".370		25.045
49	20 73 .308		27.983	32	20 61 .720		16.395
67	20 52 .908		7.583	44	20 45 .370		0.045
38	20 48 .448		3.123	2	20 29 .273	16.052	
				7	20 02 .773	42.552	
13	20 11 .330	33.995		25	20 15 .623	29.702	
39	20 65 .383		20.058	33	20 10 .473	34.852	
51	20 49 .533		4.208	45	20 15 .723	29.602	
69	20 52 .183		6.858	63	20 10 .173	35.152	
70	20 58 .935		13.610	3	20 50 .655		5.330
				8	20 05 .155	40.170	
20	20 52 .184		6.859	34	20 64 .405		19.080
28	20 20 .684	24.641		46	20 56 .855		11.530
16	20 23 .633	21.692		4	21 43 .170		57.845
21	20 21 .933	23.392		9	20 50 .320		4.995
26	20 40 .983	4.342		35	20 59 .870		14.545
29	20 45 .194	0.131		47	20 74 .070		28.745
42	20 10 .783	34.542					
				5	20 49 .608		4.283
1	20 18 .874	26.451		10	20 66 .208		20.883
17	20 17 .943	27.382		36	20 28 .158	17.167	
22	20 26 .193	19.132		46	321°20'45".325	487.331	487.349
30	20 18 .943	26.382					
18	20 75 .871		30.546		$[vv] = 28665.5$		
23	20 90 .671		45.346		$m^2 = 637.0$		
31	20 86 .471		41.146		$m = \pm 25''.2$		
43	20 67 .771		22.446		$M^2 = 637.0 : 46 = 13.85$		
62	20 70 .621		25.296		$M = \pm 3''.7$		

On page 22 of [65] Gauss mentions for the azimuth Bremerlehe-Varel $58^{\circ}15'58''.861$ (see also Fig. 17 in section 18). From this azimuth an azimuth Jever-Varel = $321^{\circ}20'35''.5$ can be computed as Van der Plaats showed already on page 294 of his paper [20]. The small difference ($2''.2$) with the amount $321^{\circ}20'37''.69$ mentioned above in my opinion shows the good harmony between the orientation of the Oldenburg-triangulation and that of Krayenhoff's network, adjusted according to the least squares and adapted as well as possible at identical points of the R. D. The good agreement between the adjusted angles and the side lengths in the two networks was already shown in section 23.

For the original computation of his network Krayenhoff started, as already remarked before, from the azimuth $343^{\circ}13'32''$ 703 of the side Duinkerken-Mont-Cassel, found from Delambre's measurements. If the same azimuth is computed from the coordinates X'' Y'' in column 7 and 8 of table 26 one finds $343^{\circ}13'32''.507$, a difference of only $0''.2$. According to page 187 of the Précis Historique Krayenhoff's computation of this azimuth - for this computation he started from his own determination of the azimuth Amsterdam-Utrecht - is $343^{\circ}13'33''.569$. Here too the difference with my adjustment of the triangulation is very small ($1''.062$).

30. Instrumental errors, affecting the accuracy of the determination of azimuths

From the sections 28 and 29 it appears that the internal accuracy of the determinations of azimuths in Amsterdam and at Jever is very bad indeed. These bad results will partially be due to errors of the transit instrument used. As it is probably lost we know no more of it than Krayenhoff says in the only sentence already quoted in section 28.

It will be clear that the method used for the determination of the sun's azimuth implicates that, after having pointed with the telescope at the terrestrial object, its line of sight must move in a vertical plane through the station when the telescope is turned in the sun's direction. This can be attained when:

a the horizontal axis of the instrument is horizontal indeed;

b the line of sight of the telescope is perpendicular to the horizontal axis.

The realization of the horizontal axis can be obtained by a level on this axis. Its bubble tube axis must be parallel to the connecting line of the two supporting points at a distance of 0.772 m from each other. When the bubble is centred, the axis will be horizontal. Nowhere in the Précis Historique, however, it is to be found whether Krayenhoff investigated the instrument on these important conditions. On the level used he only says that it was good. On its sensitivity, however, important for a correct horizontal position of the axis, nothing is known.

If condition b is satisfied but the bubble tube axis and the "horizontal" axis deviate the small angle α , the line of sight of the telescope will move in a plane that is not vertical. Then in azimuthal sense, as can be easily derived, there is an error:

$$\Delta_1 = \alpha'' \cot z,$$

in which z is the sun's zenith distance at the moment of its observation. As for the observation of the azimuth to Nieuwkoop on April 26th, 1811 (series 51 in table 36) $z \simeq 40^o$, $\Delta_1 \simeq 1.2\alpha''$.

For an assumed amount $\alpha = 10''$ — the sensitivity of the levels of the repetition circle in Figures 3 and 4 is about $20''$ per 2mm — $\Delta_1 \simeq 12''$. The error is of a systematic character. It is minimum (zero) when the sun rises or sets in the direction of the terrestrial object, maximum when the sun (the terrestrial point) is in the meridian. The terrestrial point Etzel (azimuth about $359^o 21'$, see table 41) is therefore badly chosen. For the series 41 on August 24th, 1811 $z \simeq \varphi - \delta$ (φ is the latitude of Jever and δ the sun's declination) is about 42^o so that $\Delta_1 \simeq 11''$.

If condition b is not satisfied and the angle between the sighting line of the telescope and the horizontal axis is $90^o - \beta$ instead of 90^o, the small error β manifests itself in an azimuthal sense as:

$$\Delta_2 = \beta'' \cot z \tan \tfrac{1}{2} (90^o - z)$$

Here too the systematic error is minimum for $z = 90^o$ and maximum when the sun (the terrestrial object) is in the meridian. For series 51 to Nieuwkoop one finds:

$$\Delta_2 = 0.56\, \beta'' ,$$

for series 41 to Etzel:

$$\Delta_2 = 0.49\, \beta'' .$$

The amount β is unknown. An estimation may be found if one remarks that the realization of the line of sight perpendicular to the horizontal axis is dependent on the accuracy with which in Krayenhoff's time the vertical cross-wire could be shifted with the horizontal correction screw of the reticule in a direction perpendicular to the line of sight. If we assume that this accuracy is about 0.15 mm, then β for the 1.03 m long telescope is about 0.00015 radians or about $30''$. Δ_2 for series 51 in Amsterdam (Nieuwkoop) and series 41 at Jever (Etzel) is then about $18''$ and $15''$ respectively.

The considerable amounts Δ_1 and Δ_2 in these considerations are very rough estimates . As Krayenhoff left us in uncertainty about all concerning the investigation of his instrument and its eventual adjustment the considerations remained unfortunately rather speculative. In the next section I can be more positive.

31. Determination of standard deviations in azimuths

The standard deviation in an azimuth A because of the standard deviation m_t in the measured hour angle t and the standard deviation m_φ in the measured latitude φ can be determined by application of the law of propagation of errors to the formula:

$$\cot A = \frac{\sin\varphi \cos t - \cos\varphi \tan\delta}{\sin t} = F \quad \ldots\ldots\ldots\ldots\ldots \quad (22)$$

from which A, counted from the south in a clockwise direction, can be computed from the data φ, t and δ (see Fig. 29). A standard deviation m_δ in the sun's declination δ will be left out of consideration as Krayenhoff borrowed δ from the Connaissance des Temps.

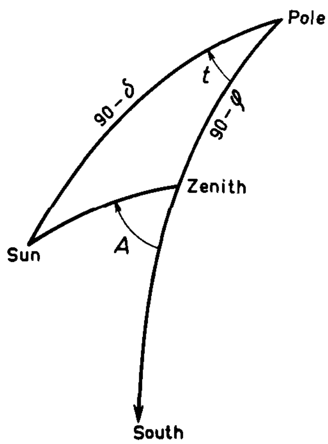


Fig. 29

The law runs as follows:

$$\left(\frac{\partial F}{\partial A}\right)^2 m_A^2 = \left(\frac{\partial F}{\partial t}\right)^2 m_t^2 + \left(\frac{\partial F}{\partial \varphi}\right)^2 m_\varphi^2 .$$

As

$$\frac{\partial F}{\partial A} = -\frac{1}{\sin^2 A},$$

$$\frac{\partial F}{\partial t} = -(\sin\varphi + \cot A \cot t) \text{ and}$$

$$\frac{\partial F}{\partial \varphi} = \frac{\cos\varphi \cos t + \sin\varphi \tan\delta}{\sin t}$$

one finds:

$$\left(\frac{1}{\sin^2 A}\right)^2 m_A^2 = (\sin\varphi + \cot A \cot t)^2 m_t^2 + \left(\frac{\cos\varphi \cos t + \sin\varphi \tan\delta}{\sin t}\right)^2 m_\varphi^2$$

$$m_A^2 = \left\{\sin^2 A\,(\sin\varphi + \cot A \cot t)\right\}^2 m_t^2 + \left\{\frac{\sin^2 A\,(\cos\varphi \cos t + \sin\varphi \tan\delta)}{\sin t}\right\}^2 m_\varphi^2$$

$$\ldots\ldots\ldots\ldots \quad (23)$$

As in the term with m_φ^2 :

$$\frac{\sin A}{\sin t} = \frac{\sin (180^\circ - A)}{\sin t} = \frac{\cos \delta}{\sin z} \; ,$$

one can also write this term as:

$$\left\{ \frac{\sin A \, (\cos \varphi \, \cos \delta \cos t + \sin \varphi \, \sin \delta)}{\sin z} \right\}^2 m_\varphi^2 = (\sin A \cot z)^2 \, m_\varphi^2 .$$

Applying the same trick to the term with m_t^2 one finds:

$$\left\{ \sin^2 A \, (\sin \varphi + \cot A \cot t) \right\}^2 m_t^2 =$$

$$\left\{ \sin A \, \frac{\cos \delta}{\sin z} (\sin \varphi \; \sin t + \cot A \cos t) \right\}^2 m_t^2 =$$

$$\left\{ \frac{\cos \delta \, (\sin \varphi \; \sin A \sin t + \cos A \cos t)}{\sin z} \right\}^2 m_t^2 .$$

At pleasure (23) can therefore be written as:

$$m_A^2 = \left\{ \sin^2 A \, (\sin \varphi + \cot A \cot t) \right\}^2 m_t^2 + (\sin A \cot z)^2 \, m_\varphi^2 \; . \; . \; . \; . \quad (24)$$

or as:

$$m_A^2 = \left\{ \frac{\cos \delta \, (\sin \varphi \sin A \sin t + \cos A \cos t)}{\sin z} \right\}^2 m_t^2 + (\sin A \cot z)^2 \, m_\varphi^2 \; . \; . \; . \; . \quad (25)$$

In (24) the computation of the term with m_t^2 is easier than in (25). For $A = t = 0$, however, the formula cannot be used. According to (25), however, the coefficient is $\left(\frac{\cos \delta}{\sin z} \right)^2$ or, as in that case $z = \varphi - \delta$:

$$\left(\frac{\cos \delta}{\sin (\varphi - \delta)} \right)^2 .$$

As Etzel (see table 41) lies almost in the meridian of Jever, the coefficient of m_t^2 for the observation 41 on August 24th, 1811 is about:

$$\left\{ \frac{\cos 11^\circ}{\sin (53^\circ - 11^\circ)} \right\}^2 = \left(\frac{0.98}{0.67} \right)^2 = (1.5)^2$$

so that:

$$m_A \simeq 1.5 \, m_t \; ,$$

for the coefficient of m_φ^2 in this case is zero. For the stations Amsterdam and Jever $(\sin A \cot z)^2$ will always remain very small as a small z (cot z large) implicates a small A (sin A small) and a large A (sin A large) a large z (cot z small).

If the formulae (24) and/or (25) are applied to the example of the measured azimuth Amsterdam-Nieuwkoop, series 51 (see table 36 and the text belonging to that table), one finds:

$$m_A^2 = (1.49)^2 \, m_t^2 + (0.34)^2 \, m_\varphi^2 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (26)$$

For an estimate of m_φ one can use the results of the determination of the latitudes of Amsterdam and Jever in sections 26 and 27 (see table 32 and 33).

As the standard deviations in the mean of 11 series in Amsterdam in 1810 and in the mean of 13 series in 1811 is $M = \pm 0''.328$ and $M = \pm 0''.957$ respectively and those in Jever in 7 and 6 series $M = \pm 0''.319$ and $M = \pm 0''.569$ respectively, m_φ can be estimated at about $0''.6$.

I think the standard deviation in a chronometer reading "by calling out" can be estimated at about 0.5 sec = $7''.5$. In this standard deviation the accidental part of the error made in the determination of the moment the sun's limb touches the vertical cross-wire is included. We know nothing, however, of the systematic part of this error, caused by the psychological and physical disposition of the (two) observers to judge, either too early or too late, the real moment of contact of the cross-wire with the sun's disc. It may influence the result of the determination of t in a considerable manner.

As Krayenhoff determined the azimuth to the terrestrial object from the mean of two observations R and L (see Fig. 26), m_t is about $7''.5 : \sqrt{2} \simeq 5''.3$ ($m_t^2 \simeq 28$). Because of the small influence of the term $(\sin A \cot z)^2 m_\varphi^2$ in (24) and (25) on m_A^2 it can be neglected so that:

$$m_A = \sin^2 A\ (\sin\varphi + \cot A \cot t) m_t, \text{ or}$$

$$m_A = \frac{\cos\delta\ (\sin\varphi \sin A \sin t + \cos A \cos t)}{\sin z} m_t$$

or, for the example Nieuwkoop in Fig. 27:

$$m_A = 1.49\ m_t = \pm 7''.9$$

and for the azimuth to Etzel (Fig. 28):

$$m_A = 1.5\ m_t = \pm 8''.0.$$

As for the small hour angles t the zenith distances z of the sun are also small, the unfavourable influence of the determination of time on the sun's azimuth is once again increased by the systematic error due to the disadjustment of the instrument used. It is incomprehensible, that Krayenhoff did not see this and, both in Amsterdam (Fig. 27) and at Jever (Fig. 28), used so many sighting points for his determination of azimuths in southern directions and so few at the west side of the horizon. Instead of measuring the very bad azimuths in Amsterdam to Wilnis and Mijdrecht ($355°48'$ and $3°20'$ respectively) Krayenhoff should at any rate have computed the azimuths to Muiderberg, Naarden, Weesp, Heemstede and Haarlem measured between April 27th and May 23rd, 1811 which he let uncomputed. Because of the much smaller coefficient of m_t^2 in (24) or (25) it would have improved the accuracy of the determination also because of the small influence of the systematic error due to the disadjustment of the instrument. For a tower with an azimuth $A = 90^o$ and on March 21st and September 23rd ($\delta = 0$, $z = 90^o$) the coefficient of m_t^2 is $\sin^2\varphi$ so that $m_A = \sin\varphi\ m_t$. For Amsterdam it amounts to $m_A = 0.79\ m_t = 4''.2$. The systematic errors Δ_1 and Δ_2, discussed in section 30 are zero.

It will be clear that for the determination of the standard deviation in the azimuth to Utrecht (in Amsterdam) and to Varel (at Jever) the standard deviation in the horizontal angular measurement must be taken into account. In order to reduce e.g. the measured azimuth Amsterdam-Nieuwkoop to the azimuth Amsterdam-Utrecht, the first must be diminished with the angle Nieuwkoop-Amsterdam-Utrecht (160). As it has a standard deviation $m_\alpha \simeq 1''.6$ (see section 8) the astronomical azimuth reduced to Utrecht would have a standard deviation:

$$m_A \simeq \sqrt{7.9^2 + 1.6^2} \simeq \pm\ 8''.1$$

It is but hardly larger than the dominating amount $7''.9$.
For the reduction of the measured azimuth to Heemstede to that to Utrecht three horizontal angles are necessary (the angles Haarlem-Amsterdam-Heemstede, Haarlem-Amsterdam-Nieuwkoop, and Nieuwkoop-Amsterdam-Utrecht, so that, instead of m_α^2 the amount $3m_\alpha^2$ must be superposed at the square of the standard deviation in the astronimical measurement. Here too the latter amount dominates in the final result.

In the example of the fictitious tower with an azimuth $A = 90^o$ (see above) measured on March 21st or September 23rd, m_A would be:

$$m_A = \sqrt{\sin^2\varphi\, m_t^2 + 3\, m_\alpha^2} = \sqrt{4.2^2 + 7.7} \simeq 5''.0.$$

According to column 7 of the tables 39 and 41 all the standard deviations in the measurement of an azimuth are much higher than the amounts found in this section. Neither in Amsterdam nor at Jever an obvious improvement of the accuracy of the azimuths at the west side or the east side of the horizon is perceptible. The observations 14, 72 and 15 at Jever to Wittmund and Burhafe were even rejected. The systematic instrumental errors discussed in section 30 must therefore have been of paramount influence.

The remaining errors v in columns 3 and 4 of tables 40 and 43 don't give the impression to be accidental. This can be seen very clearly in table 43: at Varel (4 series 11-67), Eckwarden (5 series 18-62), Kniphausen (3 series 24-44) and Marienhausen (4 series 4-47) all the amounts v are negative; at Sillenstede (5 series 16-42), Fedderwarden (4 series 1-30) and Niende (6 series 2-63) all v's are positive. I can't see it otherwise than that, both in Amsterdam and at Jever, the excellent external results of Krayenhoff's determination of azimuths must only be ascribed to chance. From the very bad measurement such excellent results cannot possibly be predicted.

