INVESTIGATION OF THE ACCURACY OF STAMKART'S TRIANGULATION (1866-1881) IN THE NETHERLANDS

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INVESTIGATION OF THE ACCURACY OF STAMKART'S TRIANGULATION (1866-1881) IN THE NETHERLANDS

by

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INVESTIGATION OF THE ACCURACY OF STAMKART'S TRIANGULATION (1866-1881) IN THE NETHERLANDS

1. *INTRODUCTION*

The motive for Stamkart's triangulation carried out in the Netherlands, between 1866 and 1881, is already mentioned in the first section of Haasbroek's publication on the accuracy of Krayenhoff's triangulation [1] and in the very detailed paper [2] by Mr. N. van der Schraaf, assistant secretary of the Netherlands Geodetic Commission, in the professional journal Nederlands Geodetisch Tijdschrift. The motive was based on an invitation dated July 16th 1861, by the Prussian ambassador at the Hague to the Dutch government for the participation in a Middle European triangulation, drawn up by the Prussian general Baeyer [3].

At first it seemed possible that a re-computation of Krayenhoff's triangulation [1], completed with a base line and new astronomical measurements, could be used for this purpose. A thorough investigation [4], published in 1864 by F. Kaiser [5] and L. Cohen Stuart [6], however, showed that Krayenhoff's work should be rejected "as the measurements are far too inaccurate for the Middle European triangulation". In fact the investigation was done only by Cohen Stuart. Kaiser, the well-known astronomer, gave only his name to it.

In his letter dated January 15th 1864 to Kaiser [7], Cohen Stuart suggested a new triangulation. It could consist of double chains of triangles, connecting the astronomical station Leiden with the existing triangulations in Belgium, Hannover, and possibly Prussia. "It seems desirable that our part in the Middle European triangulation should be made an independent whole by measuring a base line". Cohen Stuart thought that the measurements could be done in the summer seasons of three or four years by two "experts" and their assistants. The computation of the triangulation would take about one year extra so that the total duration would be about four or five years.

The total expenses for the geodetic part were estimated at about fifteen thousand guilders, exclusive of the base measurement apparatus that might be borrowed from the Prussian government. Another possibility was to use the apparatus ordered for the "Dutch East Indies". The expenses for the astronomical part were estimated by Kaiser at five thousand guilders, exclusive of two thousand guilders for some new instruments.

In his letter dated January 28th 1864 [8] to the Netherlands government, Kaiser proposed that Cohen Stuart should be charged with the geodetic part of the triangulation. He himself was willing to do the astronomical part.

Only on August 19th 1865 Kaiser was informed that for the year 1865 five thousand guilders were allocated for the triangulation. Cohen Stuart's appointment as the director of the geodetic part, however, could not be realized. Because of his activities as the director of the Delft Polytechnical school he had the Minister informed that he could not accept this task. Thereafter Kaiser proposed to appoint dr. M. Hoek in Cohen Stuart's place. Dr. Hoek was the director of the Utrecht Observatory. But also Hoek refused, according to his letter to Kaiser, for rancorous reasons [9].

Already in a letter dated December 1st 1865, the Minister informed Kaiser on Hoek's refusal but in the same letter he wrote that Dr. F. J. Stamkart, inspector of weights and measures in Amsterdam, was willing to accept the appointment.

Kaiser was not very happy with Cohen Stuart's and Hoek's refusal as may be derived from his letter to Stamkart dated December 13th 1865 [10]. In this letter he wrote:"It is remarkable that the younger generation, though talking big, withdraws itself when there is something to do and leaves the work to the old age". Kaiser was then 57 years old, Stamkart even 60.

According to Van der Schraaf's paper [2] from which several details in this "Introduction" were borrowed, it is not very likely that Stamkart himself sought this new position. It is not clear who drew the attention of the Minister to him. In a letter dated September 1885 [11], Van de Sande Bakhuyzen wrote to Schols "that it could not have been Kaiser. Was it possibly Stuart?".

From the official correspondence in the archives of the Netherlands Geodetic Commission, according to Van der Schraaf, this question cannot be solved. It might be possible, however, that inofficially Kaiser drew the Minister's attention to his (Kaiser's) old friend Stamkart or that the Minister himself chose him as in 1865 Stamkart and Hoek reported to the Royal Netherlands Academy of Sciences on Kaiser's and Cohen Stuart's "Eischen der medewerking" mentioned before.

Thanks to Van der Schraaf's paper the backgrounds of Stamkart's appointment as the director of the triangulation are now known in detail. The triangulation itself, however, is little known as, when Stamkart died in January 1882, the measurements were not quite completed. The computations even had to be started. The triangulation had then already taken sixteen years, much longer than the four or five years Cohen Stuart had estimated in 1864.

Already in 1878 the Minister grew impatient on the long duration of the work as may appear from his letter dated June 19th of that year. He remarks there "that three times as much time and twice as much money has been spent on the triangulation as was estimated". Before the money for a subsidy in 1879 could be put on the budget for that year the Minister asked the advice of the Royal Academy of Sciences whether the continuation of the triangulation was desirable. At the same time Stamkart was instructed to give all the informations necessary for this advice to the Academy. From the Royal Decree dated February 20th 1879, it appeared that the triangulation would be continued under the supervision of a commission, The Netherlands Commission for Triangulation and Levelling. Stamkart himself was appointed as its first president.

After his death all the documents relating to the triangulation were turned over to the commission. They were studied thoroughly by their secretary Schols [12] . He concluded that "since before 1866 Stamkart was never engaged with accurate angle measurements the results of his angle measurements were far below the precision adopted for the European triangulation and even less good than Krayenhoff's measurements, rejected previously as insufficient". This judgement, officially redacted by Schols and Oudemans [13] , but in fact only by Schols - the draught in Schols' handwriting is in the Archives of the Netherlands Geodetic Commission - was communicated in a letter dated February 28th 1885 to the minister of home affairs and to the members of the second Chamber of the States General. It was signed by the entire commission, H. G. van de Sande Bakhuyzen (the Leiden astronomer, president), Ch. M. Schols (secretary), and the three members J. Bosscha, J. A. C. Oudemans and G. van Diessen. The text of the letter is published in the professional journal Tijdschrift voor Kadaster en Landmeetkunde [14] . The above underlined quotation is an excerpt from the letter. It was the death-blow to Stamkart's work and it can be considered as the motive for the new first order triangulation by the Netherlands Commission for Triangulation and Levelling.

Since that moment the extensive files of a sixteen years work lie buried in the archives of the Netherlands Geodetic Commission.

After Schols' investigation Mr. C. W. Moor was about the first who - in 1953 - consulted them again for his thesis Triangulaties in Nederland na 1800 (Triangulations in the Netherlands after 1800) [15] . His conclusions correspond with those already given by Schols and they relate mainly to the insufficient investigation of the instruments used, e. g. the systematic errors in the calibration of the limb of the theodolite, the bad state of the microscopes (especially microscope B), the systematic errors in the slow motion screws, and the incorrect execution of the measurements of the directions in the series.

It is true that Schols in his condemnation of Stamkart's work had no other data at his disposal, but Moor might have compared the angles of some triangles in Stamkart's network with those of the identical triangles of the excellent R(ijks) D(riehoeksmeting)-network [16] .

This publication will be an attempt to investigate Stamkart's long-forgotten work.

2. *STAMKART'S BIOGRAPHY*

Franciscus Johannes Stamkart was born January 25th 1805 in Amsterdam as the eldest of four children from the marriage of Johan Hendrik Stamkart and Maria Martijntje Hoogveldt [17] , [18] , [19] . His portrait with his signature (fig. 1) is a reproduction from [19] page 200. His birth-place is situated at No. 43 Rechtboomssloot.

He lived some time with his parents in Eibergen in the province of Gelderland, but the family returned soon to Amsterdam. His mother died there at the age of 81 on December 27th 1862. His father should have liked to be a clergyman, but his guardian "thought him too well-off for this profession". During the French revolution, however, he lost all his money and was forced to put up with the position of book-keeper.

The young Stamkart never had formal schooling. He got his first lessons from his father. When he was about twelve years old he received, together with his young friend F. Kaiser (1808-1872), the later famous Leiden astronomer, his first lessons in mathematics and astronomy from J. W. Kaiser, the uncle of his friend. His Latin teacher was the reverend Sartorius.

Already at the age of eighteen he was a member of the mathematical society "Een onvermoeide arbeid komt alles te boven" (A tireless labour overcomes everything). In the same year (1823) he even solved seven prize questions of this society. In 1824 he received an "encouragement prize" and the certificate of "member of merit". In 1824 he also passed an examination in mathematics.

Because of the possibility of an appointment as a teacher in mathematics and the art of navigation at Antwerp he went on a training voyage with a cargo-boat. It began on February 18th 1825, and on March 15th the boat ran aground at the rocky isle of Cocchino, west of the isle Khio (Chio) in the Aegean See [20] . According to [19] Stamkart came back in Holland in August 1825.

In 1826 he was indeed appointed as a teacher at Antwerp at a salary of eight hundred guilders per year. In 1827 he married there Carolina Gabriella de Bock.

Because of the Belgian troubles he sent his wife and his two children to Amsterdam on October 24th 1830. He himself left Antwerp as a pilot on board of a Danish ship. As all his possessions had to be left behind and his Antwerp half-pay was only f 400.-- per year during three years, Stamkart's financial position was not very rose-coloured. In Amsterdam, where he lived with his family at Vlakkeveld (the present 3rd Weteringdwarsstraat [21]), he tried therefore to supply his poor financial circumstances with private lessons in mathematics and the art of navigation.

Two other children were born in Amsterdam, a son and a daughter. The girl already died at the age of one. The eldest son, J. A. Stamkart, a doctor in the Oriental languages and in mathematics and physics, died soon after his father; the youngest son, Franciscus Hendrik (July 15th 1836 - November 21st 1892) was a well-known clock- and chronometer maker in Amsterdam. The second son was a captain of the Royal Dutch Steam navigation company. He died at the age of 36 in 1869.

In 1833 Stamkart was nominated inspector of weights and measures at Alkmaar. Two years later, in 1835, he changed this post for Amsterdam. Thirtyfour years - till 1867 - he was active in this profession. Stamkart was a man who could not live without working. He therefore not only took interest in matters relating to his profession but also in a great number of other subjects he came across. His interest and his knowledge of the art of navigation was already mentioned before. It resulted in a great many papers on this subject. He was therefore well-known in the merchant marine. It led to the appointment of fellow-director of the Kweekschool voor Zeevaart(Nautical College) in Amsterdam in 1857. The apparatus he designed for the measurement of the horizontal and the vertical component of the earth magnetism on board of seaships had not the success he expected.

His studies in the field of life insurance and life annuity were also well-known by insurance companies and the General Company for life insurance and life annuity rightly published the paper on the pioneer Stamkart already mentioned in [19] . The Dutch government also acknowledged Stamkart's abilities in this field. His advices regarding the approval of the articles of association of life insurance companies were almost always followed implicitly. It is incomprehensible that the government never paid any compensation for these important advices and the modest Stamkart apparently made no move in this direction.

The few examples given above show something of Stamkart's interest in a broad field of sciences. His extraordinary capacities, only obtained by self-tuition, led to his promotion doctor honoris causa in mathematics and physics at Leiden

University in 1844. In 1845 he became a fellow-member of the Koninklijk Nederlandsch Instituut van Wetenschappen (Royal Netherlands Institute of Sciences). During Stamkart's membership the institute was replaced by the Koninklijke Academie van Wetenschappen (Royal Academy of Sciences).

On December 1st 1865 Stamkart was charged with the geodetic measurements in the Netherland's part of the Middle European triangulation. In his letter to the minister of home affairs, dated November 14th 1865 [22] , he already wrote that he was willing to accept this charge. The astronomical part of the triangulation was to be done by his youth friend F. Kaiser, the Leiden astronomer. Till his death Stamkart was engaged with this task, next to the other activities he was charged with, e.g. the introduction of the new standards of the metre and the kilogram. Just like as for his advice in the field of the life insurances the Dutch government never paid him any compensation for the execution of the triangulation to which he devoted his energy for sixteen years. The only payments were for the hotel and travelling expenses made.