32. Survey of the geographical coordinates φ and λ of all the points of the triangulation network and the azimuths of all the sides and, for the common points and sides, a comparison with the R. D.-results

As I already said in section 25 Krayenhoff computed from his triangulation the geographical coordinates φ and λ of all the stations of his network and the azimuths A of all the sides, once starting from the latitude φ and the longitude λ of Duinkerken (No. 1) - the latter with respect to Paris - and the azimuth Duinkerken (No. 1)-Mont Cassel (No. 2), already determined by Delambre, once from his own determination of the latitude of Amsterdam (No. 40) and his azimuth Amsterdam (No. 40)-Utrecht (No. 36). In the latter computation on the ellipsoid with a radius of the equator $\underline{a}$ = 6356356.1 m, and a flattening p = 0.003229489 (about 1 : 309.65) the longitudes are determined with respect to Amsterdam. The results of the two computations are to be found in tableau IV and V respectively of the Précis Historique.

Nowhere in his book, however, Krayenhoff mentions how the various latitudes, longitudes and azimuths were computed and even the computation of the results with respect to Amsterdam cannot be found in his registers [85] . It will be clear that, here too, he made use of Delambre's formulae, mentioned in his Méthodes analytiques.

It lies beyond the scope of this book to derive these formulae. The interested reader can find the derivation on the pages 59 and following of Delambre's book where - on page 78 - he also refers to Legendre's studies in this field, already published in the Mémoires de l'Académie of the year 1787.

For $\varphi_Q - \varphi_P$, $\lambda_Q - \lambda_P$ and $A_{QP} - (180^o \pm A_{PQ})$ —P, Q (and R) are successive angular points of a triangle of the network — Delambre finds on page 83 of his book:

$$\left.\begin{aligned}
(\varphi_Q - \varphi_P)'' &= -(\delta'' \cos A_{PQ} + \tfrac{1}{2}\delta'' \sin\delta \sin^2 A_{PQ} \operatorname{tg}\varphi_P)(1 + e^2\cos^2\varphi_P), \\
(\lambda_Q - \lambda_P)'' &= -\frac{\delta'' \sin A_{PQ}}{\cos\varphi_Q} \\
\text{and:} \\
\left\{(A_{QP} \pm 180^o) - A_{PQ}\right\}'' &= -(\delta'' \sin A_{PQ} \operatorname{tg}\varphi_Q + \tfrac{1}{4}\delta'' \sin\delta \sin 2A_{PQ})
\end{aligned}\right\} \quad \text{.. (27)}$$

with:

$$\delta'' = \rho''\frac{K}{a}(1 + \tfrac{1}{2}e^2\sin^2\varphi_P) \quad [86].$$

In the latter expression K is the length of the chord PQ found from the computation in tableau III of the Précis Historique, $\underline{a}$ the radius of the equator (6356356.1 m) and $\underline{e}$ the eccentricity of the earth ellipsoid, determined by:

$$e^2 = \frac{a^2 - b^2}{a^2} \text{ with } \frac{a - b}{a} = p = 1 : 309.65.$$

The azimuths in the formulae are counted from the south in a clockwise direction.

Delambre's formulae deviate considerably from those used nowadays for analogous computations. I give them underneath and I borrowed them from a publication of the Netherlands' Rijksdriehoeksmeting. The translation of the Dutch title runs: Formulae and tables for the computation of the geographical latitudes and longitudes of the angular points and the azimuths of the sides of the triangulation network (Delft, 1903). The derivation of the various formulae may be found in geodetic textbooks. The formulae run (azimuths from the north in a clockwise direction):

$$s \sin A_{PQ} = s_1 \qquad s \cos A_{PQ} = s_2$$

$$(\varphi_Q - \varphi_P)'' = [1]\, s_2 - [2]\, s_1^2 - [3]\, s_2^2 - [4]\, s_1^2 s_2$$

$$(\lambda_Q - \lambda_P)'' = [5] \sec \varphi_P\, s_1 + [6]\, s_1 s_2 + [7]\, s_1 s_2^2 - [8]\, s_1^3$$

$$\left\{(A_{QP} \pm 180^o) - A_{PQ}\right\}'' = [5] \tan \varphi_P s_1 + [9]\, s_1 s_2 + [10]\, s_1 s_2^2 - [11]\, s_1^3.$$

s in these formulae is the length (in metres) of the geodesic between the points P and Q,

$$[1] = \rho'' : R$$

$$[2] = \rho'' \tan \varphi_P : 2\,RN$$

$$[3] = 3\rho'' \delta \tan \varphi_P \cos^2 \varphi_P : 2\,RN$$

$$[4] = \rho'' (1 + 3 \tan^2 \varphi_P + \delta \cos^2 \varphi_P - 9\, \delta \tan^2 \varphi_P \cos^2 \varphi_P) : 6\,RN^2$$

$$[5] = \rho'' : N$$

$$[6] = \rho'' \sec \varphi_P \tan \varphi_P : N^2$$

$$[7] = \rho'' \sec \varphi_P (1 + 3 \tan^2 \varphi_P + \delta \cos^2 \varphi_P) : 3\,N^3$$

$$[8] = \rho'' \sec \varphi_P \tan^2 \varphi_P : 3N^3$$

$$[9] = \rho'' (1 + 2 \tan^2 \varphi_P + \delta \cos^2 \varphi_P) : 2\,N^2$$

$$[10] = \rho'' \tan \varphi_P (5 + 6 \tan^2 \varphi_P + \delta \cos^2 \varphi_P) : 6\,N^3$$

$$[11] = \rho'' \tan \varphi_P (1 + 2 \tan^2 \varphi_P + \delta \cos^2 \varphi_P) : 6\,N^3$$

in which:

$$R = a(1 - e^2) : \left(1 - e^2 \sin^2 \varphi_P\right)^{\frac{3}{2}}$$
$$N = a : (1 - e^2 \sin^2 \varphi_P)^{\frac{1}{2}} \qquad \text{and}$$
$$\delta = e^2 : (1 - e^2).$$

It will be clear that δ is another δ than that used in Delambre's formulae. $\underline{a}$, $\underline{e}$ and φ_P are the same quantities as those of Delambre.

With φ_i as an argument and for any arbitrary ellipsoid given by its $\underline{a}$ and $\underline{b}$ or its $\underline{a}$ and $\underline{p}$ the coefficients [1] up to and including [11] can be tabulated. The accuracy of the table in the said R. D.-publication is as such, that a correct computation of $\varphi_Q - \varphi_P$, etc. in thousandths of a second is found.

With the fourth and the first of the formulae (27) Krayenhoff computed first from the known latitude φ_P (Amsterdam, No. 40) and the azimuth A_{PQ} (Amsterdam-Utrecht) the latitude φ_Q of Utrecht (No. 36) and after this with the second and the third of the formulae the longitude λ_Q of Utrecht and the azimuth A_{QP} at Utrecht to Amsterdam. By adding the adjusted spherical angle 56 to the azimuth A_{PQ} he could find the azimuth A_{PR} in Amsterdam to Nieuwkoop (No. 35) and, with the same formulae used for the determination of Utrecht (Q), the coordinates φ_{35}, λ_{35} of the angular point R (Nieuwkoop) and the azimuth A_{RP} at Nieuwkoop to Amsterdam. From the now known azimuths Amsterdam-Utrecht, Utrecht-Amsterdam, Amsterdam-Nieuwkoop and Nieuwkoop-Amsterdam and the adjusted angles of the network the geographical coordinates of Naarden (in triangle 57), Gouda (triangle 47) and Haarlem (triangle 55) can be determined in the same way, etc. A check is even possible - and Krayenhoff computed every point twice indeed - when e.g. Naarden is computed from the coordinates of Amsterdam and those of Utrecht. As the φ's and λ's of arbitrary points, however, are dependent on the route of the computation in the not-closing network, the results of the two computations will differ, because of a small inaccuracy of the used formulae (27) and the influences of the "adjustment" of the triangulation. Krayenhoff could have separated these two influences - but he did not do so - by computing in his triangle Amsterdam (P)-Utrecht (Q)-Nieuwkoop (R) (and in every other triangle of the network) the successive differences:

$$\varphi_Q - \varphi_P,\ \varphi_R - \varphi_Q,\ \varphi_P - \varphi_R,\ \lambda_Q - \lambda_P,$$
$$\lambda_R - \lambda_Q,\ \lambda_P - \lambda_R,\ \left\{(A_{QP} \pm 180^\circ) - A_{QP}\right\},$$
$$\left\{(A_{RQ} \pm 180^\circ) - A_{QR}\right\} \text{ and } \left\{(A_{PR} \pm 180^\circ) - A_{RP}\right\}, \text{ respectively.}$$

Both the sum of the φ-differences and the λ-differences must be zero. That of the A-differences, diminished with the spherical excess E of the triangle concerned must also be zero.

Table 44

Points i		Latitudes φ_i according to			Diff. v''_i		Longitudes λ_i (Amsterdam = 0) according to			Diff. v''_i	
No.	Name	Adjustment acc. least sq.	R.D.	P.H.	4–3	3–5	Adjustment acc. least sq.	R.D.	P.H.	9–8	8–10
1	2	3	4	5	6	7	8	9	10	11	12
1	Duinkerken	51° 02′ 11″.302	——	09″.65	——	+1″.65	-2° 30′ 27″.721	——	28″.35	——	+0″.63
2	Mont Cassel	50 47 60 .207	——	58 .58	——	+1 .63	-2 23 42 .994	——	43 .59	——	+0 .60
3	Hondschoote	50 58 56 .711	——	55 .07	——	+1 .64	-2 17 51 .503	——	52 .07	——	+0 .57
4	Nieuwpoort	51 07 47 .925	——	46 .27	——	+1 .65	-2 07 58 .252	——	58 .79	——	+0 .54
5	Diksmuide	51 02 05 .201	——	03 .54	——	+1 .66	-2 01 10 .314	——	10 .82	——	+0 .51
6	Oostende	51 13 49 .909	——	48 .24	——	+1 .67	-1 57 48 .375	——	48.87	——	+0 .50
7	Brugge	51 12 32 .742	——	31 .06	——	+1 .68	-1 39 32 .219	——	32 .64	——	+0 .42
8	Hooglede	50 58 44 .802	——	43 .14	——	+1 .66	-1 48 06 .059	——	06 .51	——	+0 .45
9	Tielt	51 00 04 .347	——	02 .67	——	+1 .68	-1 33 24 .060	——	24 .44	——	+0 .38
10	Gent	51 03 14 .239	14″.226	12 .53	-0″.013	+1 .71	-1 09 25 .843	25 .882	26 .11	-0 .039	+0 .27
11	Aardenburg	51 16 26 .259	26 .271	24 .57	+0 .012	+1 .69	-1 26 08 .774	08 .793	09 .15	-0 .019	+0 .38
12	Assenede	51 13 43 .216	43 .211	41 .50	-0 .005	+1 .72	-1 07 49 .049	49 .069	49 .31	-0 .020	+0 .26
13	Middelburg	51 30 01 .721	01 .704	00 .02	-0 .017	+1 .70	-1 16 08 .154	08 .171	08 .48	-0 .017	+0 .33
14	Hulst	51 16 53 .418	53 .404	51 .70	-0 .014	+1 .72	-0 49 46 .416	46 .396	46 .62	-0 .020	+0 .20
15	Antwerpen	51 13 16 .868	16 .872	15 .16	+0 .004	+1 .71	-0 28 58 .643	58 .632	58 .76	+0 .011	+0 .12
16	Zierikzee	51 39 04 .357	04 .345	02 .64	-0 .012	+1 .72	-0 58 08 .004	08 .011	08 .25	-0 .007	+0 .25
17	Bergen op Zoom	51 29 43 .306	43 .297	41 .56	-0 .009	+1 .75	-0 35 44 .347	44 .351	44 .49	-0 .004	+0 .14
18	Hoogstraten	51 24 06 .366	06 .352	04 .63	-0 .014	+1 .74	-0 07 19 .486	19 .485	19 .51	+0 .001	+0 .02
19	Lommel	51 13 45 .714	45 .701	43 .97	-0 .013	+1 .74	+0 25 51 .928	51 .937	52 .04	+0 .009	-0 .11
20	Nederweert	51 17 12 .993	12 .990	11 .26	-0 .003	+1 .73	+0 51 52 .747	52 .748	52 .99	+0 .001	-0 .24
21	Brielle	51 54 12 .936	12 .940	11 .16	+0 .004	+1 .78	-0 43 17 .052	17 .067	17 .25	-0 .015	+0 .20
22	Willemstad	51 41 34 .496	34 .481	32 .74	-0 .015	+1 .76	-0 26 44 .483	44 .480	44 .60	+0 .003	+0 .12
23	Breda	51 35 23 .886	23 .872	22 .13	-0 .014	+1 .76	-0 06 31 .435	31 .419	31 .46	+0 .016	+0 .02
24	Hilvarenbeek	51 29 10 .002	09 .979	08 .24	-0 .023	+1 .78	+0 15 10 .010	10 .018	10 .08	+0 .008	-0 .07
25	Helmond	51 28 45 .954	45 .941	44 .19	-0 .013	+1 .76	+0 46 21 .589	21 .603	21 .80	+0 .014	-0 .21
26	Vierlingsbeek	51 35 50 .051	50 .034	48 .30	-0 .017	+1 .75	+1 07 38 .358	38 .345	38 .66	-0 .013	-0 .30
27	den Haag	52 04 41 .999	42 .016	40 .21	+0 .017	+1 .79	-0 34 37 .213	37 .223	37 .39	-0 .010	+0 .18
28	Rotterdam	51 55 20 .758	20 .762	18 .98	+0 .004	+1 .78	-0 23 55 .024	55 .017	55 .14	+0 .007	+0 .12
29	Dordrecht	51 48 54 .577	——	52 .79	——	+1 .79	-0 13 25 .090	——	25 .16	——	+0 .07
30	Leiden	52 09 25 .275	25 .274	23 .48	-0 .001	+1 .80	-0 23 31 .037	31 .050	31 .17	-0 .013	+0 .13

Table 44 (continued)

1	2	3	4	5	6	7	8	9	10	11	12
31	Gouda	52 00 42 .103	42 .100	40 .30	-0 .003	+1 .80	-0 10 22 .538	22 .540	22 .60	-0 .002	+0 .06
32	Gorinchem	51 49 49 .737	49 .720	47 .95	-0 .017	+1 .79	+0 05 20 .848	20 .854	20 .86	+0 .006	-0 .01
33	's-Hertogenbosch	51 41 20 .255	20 .246	18 .49	-0 .009	+1 .77	+0 25 27 .420	27 .426	27 .52	+0 .006	-0 .10
34	Grave	51 45 39 .638	—	37 .88	—	+1 .76	+0 51 23 .781	—	24 .00	—	-0 .22
35	Nieuwkoop	52 09 05 .227	—	03 .42	—	+1 .81	-0 06 13 .545	—	13 .58	—	+0 .04
36	Utrecht	52 05 29 .859	29 .868	28 .06	+0 .009	+1 .80	+0 14 16 .166	16 .159	16 .22	-0 .007	-0 .05
37	Rhenen	51 57 28 .499	28 .499	26 .73	0 .000	+1 .77	+0 40 50 .674	50 .661	50 .82	-0 .013	-0 .15
38	Nijmegen	51 50 56 .123	56 .111	54 .35	-0 .012	+1 .77	+0 58 44 .076	44 .076	44 .33	0 .000	-0 .25
39	Haarlem	52 22 55 .522	55 .529	53 .71	+0 .007	+1 .81	-0 14 46 .895	46 .912	46 .97	-0 .017	+0 .08
40	Amsterdam	52 22 31 .956	31 .957	30 .13	+0 .001	+1 .83	+0 00 00 .011	00 .000	00 .00	-0 .011	+0 .01
41	Naarden	52 17 48 .161	48 .183	46 .35	+0 .022	+1 .81	+0 16 43 .590	43 .568	43 .65	-0 .022	-0 .06
42	Amersfoort	52 09 22 .168	22 .178	20 .38	+0 .010	+1 .79	+0 30 13 .518	13 .522	13 .65	+0 .004	-0 .13
43	Imbosch	52 02 16 .384	—	14 .64	—	+1 .74	+1 06 38 .666	—	38 .97	—	-0 .30
44	Hettenheuvel	51 55 19 .401	—	17 .64	—	+1 .76	+1 21 04 .158	—	04 .55	—	-0 .39
45	Bocholt	51 50 20 .832	—	19 .08	—	+1 .75	+1 43 49 .132	—	49 .65	—	-0 .52
46	Harderwijk	52 20 60 .114	—	58 .34	—	+1 .77	+0 43 58 .933	—	59 .14	—	-0 .21
47	Veluwe	52 14 08 .421	08 .449	06 .65	+0 .028	+1 .77	+0 58 28 .030	28 .025	28 .33	-0 .005	-0 .30
48	Zutphen	52 08 26 .290	26 .306	24 .54	+0 .016	+1 .75	+1 18 42 .934	42 .940	43 .33	+0 .006	-0 .40
49	Groenlo	52 02 35 .962	35 .990	34 .22	+0 .028	+1 .74	+1 44 04 .176	04 .221	04 .71	+0 .045	-0 .53
50	Harikerberg	52 14 12 .038	—	10 .32	—	+1 .72	+1 39 23 .937	—	24 .48	—	-0 .54
51	Ahaus	52 04 36 .608	—	34 .88	—	+1 .73	+2 07 23 .486	—	24 .10	—	-0 .61
52	Alkmaar	52 37 56 .586	56 .591	54 .76	+0 .005	+1 .83	-0 08 00 .372	00 .351	00 .43	+0 .021	+0 .06
53	Edam	52 30 48 .097	—	46 .28	—	+1 .82	+0 09 48 .368	—	48 .39	—	-0 .02
54	Hoorn	52 38 29 .267	—	27 .52	—	+1 .75	+0 10 34 .640	—	34 .60	—	+0 .04
55	Schagen	52 47 15 .697	—	13 .93	—	+1 .77	-0 05 14 .758	—	14 .89	—	+0 .13
56	Medemblik	52 46 27 .180	27 .208	25 .43	+0 .028	+1 .73	+0 13 11 .560	11 .585	11 .49	+0 .025	+0 .07
57	Enkhuizen	52 42 17 .281	17 .283	15 .54	+0 .002	+1 .74	+0 24 32 .834	32 .842	32 .85	+0 .008	-0 .02
58	Urk	52 39 48 .091	48 .107	46 .36	+0 .016	+1 .73	+0 42 35 .594	35 .624	35 .74	+0 .030	-0 .15
59	Kampen	52 33 36 .371	36 .373	34 .64	+0 .002	+1 .73	+1 01 57 .988	57 .996	58 .27	+0 .008	-0 .28
60	Lemelerberg	52 28 26 .349	—	24 .65	—	+1 .70	+1 31 27 .061	—	27 .55	—	-0 .49
61	Oldenzaal	52 18 48 .148	48 .185	46 .44	+0 .037	+1 .71	+2 02 42 .574	42 .554	43 .27	-0 .020	-0 .70
62	Bentheim	52 18 12 .148	12 .197	10 .48	+0 .049	+1 .67	+2 16 25 .250	25 .221	26 .05	-0 .029	-0 .80
63	Uelsen	52 30 49 .062	—	47 .39	—	+1 .67	+1 59 11 .809	—	12 .48	—	-0 .67
64	Kirch Hesepe	52 37 33 .698	33 .732	32 .06	+0 .034	+1 .64	+2 21 11 .160	11 .084	12 .00	-0 .076	-0 .84
65	Kijkduin	52 57 07 .603	—	05 .81	—	+1 .79	-0 09 43 .164	—	43 .40	—	+0 .24
66	Oosterland	52 55 56 .854	56 .869	55 .09	+0 .015	+1 .76	+0 07 37 .196	37 .232	37 .04	+0 .036	+0 .16
67	Staveren	52 52 58 .842	—	57 .12	—	+1 .72	+0 28 36 .694	—	36 .68	—	+0 .01
68	Lemmer	52 50 45 .278	45 .295	43 .58	+0 .017	+1 .70	+0 49 39 .437	39 .434	39 .59	-0 .003	-0 .15