It will be clear that, because of his small income, Stamkart's way of living could not have been very luxurious. It was rather poor indeed. The location of the houses where he lived in Amsterdam - first at Kattenburg, then at Bloemgracht, and later on at Achtergracht 808 (at present numbered Frederiksplein 2) - does not agree at all with the status of a man of his scientific level [23] . The only appreciation of the government was Stamkart's nomination to Knight in the Order of the "Nederlandse Leeuw" and professor of pure and applied mathematics and the verification of weights and measures at the Polytechnical School at Delft in 1867 [24] .

His appointment at Delft caused a move to this town in November 1867. His financial circumstances were a little better now so that he could live in a more spacious house than in Amsterdam (at Binnenwatersloot 14 and Oostsingel respectively). But he apparently could never afford the purchase of a house for himself. Mr. H. Posthuma, engineer of the Cadastral landsurveying department at the Hague informed me that Stamkart never owned any real property at Delft.

Also during his professorship, however, the triangulation remained his most beloved activity and when this activity appeared to be incompatible with his paid post as a professor he asked to be discharged from this task. It was granted or July 17th 1878. He was then 73 years old. And with this voluntary discharge from a paid function he came back in the unpaid position of "his" triangulation. It is true that he received a small pension, but it appeared to be no more than the contribution he had paid himself to the pension fund.

On May 7th 1879 he moved again to Amsterdam, where he lived in the upper part of the house Prinsengracht 382 [25] near Passeerdersgracht. He remained there working on the triangulation as a member and as the president of the Netherlands Commission for Triangulation and Levelling, established on the proposal of the Royal Academy of Sciences by Royal Decree dated February 20th 1879. His eye-sight and his health declined in those years: I caught him several times on pointings at wrong towers and at Groningen [26] he had to visit the physician dr. J. Baart de la Faille because of heart complaints. They must have been inevitable for a man of 74 years old who, very often several times a day, mounted the stairs of the Martini tower, even to the fourth gallery, in order to execute his measurements.

During the measurements at Emden in August 1881 Stamkart sent a telegram to his assistant Van Hees that he was ill and could not come. The telegram was followed by a letter dated August 10th 1881 [27] in which he writes: ". . . . The illness is not, as I was afraid of, connected with the breast as in former years but it has another cause that the doctor knows better than I do. The doctor, however, does not find it alarming, though it looked like it in the beginning. I therefore hope that I can come back in a few days . . . . . ". He returned to Emden indeed on Saturday (!) August 13th and he remained there till Thursday, September 22nd. The measurements at Emden were completed the previous day. The last week no observations could be made because of the very bad weather conditions.

In a heavy storm and with reefed sails the journey to Delfzijl was made in a small boat. Because of the very low tide it ran there aground in the mud so that the instruments and the other things could not be brought ashore. This had to be done the next day at half past five in the morning. The same day - Friday 23rd - Stamkart and Van Hees travelled by train to Harlingen. Saturday 24th, finally, the boat for Amsterdam left Harlingen at ten o'clock in the morning; it arrived in Amsterdam at half past five p. m. The measuring season 1881 was closed . . . . .

Stamkart passed away, almost 77 years old, in Amsterdam on January 15th 1882. The accommodation in his house was so small that the funeral could not take place from Prinsengracht 382. As Stamkart's pension expired with his death and as there was no widows' fund, Mrs. Stamkart was left behind under rather precarious financial circumstances.

In the weekly paper "De Amsterdammer" No. 243 of February 19th 1882, A. L. J. Bouten wrote an article on Stamkart's extraordinary gifts of head and heart. In the next section Mr. Bouten's name will be mentioned several times.

By a lucky chance and by the highly appreciated collaboration of Mr. H. Jutte in Amsterdam, who gave the greater part of the following details, Stamkart's lineal descendants could be traced till the present. They are descended from Stamkart's youngest son, the clock- and chronometermaker already mentioned in the beginning of this section.

From his first marriage to Anna Maria Kley a daughter was born. In 1904 she was a nurse in the former Dutch East Indies. His second marriage on April 21st 1871 was to Hendrika Stoets. From this marriage a son, Franciscus Johannes Hendrik, was born in Amsterdam on October 27th 1874. He became a painter. In 1897 he married Elene Emilie Wissmann, also a painter. From the marriage two sons were born, Walter Emil Stamkart, on June 27th 1900 in Amsterdam, and Werner Stamkart, on June 9th 1904 at Hilversum.

After the divorce of their parents in 1907 Werner left with his mother for Germany. Mr. Jutte's researches into this German branch of the Stamkart-family were unsuccessful. Walter Emil, however, raised by his father, became a planter in 1925. He lived in East Java in the former Dutch East Indies. On February 26th 1935 he married Christina Poel at Bussum. The young couple left for Java where three children were born in their residence at Djember, two sons and a daughter. The daughter, Femmeke Marijke Elene, born July 9th 1940, the present Mrs. Hulst-Stamkart, lives at Oegstgeest near Leiden. The two sons, Wouter Werner, born April 9th 1936, and Peter Frank Marius, born February 8th 1938, live at the Hague and at Spijkenisse (near Rotterdam) respectively. In 1944 during world war 2 their father, Walter Emil, perished on board of a troopship with prisoners of war. On its way to Birma the ship was torpedoed.

On June 11th 1963 Wouter Werner married Sonja Feller. They have three children: two girls, born on August 8th 1965 and June 24th 1969 respectively, and a son, Walter Emil, born at the Hague on June 26th 1967.

Peter Frank Marius married Miss T. A. P. Stienstra on May 12th 1962. They have two children: a boy, Eric Willem, born at the Hague on March 26th 1964, and a daughter, born February 2nd 1967.

3. *STAMKART AND HIS COLLABORATORS*

It will be clear that the enormous task laid on the shoulders of Stamkart could not be executed without the aid of assistants who could help him with the measurements during the summer and with the computations during the winter.

The expression "assistants" (plural) is used here in an euphemistic sense: actually, apart from the local workmen at the several stations, there was always only one collaborator on scientific level in the summer season. When he left the triangulation, his place was taken by a new one. As Stamkart was always overloaded with work and consequently had to leave several operations to his assistants, it seems logical to mention them in this publication. Their devotion and their accuracy influenced the results of the triangulation.

The first collaborator, dr. A. L. Boeck, a professional army officer and a doctor in mathematics and physics, was Stamkart's companion during the reconnaissance of the triangulation network in 1866. In 1867, during his geodetic practice at Leiden observatory he was taken ill and never resumed his work. It will be clear that his contribution to the triangulation can be neglected.

In his place came Mr. A. N. J. van Hees, a Delft civil engineer. In my opinion he was Stamkart's most devoted collaborator. His correspondence with Stamkart is kept in a special file (file He) in the archives of the Netherlands Geodetic Commission. It is interesting to read these letters, written between 1867 and 1881. In their sequence a development is perceptible in the relation between the two men. In the first letter (July 26th 1867; He-1) the humble tone of a young man to a forty years older "authority", but already in the tenth letter (September 1870; He-10) a more human note and the signature: "your obedient servant and friend A. N. J. van Hees". It is hardly believable that they were written only about a century ago, when, in September 1867, Van Hees' "little brother died of the terrible disease which harasses Delft" (letters He-5 and 6) [28] , and that, in his letter dated September 1st 1869 (He-8) Van Hees writes that "because of an ulcer in the mouth, I cannot go out before tomorrow. Dr. Soutendam said to me that it was a blood ulcer, but one of a big size; that, however, I should not put poultice on it but raw gingerbread paste. He expected that with this paste it would burst this night".

Van Hees joined the triangulation August 1st 1867, and remained till March 1st 1872. He then entered the service of the company charged with the setting out of the local railway Groningen-Marum-Drachten-Beetsterzwaag-Heerenveen, the so-called Frisian line, in the northeast of the Netherlands. On September 1st 1872 this job was completed, and soon thereafter he left for Sumatra in the former Dutch East Indies. In a letter dated October 10th 1873 (He-24) he writes in detail about his voyage to this country, about the enormous heat in the Red Sea, about the death of two children because of this heat and, finally, about his activities in the field of landsurveying, especially in the determination of heights with barometers and tacheometry.

On August 23rd 1879, more than seven years after his departure in 1872, Van Hees re-entered the triangulation and carried on till Stamkart's death in 1882. He also rounded-off the computations and handed over the papers relating to the triangulation to the Netherlands Commission for Triangulation and Levelling, already mentioned in Stamkart's biography. As remarked before, Van Hees maintained his relations with Stamkart also in the years he worked in other services. In his letter to Stamkart dated April 12th 1872 (letter He-20), he mentions the artillery officer mr. P.C. de Wilde as a possible successor for the position he himself left six weeks before. On May 20th 1872 (letter He-22) he even mentions another replacement, mr. O. Gleuns, then a cadastral landsurveyor at Sappemeer in the province of Groningen. At that time, however, De Wilde's appointment as Stamkart's assistant was already a fact: according to a letter dated May 14th 1872, the minister of war gave him short leave during the five months of the measuring season 1872. Also in the season 1873 De Wilde assisted Stamkart with the exception, however, of the 46 days from July 1st till August 15th, according to the minister, essential for the fulfilment of military duties [29]. During these 46 days the just mentioned mr. O.Gleuns took his place with the permission of the minister of finance [30]. According to the expense account of the triangulation over the year 1873 mr. Gleuns' compensation for this period was 46 x f 5.-- = 230 guilders.

De Wilde left the triangulation on July 14th 1874 "because of his appointment at the Military School at Breda" (the later Royal Military Academy) "in order to teach there the artillery sciences" [31].

Mr. H. van Schevichaven, his successor, arrived on the same day. "After his studies at the Polytechnical School at Delft and after a successful examination C in this year, mr. van Schevichaven has now the title of civil engineer". He served the triangulation till March 1876, for on April 1st 1876 he was nominated in a permanent post as an engineer of the Provincial "Waterstaat" at 's-Hertogenbosch.

A successor was not easy to find. Through the intermediary of mr. de Wilde (see above) the attention fell on the lieutenant A.R. Krayenhoff van de Leur, but the interests of the military forces did not allow that this plan could be effectuated [32]. Finally, after the examinations at the Delft Polytechnical School, mr. A.L.J. Bouten entered the triangulation through the intermediary of Cohen Stuart. As Stamkart remarks in his diary, he passed his final examination with much praise.

Because of the departure of Van Schevichaven and the late arrival of Bouten in the measuring season the field activities in 1876 were very limited. Only the stations Lemmer and Meppel could be visited. In his report to the minister of home affairs over the year 1876 Stamkart proposes to raise the salary of the assistant to six guilders per day (180 guilders per month) during the measuring season (five months). During the seven remaining months, in which "office-work" (computations) had to be done, the salary remained the same as in former years (two guilders per day). According to Stamkart "the compensation of six guilders per day is still small for the scientific labour that must be done". In 1874 it was still five guilders but it was raised to f 5.50 during 108 days in the season 1875.

Stamkart was very satisfied with Bouten's assistance: for the year 1877 his salary for "office-work" could be raised to f 2.50 per day (f 75.-- per month) during eight months. The amount six guilders per day for the measuring season (115 days), however, remained unaltered.

Bouten served the triangulation till August 1879. As from the course 1879-1880 he was appointed as a teacher at the secondary school at Dordrecht. It was he who wrote the obituary in "De Amsterdammer" already mentioned in the previous section.

As mentioned before Van Hees took over Bouten's place in the triangulation on August 23rd 1879 and remained there till Stamkart's death in 1882. From Stamkart's annual reports to the minister of home affairs it appears that also a compensation was paid to his son Dr. J.A. Stamkart, already mentioned in the preceding section. During the "office months" he verified the copies of the observations and the computations made by Stamkart's assistants "in order to discover possible errors in so many thousands of ciphers" [33] .

GEODETIC PART OF THE TRIANGULATION

4. *GENERAL SURVEY OF THE TRIANGULATION*

A general survey of the triangulation is given in fig. 2 (see the enclosed map). Conformably to Cohen Stuart's suggestion in section 1 a double chain of triangles starts from the astronomical station Leiden (No. 14) in a southeastern direction to the stations Lommel (No. 1) and Nederweert (No. 2) situated on Belgian and Dutch territory respectively. It was Stamkart's intention to proceed the triangulation as far as Peer, an angular point of the Belgian primary network. This station could be seen from Nederweert but not from Lommel because of an obstacle [34] which could have been avoided if the reconnaissance would have been better. Also at other places of the triangulation network the bad reconnaissance is perceptible and Moor [35] rightly objects to this carelessness.

Two other, partly double, chains of triangles start from Leiden in a northern and an eastern direction respectively; the first west of the former Zuiderzee, the second east of it. They have the side Hindeloopen (No. 26) - Lemmer (No. 27) in common. From this side the triangulation goes as a single chain in an eastnortheastern direction to Leer, Emden (No. 46), and Pilsum (No. 47) in northwestern Germany.

The single chain is in contradistinction to Cohen Stuart's suggestion. The bad reconnaissance in Oldeholtpa (No. 30) is the reason that a double chain could not be realized. It appeared that at the same place on that tower where Krayenhoff in 1810 pointed at Meppel (No. 31), Blokzijl (No. 29), Lemmer (No. 27), Sneek (No. 36), Drachten (No. 39), Oosterwolde (and Beilen), the towers of Sneek, Drachten, and Oosterwolde could no longer be seen in 1877 [36], probably because of the increased height of the trees in the 67 years between the two measurements. The planned double chain of triangles at the east side bordered by the line Oldeholtpa - Oosterwolde - Groningen (see the dotted lines in fig. 2) had therefore to be changed into the single chain bordered by the stations Lemmer - Sneek - - Drachten - Groningen.

In the utmost northeastern part of the network the measurements were not quite ready when Stamkart died in January 1882. The directions in Onstwedde to Groningen, Midwolda, and Leer, in Leer to Onstwedde, Midwolda, and Emden, and those in Pilsum (No. 47) to Emden, Holwierde, and Uithuizermeden had still to be measured. On the map they are marked with dashed lines.

The connection of Stamkart's network with the Prussian triangulation in the neighbourhood of Bentheim was not realized. Cohen Stuart's proposal to Kaiser, mentioned in the Introduction of this publication,was not strictly necessary for this connection.

The advice for the measurement of a base line was followed. It is situated in triangle 22 of fig. 2, south of the line Haarlem (No. 17) - Amsterdam (No. 18). Its length is about 6 km. The distance Haarlem - Amsterdam of the triangulation network could be computed from it.

Almost all the angular points of the network are church towers and almost all of them are identical with those of Krayenhoff's triangulation. Krayenhoff's station The Hague, however, was replaced by Delft (No. 13) and Krayenhoff's Leiden Saaihal (Sergehall) by Leiden Stadhuis (Townhall) (No. 14). Harderwijk (No. 33), in Krayenhoff's triangulation a long fir tree on the roof of the reformed church [37] , is the church tower in Stamkart's measurements. Krayenhoff's station Staveren was replaced by Hindeloopen (No. 26), about 7.3 km northnortheast of Staveren. After several pointings at Staveren (from Enkhuizen, Medemblik, and Urk) Stamkart found out that the tower he had pointed at was not Krayenhoff's station. It appeared that it was pulled down and replaced by an other church tower at about the same place. It was, however, unsuitable for the execution of measurements. A better reconnaissance would have prevented the measurement of the unnecessary directions.

The station Dokkum of Krayenhoff's network, pulled down in 1826,finally, had to be replaced by Kollum (No. 40), about 11.4 km eastsoutheast of Dokkum.

Only one angular point of Stamkart's network is not a tower. It is the stone Veluwe (No. 34), already used in 1805 as the station "Observatoire" in Krayenhoff's triangulation [37] and found again by Stamkart in 1874. A detailed description of Stamkart's dealings with the stone is given in section 16, page 98.

In contradistinction to Krayenhoff, who measured angles, Stamkart measured directions at his stations. The numbering of these directions is shown in fig. 2. The dotted directions in Gorinchem (No. 10), Utrecht (No. 16), and Amersfoort (No. 35) to the Cunera tower at Rhenen could not be used in the triangulation as the small number of observations to heliotropes on the balustrade and the lack of centering data at Rhenen make it impossible to determine the accurate position of the spire with respect to the network. The un-numbered directions at the station Emden (No. 46) to Aurich and Hage were apparently meant for a comparison of the angles Pilsum - Emden - Hage (angle 430), Hage - Emden - Aurich (angle 433), and Aurich - Emden - Leer (angle 437) of Krayenhoff's triangulation [38] .

5. *DESCRIPTION OF THE INSTRUMENTS USED*

Apart from the apparatus for the base line measurement which will be described in section 9, the instruments used for the triangulation were two theodolites. They were made by the German firm Pistor and Martins in Berlin. Cohen Stuart had suggested that Repsold at Hamburg should be charged with the manufacturing of the instruments required, but the time of delivery by that firm was too long in Stamkart's opinion.

The two instruments have been preserved. They belong to the collection of the Netherlands Geodetic Commission at Delft. The small instrument, used for the reconnaissance of the network and for secondary local measurements is lent to the "Rijksmuseum voor de geschiedenis der natuurwetenschappen" (National Museum for the history of natural sciences) at Leiden. Its catalogue number there is 88. The diameter of its horizontal limb is 15.5 cm. Each part of the limb is 20'. The unit of the verniers on the horizontal limb is $\frac{1}{2}'$. That of the verniers on the vertical limb (diameter 12 cm) is 1'. The height of the horizontal axis of the instrument with respect to its groundplate is about 17 cm. The telescope, about 27 cm long, can only change face if it is taken from its supports and reversed. The instrument has two levels, one on the horizontal limb, the other on the telescope. It can therefore also be used as a simple levelling instrument [39] .

The big instrument, used for the measurement of the triangulation network, is pictured in fig. 3. It was ordered March 1866 and already delivered in October of the same year. Its price was 668 Prussian thalers (two thousand Mark) and seven thalers extra "for a careful packing" [40]. It is in the collection of historical instruments of the subdepartment of geodesy of the Delft University of Technology. Mr. Pouls of the subdepartment was very helpful to me in investigating it. The telescopes, the limbs, and the reading mechanism of the instrument are in a very poor condition and in the levels a and b on the telescope and on the vertical limb the level liquid is missing. Of the striding level on the horizontal axis (not on the photos) even the tube is missing.

The horizontal limb c has a diameter of about 27 cm. It is calibrated to the right in 360 degrees. They are marked with reversed ciphers. Each degree is divided

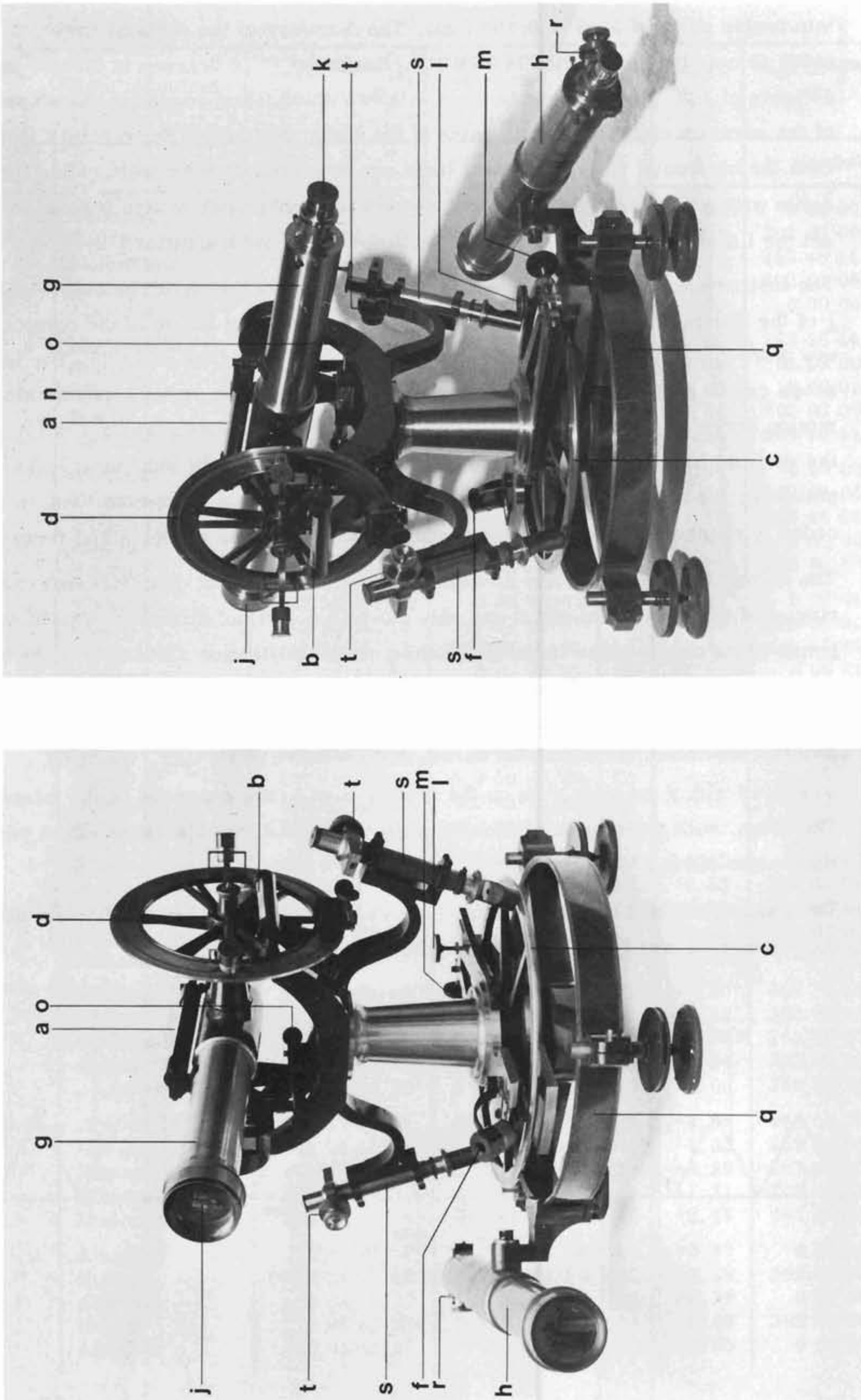


Fig. 3

into twelve parts of 5' (5' = 0.196 mm). The diameter of the vertical limb d is about 19 cm. It is calibrated in 36 x 10^{o}. Each part of 10 degrees is divided into 60 parts of 10'. The limb can be read with two diametrical verniers. As 60 parts of the verniers coincide with 59 parts of the limb, the unit of the verniers is 10". Both the horizontal and the vertical limb can be turned by hand (reiteration theodolite with a friction-tight circle). A second horizontal limb makes it possible to set the limb in any arbitrary position with the aid of the magnifier f.

The instrument has two telescopes, g and h. The focus length of the object-glass j of the "normal" telescope g is about 510 mm. The focus length of the compound eye piece k is about 15.4 mm. The magnification is therefore about 33. The telescope can be pointed at the sighting point with the clamping screw l and the slow motion screw m of the horizontal limb and the corresponding screws n and o of the vertical limb. Focussing at the object can only be done by hand as it lacks a pinion for the movement of the eye piece. From the photos it appears that, in order to change face, the telescope must be taken from its supports and reversed.

The second telescope h is the so-called reference telescope. It is fastened to the ring q of the instrument and it can only move in a vertical direction. The focus length of its object-glass is about 510 mm, its magnification about 25. At the beginning of a series the telescope was pointed at an arbitrary object. It had to remain pointed at that object during the whole measurement of the series. A possible deviation in horizontal sense, e.g. because of torsion, could be measured with a movable wire on the drum r next to the eye piece of the telescope. The drum, with a diameter of about 33 mm, is divided into 100 parts. Each part represents about 1".02.

The reading on the horizontal limb c is done with the two diametrical micrometer microscopes s and the saw-shaped mechanism pictured in fig. 4.