Table 44 (continued)

1	2	3	4	5	6	7	8	9	10	11	12
69	Blokzijl	52 43 41 . 359	41 . 376	39 . 66	+0 . 017	+1 . 70	+1 04 38 . 894	38 . 910	39 . 17	+0 . 016	-0 . 28
70	Meppel	52 41 54 . 199	54 . 204	52 . 51	+0 . 005	+1 . 69	+1 18 23 . 343	23 . 343	23 . 71	0 . 000	-0 . 37
71	Oosterend	53 05 06 . 832	06 . 863	05 . 07	+0 . 031	+1 . 76	-0 00 37 . 798	37 . 761	38 . 06	+0 . 037	+0 . 26
72	Robbezand	53 03 48 . 290	—	46 . 56	—	+1 . 73	+0 15 54 . 045	—	53 . 90	—	+0 . 14
73	Vlieland	53 17 49 . 172	49 . 154	47 . 46	-0 . 018	+1 . 71	+0 10 28 . 915	29 . 018	28 . 66	+0 . 103	+0 . 26
74	Harlingen	53 10 30 . 816	30 . 806	29 . 11	-0 . 010	+1 . 71	+0 31 42 . 834	42 . 871	42 . 73	+0 . 037	+0 . 10
75	Sneek	53 01 58 . 669	58 . 670	56 . 98	+0 . 001	+1 . 69	+0 46 30 . 583	30 . 594	30 . 63	+0 . 011	-0 . 05
76	Oldeholtpa	52 53 46 . 000	45 . 984	44 . 33	-0 . 016	+1 . 67	+1 10 05 . 834	05 . 852	06 . 11	+0 . 018	-0 . 28
77	Midsland	53 23 02 . 849	—	01 . 16	—	+1 . 69	+0 24 04 . 831	—	04 . 63	—	+0 . 20
78	Ballum	53 26 32 . 621	—	30 . 96	—	+1 . 66	+0 48 15 . 996	—	15 . 90	—	+0 . 10
79	Leeuwarden	53 12 14 . 750	14 . 750	13 . 09	0 . 000	+1 . 66	+0 54 22 . 661	22 . 677	22 . 66	+0 . 016	0 . 00
80	Dokkum	53 19 40 . 876	—	39 . 27	—	+1 . 61	+1 06 51 . 202	—	51 . 28	—	-0 . 08
81	Drachten	53 06 26 . 792	26 . 774	25 . 16	-0 . 018	+1 . 63	+1 13 03 . 260	03 . 256	03 . 48	-0 . 004	-0 . 22
82	Oosterwolde	52 59 41 . 567	41 . 556	39 . 93	-0 . 011	+1 . 64	+1 24 35 . 260	35 . 263	35 . 61	+0 . 003	-0 . 35
83	Beilen	52 51 32 . 664	32 . 648	31 . 01	-0 . 016	+1 . 65	+1 37 57 . 865	57 . 853	58 . 34	-0 . 012	-0 . 48
84	Coevorden	52 39 41 . 699	41 . 702	40 . 05	+0 . 003	+1 . 65	+1 51 23 . 103	23 . 081	23 . 71	-0 . 022	-0 . 61
85	Hornhuizen	53 23 20 . 457	20 . 466	18 . 94	+0 . 009	+1 . 52	+1 28 33 . 806	33 . 800	34 . 02	-0 . 006	-0 . 21
86	Groningen	53 13 13 . 774	13 . 783	12 . 23	+0 . 009	+1 . 54	+1 41 05 . 760	05 . 731	06 . 15	-0 . 029	-0 . 39
87	Rolde	52 59 23 . 896	23 . 880	22 . 29	-0 . 016	+1 . 61	+1 45 44 . 462	44 . 463	44 . 98	+0 . 001	-0 . 52
88	Sleen	52 46 37 . 330	37 . 333	35 . 70	+0 . 003	+1 . 63	+1 55 09 . 508	09 . 494	10 . 12	-0 . 014	-0 . 61
89	Uithuizermeden	53 24 33 . 970	33 . 979	32 . 51	+0 . 009	+1 . 46	+1 49 35 . 230	35 . 212	35 . 61	-0 . 018	-0 . 38
90	Holwierde	53 21 30 . 573	—	29 . 13	—	+1 . 44	+1 59 15 . 013	—	15 . 51	—	-0 . 50
91	Midwolda	53 11 43 . 026	43 . 007	41 . 53	-0 . 019	+1 . 50	+2 07 36 . 417	36 . 373	37 . 07	-0 . 044	-0 . 65
92	Onstwedde	53 01 53 . 941	53 . 919	52 . 38	-0 . 022	+1 . 56	+2 09 39 . 839	39 . 802	40 . 56	-0 . 037	-0 . 72
93	Pilsum	53 29 03 . 073	03 . 055	01 . 71	-0 . 018	+1 . 36	+2 10 46 . 711	46 . 758	47 . 30	+0 . 047	-0 . 59
94	Emden	53 22 04 . 707	—	03 . 31	—	+1 . 40	+2 19 24 . 610	—	25 . 33	—	-0 . 72
95	Leer	53 13 47 . 317	47 . 277	45 . 90	-0 . 040	+1 . 42	+2 33 59 . 783	59 . 746	60 . 70	-0 . 037	-0 . 92
96	Barssel	53 10 13 . 244	—	11 . 85	—	+1 . 39	+2 51 33 . 802	—	34 . 93	—	-1 . 13
97	Hage	53 36 12 . 551	—	11 . 27	—	+1 . 28	+2 24 01 . 932	—	02 . 63	—	-0 . 70
98	Aurich	53 28 14 . 350	—	13 . 03	—	+1 . 32	+2 35 48 . 250	—	49 . 12	—	-0 . 87
99	Strakholt	53 22 12 . 098	—	10 . 77	—	+1 . 33	+2 45 26 . 214	—	27 . 22	—	-1 . 01
100	Westerstede	53 15 30 . 998	—	29 . 67	—	+1 . 33	+3 02 41 . 309	—	42 . 53	—	-1 . 22
101	Esens	53 38 55 . 290	—	54 . 08	—	+1 . 21	+2 43 46 . 584	—	47 . 51	—	-0 . 93
102	Jever	53 34 23 . 929	—	22 . 71	—	+1 . 22	+3 01 11 . 070	—	12 . 22	—	-1 . 15
103	Varel	53 23 54 . 670	—	═	—	═	+3 15 12 . 419	—	═	—	═
104	Herentals	51 10 31 . 492	31 . 468	29 . 77	-0 . 024	+1 . 72	-0 02 51 . 906	51 . 886	51 . 92	+0 . 020	+0 . 01
105	Biesselt	51 45 57 . 757	—	55 . 99	—	+1 . 77	+1 01 08 . 070	—	08 . 33	—	-0 . 26
106	Borkum	53 35 20 . 481	—	19 . 07	—	+1 . 41	+1 47 14 . 547	—	14 . 84	—	-0 . 29

The amounts φ_i and λ_i (i = 1, 2 .... 105, 106), published in tableau V of the Précis Historique, are mentioned in columns 5 and 10 respectively of table 44. They relate to the stations i in columns 1 and 2 of the table. I don't know why Varel (No. 103) is missing in this list. Its coordinates are mentioned indeed in tableau IV of the book.

Columns 4 and 3 give the latitudes φ of the stations according to the R. D. and to my own adjustment of Krayenhoff's triangulation. The longitudes (Amsterdam=0) can be found in columns 9 and 8 respectively. I computed - that's to say the computer computed for me - these latitudes and longitudes from the coordinates $X_i'Y_i'$ and $X_i''Y_i''$ in columns 5-6 and 7-8 respectively of table 26. De Groot's formulae used for the computation can be found in [82] , page 6. They run as follows:

$$\varphi_i = 52^\circ 09' 22''.178 + 3236.033\, Y_i' - 32.592\, X_i'^2 - 0.247\, Y_i'^2 - 0.850\, X_i'^2 Y_i' - 0.065\, Y_i'^3 + 0.005\, X_i'^4 - 0.017\, X_i'^2 Y_i'^2 ,$$

$$\lambda_i = 0^\circ 30' 13''.522 + 5261.305\, X_i' + 105.979\, X_i' Y_i' + 2.458\, X_i' Y_i'^2 - 0.819\, X_i'^3 + 0.056\, X_i' Y_i'^3 - 0.056\, X_i'^3 Y_i' .$$

For the computation of the φ's and λ's of Krayenhoff's network, adjusted according to the least squares and adapted as well as possible to the R. D. -system $X'Y'$, the coordinates $X''Y''$ must of course be used. In the given formulae $X_i'Y_i'$ ($X_i''Y_i''$) have a unit of length of 100 km. It will be clear that the φ's and λ's relate to a position on Bessel's ellipsoid. For the longitudes with respect to Greenwich the λ_i's must be augmented with $4^\circ 53' 01''.978$.

It is interesting to consider the longitude of Duinkerken: $\lambda_1 = -2^\circ 30' 27''.721 + 4^\circ 53' 01''.978 = +2^\circ 22' 34''.257$ with respect to Greenwich, found from my adjustment of Krayenhoff's triangulation and the amount $0^\circ 02' 23''.000$ east of Paris found by Delambre. As, according to the Astronomical Ephemeris 1971, Cassini's meridian of Paris lies $0^h 09^m 20^s.91$ east of Greenwich, Delambre would have found for Duinkerken $\lambda_1 = +2^\circ 22' 36''.6$ with respect to Greenwich. It differs $2''.3$ or $0^s.15$ from the amount just mentioned. In this difference, in my opinion very small, not only the inaccuracies of Delambre's measurement between Paris and Duinkerken are included and those of Krayenhoff's triangulation between Amsterdam and Duinkerken, but also those in the determination of the differences in longitude between Greenwich and Paris and Greenwich and Amersfoort respectively on two different ellipsoids. For the latter difference the difference $0^h 17^m 56^s.15 = 4^\circ 29' 02''.250$ between Greenwich and the Leiden Observatory was already determined in 1880 and 1881 by H. G. van de Sande Bakhuyzen [87] . That between the meridian circle at Leiden ($X' = -61832.511$, $Y' = +346.653$) and Amersfoort ($X' = Y' = 0.000$) is $0^{\circ}54'13''.228$. It was computed in 1897 from the R. D.-triangulation network. The sum of the two amounts $5^{\circ}23'15''.478$, the longitude of Amersfoort, was rounded off by the R. D. at $5^{\circ}23'15''.500$.

The latitude $\varphi_1 = 51^{\circ}02'11''.302$ of Duinkerken, computed from my adjustment of the network, differs $2''.57$ from the result $51^{\circ}02'08''.730$ of Delambre's measurement and $1''.65$ from Krayenhoff's result $51^{\circ}02'09''.65$ in tableau V of the Précis Historique. The latter difference, however, must be imputed to Amsterdam's latitude $\varphi_{40} = 52^{\circ}22'30''.13$ from which he started his computation (column 5 of table 44). If he had used the correct latitude $52^{\circ}22'31''.96$ (column 4), he would have found about $1''.83$ less or $-0''.18$. As the difference $1''.83$ holds for all the latitudes in Krayenhoff's computation of the network, it will be clear that the amounts in column 7 of the table are all of them positive. For the small projections in the direction north-south of the vectors in Fig. 20 (section 21) represent but fractions of a second of arc in latitude.

As Krayenhoff's computation of the sides of his network, however, is influenced by a gradually changing scale factor, this influence must be perceptible in the latitudes. If one diminishes the differences in column 7 with the just mentioned amount $1''.83$ in Amsterdam, one finds, as already said, for Duinkerken $-0''.18$ but for Jever $+0''.61$. They demonstrate once again that in the southern part of the network Krayenhoff's side lengths are too short and in the northern part too long.

Analogous considerations may be held for the comparison between the λ's in column 8 and those in column 10 (table 44). Their differences in column 11 are affected by the scale factors already mentioned before. In table 45 I give an example both for an almost constant longitude (φ-influence, left side of the table) and an almost constant latitude (λ-influence, right side). In both parts of the table the connection between the columns 3 and 4 is very clear.

Table 45

Stations		Scale factors table 28	Column 7 table 44 minus 1''.83	Stations		Scale factors table 28	Column 12 table 44 minus 0''.01
No.	Name			No.	Name		
1	2	3	4	1	2	3	4
49	Groenlo	1.000023	$-0''.09$	74	Harlingen	1.000046	$+0''.09$
50	Harikerberg	1.000018	-0.11	79	Leeuwarden	1.000030	-0.01
60	Lemelerberg	1.000018	-0.13	86	Groningen	0.999985	-0.40
83	Beilen	1.000008	-0.18	91	Midwolda	0.999957	-0.66
87	Rolde	0.999986	-0.22	99	Strakholt	0.999935	-1.02
86	Groningen	0.999985	-0.29				
89	Uithuizermeden	0.999980	-0.37				

For a comparison of the geographical azimuths of the triangulation network in the various systems I made table 46. In column 6 are Krayenhoff's azimuths; they are borrowed from tableau V of the Précis Historique. The azimuths in Duinkerken (station No. 1) are in No. 1 of the table. That one to Nieuwpoort (station No. 4) is $248^{\circ}14'55''.97$, etc. In column 4, I give the azimuths according to my own adjustment of the network and - where possible - in column 5 the azimuths according to the R. D.-triangulation. Those in columns 4 and 5 are computed from the coordinates $X''Y''$ and $X'Y'$ respectively in table 26.

An example of the computation of the R. D.-azimuth Rhenen (No. 37)-Gorinchem (No. 32) is illustrated in Fig. 30.