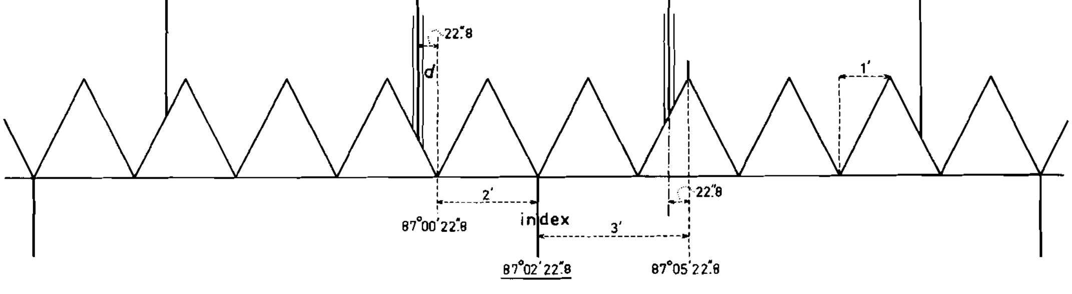


Fig. 4

The part of the limb between about $86^{o}55'$ and $87^{o}10'$ as it is seen in one of the microscopes ("normal" ciphers in an anti-clockwise direction) is drawn with respect to the teeth of the saw. Each of the teeth represents 2' (0. 078 mm), half a tooth 1' (0. 039 mm). The bottom side of the tooth in the centre of the field of view of the microscope is the index. It is marked with a dash. The reading of the limb with respect to the index is $87^{o}00' + 2'$ plus the distance d. This distance can be measured on the calibrated drum t with the aid of a movable double wire. The drum is divided into 2 x 60". If the reading on the drum is zero, the double wire is in the bottom point of a tooth. Via the readings 1-2-3. . . . . -57-58-59 (the wire moves then to the left) and the reading of the calibration line $87^{o}00'$ on the limb, it reaches the top of a tooth, marked on the drum with a dash (60"). One second on the drum represents about 0. 86 mm, sothat tenths of a second can be estimated. If the reading of the calibration line $87^{o}00'$ is $22''.8$, the reading of the limb is:

$$87^{o}00' + 2' + 22''.8 = 87^{o}02'22''.8 .$$

It will be clear that the reading $22''.8$ of the calibration line $87^{o}05'$ falls on the second half of the drum, marked: dash -1-2 . . . . . . -58-59-0. As $0''.1$ on the limb of the theodolite represents only 0. 000065 mm, the mechanical requirements for the instrument and its limb must be extremely high in order to rely on readings in tenths of a second.

It must be stated that Stamkart never investigated the accuracy of his instruments in a serious manner. After his death this investigation was done by Schols. He concluded that the limb of the first order theodolite showed imperfections which could only be removed by measuring with special precautions and that the state of the screws of the microscopes was very poor, even so poor, that an accurate measurement with these screws was impossible [41]. Stamkart himself had also difficulties with them, especially with the screw on microscope B [42]. It was even sent back to the factory after the measurements in 1873 but it seems that the repair was of little use. During a short time Stamkart even used a special method for the reading of B without the use of its micrometer screw [43]. The reading of B was determined by the measurement of the difference between the readings of A and B with the micrometer screw of A. The method had the drawback, that the slow motion screw m of the horizontal limb had to be used for it. This method was therefore soon abandoned.

In some cases Stamkart used heliotropes for his measurements. Three of these instruments were made by the instrument maker L. J. Harri "after a model of the Delft Polytechnical School". They cost together f 220. -- [44] . An objection of the heliotropes was that they were difficult to handle by the unskilled workmen. During the measurements of a series it often happened that, as Stamkart says in his diary, "the light disappeared", so that the direction had to be omitted. It will be clear that the poor contact in those days between the measuring station and the heliotropist did not foster a quick restoration of the lost optical communication.

6. *EXECUTION OF THE ANGULAR MEASUREMENT*

For the measurement of the directions at the stations the theodolite was set up in an eccentric point on a galery of the tower where the directions could be seen. It will be clear that in several cases more than one eccentric point was necessary as the towers to be pointed at were distributed all over the horizon.

Stamkart gives very little information on the provisions which had to be taken in order to make a stable construction for the place of the instrument. Only here and there he mentions the effect of an apparent unstable place, e.g. in Hoofddorp (see fig. 14 in section 11): "Mr. Huet and Mr. Amersfoordt" (Mr. Amersfoordt was the burgomaster of Haarlemmermeer; Hoofddorp is the main village in this municipality) "being in the tower since the beginning of the measurement, make their departure; the bubble of the level is no longer centered. At the beginning it was centered. Their weight, no longer pressing" (on the floor of the tower) "makes that the bubble comes higher at that side" [45].

Before the measurement in a series began, the reference telescope was pointed at an arbitrary point (spire of a tower) that could be seen. It remained pointed at that same object during the whole measurement. It was the intention to check after every pointing whether the position of the reference telescope had not changed. A small deviation in horizontal sense could, as already remarked in section 5, be measured on the drum of the telescope and be given as a correction to the corresponding direction in the series. This, however, was never done systematically. The use of the reference telescope therefore lost the meaning for which it was meant. At the most it was a rather rough check on an alteration in the orientation of the series.

The readings on the limb of the theodolite were done by two observers, one for microscope A, the other for microscope B. The pointing at a station and the reading of that pointing with the two microscopes was repeated several times following each other immediately. Some times it was done five times, in other cases, however, two or three times, in still other cases, when the object pointed at was difficult to see, even eleven times [46]. From the readings in tenth or twentieth parts of a second the mean was computed in hundredths of a second. Without a possible correction to be derived from the reading on the reference telescope this mean was used as only one observation in the series. In some exceptional cases when the object disappeared already after the first pointing the mean could only be determined from the reading on each of the microscopes A and B. It will be clear that in every series the reliability of "a" pointing is therefore often very variable.

In table 1 an example is given of the measurement of three series (the numbers 6, 7, 8) at the station Schagen of the main triangulation network on September 5th and September 6th 1871, at 9.45 - 10.18 a.m. (van Hees), 8.30 - 9.20 a.m. (Stamkart), and 9.25 - 10.15 a.m. (van Hees) respectively [47] . The names between brackets are the observers in the series. The readings are shown in the columns 3 and 4 of the table, the mean readings in column 5. Column 6 gives the incidental readings on the drum of the reference telescope, and column 7 the weather conditions during the measurements.

Table 1

Series	Sighting points	Microscope A	Microscope B	Mean reading	Ref. telesc.	Remarks
1	2	3	4	5	6	7
6	Alkmaar	197° 49' 42."3	49."4	197° 49' 46."08	43."4	very good
		43.05	49.85			
		41.9	50.0			
	Hoorn	139 52 46.05	48.35	139 52 46.12	43.4	rather good
		45.7	46.45			
		43.9	46.25		41.3	
	Medemblik	101 36 30.1	28.95	101 36 32.76		not clear
		34.45	33.1			
		30.75	30.0			
		36.25	34.85			the best one
		35.1	34.1		41.3	also good
	Hoorn	139 52 46.0	47.35	139 52 46.58		very good
		45.5	46.7			
		45.7	48.25			
	Alkmaar	197 49 43.25	50.65	197 49 45.42	39.6	faint but can be seen with certainty
		40.65	48.0			
		40.85	49.15			

Table 1 (continued)

1	2	3	4	5	6	7
		° ′ ″	″	° ′ ″	″	
7	Alkmaar	257 53 48.3	52.0	257 53 48.64	37.5	Wind east;
		43.95	50.3			clear sky, sun;
		45.85	51.55			beautiful
		46.0	51.2			weather
	Hoorn	199 56 50.1	57.35	199 56 50.81	37.5	
		47.55	54.3			
		44.7	50.85			
	Medemblik	161 40 33.0	37.8	161 40 34.46		
		29.45	35.6			
		31.75	38.95		39.0	
		31.2	37.95			
	Hoorn	199 56 43.75	49.15	199 56 45.75		
		42.35	49.05		35.5	
		41.9	48.3			
	Alkmaar	257 53 43.45	48.1	257 53 46.22		
		44.65	49.45			
		43.25	48.45		35.5	
8	Medemblik	262 05 29.45	32.95	262 05 29.84	69.2	very faint
		26.6	30.3			
		24.45	31.4			good
		28.7	34.85			
	Hoorn	—	—		69.2	not visible
	Alkmaar	358 18 50.15	46.25	358 18 47.92		very faint
		48.35	44.25			and
		51.55	46.95		67.3	shimmering
	Medemblik	262 05 20.7	25.1	262 05 24.27		faint
		22.0	26.95			
		22.25	28.6		66.7	
	Alkmaar	358 18 42.4	39.25	358 18 41.38		very faint
		43.35	40.3			
		44.2	38.8		63.8	
	Medemblik	262 05 23.25	27.5	262 05 25.45		very faint
		24.8	30.0			rather good
		20.35	26.8		64.2	

The amounts in column 5 are once again given in column 3 of table 2, where all mean readings for the directions Alkmaar, Hoorn, and Medemblik at the station Schagen are mentioned in the sequence of the series. The meaning of the columns 4-7 of that table will be treated in section 7.

Table 2

Series j	Sighting points	Directions α_i (measured)	Directions $\alpha_i + 0'_j$ (Alkmaar=0)	i	Corr. p_i	Adj. directions $\alpha_i + 0'_j + p_i$
1	2	3	4	5	6	4+6=7
		° ' "	° ' "		"	° ' "
1	Alkmaar	318 09 17.41	0 00 00.00	1	+0.04	0 00 00.04
	Hoorn	260 12 20.07	302 03 02.66	2	-2.61	302 03 00.05
	Medemblik	221 55 58.32	263 46 40.91	3	+3.60	263 46 44.51
	Hoorn	260 12 17.60	302 03 00.19	4	-0.14	302 03 00.05
	Alkmaar	318 09 18.35	0 00 00.94	5	-0.90	0 00 00.04
2	Medemblik	280 26 04.42	263 46 45.47	6	-0.66	263 46 44.81
	Hoorn	318 42 18.83	302 02 59.88	7	+0.48	302 03 00.36
	Alkmaar	16 39 18.95	0 00 00.00	8	+0.35	0 00 00.35
	Hoorn	318 42 17.91	302 02 58.96	9	+1.40	302 03 00.36
	Medemblik	280 26 05.34	263 46 46.39	10	-1.58	263 46 44.81
3	Alkmaar	77 32 05.82	0 00 00.00	11	+0.27	0 00 00.27
	Hoorn	19 35 07.66	302 03 01.84	12	-1.56	302 03 00.28
	Medemblik	341 18 52.28	263 46 46.46	13	-1.73	263 46 44.73
	Alkmaar	77 32 07.02	0 00 01.20	14	-0.93	0 00 00.27
	Hoorn	19 35 02.14	302 02 56.32	15	+3.96	302 03 00.28
4	Alkmaar	137 52 09.54	. 0 00 00.00	16	+1.67	0 00 01.67
	Hoorn	79 55 12.51	302 03 02.97	17	-1.29	302 03 01.68
	Medemblik	41 38 55.46	263 46 45.92	18	+0.21	263 46 46.13
	Alkmaar	137 52 11.81	0 00 02.27	19	-0.60	0 00 01.67
5	Alkmaar	197 49 61.89	0 00 00.00	20	-6.45	359 59 53.55
	Medemblik	101 36 45.37	263 46 43.48	21	-5.47	263 46 38.01
	Hoorn	139 52 52.91	302 02 51.02	22	+2.54	302 02 53.56
	Alkmaar	197 49 51.92	359 59 50.03	23	+3.52	359 59 53.55
	Hoorn	139 52 55.47	302 02 53.58	24	-0.02	302 02 53.56
	Medemblik	101 36 34.03	263 46 32.14	25	+5.87	263 46 38.01
6	Alkmaar	197 49 46.08	0 00 00.00	26	+0.42	0 00 00.42
	Hoorn	139 52 46.12	302 03 00.04	27	+0.38	302 03 00.42
	Medemblik	101 36 32.76	263 46 46.68	28	-1.80	263 46 44.88
	Hoorn	139 52 46.58	302 03 00.50	29	-0.08	302 03 00.42
	Alkmaar	197 49 45.42	359 59 59.34	30	+1.08	0 00 00.42
7	Alkmaar	257 53 48.64	0 00 00.00	31	-0.36	359 59 59.64
	Hoorn	199 56 50.81	302 02 62.17	32	-2.52	302 02 59.65
	Medemblik	161 40 34.46	263 46 45.82	33	-1.72	263 46 44.10
	Hoorn	199 56 45.75	302 02 57.11	34	+2.54	302 02 59.65
	Alkmaar	257 53 46.22	359 59 57.58	35	+2.06	359 59 59.64
8	Medemblik	262 05 29.84	263 46 41.92	36	-2.28	263 46 39.64
	Alkmaar	358 18 47.92	0 00 00.00	37	-4.83	359 59 55.17
	Medemblik	262 05 24.27	263 46 36.35	38	+3.29	263 46 39.64
	Alkmaar	358 18 41.38	359 59 53.46	39	+1.71	359 59 55.17
	Medemblik	262 05 25.45	263 46 37.53	40	+2.11	263 46 39.64
9	Alkmaar	65 05 42.36	0 00 00.00	41	+0.47	0 00 00.47
	Hoorn	7 08 42.26	302 02 59.90	42	+0.58	302 03 00.48
	Alkmaar	65 05 43.73	0 00 01.37	43	-0.90	0 00 00.47
	Hoorn	7 08 42.93	302 02 60.57	44	-0.09	302 03 00.48
	Alkmaar	65 05 42.88	0 00 00.52	45	-0.05	0 00 00.47

Table 2 (continued)

1	2	3	4	5	6	4+6=7
		° ′ ″	° ′ ″		″	° ′ ″
10	Medemblik	21 39 05.22	263 46 46.07	46	+1.10	263 46 47.17
	Hoorn	59 55 24.24	302 03 05.09	47	-2.37	302 03 02.72
	Alkmaar	117 52 19.15	0 00 00.00	48	+2.71	0 00 02.71
	Hoorn	59 55 23.00	302 03 03.85	49	-1.13	302 03 02.72
	Medemblik	21 39 06.62	263 46 47.47	50	-0.30	263 46 47.17
11	Medemblik	86 59 23.76	263 46 43.59	51	+1.57	263 46 45.16
	Hoorn	125 15 38.32	302 02 58.15	52	+2.56	302 03 00.71
	Alkmaar	183 12 40.17	0 00 00.00	53	+0.70	0 00 00.70
	Hoorn	125 15 43.49	302 02 63.32	54	-2.61	302 03 00.71
	Medemblik	86 59 27.54	263 46 47.37	55	-2.21	263 46 45.16

$m^2 = [pp] : \{(55 - 3) - (11 - 1)\} = [pp] : 42. \quad m = \pm 2''.62$

The reader will see that Stamkart's measuring method deviates considerably from the one we use nowadays. It is striking that in the first series the first sighting point Alkmaar gives the arbitrary reading 318° on the limb of the theodolite, whereas in general a reading 0° is usual. The sequence Hoorn-Medemblik, an anti-clockwise direction, is also unusual.

It is incorrect to suffice with only one observation at Medemblik in the first series. Stamkart should have done a second observation with another reading on the limb . The same can be said of the other ten series. A regular distribution of the observations all over the limb of the instrument - and this is necessary for the elimination of errors in the calibration - could therefore have been realized in the best way by turning the limb over an angle $\pi : 22 \approx 8^\circ 11'$ after each half-series. It must be said, however, that in Stamkart's measuring system this was not practicable as in the beginning of the measurement the number of series could not yet be predicted. He continued his measurements till the moment that, in his opinion, the number of pointings at every sighting point would suffice.

Apart from these considerations his insight into the distribution of the measurements over the limb must have been very bad. As series 4 is measured at the same place of the limb as series 1 - only microscope A in the first series is B in the fourth - and, in the same way, 5 = 6 = 2 and 7 = 3, there remain only the places 3° (series 11), 17° (series 2,5,6), 65° (series 9), 77° (series 3,7), 117° (series 10), 138° (series 1,4), and 178° (series 8) where, either with microscope A or B, the direction Alkmaar was read. It is a very poor distribution indeed of the observations over the (semi) circle.

The reader will have noticed that in series 8 (see table 1) Hoorn fails. It could not be seen and was therefore skipped. The same holds for Hoorn in the second half of the series 4 (see table 2). In the observation register nothing is said about the missing direction Medemblik in series 9. Possibly this series was measured in order to complete the supposed number of missing observations to Hoorn in the just mentioned series 4 and 8. For this reason the number of series could not be determined at the beginning of the measurement. It was dependent on the circumstances under which the measurements passed off.

A very serious objection to Stamkart's measuring scheme is that apparently he did not realize that the instrumental errors (vertical axis not perpendicular to horizontal axis, and horizontal axis not perpendicular to line of sight) affect the accuracy of the observation. Nowhere in his triangulation network appears that, by a judicious measuring scheme, these errors could be eliminated. With a "normal" theodolite this can be done by changing face after the first half of a series and measure the second half with the vertical limb at the right instead of the left side of the observer. It is true that the telescope of Stamkart's instrument could not change face, but the same effect could have been obtained if he had taken the telescope from its supports and had reversed it. This, however, was never done. This lack of elementary geodetic knowledge shows that Stamkart has apparently not prepared himself sufficiently for the geodetic task he was charged with. In a flat country like the Netherlands, however, the consequences of this enormous theoretical blunder must not be exaggerated. As the inclination h of the line of sight was always about zero and the influence of the instrumental errors in question is proportional to tan h and tan h tan $\frac{1}{2}$ h respectively, the errors made can not be very big.

The objections against Stamkart's measuring method, amply given for Schagen, hold for all his other stations, though in Uithuizermeden, where he measured 12 series, the distribution of the series over the limb b is adequate; after every series the limb was turned over an angle $180^{\circ} : 12 = 15^{\circ}$ [48].

Finally it must be said that several series in Stamkart's measurements are much too long. They occur of course only at stations with a "large" number of sighting points. At Lemmer e. g. (6 sighting points) each of the series 1 and 2, measured on September 24th 1875, lasted $2\frac{1}{4}$ hours [49]. It will be clear that they affect the results of the measurements in an unfavourable way.

7. *COMPUTATION OF THE ADJUSTED DIRECTIONS AT A STATION AND OF THE STANDARD DEVIATIONS IN THESE DIRECTIONS*

The observations in the various series at a station, discussed in the previous section, must of course be arranged in one series with the adjusted directions. They must deviate as little as possible from the observations. The magnitude of the deviations determines the accuracy of the measurement. For that purpose Stamkart copied the mean values of column 5 of table 1 in a "couvert A", forming part of a file relating to the station of the network. The files, numbered in sequence of the stations visited, have roman figures. Schagen e.g. is numbered VIII. In this publication the roman figures are changed into arabic figures. The result is given in column 3 of table 2 already mentioned before. Subsequently Stamkart reduced every series in the "couvert A" to a series with another orientation in such a way that a common direction in all series got the "reading" zero (the first pointing at Alkmaar in series 1, table 2, column 4). This series was reduced for a second time to Alkmaar = 0 as the direction Alkmaar in this series has two pointings with a mean $0^{\circ}00'00''.47$. The result after this reduction is given in table 3. The mean amounts for the three directions are given on the bottom row [50] .

Table 3

Series	Alkmaar	Medemblik	Hoorn
1	2	3	4
1	$0^{\circ}00'00''.00$	$263^{\circ}46'40''.44$	$302^{\circ}02'60''.95$
2	00.00	45.93	59.42
3	00.00	45.86	58.48
4	00.00	44.78	61.83
5	00.00	42.80	57.29
6	00.00	47.01	60.60
7	00.00	47.03	60.85
8	00.00	41.87	———
9	00.00	———	59.61
10	00.00	46.77	64.47
11	00.00	45.48	60.73
	$0^{\circ}00'00''.00$	$263^{\circ}46'44''.80$	$302^{\circ}03'00''.42$

It is important in Stamkart's adjusting method that all series at a station have indeed a common direction. For a station where the observations were done in only one eccentric point on the tower, this common direction could even be an arbitrary point that forms no part of the triangulation network. Its distance to the station even was not necessary. In Hornhuizen (file 44) e.g. this common sighting point was the spire of the tower in the village of Leens, situated at about 3.3 km southsoutheast of Hornhuizen. Assuming that centering errors of the instrument during the period of the measurement(Aug. 1-11,1879) can be neglected and that the pointings at Leens have the same accuracy as those at Uithuizermeden (23.4 km), Groningen (23.4 km), and Kollum (18.5 km) of the triangulation network, then the method used appears to be admissable. The advantage of a reference point at not too large a distance is of course that it is very likely that it can be seen in every series; the drawback in general, however, that the relative great number of pointings at such a reference point (in Hornhuizen 37 out of 117) contributes little or anything at all to the accuracy of the angles of the network.

It will be clear that Stamkart's method of adjustment of the directions is wrong. For the determination of the several means in a series the number of pointings at a station was not taken into account. The same can be said of the computation of the mean of the series. Though the common direction Alkmaar in the example of table 2 is "adjusted" indeed in the series (except in the series 2, 10, and 11), it remains unchanged and can possibly not be altered in the mean of the series. His determination of standard deviations in his measurements is therefore also wrong. Moreover he apparently did not know that, because of the measurement of irregular and incomplete series, the results are correlated. In the example of the station Schagen in tables 2 and 3 it is therefore not allowed - as he did in analogous cases - to determine the standard deviation in the mean direction Hoorn = $302^{\circ}\ 03'\ 00''.42$ by dividing m, the standard deviation in every pointing at the station, through $\sqrt{19}$ (19 is the number of pointings at Hoorn).

In order to get a reliable impression of m the observations should have been adjusted according to the method of the least squares, though it is true that, by their great number, the practical solution of this problem would have been impossible in Stamkart's time.

The adjustment of the directions at the station Schagen - with the exception of Nederweert (file 18), the station with the fewest number of pointings - is given in table 2. The 55 directions measured at Schagen are numbered in column 5. For the first series holds (see column 4):

$0 + p_1 = 0.94 + p_5$ or $p_1 - p_5 = + 0.94$,

$2.66 + p_2 = 0.19 + p_4$ or $p_2 - p_4 = - 2.47$,

and for the second series:

$45.47 + p_6 = 46.39 + p_{10}$ or $p_6 - p_{10} = + 0.92$,

$59.88 + p_7 = 58.96 + p_9$ or $p_7 - p_9 = -0.92$, etc.

Moreover in all series the adjusted amounts Hoorn minus Alkmaar and Medemblik minus Alkmaar should be equal. For Hoorn minus Alkmaar in the first series it is (only the seconds are given):

$2.66 + p_2 - p_1$, in the second series:

$-0.12 + p_7 - p_8$, and in the third series:

$1.84 + p_{12} - p_{11}$, etc, sothat:

$2.66 + p_2 - p_1 = - 0.12 + p_7 - p_8$ or $- p_1 + p_2 - p_7 + p_8 = - 2.78$,

$2.66 + p_2 - p_1 = 1.84 + p_{12} - p_{11}$ or $- p_1 + p_2 + p_{11} - p_{12} = - 0.82$, etc.

As there are 55 observations in 11 series and the mutual position of the three directions Alkmaar, Medemblik, and Hoorn is already known from the first series, the number of redundant observations is (55 - 3) - (11 - 1) = 42. There are therefore 42 conditions the corrections p_1 up to and including p_{55} have to comply with. Six of them are mentioned above. For [pp] = minimum the p's are mentioned in column 6 of the table. They were computed with the IBM 360/65-computer of the Delft University of Technology. For every series [p] is of course zero (check). $m = \sqrt{[pp]:42} = \pm 2''.62$ is the standard deviation in each of the 55 pointings at the station.

The adjusted directions are mentioned in column 7. They can all be reduced to:

Alkmaar	0° 00' 00".00
Medemblik	263 46 44.47
Hoorn	302 03 00.01 (check).

Medemblik and Hoorn differ - 0".33 and - 0".41 respectively from the results of Stamkart's primitive adjustment in table 3.