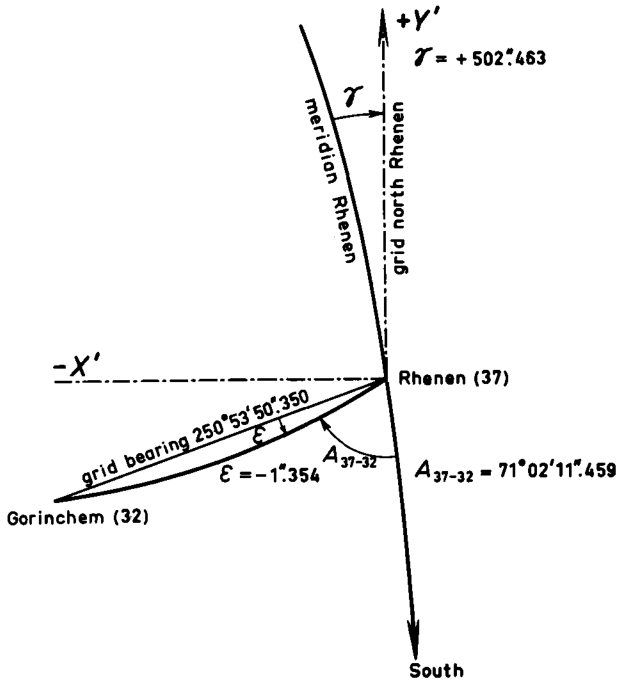


Fig. 30

In Rhenen the line RY' is parallel to the Y'-axis of the R. D.-coordinate system, the direction of the astronomical north at Amersfoort. The angle γ between the meridian of Rhenen and the Y'-axis, counted from the meridian in a clockwise direction, is the convergence of the meridians. It can be computed with De Groot's formula on page 6 of the H.T.W. [49] :

$$\gamma''_{37} = +4154.7761\, X'_{37} + 109.0111\, X'_{37}Y'_{37} + 2.4507\, X'_{37}Y'^{2}_{37} - 0.8168\, X'^{3}_{37} + $$
$$+0.0561\, X'_{37}Y'^{3}_{37} - 0.0561\, X'^{3}_{37}Y'_{37} = +502''.463.$$

Table 46

No.	From	No.	Geographical Azimuths			Differences v″	
	to		Adj. least sq.	R.D.	P.H.	5–4	4–6
1	2	3	4	5	6	7	8
1	Duinkerken				′ ″		
	Nieuwpoort	4	248 14 54.74	—	14 55.97	—	-1.25
	Hondschoote	3	292 06 29.58	—	06 30.21	—	-0.63
	Mont Cassel	2	343 13 32.51	—	13 33.57	—	-1.06
2	Mont Cassel						
	Duinkerken	1	163 18 46.67	—	18 47.76	—	-1.09
	Hondschoote	3	198 40 09.64	—	40 10.48	—	-0.84
3	Hondschoote						
	Mont Cassel	2	18 44 42.38	—	44 43.24	—	-0.86
	Duinkerken	1	112 16 17.35	—	16 18.02	—	-0.67
	Nieuwpoort	4	215 04 28.27	—	04 29.18	—	-0.91
	Diksmuide	5	253 16 32.52	—	16 33.47	—	-0.95
4	Nieuwpoort						
	Hondschoote	3	35 12 09.68	—	12 10.61	—	-0.93
	Duinkerken	1	68 32 24.71	—	32 26.00	—	-1.29
	Oostende	6	226 34 26.55	—	34 27.41	—	-0.86
	Diksmuide	5	323 05 46.30	—	05 47.40	—	-1.10
5	Diksmuide						
	Hondschoote	3	73 29 30.69	—	29 31.69	—	-1.00
	Nieuwpoort	4	143 11 03.69	—	11 04.84	—	-1.15
	Oostende	6	190 11 54.11	—	11 55.07	—	-0.96
	Brugge	7	232 19 44.82	—	19 45.81	—	-0.99
	Hooglede	8	291 57 51.97	—	57 52.90	—	-0.93
6	Oostende						
	Diksmuide	5	10 14 31.34	—	14 32.30	—	-0.96
	Nieuwpoort	4	46 42 21.72	—	42 22.61	—	-0.89
	Brugge	7	276 16 39.02	—	16 40.05	—	-1.03
7	Brugge						
	Hooglede	8	21 24 05.59	—	24 06.62	—	-1.03
	Diksmuide	5	52 36 35.37	—	36 36.40	—	-1.03
	Oostende	6	96 30 53.54	—	30 54.61	—	-1.07
	Aardenburg	11	245 04 01.14	—	04 01.86	—	-0.72
	Gent	10	295 58 26.11	—	58 27.30	—	-1.19
	Tielt	9	342 45 08.93	—	45 09.95	—	-1.02
8	Hooglede						
	Diksmuide	5	112 08 01.51	—	08 02.47	—	-0.96
	Brugge	7	201 17 25.73	—	17 26.75	—	-1.02
	Tielt	9	261 46 20.65	—	46 21.65	—	-1.00
9	Tielt						
	Hooglede	8	81 57 46.00	—	57 47.05	—	-1.05
	Brugge	7	162 49 55.47	—	49 56.52	—	-1.05
	Gent	10	258 01 10.23	—	01 11.15	—	-0.92
10	Gent						
	Tielt	9	78 19 48.37	—	19 49.37	—	-1.00
	Brugge	7	116 21 52.54	—	21 53.83	—	-1.29
	Aardenburg	11	141 34 56.05	34 61.23	34 57.09	+5.18	-1.04
	Assenede	12	185 31 08.16	31 11.81	31 09.27	+3.65	-1.11
	Hulst	14	222 01 23.70	01 28.92	01 24.20	+5.22	-0.50
	Antwerpen	15	248 12 03.13	12 02.57	12 03.78	-0.56	-0.65

Table 46 (continued)

1	2	3	4	5	6	7	8
11	Aardenburg						
	Brugge	7	65 14 27.66	———	14 28.39	———	-0.73
	Middelburg	13	204 40 37.46	40 40.53	40 38.00	+3.07	-0.54
	Assenede	12	283 10 23.31	10 28.25	10 24.08	+4.94	-0.77
	Gent	10	321 21 54.82	21 60.01	21 55.78	+5.19	-0.96
12	Assenede						
	Gent	10	5 32 23.53	32 27.19	32 24.63	+3.66	-1.10
	Aardenburg	11	103 24 40.99	24 45.93	24 41.83	+4.94	-0.84
	Middelburg	13	162 20 44.58	20 44.16	20 45.48	-0.42	-0.90
	Hulst	14	254 14 30.76	14 35.14	14 31.36	+4.38	-0.60
13	Middelburg						
	Aardenburg	11	24 48 26.77	48 29.84	48 27.33	+3.07	-0.56
	Zierikzee	16	231 00 17.29	00 17.40	00 18.81	+0.11	-1.52
	Hulst	14	308 22 07.93	22 05.11	22 09.13	-2.82	-1.20
	Assenede	12	342 14 14.71	14 14.30	14 15.56	-0.41	-0.85
14	Hulst						
	Gent	10	42 16 42.45	16 47.72	16 42.98	+5.27	-0.53
	Assenede	12	74 28 35.15	28 39.56	28 35.78	+4.41	-0.63
	Middelburg	13	128 42 43.94	42 41.15	42 45.23	-2.79	-1.29
	Zierikzee	16	166 48 31.34	48 28.86	48 32.08	-2.48	-0.74
	Bergen op Zoom	17	214 17 37.24	17 33.90	17 37.48	-3.34	-0.24
	Antwerpen	15	285 19 25.02	19 21.02	19 25.70	-4.00	-0.68
15	Antwerpen						
	Gent	10	68 43 33.10	43 32.57	43 33.81	-0.53	-0.71
	Hulst	14	105 35 38.16	35 34.16	35 38.89	-4.00	-0.73
	Bergen op Zoom	17	165 36 15.22	36 12.64	36 15.14	-2.58	+0.08
	Hoogstraten	18	231 16 47.88	16 50.01	16 48.70	+2.13	-0.82
	Herentals	104	279 22 02.24	22 07.80	22 03.51	+5.56	-1.27
16	Zierikzee						
	Middelburg	13	51 14 23.52	14 23.63	14 25.08	+0.11	-1.56
	Brielle	21	211 12 50.16	12 47.76	12 48.84	-2.40	+1.32
	Willemstad	22	262 29 21.27	29 21.82	29 20.82	+0.55	+0.45
	Bergen op Zoom	17	303 40 46.86	40 46.04	40 47.32	-0.82	-0.46
	Hulst	14	346 41 58.98	41 56.47	41 59.67	-2.51	-0.69
17	Bergen op Zoom						
	Hulst	14	34 28 35.23	28 31.87	28 35.50	-3.36	-0.27
	Zierikzee	16	123 58 19.49	58 18.68	58 20.02	-0.81	-0.53
	Willemstad	22	205 14 45.19	14 46.88	14 44.93	+1.69	+0.26
	Breda	23	252 30 09.90	30 11.45	30 10.02	+1.55	-0.12
	Hoogstraten	18	287 21 59.81	21 60.67	21 60.33	+0.86	-0.52
	Antwerpen	15	345 30 58.34	30 55.73	30 58.24	-2.61	+0.10
18	Hoogstraten						
	Antwerpen	15	51 53 41.95	33 44.07	33 42.84	+2.12	-0.89
	Bergen op Zoom	17	107 44 13.10	44 13.96	44 13.70	+0.86	-0.60
	Breda	23	182 31 46.25	31 49.22	31 46.34	+2.97	-0.09
	Hilvarenbeek	24	250 03 07.15	03 09.13	03 07.17	+1.98	-0.02
	Lommel	19	296 13 35.70	13 35.14	13 35.48	-0.56	+0.22
	Herentals	104	348 20 10.78	20 08.29	20 11.19	-2.49	-0.41
19	Lommel						
	Herentals	104	80 01 08.65	01 06.50	01 09.54	-2.15	-0.89
	Hoogstraten	18	116 39 30.20	39 29.65	39 30.10	-0.55	+0.10
	Hilvarenbeek	24	156 34 09.00	34 08.12	34 08.98	-0.88	+0.02
	Helmond	25	220 24 20.52	24 20.91	24 19.82	+0.39	+0.70
20	Nederweert						
	Lommel	19	78 13 08.06	13 05.68	13 07.57	-2.38	+0.49
	Helmond	25	163 23 14.78	23 16.18	23 13.32	+1.40	+1.46
	Vierlingsbeek	26	207 46 37.04	46 36.98	46 36.39	-0.06	+0.65

Table 46 (continued)

1	2	3	4	5	6	7	8
21	Brielle						
	Zierikzee	16	31 24 30.10	24 27.70	24 28.79	-2.40	+1.31
	den Haag	27	206 58 19.77	58 18.75	58 16.29	-1.02	+3.48
	Rotterdam	28	264 28 53.79	28 54.36	28 52.85	+0.57	+0.94
	Willemstad	22	320 50 06.06	50 06.82	50 05.82	+0.76	+0.24
22	Willemstad						
	Bergen op Zoom	17	25 21 48.24	21 49.94	21 47.99	+1.70	+0.25
	Zierikzee	16	82 53 58.85	53 59.40	53 58.47	+0.55	+0.38
	Brielle	21	141 03 06.06	03 06.83	03 05.88	+0.77	+0.18
	Rotterdam	28	187 13 36.89	13 36.84	13 35.74	-0.05	+1.15
	Dordrecht	29	228 20 20.05	———	20 20.03	———	+0.02
	Breda	23	296 01 14.92	01 13.84	01 14.56	-1.08	+0.36
23	Breda						
	Hoogstraten	18	2 32 23.85	32 26.83	32 23.94	+2.98	-0.09
	Bergen op Zoom	17	72 53 02.56	53 04.12	53 02.76	+1.56	-0.20
	Willemstad	22	116 17 06.12	17 05.05	17 05.84	-1.07	+0.28
	Dordrecht	29	162 27 15.86	———	27 14.93	———	+0.93
	Gorinchem	32	206 59 30.80	59 29.90	59 29.91	-0.90	+0.89
	's-Hertogenb.	33	253 10 18.21	10 17.04	10 17.70	-1.17	+0.51
	Hilvarenbeek	24	294 35 34.26	35 36.50	35 33.63	+2.24	+0.63
24	Hilvarenbeek						
	Hoogstraten	18	70 20 42.45	20 44.44	20 42.56	+1.99	-0.11
	Breda	23	114 52 33.32	52 35.55	52 32.77	+2.23	+0.55
	's-Hertogenb.	33	207 42 26.45	42 24.51	42 25.03	-1.94	+1.42
	Helmond	25	270 58 31.78	58 30.20	58 31.53	-1.58	+0.25
	Lommel	19	336 25 47.63	25 46.74	25 47.57	-0.89	+0.06
25	Helmond						
	Lommel	19	40 40 20.92	40 21.31	40 20.26	+0.39	+0.66
	Hilvarenbeek	24	91 22 56.15	22 54.57	22 55.97	-1.58	+0.18
	's-Hertogenb.	33	134 07 42.47	07 42.44	07 41.76	-0.03	+0.71
	Grave	34	190 28 47.43	———	28 46.89	———	+0.54
	Vierlingsbeek	26	241 49 09.31	49 08.34	49 08.98	-0.97	+0.33
	Nederweert	20	343 18 56.03	18 57.44	18 54.55	+1.41	+1.48
26	Vierlingsbeek						
	Nederweert	20	27 58 56.50	58 56.42	58 55.91	-0.08	+0.59
	Helmond	25	62 05 49.05	05 48.07	05 48.82	-0.98	+0.23
	Grave	34	134 19 34.00	———	19 33.58	———	+0.42
	Biesselt	105	158 16 44.52	———	16 44.40	———	+0.12
27	den Haag						
	Brielle	21	27 05 09.36	05 08.34	05 05.89	-1.02	+3.47
	Leiden	30	235 17 31.32	17 37.16	17 28.13	+5.84	+3.19
	Rotterdam	28	324 41 53.34	41 53.17	41 51.23	-0.17	+2.11
28	Rotterdam						
	Willemstad	22	7 15 50.07	15 50.03	15 48.93	-0.04	+1.14
	Brielle	21	84 44 08.39	44 08.98	44 07.51	+0.59	+0.88
	den Haag	27	144 50 19.39	50 19.24	50 17.33	-0.15	+2.06
	Leiden	30	181 00 03.01	00 00.20	00 00.86	-2.81	+2.15
	Gouda	31	237 16 45.94	16 46.89	16 45.79	-0.95	+0.15
	Dordrecht	29	314 39 08.48	———	39 07.99	———	+0.49
29	Dordrecht						
	Willemstad	22	48 30 47.86	———	30 47.89	———	-0.03
	Rotterdam	28	134 47 23.98	———	47 23.54	———	+0.44
	Gouda	31	189 02 45.35	———	02 44.52	———	+0.83
	Gorinchem	32	265 21 24.53	———	21 23.86	———	+0.67
	Breda	23	342 21 51.22	———	21 50.27	———	+0.95

Table 46 (continued

1	2	3	4	5	6	7	8
30	Leiden						
	Rotterdam	28	1 00 21.92	00 19.10	00 19.76	-2.82	+2.16
	den Haag	27	55 26 17.12	26 22.95	26 13.96	+5.83	+3.16
	Haarlem	39	201 35 18.83	35 17.50	35 20.72	-1.33	-1.89
	Nieuwkoop	35	271 41 08.06	———	41 08.92	———	-0.86
	Gouda	31	317 02 13.17	02 12.27	02 12.25	-0.90	+0.92
31	Gouda						
	Dordrecht	29	9 05 09.04	———	05 08.20	———	+0.84
	Rotterdam	28	57 27 25.90	27 26.84	27 25.80	+0.94	+0.10
	Leiden	30	137 12 35.23	12 34.34	12 34.37	-0.89	+0.86
	Nieuwkoop	35	196 55 45.53	———	55 43.92	———	+1.61
	Utrecht	36	252 19 12.02	19 09.43	19 10.76	-2.59	+1.26
	Gorinchem	32	318 05 42.57	05 43.82	05 42.20	+1.25	+0.37
32	Gorinchem						
	Breda	23	27 08 49.86	08 48.96	08 49.01	-0.90	+0.85
	Dordrecht	29	85 36 09.63	———	36 09.02	———	+0.61
	Gouda	31	138 18 05.17	18 06.42	18 04.85	+1.25	+0.32
	Utrecht	36	199 19 17.72	19 14.39	19 16.79	-3.33	+0.93
	Rhenen	37	250 34 18.54	34 15.57	34 16.61	-2.97	+1.93
	's-Hertogenb.	33	304 06 05.91	06 04.35	06 04.48	-1.56	+1.43
33	's-Hertogenbosch						
	Hilvarenbeek	24	27 50 30.23	50 28.28	50 28.83	-1.95	+1.40
	Breda	23	73 35 22.83	35 21.65	35 22.42	-1.16	+0.41
	Gorinchem	32	124 21 53.58	21 52.02	21 52.21	-1.56	+1.37
	Rhenen	37	210 28 37.99	28 35.37	28 36.81	-2.62	+1.18
	Grave	34	254 48 30.51	———	48 29.68	———	+0.83
	Helmond	25	313 51 19.80	51 19.76	51 19.04	-0.04	+0.76
34	Grave						
	Helmond	25	10 32 44.32	———	32 43.83	———	+0.49
	's-Hertogenb.	33	75 08 52.32	———	08 51.59	———	+0.73
	Rhenen	37	151 07 34.63	———	07 34.14	———	+0.49
	Nijmegen	38	220 43 40.65	———	43 40.52	———	+0.13
	Biesselt	105	267 04 30.57	———	04 30.38	———	+0.19
	Vierlingsbeek	26	314 06 49.39	———	06 48.93	———	+0.46
35	Nieuwkoop						
	Gouda	31	16 59 01.96	———	59 00.34	———	+1.62
	Leiden	30	91 54 47.33	———	54 48.25	———	-0.92
	Haarlem	39	159 16 59.82	———	16 61.24	———	-1.42
	Amsterdam	40	195 49 10.64	———	49 10.16	———	+0.48
	Utrecht	36	285 44 50.98	———	44 49.91	———	+1.07
36	Utrecht						
	Gorinchem	32	19 26 19.33	26 15.99	26 18.43	-3.34	+0.90
	Gouda	31	72 38 38.08	38 35.49	38 36.90	-2.59	+1.18
	Nieuwkoop	35	106 01 01.61	———	01 00.62	———	+0.99
	Amsterdam	40	152 52 37.36	52 36.26	52 36.79	-1.10	+0.57
	Naarden	41	186 58 46.63	58 43.79	58 44.88	-2.84	+1.75
	Amersfoort	42	248 22 44.02	22 44.20	22 41.54	+0.18	+2.48
	Rhenen	37	295 54 04.08	54 05.81	54 01.50	+1.73	+2.58
37	Rhenen						
	's-Hertogenb.	33	30 40 43.77	40 41.14	40 42.64	-2.63	+1.13
	Gorinchem	32	71 02 14.45	02 11.46	02 12.62	-2.99	+1.83
	Utrecht	36	116 14 61.00	14 62.72	14 58.50	+1.72	+2.50
	Amersfoort	42	151 14 36.95	14 40.50	14 34.07	+3.55	+2.88
	Veluwe	47	212 57 32.07	57 30.13	57 31.07	-1.94	+1.00
	Imbosch	43	253 03 49.12	———	03 46.31	———	+2.81
	Nijmegen	38	300 27 49.35	27 50.96	27 48.95	+1.61	+0.40
	Grave	34	330 59 16.69	———	59 16.16	———	+0.53

Table 46 (continued)