Not only for Schagen, but also for the other stations of Stamkart's triangulation network these extensive computations were executed. The results of the amounts m can be found in table 4. The stations are mentioned in the sequence of the file numbers. As already remarked before this file number indicates mostly the sequence in which the stations were measured. Amsterdam e.g. (file 7) was

Table 4

File	Stations Name	Stations No.	Directions	Series	Pointings	Stand. dev.	Date
1	2	3	4	5	6	7	8
1	Haarlem	17	4	18	79	2.″69	Oct. 14 - Oct. 21, 1870 Aug. 11 -Aug. 15, 1871
2	Delft	13	6	18	127	2.18	June 15 - June 26, 1871
3	Rotterdam	11	6	12	90	2.15	June 30 - July 5, 1871
4	Gouda	12	8	18	121	2.41	July 14 - July 24, 1871
5	Nieuwkoop	15	5	17	134	2.58	July 28 - Aug. 4, 1871
6	Alkmaar	21	5	17	90	2.88	Aug. 22 - Aug. 29, 1871
7	Amsterdam	18	6	37	154	2.03	Sept. 5 - Sept. 23, 1871
8	Schagen	23	3	11	55	2.62	Sept. 4 - Sept. 11, 1871
9	Hoorn	22	5	16	98	2.59	Sept. 14 - Sept. 20, 1871
10	Edam	20	5	25	130	2.39	Sept. 22 - Oct. 12, 1871
11	Leiden townhall	14	4	18	85	2.74	June 22 - July 13, 1872
	" Petruschurch		3	12	58	1.64	
12	Utrecht	16	6	21	107	2.11	July 16 - July 25, 1872
13	Gorinchem	10	5	25	113	2.07	July 30 - Aug. 14, 1872
14	Dordrecht	9	5	16	105	2.65	Aug. 16 - Aug. 21, 1872
15	's-Hertogenbosch	7	4	17	84	2.54	Sept. 6 - Sept. 10, 1872 June 19 - June 28, 1873
16	Breda	6	6	22	132	2.82	July 7 - July 16, 1873
17	Helmond	3	4	19	105	2.54	July 29 - Aug. 7, 1873
18	Nederweert	2	3	11	51	1.64	Aug. 14 - Aug. 16, 1873
19	Hilvarenbeek	4	5	19	124	2.31	Aug. 25 - Sept. 5, 1873
20	Hoogstraten	5	5	34	143	1.96	Sept. 12 - Sept. 29, 1873
21	Lommel	1	5	22	109	1.80	June 27 - July 6, 1874
22	Naarden	19	6	19	130	2.13	July 14 - July 21, 1874
23	Willemstad	8	5	25	108	2.25	Aug. 28 - Sept. 2, 1872 Oct. 5 - Oct. 16, 1873
24	Amersfoort	35	4	14	69	2.29	July 28 - Aug. 9, 1874
25	Veluwe	34	3	23	82	2.58	Aug. 13 - Aug. 28, 1874
26	Harderwijk	33	6	29	131	2.74	Sept. 8 - Sept. 24, 1874
27	Kampen	32	6	44	186	2.65	Sept. 29 - Oct. 17, 1874 June 28 - July 2, 1875
28	Blokzijl	29	5	17	101	2.17	July 7 - July 18, 1875
29	Urk	28	6	19	93	1.52	July 24 - July 29, 1875

Table 4 (continued)

1	2	3	4	5	6	7	8
30	Enkhuizen	25	7	29	170	1."96	Aug. 3 - Aug. 18, 1875
31	Medemblik	24	5	23	100	1.89	Aug. 20 - Sept. 2, 1875
32	Hindeloopen	26	6	36	181	2.09	Sept. 7 - Sept. 16, 1875 Sept. 2 - Sept. 3 , 1878
33	Lemmer	27	6	33	174	2.83	Sept. 23 - Oct. 8, 1875 Aug. 7 - Aug. 25, 1876
34	Meppel	31	4	24	107	2.13	Sept. 22 - Oct. 11, 1876
35	Oldeholtpa	30	4	16	80	2.06	July 18 - July 28, 1877
36	Sneek	36	6	33	154	2.88	Aug. 1 - Aug. 14, 1877
37	Leeuwarden	38	5	25	160	2.01	Aug. 20 - Sept. 3, 1877
38	Drachten	39	5	25	161	2.02	Sept. 4 - Sept. 22, 1877
39	Kollum	40	5	27	146	2.45	Sept. 26 - Oct. 7, 1877
40	Amsterdam	18					
41	Harlingen	37	4	21	96	2.54	Aug. 19 - Aug. 24, 1878
42	Groningen	41	8	66	397	2.98	Sept. 11 - Oct. 6, 1878
43	"						June 26 - July 26, 1879
44	Hornhuizen	42	4	20	117	1.48	Aug. 1 - Aug. 11, 1879
45	Uithuizermeden	43	4	12	83	1.57	July 18 - Aug. 1, 1880
46	Holwierde	44	6	23	190	2.16	Aug. 10 - Aug. 24, 1880
47	Midwolda	45	6	13	132	1.96	Sept. 5 - Sept. 29, 1880
48	Emden	46	7	26	239	2.60	July 23 - Sept. 15, 1881
			241	1067	5881	2.19	

measured from September 5th till September 23rd 1871 (column 8). Six directions were measured (column 4) in 37 series (column 5). The number of pointings at the six sighting points was 154 (column 6), the standard deviation in each of these pointings is $\pm 2''.03$ (column 7). It is true that file 40 also refers to Amsterdam but as the measurements mentioned in that file relate to the base line extension network (see section 11) and not to the triangulation of the country, they were omitted in table 4.

Groningen takes up two files (42 and 43). The measurements at this station were very extensive: 397 pointings in the years 1878 and 1879. As can be seen, file 11 refers to two stations at Leiden, the townhall and the Petruschurch with 85 and 58 pointings respectively and standard deviations $m = \pm 2''.74$ and $m = \pm 1''.64$.

In some exceptional cases Stamkart rejected a series. Their number is so small that it can be neglected. These series and the number of pointings in these series are not inserted in the table.

In my adjustment of the observations each of some very long series (e.g. the two series in Lemmer mentioned at the end of section 6) were split sometimes into two other series. In that case the number of series in column 5 is greater than the number of series measured by Stamkart. The standard deviations range from $m = \pm 1''.48$ (Hornhuizen, file 44) to $m = \pm 2''.98$ (Groningen, files 42 and 43), in my opinion between narrow limits. The mean m is $\pm 2''.19$. As the total number of pointings (column 6) is 5881 and the total number of sighting points (column 4) is 241, the mean number of pointings at every sighting point is about 25. A rough estimate of the standard deviation in an adjusted direction - the correlation already discussed before was not taken into consideration - is therefore about $2''.19 : 5 \approx \pm 0''.44$. This is a very reasonable amount.

The adjusted amounts for the several directions at the stations of the triangulation network are mentioned in table 14. They are arranged there in sequence of the numbers of the stations. Schagen has the number 23. According to column 4 the number of pointings at Alkmaar, Medemblik, and Hoorn is 20, 16, and 19 respectively. The adjusted directions in column 5 do not agree with those on page 32 as the latter amounts in the eccentric point of the station are reduced here to directions in the spire of the tower to the spires of the surrounding sighting points. Stamkart's determination of these reductions will be treated in the next section.

8. *REDUCTION OF THE DIRECTIONS MEASURED TO CENTRE*

The directions measured in an (the) eccentric point(s) at a station had of course to be reduced to a centre, serving as the angular point of the triangles in the triangulation network meeting at that station.

In Krayenhoff's triangulation there was doubt which point had to be considered as the centre. For his measurements at a station he reduced the angles measured to the centre of the frame of the tower at the height where the measurements were done. On the pages 44, 48, and 50 of [1] some examples are given for the stations Amsterdam and Rhenen. For the determination of the other angles of the triangles which meet e.g. in Amsterdam, however, the spire of the station (Amsterdam) was used as the angular point of the triangles.

Stamkart rightly made a difference between these "two centres" which can deviate considerably for not quite a vertical tower (in Amsterdam about 0.63 m) and chose as "his" centre the centre of the tower. As, however, he also used the spires as the sighting points in his triangulation (and, in exceptional cases, heliotropes), the situation of these spires (heliotropes) with respect to the centre of the tower had to be determined. In fig. 5 this is illustrated with an example.

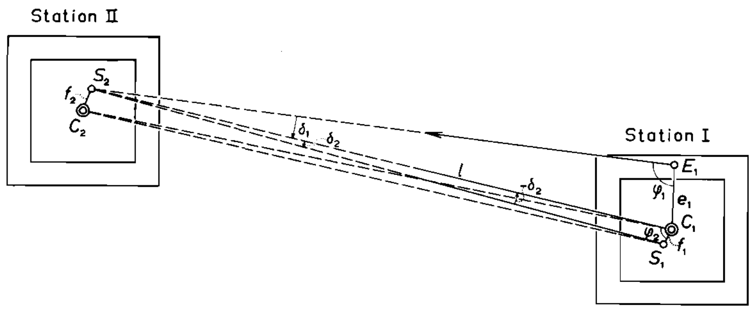


Fig. 5

In the eccentric point E_1 of station I the directions to the spires of several surrounding stations are supposed to be measured, among them the direction to the spire S_2 of the station II. C_1 is the cpntre of the station I, S_1 the spire. For the determination of the reductions δ_1 and δ_2 the distances e_1, f_1 and l must be known and the angles φ_1 and φ_2. As e_1 and f_1 are small with respect to l, approximate values of φ_1 and φ_2 will of course do. For an accurate computation of δ_1 and δ_2 holds:

$$\delta_1'' = \rho \frac{e_1 \sin \varphi_1}{l} \quad \text{and} \quad \delta_2'' = \rho \frac{f_1 \sin \varphi_2}{l}$$

with $\rho = 206264''.8$.

As almost all stations of Stamkart's network are the same as those of Krayenhoff's triangulation, the distances l in the above reductions could be borrowed from tableau III of Krayenhoff's Précis historique [51]. The amounts e, f, and φ in the formulae had to be determined by measurements. They can, for each of the files, be found in the so-called "Couvert C". It has the heading "Bepaling van punten op den toren te ..... " (Determination of points on the tower at ....) and contains a detailed survey of the tower at the height of the gallery where the measurements were done.

In fig. 6 an example is given of the cross section of the first gallery of the St. Bavo church tower at Haarlem (file 1). The general form is a regular octagon with a side length 2. 890 m and a distance 6. 978 m between two parallel sides. C is the centre of the octagon and Stamkart's centre of the tower.

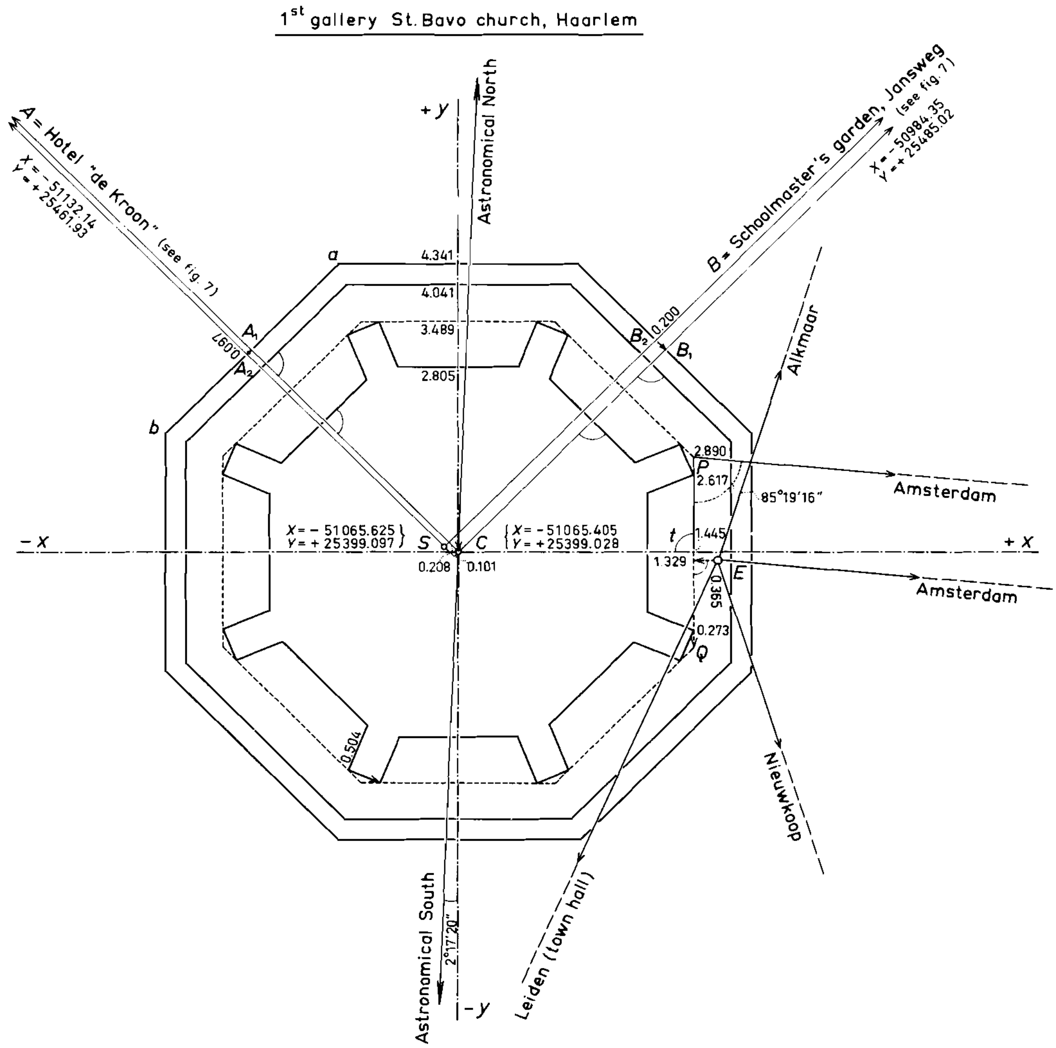


Fig. 6

In 1870 the directions to Alkmaar, Amsterdam, Nieuwkoop, and Leiden (townhall) were measured in the point E of the gallery. If C is the origin of a local coordinate system xy and Ct the positive x-axis (t is the middle of the eastern side of the octagon), then the coordinates of E are:

$$x_E = 3.489 + 0.365 = +3.854, \qquad y_E = -0.116$$

From these coordinates it follows that CE = 3.856 m and that - Stamkart's computation method is used here - angle tCE = arc tan(0.116 : 3.854) ≈ $1^{\circ}43'30''$.

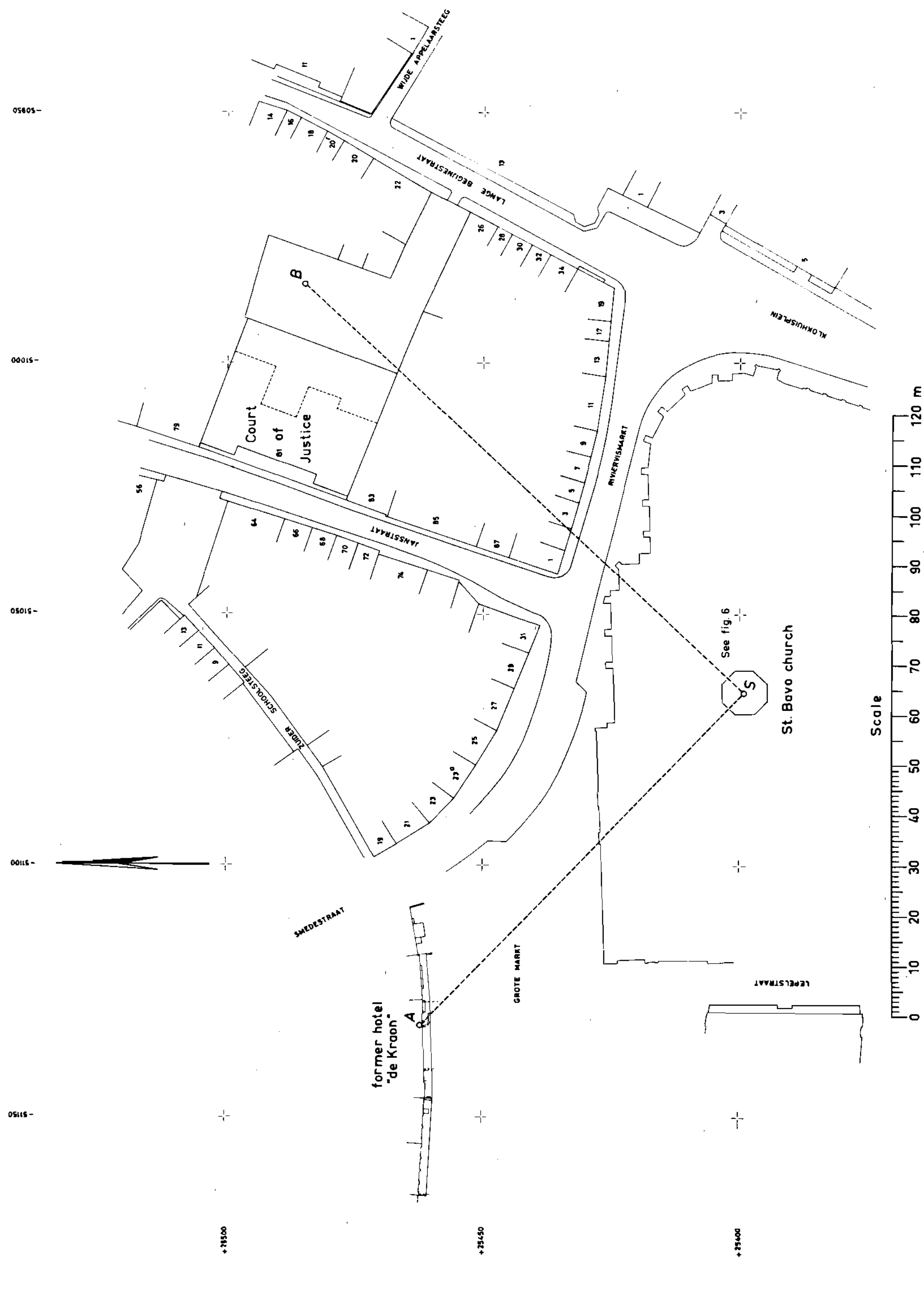

B
Court of Justice
S
See fig. 6
St. Bavo church
A
former hotel "de Kroon"
SMEDESTRAAT
GROTE MARKT
JANSSTRAAT
LANGE BEGIJNESTRAAT
WIJDE APPELAARSTEEG
ZUIDER SCHOOLSTEEG
Scale
0 10 20 30 40 50 60 70 80 90 100 110 120 m

Fig. 7

For the determination of the spire S in the local coordinate system Stamkart used - not only in Haarlem, but at every station of his network - a method which seems very out of date nowadays. It was, however, frequently applied in former times. It is shown in the figures 6 and 7. The latter figure could be reproduced thanks to the kind collaboration of Mr. P. J. Toussaint, chief engineer of the Cadastral landsurveying department at Haarlem and to the Municipal landsurveying department of that town.

A in these figures - the scale is mentioned on the map - is a point where the small theodolite was set up in the forefront of the second floor of the then "Hotel de Kroon" (Kroon = Crown). Later on the cinema Rembrandt Theatre was established in the building, but nowadays it is empty. The number of the cadastral lot is C 5323. The point was chosen in line with CA_1, on fig. 6 (A_1 midway between the angular points a and b of the balustrade, CA_1 perpendicular to ab, angle $tCA_1 = 135^{\circ}$). A_2S is the intersection of the vertical plane through A and the spire of the tower with the first gallery. This plane can be realized by the line of sight of the adjusted instrument when the telescope is turned around the horizontal axis. If A_2 is the intersection of this plane with the balustrade, the distance A_1A_2 is a measure for the deviation of the spire in a direction perpendicular to CA if at least the distance CA is known. According to Stamkart it is 91.71 m [52]. As $CA_1 = 4.34$ m and $A_1A_2 = 0.097$ m, the deviation is 0.101 m.

In the same way B, in line with CB_1, (CB_1 perpendicular to the balustrade, angle $tCB_1 = 45^{\circ}$) is a "point in the schoolmaster's garden in the Jansstreet". The school, indicated with a dashed line, was pulled down in 1888. In 1890 it was replaced by the present Court of Justice. Its cadastral number is D 6451. As, according to Stamkart, CB = 118.17 m [53], the deviation between C and S in a direction perpendicular to CB is 0.208 m. From these measures follows the distance CS = 0.231 m and the angle $tCS = 160^{\circ}54'$.

For the computation of the angles φ_1 and φ_2 in fig. 5 Stamkart used the astronomical azimuths published by Krayenhoff in the Table alphabétique des azimuths of his Précis Historique. For the station Haarlem these azimuths - counted from the south in a clockwise direction - can also be found on page 205 (column 6) of [1] . That to Alkmaar e.g. is $195^{\circ}20'47''.85$ and that to Amsterdam $272^{\circ}23'20''.00$.

For the computation of the φ's the angle between the positive y-axis in fig. 6 and the astronomical north is necessary. It could be found by measuring in P the angle $85^{\circ}19'16''$ between Amsterdam and Q [54]. As the astronomical azimuth PQ is $272^{\circ}23'20'' + 85^{\circ}19'16'' = 357^{\circ}42'36''$, the positive y-axis must turn about $2^{\circ}17\frac{1}{3}'$

to the right in order to coincide with the astronomical North. From this amount and the angle tCE $\approx 1^{\circ}43\frac{1}{2}'$ the astronomical azimuth EC $\approx 89^{\circ}26'$ can be derived and, as that to Alkmaar $\approx 195^{\circ}21'$, $\varphi_1 \approx 105^{\circ}55'$ and

$$\delta_1'' = \rho'' \frac{3.856 \sin \varphi_1}{28882.63} = +26''.48 .$$

In the same way the reduction from E to C for the direction to Amsterdam is

$$\delta_1'' = \rho'' \frac{3.856 \sin 182^{\circ}57'}{16789.28} = -2''.44 .$$

The reductions for Nieuwkoop and Leiden follow in the same way.

The reductions δ_2 from C to S can be computed analogously. As the astronomical azimuth CS is about $106^{\circ}49'$ [55] the reduction to Alkmaar is $\delta_2 = +1''.65$ and that to Amsterdam $\delta_2 = +0''.71$.

As in Alkmaar and in Amsterdam was pointed at the spire S at Haarlem the directions reduced to the centres of the towers of Alkmaar and Amsterdam had to be reduced for a second time with $-1''.65$ and $-0''.71$ respectively in order to find the directions Alkmaar (centre) - Haarlem (centre) and Amsterdam (centre) - Haarlem (centre): because of the always very small distances f_2 in station II of fig. 5 and the large distances l the very small angles $C_1S_2S_1$ and $C_1C_2S_1$ can assumed to be the same.

The method described demonstrates a lack of insight into the measurement of a triangulation network: its sides are the connecting lines of invisible points which might be considered as the station marks of the spires. It is strange that in his thesis Moor does not say a word about this question. However, the place of these "station marks" is very bad as they are much too high in the towers. They lose their meaning when the tower sags on one side.

In my considerations on the accuracy of Stamkart's triangulation network and for its comparison with the results of the R. D. all directions measured are reduced to the spires of the stations, and not the "centres". As was already remarked at the end of section 7,column 5 of table 14 gives a survey of these adjusted directions.

It will be clear that an accurate determination of the mutual situation of C and S in fig. 6 depends in the first place on the collinearity of the points A, A_1, and C (B, B_1, and C) and on the accuracy of the distances A_1A_2 and B_1B_2. The accuracy of CA and CB is less important as CA_1 and CB_1 are small. Stamkart gives neither details how he determined A and B nor how he realized the vertical planes through these points and S, necessary for an accurate determination of A_2 and B_2. It is of course necessary that this must be done twice, once with face right and once with face left of the vertical limb. In the mean of the two values the errors of adjustment of the theodolite are eliminated but it cannot be traced whether he acted indeed in this way. The determination of the length CB on the pages 2 and 4 of the "cahier C" makes it even plausible that he used the telescope only in one position with respect to the vertical limb, though it could be taken from its supports and reversed.