1	2	3	4	5	6	7	8
38	Nijmegen						
	Grave	34	40 49 26.68	———	49 26.54	———	+0.14
	Rhenen	37	120 41 54.09	41 55.71	41 53.71	+1.62	+0.38
	Imbosch	43	203 16 19.03	———	16 17.77	———	+1.26
	Hettenheuvel	44	252 14 23.18	———	14 20.76	———	+2.42
	Biesselt	105	343 19 44.28	———	19 43.67	———	+0.61
39	Haarlem						
	Leiden	30	21 42 13.37	42 12.03	42 15.30	-1.34	-1.93
	Alkmaar	52	195 20 48.25	20 53.30	20 47.85	+5.05	+0.40
	Amsterdam	40	272 23 18.96	23 21.28	23 20.00	+2.32	-1.04
	Nieuwkoop	35	339 10 13.82	———	10 15.22	———	-1.40
40	Amsterdam						
	Nieuwkoop	35	15 54 06.06	———	54 05.61	———	+0.45
	Haarlem	39	92 35 01.45	35 03.78	35 02.54	+2.33	-1.09
	Alkmaar	52	162 27 44.79	27 49.17	27 43.43	+4.38	+1.36
	Edam	53	215 51 47.87	———	51 46.51	———	+1.36
	Naarden	41	294 40 10.81	40 05.59	40 10.01	-5.22	+0.80
	Utrecht	36	332 41 20.56	41 19.45	41 19.94	-1.11	+0.62
41	Naarden						
	Utrecht	36	7 00 43.11	00 40.25	00 41.36	-2.86	+1.75
	Amsterdam	40	114 53 25.25	53 20.02	53 24.50	-5.23	+0.75
	Edam	53	162 00 36.94	———	00 35.67	———	+1.27
	Harderwijk	46	258 58 33.32	———	58 29.61	———	+3.71
	Amersfoort	42	315 24 13.12	24 12.33	24 11.58	-0.79	+1.54
42	Amersfoort						
	Utrecht	36	68 35 19.70	35 19.89	35 17.27	+0.19	+2.43
	Naarden	41	135 34 53.32	34 52.55	34 51.83	-0.77	+1.49
	Harderwijk	46	215 52 53.70	———	52 51.63	———	+2.07
	Veluwe	47	254 26 35.34	26 31.85	26 31.93	-3.49	+3.41
	Rhenen	37	331 06 14.48	06 18.04	06 11.57	+3.56	+2.91
43	Imbosch						
	Nijmegen	38	23 22 32.72	———	22 31.56	———	+1.16
	Rhenen	37	73 24 08.92	———	24 06.22	———	+2.70
	Veluwe	47	157 04 36.54	———	04 35.16	———	+1.38
	Zutphen	48	230 15 28.33	———	15 28.12	———	+0.21
	Hettenheuvel	44	307 52 01.59	———	52 00.68	———	+0.91
44	Hettenheuvel						
	Nijmegen	38	72 31 57.53	———	31 55.23	———	+2.30
	Imbosch	43	128 03 23.42	———	03 22.53	———	+0.89
	Zutphen	48	173 41 55.27	———	41 53.76	———	+1.51
	Groenlo	49	242 43 18.83	———	43 18.99	———	-0.16
	Bocholt	45	289 18 57.95	———	18 58.38	———	-0.43
45	Bocholt						
	Hettenheuvel	44	109 36 51.81	———	36 52.34	———	-0.53
	Groenlo	49	180 43 22.72	———	43 23.96	———	-1.24
	Ahaus	51	225 26 33.35	———	26 35.41	———	-2.06
46	Harderwijk						
	Amersfoort	42	36 03 46.38	———	03 44.38	———	+2.00
	Naarden	41	79 20 07.65	———	20 04.06	———	+3.59
	Urk	58	177 25 40.39	———	25 33.98	———	+6.41
	Kampen	59	220 57 30.88	———	57 25.34	———	+5.54
	Veluwe	47	307 35 32.94	———	35 27.94	———	+5.00

Table 46 (continued

1	2	3	4	5	6	7	8
47	Veluwe						
	Rhenen	37	33 11 26.38	11 24.45	11 25.48	-1.93	+0.90
	Amersfoort	42	74 48 54.20	48 50.70	48 50.91	-3.50	+3.29
	Harderwijk	46	127 46 60.52	———	46 55.57	———	+4.95
	Kampen	59	186 15 07.56	15 09.52	15 02.25	+1.96	+5.31
	Lemelerberg	60	234 28 62.53	———	28 59.36	———	+3.17
	Zutphen	48	294 28 60.08	28 62.05	28 58.19	+1.97	+1.89
	Imbosch	43	336 58 09.19	———	58 07.80	———	+1.39
48	Zutphen						
	Imbosch	43	50 24 59.76	———	24 59.59	———	+0.17
	Veluwe	47	114 44 59.89	44 61.87	44 58.06	+1.98	+1.83
	Lemelerberg	60	201 14 12.01	———	14 10.84	———	+1.17
	Harikerberg	50	245 28 44.72	———	28 43.03	———	+1.69
	Groenlo	49	290 19 58.05	19 54.04	19 56.60	-4.01	+1.45
	Hettenheuvel	44	353 40 03.93	———	40 02.45	———	+1.48
49	Groenlo						
	Bocholt	45	0 43 34.57	———	43 35.83	———	-1.26
	Hettenheuvel	44	63 01 26.05	———	01 26.32	———	-0.27
	Zutphen	48	110 39 58.31	39 54.33	39 56.95	-3.98	+1.36
	Harikerberg	50	166 07 04.17	———	07 04.04	———	+0.13
	Ahaus	51	261 53 03.14	———	53 04.24	———	-1.10
50	Harikerberg						
	Zutphen	48	65 45 05.16	———	45 03.58	———	+1.58
	Lemelerberg	60	161 10 58.53	———	10 57.36	———	+1.17
	Oldenzaal	61	252 00 24.12	———	00 24.27	———	-0.15
	Ahaus	51	298 56 06.67	———	56 06.86	———	-0.19
	Groenlo	49	346 03 22.92	———	03 22.82	———	+0.10
51	Ahaus						
	Bocholt	45	45 45 07.25	———	45 09.44	———	-2.19
	Groenlo	49	82 11 26.72	———	11 27.93	———	-1.21
	Harikerberg	50	119 18 13.00	———	18 13.26	———	-0.26
	Oldenzaal	61	168 34 15.46	———	34 15.53	———	-0.07
	Bentheim	62	202 08 58.05	———	08 58.92	———	-0.87
52	Alkmaar						
	Haarlem	39	15 26 10.80	26 15.87	26 10.40	+5.07	+0.40
	Schagen	55	190 10 48.68	———	10 40.57	———	+8.11
	Hoorn	54	267 07 06.95	———	07 02.39	———	+4.56
	Edam	53	303 13 45.54	———	13 42.91	———	+2.63
	Amsterdam	40	342 21 23.66	21 28.06	21 22.25	+4.40	+1.41
53	Edam						
	Amsterdam	40	35 59 34.30	———	59 32.96	———	+1.34
	Alkmaar	52	123 27 54.26	———	27 51.69	———	+2.57
	Hoorn	54	183 29 33.74	———	29 29.05	———	+4.69
	Enkhuizen	57	217 53 63.83	———	53 58.00	———	+5.83
	Naarden	41	341 55 07.94	———	55 06.65	———	+1.29
54	Hoorn						
	Edam	53	3 30 10.49	———	30 05.86	———	+4.63
	Alkmaar	52	87 21 53.17	———	21 48.73	———	+4.44
	Schagen	55	132 30 11.08	———	30 04.43	———	+6.65
	Medemblik	56	191 15 39.85	———	15 32.01	———	+7.84
	Enkhuizen	57	245 47 52.63	———	47 47.13	———	+5.50
55	Schagen						
	Alkmaar	52	10 12 60.44	———	12 52.39	———	+8.05
	Kijkduin	65	164 40 69.25	———	40 55.16	———	+14.09
	Oosterland	66	221 47 27.33	———	47 15.02	———	+12.31
	Medemblik	56	274 00 51.52	———	00 42.90	———	+8.62
	Hoorn	54	312 17 35.71	———	17 28.99	———	+6.72

Table 46 (continued)

1	2	3	4	5	6	7	8
56	Medemblik						
	Hoorn	54	11 17 44.69	———	17 36.82	———	+7.87
	Schagen	55	94 15 32.51	———	15 23.93	———	+8.58
	Oosterland	66	160 28 37.45	28 38.27	28 27.92	+0.82	+9.53
	Staveren	67	234 56 42.50	———	56 32.73	———	+9.77
	Enkhuizen	57	301 04 20.21	04 31.73	04 12.48	+11.52	+7.73
57	Enkhuizen						
	Edam	53	38 05 46.55	———	05 40.81	———	+5.74
	Hoorn	54	65 58 59.15	———	58 53.69	———	+5.46
	Medemblik	56	121 13 22.43	13 33.94	13 14.76	+11.51	+7.67
	Staveren	67	192 56 52.76	———	56 44.74	———	+8.02
	Urk	58	282 39 18.69	39 13.79	39 09.81	- 4.90	+8.88
58	Urk						
	Enkhuizen	57	102 53 39.82	53 34.93	53 31.02	-4.89	+8.80
	Staveren	67	147 20 09.76	———	20 02.22	———	+7.54
	Lemmer	68	201 19 23.89	19 18.35	19 15.33	-5.54	+8.56
	Blokzijl	69	253 40 26.04	40 25.20	40 17.92	-0.84	+8.12
	Kampen	59	297 35 12.21	35 16.95	35 04.79	+4.74	+7.42
59	Kampen						
	Veluwe	47	6 17 53.91	17 55.88	17 48.60	+1.97	+5.31
	Harderwijk	46	41 11 46.44	———	11 40.94	———	+5.50
	Urk	58	117 50 35.78	50 40.51	50 28.46	+4.73	+7.32
	Blokzijl	69	189 10 18.55	10 19.24	10 12.54	+0.79	+6.01
	Meppel	70	230 11 16.06	11 14.57	11 10.17	-1.49	+5.89
	Lemelerberg	60	285 49 57.55	———	49 51.63	———	+5.92
60	Lemelerberg						
	Zutphen	48	21 24 16.66	———	24 15.59	———	+1.07
	Veluwe	47	54 55 09.55	———	55 06.52	———	+3.03
	Kampen	59	106 13 21.37	———	13 15.59	———	+5.78
	Meppel	70	149 30 30.27	———	30 23.22	———	+7.05
	Beilen	83	189 40 54.61	———	40 48.42	———	+6.19
	Coevorden	84	227 02 51.20	———	02 47.06	———	+4.14
	Uelsen	63	261 49 17.75	———	49 16.12	———	+1.63
	Oldenzaal	61	296 32 19.63	———	32 18.60	———	+1.03
	Harikerberg	50	341 04 40.94	———	04 39.74	———	+1.20
61	Oldenzaal						
	Harikerberg	50	72 18 50.38	———	18 50.63	———	-0.25
	Lemelerberg	60	116 57 05.46	———	57 04.54	———	+0.92
	Uelsen	63	169 53 14.47	———	53 13.40	———	+1.07
	Bentheim	62	273 59 35.21	59 30.54	59 34.77	-4.67	+0.44
	Ahaus	51	348 30 33.51	———	30 33.60	———	-0.09
62	Bentheim						
	Ahaus	51	22 16 06.07	———	16 07.08	———	-1.01
	Oldenzaal	61	94 10 26.20	10 21.52	10 25.88	-4.68	+0.32
	Uelsen	63	140 15 09.53	———	15 08.49	———	+1.04
	Kirch Hesepe	64	188 31 05.39	31 00.81	31 04.82	-4.58	+0.57
63	Uelsen						
	Lemelerberg	60	82 11 18.37	———	11 16.89	———	+1.48
	Coevorden	84	151 51 45.07	———	51 41.04	———	+4.03
	Kirch Hesepe	64	243 08 09.86	———	08 08.15	———	+1.71
	Bentheim	62	320 01 30.65	———	01 29.52	———	+1.13
	Oldenzaal	61	349 50 27.46	———	50 26.41	———	+1.05
64	Kirch Hesepe						
	Bentheim	62	8 34 52.11	34 47.50	34 51.57	-4.61	+0.54
	Uelsen	63	63 25 37.55	———	25 35.96	———	+1.59
	Coevorden	84	96 54 34.28	54 29.29	54 32.35	-4.99	+1.93

Table 46 (continued)

1	2	3	4	5	6	7	8
65	Kijkduin						
	Oosterend	71	214 23 56.25	———	23 38.51	———	+17.74
	Oosterland	66	276 18 27.48	———	18 15.31	———	+12.17
	Schagen	55	344 37 35.25	———	37 21.09	———	+14.16
66	Oosterland						
	Schagen	55	41 57 42.70	———	57 30.37	———	+12.33
	Kijkduin	65	96 32 17.72	———	32 05.60	———	+12.12
	Oosterend	71	151 33 33.66	33 36.26	33 22.45	+2.70	+11.21
	Robbezand	72	212 23 39.50	———	23 28.45	———	+11.05
	Staveren	67	283 00 66.73	———	00 54.72	———	+12.01
	Medemblik	56	340 24 10.93	24 11.76	24 01.34	+0.83	+9.59
67	Staveren						
	Enkhuizen	57	12 59 66.99	———	59 58.95	———	+8.04
	Medemblik	56	55 08 59.67	———	08 49.95	———	+9.72
	Oosterland	66	103 17 51.39	———	17 39.48	———	+11.91
	Robbezand	72	144 45 09.51	———	45 01.36	———	+8.15
	Harlingen	74	186 03 65.88	———	03 53.92	———	+11.96
	Sneek	75	230 06 20.52	———	06 10.65	———	+9.87
	Lemmer	68	279 46 26.83	———	46 18.75	———	+8.08
	Urk	58	327 08 61.79	———	08 54.14	———	+7.65
68	Lemmer						
	Urk	58	21 24 61.29	24 55.72	24 52.73	-5.57	+8.56
	Staveren	67	100 03 13.50	———	03 05.53	———	+7.97
	Sneek	75	170 24 16.69	24 18.61	24 06.24	+1.92	+10.45
	Oldeholtpa	76	256 10 37.74	10 47.10	10 29.14	+9.36	+8.60
	Blokzijl	69	307 45 41.97	45 39.71	45 33.75	-2.26	+8.22
69	Blokzijl						
	Kampen	59	9 12 26.45	12 27.15	12 20.43	+0.70	+6.02
	Urk	58	73 57 58.63	57 57.79	57 50.60	-0.84	+8.03
	Lemmer	68	127 57 38.29	57 36.05	57 30.16	-2.24	+8.13
	Oldeholtpa	76	198 06 14.57	06 18.27	06 07.77	+3.70	+6.80
	Meppel	70	281 59 21.83	59 27.40	59 15.03	+5.57	+6.80
70	Meppel						
	Kampen	59	50 24 19.14	24 17.65	24 13.32	-1.49	+5.82
	Blokzijl	69	102 10 17.77	10 23.33	10 11.05	+5.56	+6.72
	Oldeholtpa	76	157 05 43.00	05 43.50	05 36.66	+0.50	+6.34
	Beilen	83	230 47 10.29	47 13.07	47 05.37	+2.78	+4.92
	Lemelerberg	60	329 20 07.78	———	20 00.66	———	+7.12
71	Oosterend						
	Kijkduin	65	34 30 71.91	———	30 54.14	———	+17.77
	Vlieland	73	207 38 12.45	38 26.41	38 05.92	+13.96	+6.53
	Robbezand	72	277 22 50.30	———	22 36.97	———	+13.33
	Oosterland	66	331 26 58.30	26 60.90	26 47.00	+2.60	+11.30
72	Robbezand						
	Oosterland	66	32 30 16.29	———	30 05.24	———	+11.05
	Oosterend	71	97 35 63.20	———	35 49.94	———	+13.26
	Vlieland	73	166 77 33.48	———	57 23.14	———	+10.34
	Harlingen	74	234 42 11.08	———	42 01.73	———	+9.35
	Staveren	67	324 34 60.64	———	34 52.40	———	+8.24
73	Vlieland						
	Oosterend	71	27 46 66.25	46 80.26	46 58.97	+14.01	+7.28
	Midsland	77	237 11 38.03	———	11 27.77	———	+10.26
	Harlingen	74	299 41 38.87	41 41.84	41 30.10	+2.97	+8.77
	Robbezand	72	346 53 13.21	———	53 02.04	———	+11.17

Table 46 (continued)

1	2	3	4	5	6	7	8
74	Harlingen						
	Staveren	67	6 06 34.59	———	06 22.56	———	+12.03
	Robbezand	72	54 54 50.01	———	54 40.67	———	+9.34
	Vlieland	73	119 58 39.42	58 42.33	58 31.49	+2.91	+7.93
	Midsland	77	159 59 53.44	———	59 44.92	———	+8.52
	Ballum	78	211 37 28.85	———	37 18.29	———	+10.56
	Leeuwarden	79	262 35 47.44	35 44.90	35 35.62	-2.54	+11.82
	Sneek	75	313 41 26.14	41 26.96	41 13.61	+0.82	+12.53
75	Sneek						
	Staveren	67	50 20 37.69	———	20 27.86	———	+9.83
	Harlingen	74	133 53 16.09	53 16.90	53 03.68	+0.81	+12.41
	Leeuwarden	79	204 41 48.90	41 49.98	41 34.28	+1.08	+14.62
	Drachten	81	254 12 25.30	12 28.72	12 14.47	+3.42	+10.83
	Oldeholtpa	76	299 48 32.52	48 35.23	48 21.96	+2.71	+10.56
	Lemmer	68	350 21 45.98	21 47.91	21 35.47	+1.93	+10.51
76	Oldeholtpa						
	Blokzijl	69	18 10 35.03	10 38.73	10 28.24	+3.70	+6.79
	Lemmer	68	76 26 55.52	26 64.89	26 47.01	+9.37	+8.51
	Sneek	75	120 07 22.27	07 24.98	07 11.86	+2.71	+10.41
	Drachten	81	187 59 18.67	59 15.36	59 05.98	-3.31	+12.69
	Oosterwolde	82	235 48 13.60	48 10.76	48 04.57	-2.84	+9.03
	Beilen	83	277 19 25.26	19 25.84	19 19.42	+0.58	+10.84
	Meppel	70	336.59 06.73	59 07.25	59 00.32	+0.52	+6.41
77	Midsland						
	Vlieland	73	57 22 32.55	———	22 23.05	———	+9.50
	Ballum	78	256 14 13.34	———	14 03.60	———	+9.74
	Harlingen	74	339 53 46.32	———	53 37.72	———	+8.60
78	Ballum						
	Harlingen	74	31 50 45.24	———	50 34.67	———	+10.57
	Midsland	77	76 33 38.56	———	33 28.90	———	+9.66
	Dokkum	80	301 34 23.59	———	34 06.78	———	+16.81
	Leeuwarden	79	345 36 17.64	———	36 05.49	———	+12.15
79	Leeuwarden						
	Sneek	75	24 47 66.51	47 67.59	47 51 86	+1.08	+14.65
	Harlingen	74	82 53 56.15	53 53.59	53 44.42	-2.56	+11.73
	Ballum	78	165 40 71.72	———	40 59.65	———	+12.07
	Dokkum	80	225 04 71.85	———	04 59.02	———	+12.83
	Drachten	81	297 11 43.24	11 49.07	11 29.19	+5.83	+14.05
80	Dokkum						
	Leeuwarden	79	45 14 71.75	———	14 58.97	———	+12.78
	Ballum	78	121 49 18.73	———	49 02.05	———	+16.68
	Hornhuizen	85	254 06 65.14	———	06 46.89	———	+18.25
	Groningen	86	287 12 72.69	———	12 57.14	———	+15.55
	Drachten	81	344 14 64.72	———	14 48.95	———	+15.87
81	Drachten						
	Oldeholtpa	76	8 01 40.38	01 37.04	01 27.63	-3.34	+12.75
	Sneek	75	74 33 38.44	33 41.85	33 27.71	+3.41	+10.73
	Leeuwarden	79	117 26 40.02	26 45.84	26 26.10	+5.82	+13.92
	Dokkum	80	164 19 62.70	———	19 47.01	———	+15.69
	Groningen	86	247 53 18.22	53 12.33	53 04.98	+4.11	+13.24
	Oosterwolde	82	314 06 16.62	06 13.85	06 05.30	-2.77	+11.32
82	Oosterwolde						
	Oldeholtpa	76	55 59 47.46	59 44.60	59 38.47	-2.86	+8.99
	Drachten	81	134 15 29.65	15 26.88	15 18.42	-2.77	+11.23
	Groningen	86	216 09 49.24	09 43.53	09 35.46	-5.71	+13.78
	Rolde	87	271 10 51.79	10 53.02	10 44.17	+1.23	+7.62
	Beilen	83	315 08 22.48	08 25.40	08 15.61	+2.82	+6.87

Table 46 (continued)

1	2	3	4	5	6	7	8
83	Beilen						
	Lemelerberg	60	9 45 65.35	———	45 59.16	———	+6.19
	Meppel	70	51 02 45.57	02 48.34	02 40.72	+2.77	+4.85
	Oldeholtpa	76	97 41 38.46	41 39.01	41 32.75	+0.55	+5.71
	Oosterwolde	82	135 18 62.85	18 65.76	18 56.09	+2.91	+6.76
	Rolde	87	210 50 39.78	50 42.15	50 33.77	+2.37	+6.01
	Sleen	88	295 10 41.20	10 36.12	10 33.87	-5.08	+7.33
	Coevorden	84	325 25 07.50	25 06.07	25 00.98	-1.43	+6.52
84	Coevorden						
	Lemelerberg	60	47 18 40.95	———	18 36.90	———	+4.05
	Beilen	83	145 35 48.56	35 47.13	35 42.12	-1.43	+6.44
	Sleen	88	198 16 38.42	16 40.69	16 31.41	+2.27	+7.01
	Kirch Hesepe	64	276 30 52.99	30 48.04	30 50.90	-4.95	+2.09
	Uelsen	63	331 45 32.78	———	45 28.71	———	+4.07
85	Hornhuizen						
	Dokkum	80	74 24 30.34	———	24 12.17	———	+18.17
	Borkum	106	222 44 44.52	———	44 28.75	———	+15.77
	Uithuizermeden	89	264 17 24.22	17 24.17	17 08.95	-0.05	+15.27
	Groningen	86	323 19 30.80	19 34.07	19 17.41	+3.27	+13.39
86	Groningen						
	Oosterwolde	82	36 22 61.41	22 55.68	22 47.67	-5.73	+13.74
	Drachten	81	68 15 44.82	15 38.91	15 31.71	-5.91	+13.11
	Dokkum	80	107 40 39.44	———	40 24.11	———	+15.33
	Hornhuizen	85	143 29 33.74	29 36.99	29 20.48	+3.25	+13.26
	Uithuizermeden	89	204 05 65.24	05 67.01	05 53.34	+1.77	+11.90
	Holwierde	90	232 35 66.80	———	35 52.40	———	+14.40
	Midwolda	91	275 14 65.06	14 71.38	14 53.67	+6.32	+11.39
	Onstwedde	92	303 12 32.75	12 37.45	12 22.03	+4.70	+10.72
	Rolde	87	348 32 35.43	32 32.16	32 21.27	-3.27	+14.16
87	Rolde						
	Beilen	83	30 56 52.05	56 54.43	56 46.06	+2.38	+5.99
	Oosterwolde	82	91 27 45.32	27 46.55	27 37.84	+1.23	+7.48
	Groningen	86	168 36 18.32	36 15.08	36 04.26	-3.24	+14.06
	Onstwedde	92	260 00 28.76	00 29.20	00 20.41	+0.44	+8.35
	Sleen	88	335 54 20.77	54 20.90	54 16.67	+0.13	+4.10
88	Sleen						
	Coevorden	84	18 19 38.57	19 40.84	19 31.55	+2.27	+7.02
	Beilen	83	115 24 23.13	24 18.04	24 15.88	-5.09	+7.25
	Rolde	87	156 01 51.34	01 51.46	01 47.30	+0.12	+4.04
	Onstwedde	92	209 45 48.49	45 48.50	45 46.04	+0.01	+2.45
89	Uithuizermeden						
	Groningen	86	24 12 53.80	12 55.58	12 41.90	+1.78	+11.90
	Hornhuizen	85	84 34 16.90	34 16.84	34 01.77	-0.06	+15.13
	Borkum	106	172 37 23.57	———	37 06.72	———	+16.85
	Pilsum	93	250 19 75.52	19 85.45	19 58.75	+9.93	+16.77
	Holwierde	90	297 48 62.82	———	48 48.60	———	+14.22
90	Holwierde						
	Groningen	86	52 50 40.02	———	50 25.71	———	+14.31
	Uithuizermeden	89	117 56 48.18	———	56 34.06	———	+14.12
	Pilsum	93	222 19 21.20	———	19 10.87	———	+10.33
	Emden	94	267 09 51.05	———	09 39.27	———	+11.78
	Midwolda	91	332 50 75.00	———	50 59.31	———	+15.69
91	Midwolda						
	Groningen	86	95 36 18.88	36 25.16	36 07.69	+6.28	+11.19
	Holwierde	90	152 57 56.89	———	57 41.31	———	+15.58
	Emden	94	214 14 35.64	———	14 24.09	———	+11.55
	Leer	95	262 22 25.58	22 30.03	22 17.52	+4.45	+8.06
	Onstwedde	92	352 47 64.79	47 63.53	47 53.62	-1.26	+11.17

Table 46 (continued

1	2	3	4	5	6	7	8
92	Onstwedde						
	Sleen	88	29 57 22.70	57 22.66	57 20.31	-0.04	+2.39
	Rolde	87	80 19 35.27	19 35.65	19 27.06	+0.38	+8.21
	Groningen	86	123 35 23.94	35 28.61	35 13.47	+4.67	+10.47
	Midwolda	91	172 49 43.51	49 42.25	49 32.40	-1.26	+11.11
	Leer	95	230 44 46.41	44 49.03	44 37.38	+2.62	+9.03
93	Pilsum						
	Holwierde	90	42 28 36.66	———	28 26.40	———	+10.26
	Borkum	106	114 18 71.64	———	18 50.77	———	+20.87
	Hage	97	227 42 40.02	———	42 28.62	———	+11.40
	Emden	94	323 27 49.70	———	27 36.89	———	+12.81
94	Emden						
	Midwolda	91	34 23 63.32	———	23 51.82	———	+11.50
	Holwierde	90	87 25 61.68	———	25 50.06	———	+11.62
	Pilsum	93	143 34 45.62	———	34 32.91	———	+12.71
	Hage	97	191 00 29.58	———	00 19.19	———	+10.39
	Aurich	98	237 42 63.17	———	42 53.72	———	+9.45
	Leer	95	313 23 38.62	———	23 29.80	———	+8.82
95	Leer						
	Onstwedde	92	51 04 14.38	04 17.00	04 05.57	+2.62	+8.81
	Midwolda	91	82 43 33.65	43 38.10	43 25.88	+4.45	+7.77
	Emden	94	133 35 20.31	———	35 11.63	———	+8.68
	Aurich	98	184 15 67.34	———	15 58.47	———	+8.87
	Strakholt	99	219 05 27.44	———	05 18.61	———	+8.83
	Westerstede	100	264 04 29.49	———	04 22.03	———	+7.46
	Barssel	96	288 34 13.05	———	34 04.93	———	+8.12
96	Barssel						
	Leer	95	108 48 17.04	———	48 09.08	———	+7.96
	Westerstede	100	231 30 35.29	———	30 27.30	———	+7.99
97	Hage						
	Emden	94	11 04 12.46	———	04 02.06	———	+10.40
	Pilsum	93	47 53 19.63	———	53 08.32	———	+11.31
	Esens	101	256 51 19.22	———	51 08.65	———	+10.57
	Aurich	98	318 34 37.36	———	34 26.51	———	+10.85
98	Aurich						
	Leer	95	4 17 34.36	———	17 25.45	———	+8.91
	Emden	94	57 56 13.06	———	56 03.71	———	+9.35
	Hage	97	138 43 65.41	———	43 54.70	———	+10.71
	Esens	101	203 54 14.90	———	54 06.20	———	+8.70
	Jever	102	247 40 15.48	———	40 07.50	———	+7.98
	Strakholt	99	316 18 42.35	———	18 34.62	———	+7.73
99	Strakholt						
	Leer	95	39 14 37.80	———	14 29.03	———	+8.77
	Aurich	98	136 26 26.47	———	26 18.84	———	+7.63
	Jever	102	217 29 66.51	———	29 59.63	———	+6.88
	Westerstede	100	302 47 24.56	———	47 16.94	———	+7.62
100	Westerstede						
	Barssel	96	51 39 29.88	———	39 21.95	———	+7.93
	Leer	95	84 27 28.77	———	27 21.53	———	+7.24
	Strakholt	99	123 01 14.63	———	01 07.18	———	+7.45
	Jever	100	177 17 07.98	———	17 01.52	———	+6.46
	Varel	103	221 40 64.24	———	40 57.15	———	+7.09
101	Esens						
	Aurich	98	24 00 39.71	———	00 31.05	———	+8.66
	Hage	97	77 07 13.07	———	07 02.67	———	+10.40
	Jever	102	293 28 59.30	———	28 50.66	———	+8.64

Table 46 (continued)

1	2	3	4	5	6	7	8
102	Jever						
	Strakholt	99	37 42 45.77	———	42 38.99	———	+6.78
	Aurich	98	68 00 39.96	———	00 32.20	———	+7.76
	Esens	101	113 42 60.12	———	42 51.66	———	+8.46
	Varel	103	321 20 37.69	———	20 30.41	———	+7.28
	Westerstede	100	357 15 55.51	———	15 48.99	———	+6.62
103	Varel						
	Westerstede	100	41 51 06.68	———	———	———	———
	Jever	102	141 31 53.90	———	———	———	———

Just as for the computation of the φ- and λ-coordinates, here too the unit of length for X' and Y' is 100 km.

The small angle ϵ between chord and arc at Rhenen to Gorinchem is, as already discussed before:

$$\epsilon''_{37} = 0.0012658\,(X'_{37}Y'_{32} - X'_{32}Y'_{37}) = -1''.354.$$

It is counted from the chord to the arc in a clockwise direction (here therefore negative).

As the gridbearing of the chord Rhenen-Gorinchem is:

$$\psi_{37-32} = \text{arc tan}\,\frac{X'_{32} - X'_{37}}{Y'_{32} - Y'_{37}} = 250^{\circ}53'50''.350,$$

the geographical azimuth Rhenen-Gorinchem is:

$$A_{37-32} = (\psi_{37-32} + \nu_{37} + \epsilon_{37}) \pm 180^{\circ} = 71^{\circ}02'11''.459 \;.\;.\;.\;.\;.\;.\;.\; (28)$$

It is counted from the south in a clockwise direction.

With (28) all the azimuths were determined with the computer, those in column 4 of course with the coordinates $X''Y''$. As the latter coordinates were rounded-off by the computer at cm, a computation of ψ_{ik} in (28) in hundredths of a second of arc is not quite justified when the distances $\underline{i}\ \underline{k}$ are small [88] .

Column 7 (table 46) gives the differences v'' between the azimuths of the sides in the R. D.-system and my own computation of the network which is adapted as well as possible to that of the R. D. It guarantees, as already said before, not only an ideal base length, but also an ideal orientation of the triangulation. Like the vectors in Fig. 20, the v's in column 7 of the table give an impression of the accuracy of Krayenhoff's triangulation. Especially in the southern part of the network the agreement between the R. D.-results and my own computation is excellent. See e. g. the very small v's for the azimuths in the stations No. 16 up to and including No. 26.

In the stations No. 10 up to and including No. 15 the agreement is somewhat less. I don't know whether these larger deviations must be imputed to a less accurate measurement or to small alterations in some spires. Especially in rather short sides between "identical" stations these alterations have of course a great influence.

The rather large vector 0.98 m (see Fig. 20) at Medemblik (station No. 56), about perpendicular to the direction to Enkhuizen (station No. 57) e.g., at a distance of only 14.9 km, causes a deviation $v = +11''.52$ at Medemblik and $v = +11''.51$ at Enkhuizen. They are only surpassed by the v's $+ 13''.96$ at Oosterend (No. 71) to Vlieland (No. 73) and $v = +14''.01$ at Vlieland to Oosterend. The very large vector 1.99 m at Vlieland, about perpendicular to the direction to Oosterend caused these large deviations. Perhaps lateral refraction may have contributed to this phenomenon as for its full length of 26.6 km the connecting line Vlieland-Oosterend passes the Dutch-shallows. As one sees and as could be expected the v's in column 7 have an accidental character. In every station there are positive and negative v's.

The v's in column 8 of table 46 demonstrate only the differences in azimuth as a result of my own adjustment of the network and that of Krayenhoff. For the azimuth in Amsterdam (No. 40) to Utrecht (No. 36) this v is $+0''.62$. If the v is required between the R.D.-azimuth and the amount $332^\circ 41' 19''.94$ from which Krayenhoff started his computation, the v's in columns 7 and 8 must of course be added. The amazing result $-0''.49$ was already discussed at the end of section 28.

As Krayenhoff's network is not a closing mathematical figure, it will be clear that the azimuths of the sides are dependent on the route chosen for their computation. Any other arbitrary route would have given other results. In the southern part of the triangulation the agreement between Krayenhoff's azimuths and those found from my own computation is very good indeed. For the station Nos. 1-15 all v's in column 8 are slightly negative. A little more to the northeast their signs change into positive but up to and including the station No. 45 (Bocholt) they remain very small with the exception perhaps of Amersfoort (No. 42). It might be possible that the neighbourhood of the Zuiderzee-pentagon, not adjusted in Krayenhoff's computations, asserts here its influence upon the azimuths of the sides. In the stations Harderwijk (No. 46) and Veluwe (No. 17) the neighbourhood of the Zuiderzee-pentagon is also perceptible. The v's in these stations are rather large but more to the east (stations Nos. 48-51, 60-64) this influence decreases.

In the narrow strip between Noordzee (North Sea) and Zuiderzee where the sides of the triangles are small (stations Nos. 54-57 e.g.) the v's are very large. At

Schagen (station No. 55), e. g. the azimuth to Oosterland (No. 66) in my computation deviates even 12$''$.31 from that in the Précis Historique. Around the Zuiderzee at Staveren (No. 67), Lemmer (No. 68), Urk (No. 58), Kampen (No. 59) and Blokzijl (No. 69) the v's in column 8 remain very bad and all positive and at Drachten (station No. 81), about the centre of the part of the network criticized by Gauss [11] the v's are almost the largest of the triangulation. Only because of Krayenhoff's incorrect adjustment of his network the small amounts v in Amsterdam (No. 40) changed into the large deviations v = +13$''$.92 and v = 15$''$.69 in the azimuths to Leeuwarden (No. 79) and Dokkum (No. 80). It must be remarked once again, however, that these large deviations say nothing whatever about the accuracy of Krayenhoff's observations. At the end of section 29 I already remarked that the azimuth Jever (No. 102)-Varel (No. 103) = $321^{\circ}20'37''.69$ computed from the coordinates $X''Y''$ in table 26 agrees excellently with the amount $321^{\circ}20'35''.5$ which can be derived from Gauss' Oldenburg-triangulation (difference +2$''$.2). As already shown in table 43 Krayenhoff's direct observation of the azimuth gives $321^{\circ}20'45''.33 \pm 3''.7$ (difference - 7$''$.64).

33. Conclusions

According to Baeyer's demands for a Middle European triangulation in 1864, should Krayenhoff's measured angles be used or be rejected ? It is very difficult to give an answer to this question as it is dependent on the way one wishes to judge Krayenhoff's network. From a theoretical point of view there can be made several objections indeed. I already discussed some of them in my final consideration of the geodetic part of the triangulation (section 24).

Cohen Stuart's most serious objection is - and I agree with him - that in many cases observations were rejected which, according to Krayenhoff, did not fit in the computation of the network. The closing errors in the triangles and the central points are therefore much too small and not at all in accordance with the standard deviation in the angular measurement. Especially for the measurements in the years 1810 and 1811 in the northern part of the territory this number of rejected series was large, 41 and 51 percent respectively (see tables 3 and 4). The main reason for this rejection, however, was the less accurate instrument used in those years of which Krayenhoff complains seriously in his Précis Historique. For the measurements in the years 1802, 1803, 1805 and 1807 with the accurate instrument the number of rejected series was only about 12 percent and much too small for a serious influence on the result of the computation.

As I already remarked before, however, I cannot agree with Cohen Stuart that in principle all the rejected series (389 out of 1514) should have been used for the computation. In my opinion he did not take into account that the dynamic Krayenhoff tried to measure when the weather conditions seemed favourable. It will be clear that a great number of series which he began could not be finished because of changing circumstances or, if finished, could not be maintained because of too bad weather conditions at the end of the series (too strong wind or heat shimmer, darkness, rain, mist, haze or fog). In table 18 I illustrated this with several examples. In other cases, also mentioned in table 18 and the text of section 17 I too cannot agree with the arbitrary and inadmissable rejection of a number of series.

Another objection to the triangulation is the bad reduction of the measured angles to centre. In Fig. 9 (section 11) I illustrated the influence of the slope of the Western tower in Amsterdam (station No. 40) on the accuracy of this reduction. As it is inherent to the repetition circle used, Krayenhoff cannot be blamed for it. But he should be reproached for his method of angular measurement which always began with a reading zero on the limb of the instrument and, for the same number of repetitions of the same angle, always ended at the same place of the limb. Therefore accidental errors in the calibration of the limb could not be made harmless. The academic objection that spheroidical angles are no spherical angles (the plane through the vertical of the station P and the sighting point Q does not coincide with the plane through the vertical of the station Q and the sighting point P) need not to be taken into account; with regard to the accuracy of Krayenhoff's observations its influence is too small by far. The objection that no baseline was measured and the whole triangulation had to be built on the side Duinkerken-Mont Cassel in the utmost southwestern part (extrapolation about 500 km) was already discussed before.

From a theoretical point of view the astronomical part of the triangulation is very bad. In this part of his work Krayenhoff did not convert mean solar time, read from his chronometer, into sidereal time, necessary for the computation of circum meridian zenith distances of Polaris. In my opinion, however, he knew what he did: because of the small hour angles the errors made are so small that they can almost be neglected.

The worst part by far of the astronomical measurements is the determination of azimuths, just like the determination of latitudes, executed in the stations Amsterdam (No. 40) and Jever (No. 102) and described in sections 28 and 29. For this determination Krayenhoff could not lean upon Delambre who used a method with which Krayenhoff could not obtain satisfactory results. By the inexpert manner, however, with which he executed a method of himself - even to towers almost in the meridian of his stations - the results are very bad indeed. A great number of systematic and accidental errors could have been made harmless or reduced to much smaller amounts if he should have chosen his terrestrial points a little south of the direction of the rising or setting sun.

In section 24 I already stated that, as for the practical results of the triangulation, Krayenhoff's measured angles would have satisfied Baeyer's demands for a middle European triangulation. As one knows here I don't agree with Cohen Stuart. The proof for my statement may be found in table 29 and in Fig. 22 of section 23 where I compared the 171 angles in the 57 triangles of Krayenhoff's network which are identical or are assumed to be identical with those of the R.D., with the R.D.-results. The comparison shows that the external accuracy of an angle can be given by the amount $m \simeq \pm 3''.6$, which is worse indeed than the inner accuracy $m \simeq \pm 1''.8$ found in section 19. One should not forget, however, - see also my considerations in section 23 - that eventual errors on my part in the assumed identity of the towers in the two systems cause the external accuracy to become worse.

The deviations v in the triangles in the southern part of the network in general are better than those in the northern part. The accuracy of the instrument used will probably be the main reason for this phenomenon. Several angles in the northeast, however, - see e.g. the excellent agreement between the angles in the triangles 122, 125, 134, 138 and 143 - have very small v's. They prove that there can hardly be any talk of seriously influencing the observations. Some triangles in the south of the triangulation (see e.g. the numbers 13 and 15) are worse than might be expected.

The differences $\underline{v}$ for the side lengths between the identical points in table 29 are very small in my opinion (see e.g. the excellent v's in the triangles 17, 23, 26, 125, 132 and 134). As already said before the lengths in Krayenhoff's system were computed with an "ideal" baseline length which matches as well as possible the R.D. triangulation network. The excellent harmony between the angles of triangle 143 apparently could not be retained in the side lengths. The sides in the R.D.-system are about 27 mm per km smaller than those in Krayenhoff's adjusted network. The difference remains far beyond Baeyer's demand, 50 mm per km for long distances. The excellent relative differences 4 mm per km and 5 mm per km in the very long sides Gent-Amsterdam (167.4 km) and Amsterdam-Leer (197.4 km) respectively must of course be ascribed to the "ideal baseline" of the triangulation. As already remarked the amounts are a factor 10 better than Baeyer demanded.

From a practical point of view the results of the determination of latitudes in Amsterdam (No. 40) and Jever (No. 102) are good in my opinion. According to table 44 the latitude Krayenhoff found for Amsterdam is but 1".83 smaller than the one computed from the R. D.-coordinates X'Y' (also see the end of section 26). Krayenhoff's direct determination $\varphi = 53^\circ 34' 23''.445$ of the centre of his station Jever (see the end of section 27) differs but 0".484 from the amount $53^\circ 34' 23''.929$ found from the computation from the coordinates X"Y" of that station. The amount $53^\circ 34' 22''.71$ in tableau V of the Précis Historique (see column 5 of table 44) is less good because it is affected by the changing scale factors in Fig. 21. The computation of the latitude (and longitude) of Duinkerken (No. 1) in column 5 (and 10) is also affected by these scale factors.

The very bad internal accuracy of the determination of the astronomical azimuths Amsterdam-Utrecht and Jever-Varel appeared to give excellent practical results. The azimuth from Amsterdam to Utrecht deviates only 0".49 from that of the R. D. The one from Jever to Varel is worse: Krayenhoff's direct measurement $321^\circ 20' 45''.33$ deviates 7".64 from the amount $321^\circ 20' 37''.69$ computed from the adjusted coordinates X"Y". The latter azimuth agrees excellently with the amount $321^\circ 20' 35''.5$ which can be derived from Gauss' Oldenburg-triangulation.

The azimuth Duinkerken-Mont Cassel $343^\circ 13' 32''.51$ computed from the coordinates X"Y" differs but 0".19 from the amount found by Delambre. From this amount Krayenhoff started his computations in tableau IV of the Précis Historique.

Strictly speaking the considerations on the accuracy of the astronomical part of the triangulation have nothing to do with the question whether the results of the triangulation should be rejected or retained. Baeyer's demands related only to the accuracy of the angles in the triangles of the network. All the other operations should be done anew. The excellent agreement mentioned above justifies the supposition that a better result could hardly be expected.

It seems incredible that, after the publications of the R(ijks) D(riehoeksmeting) nobody apparently hit upon the idea to compare the excellent results of this triangulation with those of Krayenhoff's network, exuberantly praized just after the appearance of the Précis Historique and reviled by Gauss and Cohen Stuart in later days. But the work has been done at last. In some respects the results of this study may be considered a third edition of the Précis Historique. All the tables with the exception of tableau II (the computation of the provisional lengths of the sides) and tableau IV (the computation of the latitudes and the longitudes with respect to Paris) are included in this study. Should the bad criticism be maintained or, as Van der Plaats hoped and expected, ought Krayenhoff to be rehabilitated ? The reader may draw his own conclusions.

My opinion on Krayenhoff's triangulation - the first in which an attempt was made to adjust an extensive network - can be summed up by Van der Plaats' quotation from Racine's Brittannicus, already mentioned in the introduction of this book:

"J'ose dire pourtant que je n'ai mérité,
"Ni cet excès d'honneur, ni cette indignité".

REFERENCES

[1] Précis Historique des opérations géodésiques et astronomiques, faites en Hollande pour servir de base à la topographie de cet état, exécutées par le lieutenant général Baron Krayenhoff (first edition, The Hague, 1815, second edition, The Hague, 1827).

[2] J. B. J. DELAMBRE (1749-1822), French astronomer, from 1792 till 1799 occupied with the measurement of the arc of the meridian of Paris between Dunkirk and Barcelona.

[3] J. H. van SWINDEN (1746-1823), professor of mathematics and physics in Amsterdam and member of the executive Council of the Republic.

[4] H. C. SCHUMACHER (1780-1850), professor of astronomy and mathematics in Copenhagen, pupil and friend of Gauss.

[5] F.W. BESSEL (1784-1846), German astronomer, friend of Gauss.

[6] H. W. M. OLBERS (1758-1840), German physician and astronomer, friend of Gauss.

[7] CARL FRIEDRICH GAUSS (1777-1855), German mathematician and physicist.

[8] Briefwechsel mit Bessel, page 457.

[9] GAEDE: Beiträge zur Kenntniss von Gauss' praktisch-geodätischen Arbeiten (Zeitschrift fur Vermessungswesen 1885, pages 113-137, 145-157, 161-173, 177-192, 193-207, and 225-245). The quotation may be found on page 181.

[10] Briefwechsel mit Schumacher, Band I, page 349.

[11] JORDAN's Handbuch der Vermessungskunde, erster Band, Stuttgart, 1920, page 511.

[12] F. KAISER (1808-1872), Dutch astronomer.

[13] L. COHEN STUART (1827-1878), Dutch geodesist.

[14] F. KAISER en L. COHEN STUART: De eischen der medewerking aan de ontworpen graadmeting in Midden Europa voor het Koningrijk der Nederlanden, Amsterdam, 1864, (72 pages).

[15] [14] page 9.

[16] [14] page 9.

[17] [14] page 21.

[18] F.J. STAMKART (1805-1882), inspector of weights and measures from 1833 till 1867 at Alkmaar and Amsterdam. He wrote several publications in the field of mathematics, mechanics, physics and astronomy. In 1844 Leiden University appointed him doctor honoris causa in mathematics and physics. From 1867 till 1878 he was professor at the Polytechnical School at Delft, the later University of Technology. From 1865 till his death he was engaged with his unsuccessful measurement of the Netherlands' part for the Middle European triangulation.

[19] Verslag Rijkscommissie voor Graadmeting en Waterpassing over het jaar 1887 (Tijdschrift voor Kadaster en Landmeetkunde, 1888, page 116).

[20] J. D. van der PLAATS: Overzicht van de graadmetingen in Nederland (Tijdschrift voor Kadaster en Landmeetkunde, 1889, pages 217-243, 257-306, and 1891 pages 65-101, 109-133).

[21] [20], page 109.

[22] [20], pages 113-114.

[23] [20], page 123.

[24] [20], page 130.

[25] [20], page 122.

[26] Levensbijzonderheden van den luitenant generaal baron C.R.T. Krayenhoff, door hem zelven op schrift gesteld en in het licht gegeven door H.W. Tydeman (Nijmegen, 1844).

[27] Nieuw Nederlands biografisch woordenboek, vol. II, columns 719-725.

[28] N. van der SCHRAAF: Historisch overzicht van het driehoeksnet van Krayenhoff (Nederlands geodetisch tijdschrift, April 1972, pages 65-81).

[29] N.D. HAASBROEK: Gemma Frisius, Tycho Brahe and Snellius and their triangulations (Publication of the Netherlands Geodetic Commission, 1968), pages 63-66.

[30] [1], page 4.

[31] An examination of Perny's triangulation may be found in [20], page 237. It is very bad indeed though the closing errors of the triangles are small. Apparently in order to prevent contradictions in the computations the angles of the triangles Middelburg-Goes-Hulst and Hulst-Antwerpen-Lier were not even measured.

[32] [14], page 44.

[33] [20], page 271.

[34] The Dutch text of this letter was published in Algemeene Konst en Letter-Bode of April 10th, 1804. Van der Schraaf gives the text on page 71 of his paper [28].

[35] [20], page 272.

[36] C.R.T. KRAYENHOFF: Verzameling van Hydrographische en Topographische waarnemingen in Holland (Doorman en Comp., Amsterdam, 1813), page XII. Van der Schraaf gives the text on page 71 of his paper [28].

[37] In his letter dated January 31st, 1971, Baron A. Krayenhoff at Amersfoort wroteto me that his father, Baron C.R.T. Krayenhoff, presented this copy to Topografische Dienst.

[38] [20], page 276.

[39] C.W. MOOR: Triangulaties in Nederland na 1800 (Library of the Sub-Department of Geodesy of the Delft University of Technology, Delft, 1953).

[40] [39], page 9.

[41] BERTHAUT: La Carte de France 1750-1798, tome 1, pages 102-106.

[42] [1], page 12.

[43] Library Leiden University, Codex 241, octavo II, page 31, series 19.

[44] Instructie voor de geographische ingenieurs bij het depot-generaal van oorlog van het Koningrijk Holland (March 4th, 1808).

[45] JACOB de GELDER (1765-1848), especially in the first years of the triangulation Krayenhoff's assistant during the measurements, had an important part in the computation of the network. In 1819 he was appointed professor of mathematics and physics in the university of Leiden. His biography (by professor G.J. Verdam) may be found in Algemeene Konst en Letter-Bode of December, 1848.

[46] [1] , pages 7 and 8.

[47] [39] , page 10.

[48] ADRIEN MARIE LEGENDRE (1752-1833), French mathematician.

[49] On account of the great lengths of the sides the formula is somewhat more accurate than that on page 6 of the Dutch "Handleiding voor de technische werkzaamheden van het Kadaster (H. T. W. 1956)".

[50] [29] , page 101.

[51] [20] , foot-note on page 263.

[52] Archives Netherlands Geodetic Commission, file Krayenhoff, letter No. 26.

[53] [1] , pages IX-XVIII (XIV).

[54] [1] , page 103.

[55] [1] , page 12.

[56] [20] , page 279

[57] The series are mentioned on page 38 of [14] .

[58] Base du système métrique, tome 2, page 801.

[59] Also see the angles in column 5 of table 20 for the stations Nos. 22, 29, 32, 33 and 24.

[60] [20] , page 85.

[61] [11] , page 512.

[62] L. N. M. CARNOT (1753-1823), French general and military genius.

[63] ULRICH HUGUENIN (1755-1833) was a Dutch artillery officer. He was very often abroad. In Germany he met Gauss in 1798 or 1799. For some time he was Krayenhoff's assistant for his triangulation. In an interesting paper in the Dutch language "A conflict between Gauss and a Dutch mathematician" (Wiskundig tijdschrift, year 1918-1919, No. 3, pages 140-145) from which I borrowed these data, the author, the Dutch Gauss-connoisseur S. C. van Veen, writes on Huguenin's merits in the mathematical field, especially on his publication in 1803 of the construction of $2\pi:17$ of which Gauss had already given an elegant solution in 1796.

[64] Krayenhoff mentions their names on the pages 46 and 47 of the Précis Historique but forgets Huguenin in his enumeration.

[65] Generalbericht Europäische Gradmessung, year 1865, pages 22-28.

[66] Polygon condition 149; see section 13, page 57.

[67] Hk. J. HEUVELINK: Topografische kaart en Rijksdriehoeksmeting (Delft, 1920), pages 8-16.

[68] In the reconnaissance of the R. D.-triangulation one reads concerning this point: the place of the beacon-light is indicated by a block of brickwork, covered up with a free stone plate in the centre of which a bronze pin approximately indicates Krayenhoff's triangulation point.

[69] Letter dated December 21st, 1970, No. 3933.

[70] Letter dated December 2nd, 1970, No. 4663/32c with a copy of Ph. J. C. G. van HINSBERGEN: De geschiedenis van Nieuwkoop (the history of Nieuwkoop), pages 153-159.

[71] [9], page 237.

[72] Briefwechsel Gauss-Schumacher II, 1860, letter 259, page 29.

[73] log (Jever-Varel) (Prussian Rhinel. roods) 3.8200069
log reduction Rhinel. roods to toises 9.7139117
log (Jever-Varel) (toises) 4.1060952
Jever-Varel = 12767.18 toises; 1 toise = 1.949036 m
Jever-Varel = 24883.69 m.

[74] [14], page 20.

[75] Base du système métrique, tome 2, page 648; the plus-sign of the longitude means east of Paris.

[76] Base du système métrique, tome 2, pages 123 and 800.

[77] Base du système métrique, tome 3, pages 134 and 135.

[78] [1], page 12.

[79] [1], page 13.

[80] Series 8 on page 6 verso of the register Observations astronomiques, Codex 241, in Library Leiden University.

[81] Krayenhoff used for this reduction to "centre" (the point C in Fig. 9) an eccentricity e = 3.792 m which is impossible in my opinion. According to the original drawing 1:40 of Fig. 9 the distance between the middle of the balustrade and C is 4.74 m.

[82] Handleiding voor de Technische werkzaamheden van het Kadaster (H.T.W. 1956), page 6.

[83] JOHN NAPIER (1550-1617), Scotch mathematician.

[84] Gridbearing chord Amsterdam-Utrecht ($333^\circ 05' 12''.821$) plus angle between arc and chord ($+0''.876$) plus angle between astronomical north and grid north in Amsterdam ($-0^\circ 23' 54''.246$). The latter amount was computed with De Groot's formula in [49], page 6.

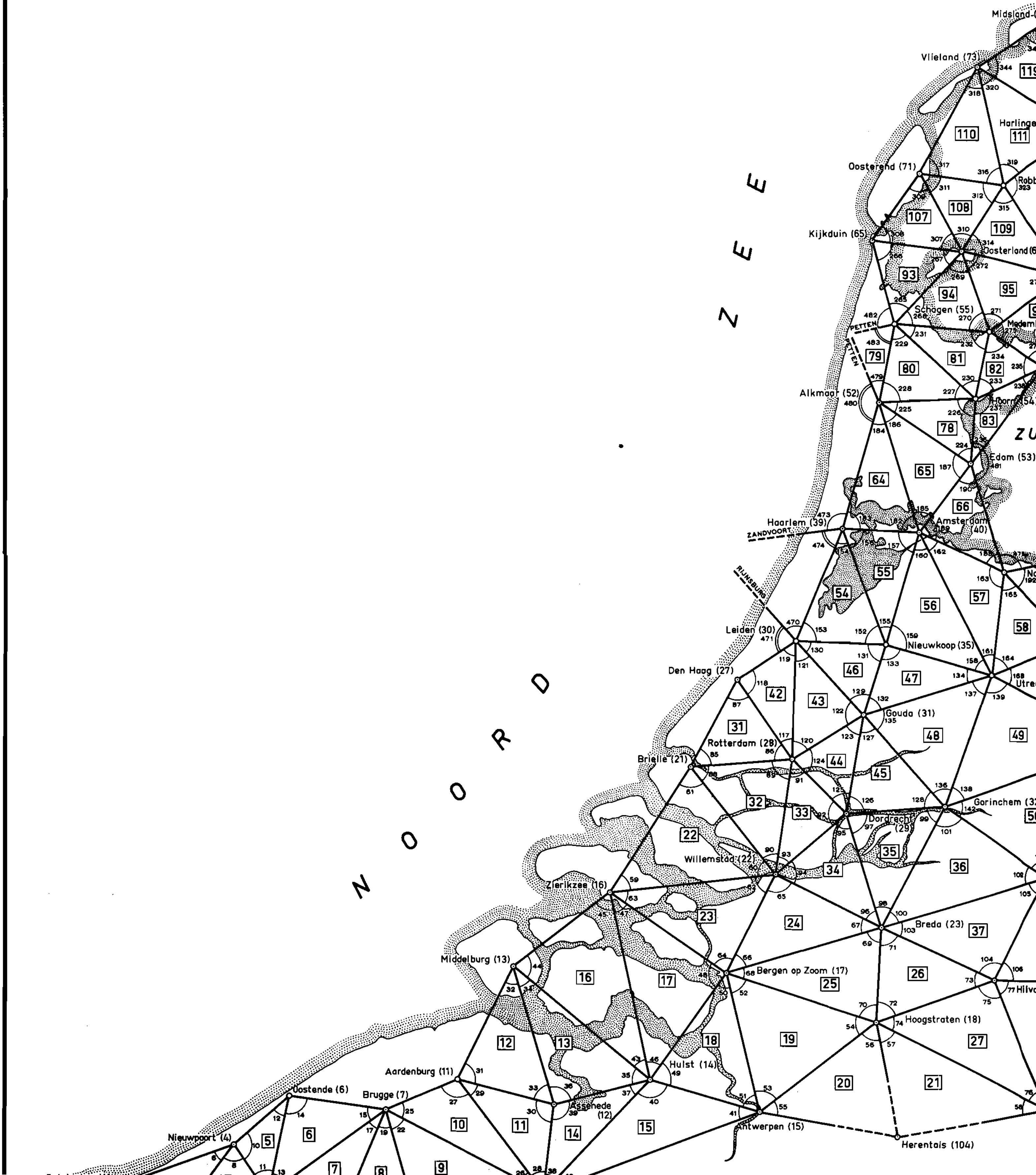
[85] Van der Plaats (page 229 of his paper [20]) thinks that the computation is in the archives of the (Dutch) Ministery of Defence.

[86] Delambre and Krayenhoff called φ_P, φ_Q, λ_P, λ_Q, A_{PQ}, ($A_{QP} \pm 180^\circ$) and $\underline{a}$: L, L', M, M', Z, Z' and R respectively. In the expressions for $\varphi_Q - \varphi_P$ and δ Krayenhoff calls $1 + e^2 \cos^2 \varphi_P = q$ and $\rho\,(1 + \frac{1}{2} e^2 \sin^2 \varphi_P)$: a = p

[87] Détermination de la différence de longitude Leyde-Greenwich, exécutée en 1880 et 1881 par H.G. et E.F. van de Sande Bakhuyzen, published in Annalen der Sternwarte Leiden, Band 7 (1897), page 245 and following.

[88] If in table 9 (column 13) one compares e.g. the adjusted angles 185, 189, 162, 160, 157 and 182 at the station Amsterdam (No. 40) with the amounts computed from the differences of the astronomical azimuths in table 46, the differences are $+0''.06$, $-0''.04$, $+0''.01$, $-0''.02$, $0''.00$ and $-0''.01$ respectively.

KRAYENHOFF's triangulation (1802 - 1811) in Belgium, The Netherlands and a part of north-western Germany



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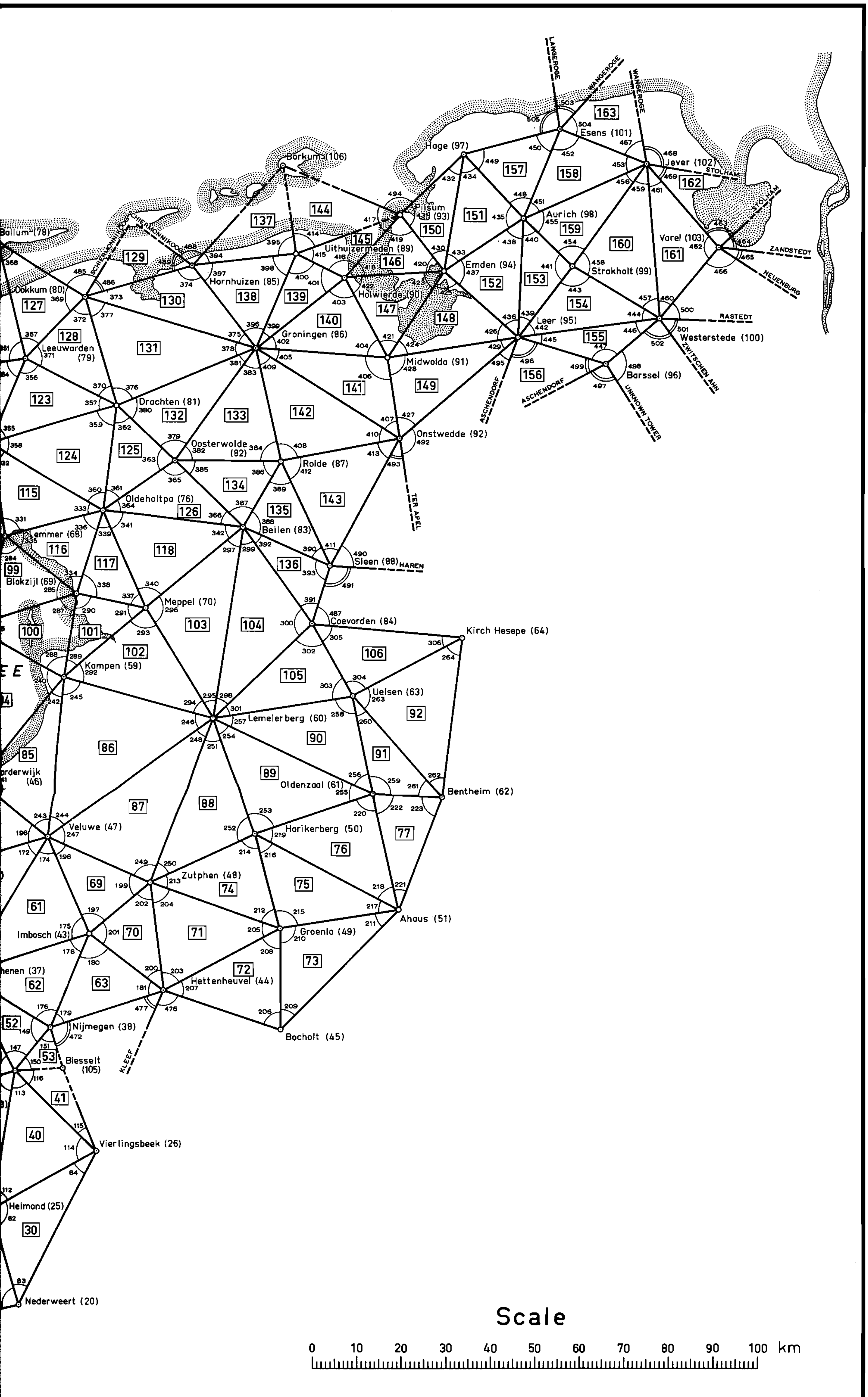
LANGEROGE
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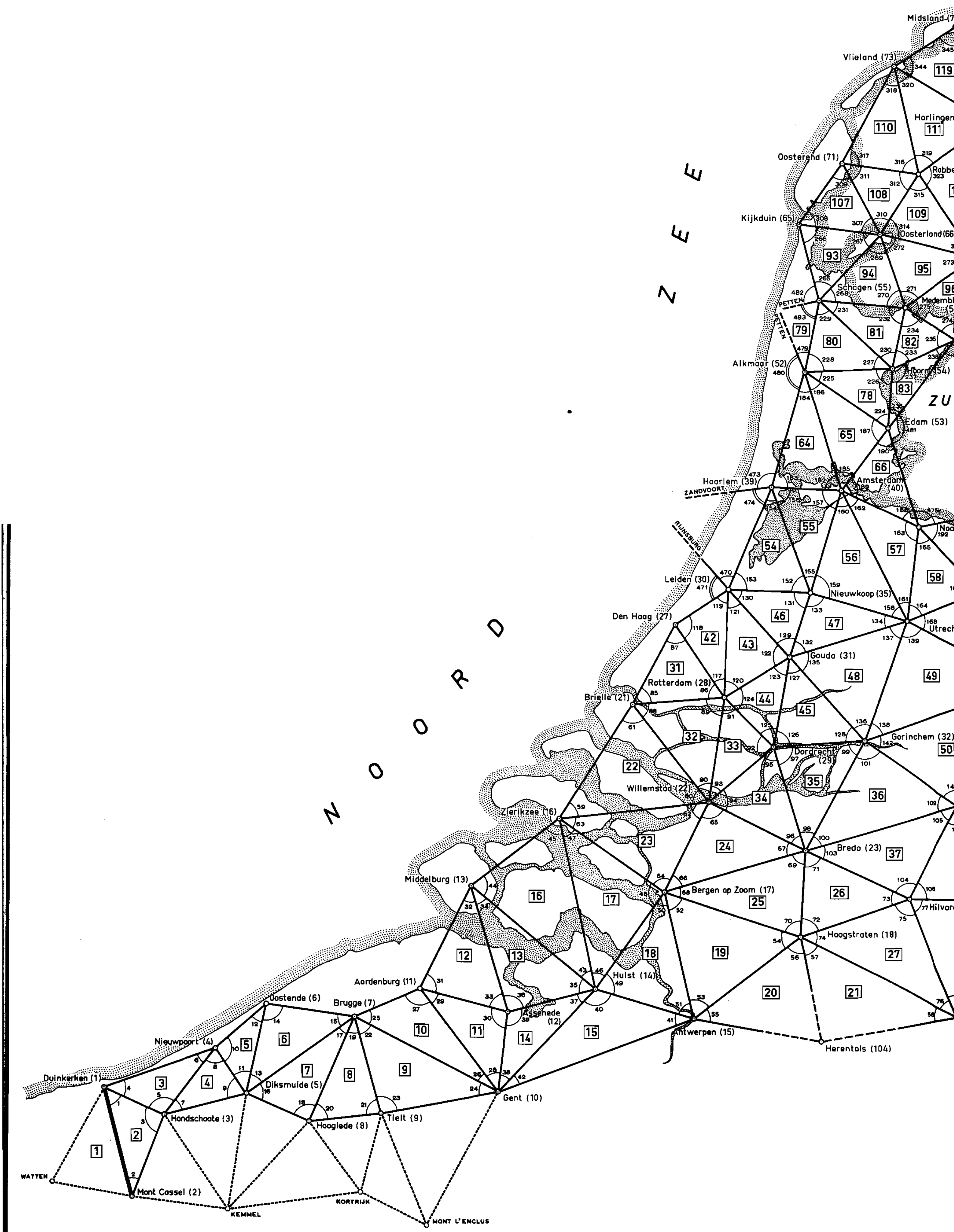
ASCHENDORF
ASCHENDORF
TER APEL
HAREN
KLEEF
SCHIERMONNIKOOG
SCHIERMONNIKOOG
Esens (101)
Hage (97)
Jever (102)
Borkum (106)
Pilsum (93)
Aurich (98)
Varel (103)
Ballum (78)
Uithuizermeden (89)
Emden (94)
Strokholt (99)
Hornhuizen (85)
Holwierde (90)
Dokkum (80)
Leer (95)
Westerstede (100)
Groningen (86)
Midwolda (91)
Leeuwarden (79)
Barssel (96)
Drachten (81)
Onstwedde (92)
Oosterwolde (82)
Rolde (87)
Oldeholtpa (76)
Beilen (83)
Lemmer (68)
Sleen (88)
Blokzijl (69)
Meppel (70)
Coevorden (84)
Kirch Hesepe (64)
Kampen (59)
Uelsen (63)
Lemelerberg (60)
Oldenzaal (61)
Bentheim (62)
Horikerberg (50)
Veluwe (47)
Zutphen (48)
Ahaus (51)
Imbosch (43)
Groenlo (49)
Hettenheuvel (44)
Nijmegen (38)
Bocholt (45)
Biesselt (105)
Vierlingsbeek (26)
Helmond (25)
Nederweert (20)
Scale
0 10 20 30 40 50 60 70 80 90 100 km

KRAYENHOFF's triangulation (1802-1811) in Belgium, The Netherlands and a part of north-western Germany



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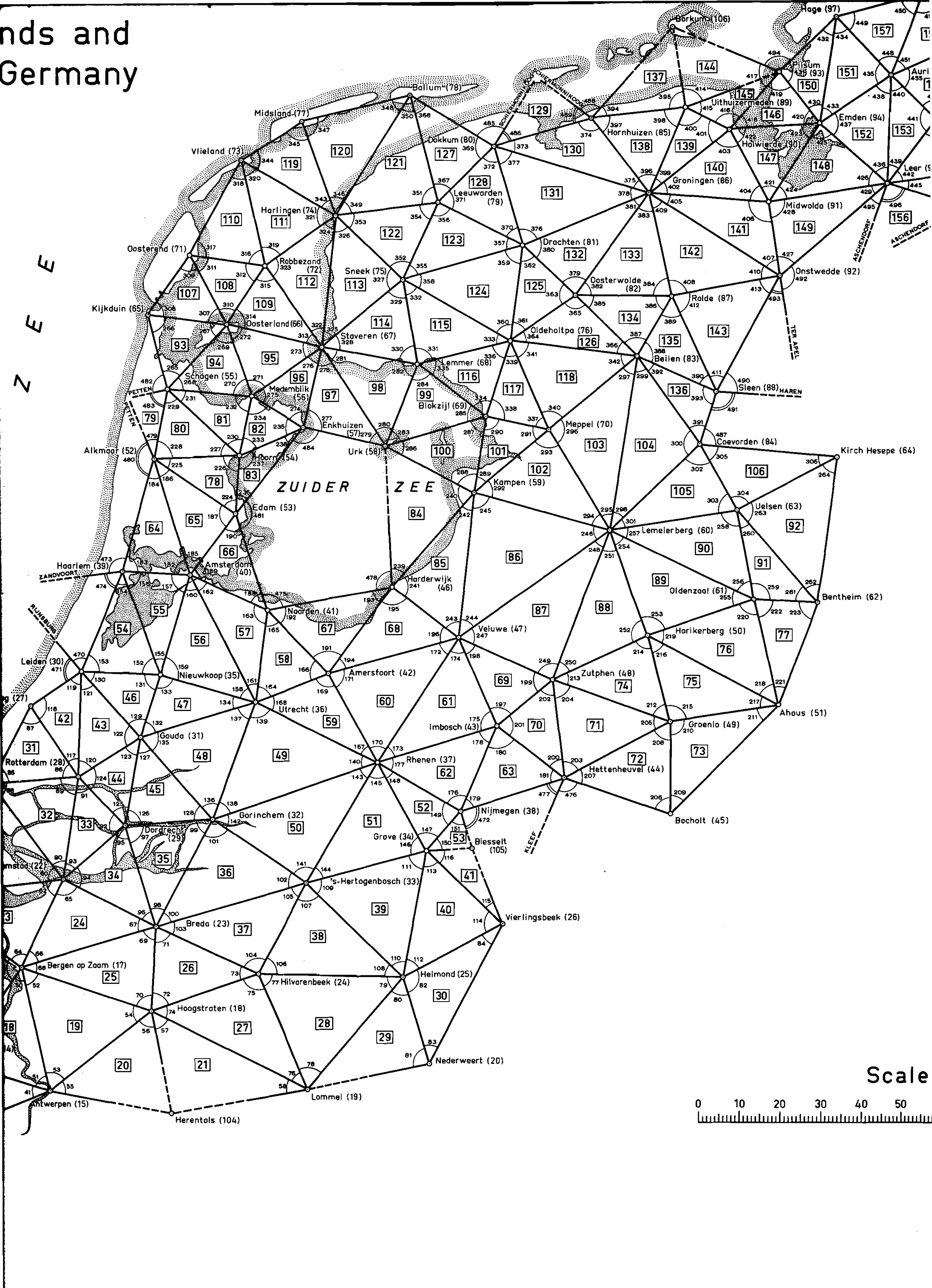

Fig. 2