B in the sketch of fig. 8 is the point in "the schoolmaster's garden". The inclination α of the line of sight BG (GC is the first gallery of the tower) is $21^{\circ}25'$+c, that of BS (S is the spire) $\beta = 29^{\circ}55'$+c. The amounts c point to an index correction for the vertical limb. It would not have been necessary to determine this correction if the instrument had been used with face right and face left (of the vertical limb). According to page 2 of "cahier C", c = $-7'$, according to page 4 c = $-2'$.

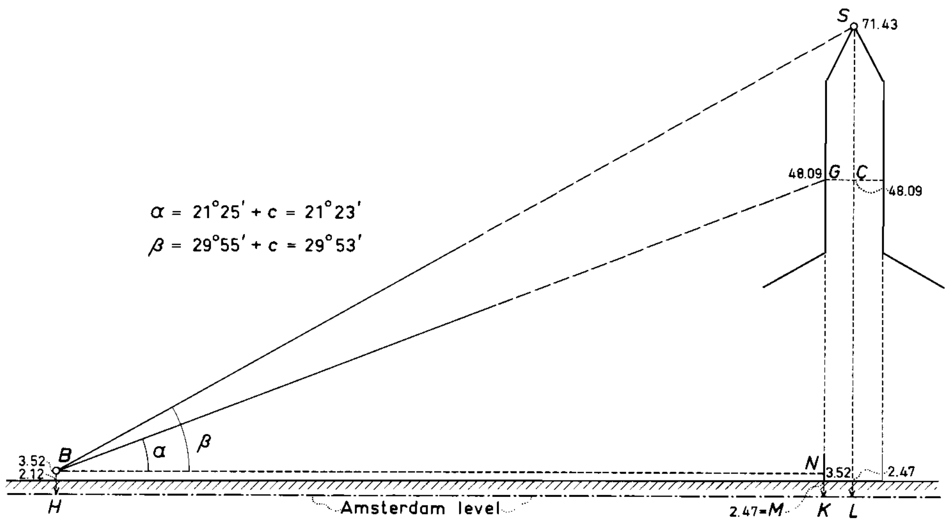


Fig. 8

Stamkart's sketch on page 2 gives the height GM = 45.62 m of the first gallery above the floor of the church and 45.62 + 2.47 = 48.09 m above A(msterdams) P(eil)) (peil = level), the levelling plane used in the Netherlands [56]. The height of M (2.47 m + AP) with respect to this plane was determined by levelling from the nearest levelling mark. The distance GM = 45.62 m was found from a direct measurement, but unfortunately Stamkart does not say how he measured this distance.

The height of the horizontal axis of the theodolite in B, also determined by levelling appeared to be 3.52 + A.P. In fig. 8 GN is therefore 44.57 m. For $\alpha = 21^{\circ}23'$ the horizontal distance BN is 113.83 m. The value agrees with that mentioned on page 4 of Stamkart's computation. As GC = 4.34 m, the horizontal distance between B and the centre of the tower is 118.17 m. This value was already used on page 39. The height of the tower above the floor of the church is therefore:

$$118.17 \tan 29^{\circ}53' + 1.05 = 68.96 \text{ m } (71.43 \text{ m} + \text{A. P.}).$$

Not only at Haarlem but also at all his triangulation stations Stamkart determined the absolute heights with respect to A. P. of the towers and the galleries where the measurements were done. Though he does not mention the aim of these vertical measurements, they might be meant for a reduction of the measurements at various heights to a same levelling plane.

In a flat country like the Netherlands this reduction is unnecessary: even in the computations of the R. D. - first order triangulation network the differences in height between the stations were never taken into account. In an alternative computation of the horizontal distance between B and the centre of the churchtower on page 2 of "cahier C" Stamkart finds GN = 48.09 - 2.47 - 1.40 = 44.22 m. For $\alpha = 21^{\circ}25' + c = 21^{\circ}18'$ the horizontal distance is 113.42 + 4.34 = 117.76. The difference of 0.41 m with the above computation does not affect the perpendicular distance 0.208 m between S and CB in fig. 6. As, however, the distances B_1B_2 = 0.200 m and A_1A_2 = 0.097 m on the balustrade are subject to some doubt because of the instrumental errors, the situation of S with respect to C might be less good than can be derived from Stamkart's inexpert measurements.

For a great number of cases it is of course impossible to compare the mutual situation of the spire and the "centre" of a tower in Stamkart's computation with the result that can be derived from the first order R. D.-measurements of the same tower, of about 20 years later. Only for Amsterdam an exception could be made as already in 1970 the Municipal landsurveying department in Amsterdam determined the mutual situation of the R. D.-spire in 1898 and the centre of the tower. It is given in fig. 9 on page 48 of [1]. Stamkart's results computed in "cahier D" of file 40 deviate only a few cm from the values in the above publication though the spire of the tower is about 42.8 m above the first gallery where the measurements took place.

This is an excellent result that of course might be attributed to chance. But perhaps it shows, notwithstanding Stamkart's poor knowledge of geodesy, something of his feeling for accuracy that can also be found furtheron in this book.

In some exceptional cases Stamkart's reductions to "centre" are very large, e.g. at the stations Urk (reformed church) (file 29) and Leiden (file 11). For Urk they must be ascribed to the large distance (about 159 m) between the lighthouse where he did his observations and the reformed church. He determined this distance by means of a baseline on the isle and the measurement of some angles in its terminal points.

For a local triangulation network in Leiden even two baselines were measured. This geometrically poor network determined the mutual situation of the spires of three churches in that town and the Observatory. Here too the distance of about 154 m between the townhall and the Petruschurch, where the measurements for the main triangulation network were done, was much too large for an accurate determination of the reductions to the centre, the townhall. The distance from the townhall to the Observatory where the astronomical measurements took place even was about 603 m. The network will be analyzed in section 13.

9. *THE MEASUREMENT OF THE BASE LINE*

As already mentioned in Krayenhoff's triangulation, the measurement of a baseline is missing in Krayenhoff's remarkable work. The lengths of all its sides were derived from the side Duinkerken - Mont Cassel of the French triangulation by Méchain and Delambre.

By accidental circumstances Stamkart - apart from some incidental reconnaissances between August 1866 and August 1867 - just started his geodetic work with the measurement of a baseline. The apparatus for such a measurement was at hand as it was bought by the Dutch government for the measurement of a baseline on the isle of Java in the former Dutch East Indies. Dr. J.A.C. Oudemans [13], charged with the triangulation of this island, found it even desirable that the apparatus was used in practice in Holland before sending it off to Batavia, the present Djakarta. It was made by the German firm Repsold and it consisted of two bars of four metres and two bars of one metre. A fifth bar of one metre, the "normal metre", belonged to the apparatus. It was used as a standard for the other bars. How the apparatus was used was explained in detail in a lecture on the base measurement that Stamkart held for the assembly of the Koninklijke Academie van Wetenschappen (Royal Academy of Sciences) in Amsterdam on December 19th 1868. The lecture was published in the Reports and Communications of the Academy of 1869 [57].

Already in September and October 1867 Stamkart and his assistant Van Hees had made an elaborate study of how the apparatus should be handled. On May 20th 1868 Stamkart consulted his friend prof. Kaiser at Leiden on the place where the baseline should be measured [58]. The Haarlem lake polder was judged to be the best place. This former lake was reclaimed between 1848 and 1852. It had long, straight roads with little traffic and it was situated not too far from a side of the triangulation network (Amsterdam-Haarlem) the length of which had to be computed from the length of the baseline.

Initially Stamkart had the intention to measure his baseline between the forts Liede and Schiphol at the northwest and the southeast end respectively of the Spaarnwouderdwarsweg, the present Schipholweg (weg = road), a distance of about 11.4 km. Their position is shown in fig. 9, a reproduction at a reduced scale of the topographic map 1 to 50,000 of the polder, printed in 1854. Later on he shortened this distance to about 6 km along the same road. The speed with which the measurement could be done and the necessity to send the apparatus to Java forced Stamkart to this decision. His letter to Oudemans, dated December 3rd 1868, gives an impression of this speed of work: "The work goes very slowly; 145 metres is the greatest distance which we measured per day and the mean is hardly 100 metres" [59].

The terminal points (see fig. 9) in the southern grassbank of the road, the northwestern point near the steam pumping station "De Lijnden" and the southeastern end about 600 m northwest of the fort Schiphol were marked-out in an excellent way in an about 1.70 m high subterranean brickwork construction that was built upon four poles, eight metres long each, in the form of a square with sides of one metre. The poles were driven into the ground with their heads more than one metre below the lowest level of the groundwater. A detailed description of the marks is given in the report of Stamkart's just-mentioned lecture. He took very much trouble to keep them in a good state and surrounded them therefore with "four stone pillars, 0.70 m above the ground, at the top connected with iron rods; they form a rectangle of 1.75 x 0.70 m". In the railings saw cuts were made. The connecting lines of the two pairs of opposite cuts intersected each other in the centre of the base terminal point. It will be clear that such a railing could easily be damaged by accidents. In [60] Stamkart writes on such an accident: a runaway horse damaged the railing around the southeastern terminal point. The owner of the horse, P. Merriboer, later on paid for the damage twenty eight guilders [61].

By the increasing traffic the accidents and the damages also increased. In 1911 the railings around the meanwhile historic base terminal points were therefore taken away.

Scale

2000 m 1000 500 0 1 2 3 4 km

Fig. 9

In 1928 the plan ripened to use the "useless points" for underground first order levelling marks of the Dutch Department of "Waterstaat". In 1929 these plans were executed. From the about 1.70 high stone superstructure of the terminals about one metre was hacked away and in the remaining lower part, having a height of about 0.70 m, a levelling mark was fixed. The foundation of the terminals at that moment was still in an excellent condition. The levelling department of "Rijkswaterstaat" informed me that, after 1932, the marks were never used again. They were not used in the last first order levelling in the Netherlands.

Stamkart, assisted by Van Hees, executed the baseline measurement in the years 1868 and 1869. A survey can be found in fig. 10. Moor gives a similar drawing on page 50 (fig. 11) of his thesis [15]. The distance between the terminal points S(outh) E(ast) and N(orth) W(est) was divided into the 16 sections A up to and including P and "bridge". Their approximate lengths in metres are indicated in the figure. The meaning of section Q near base point SE, about 100 m in length and in line with the baseline, will be treated in section 10.

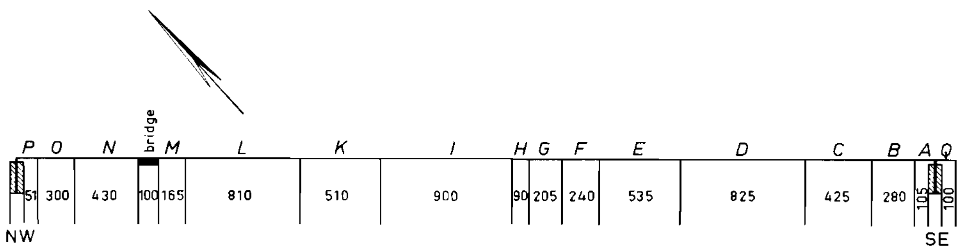


Fig. 10

The sections A up to and including M were measured once in 1868 and once in 1869, the sections N, O, and P twice in 1869. The distance of about 100 m over the bridge over the Hoofdvaart (the principal canal through the polder) was measured three times in 1869. It was, counted from the southeast, divided into three parts of about 25, 30, and 45 m respectively. The middle part was, at a distance of about 1.55 m northeast of it, parallel to the baseline. The 25 m part deviated $3^{\circ}33'23''$ from the direction of the baseline, the 45 m part $1^{\circ}58'30''$. The distance Q of about 100 m, finally, was measured twice in 1869.

It will be clear that, before the measurement could begin, many difficulties in the organizing field had to be surmounted. The heavy apparatus e.g. had to be transported to the polder and in the polder. The transport to the polder was done by boat, but as there was no communication between the water outside and inside the polder and an appropriate boat in the polder was not available, the instruments and another boat had to be dragged over the dike. Not before this was done the transport in the polder could take place.

With very much appreciation Stamkart mentions the collaboration in this field of Mr. J.P. Amersfoordt, burgomaster of Haarlemmermeer, who helped him also in other matters in the organizing field.

From Stamkart's correspondence [62] it appears that in 1868 the Amsterdam photographer P. Oosterhuis made 20 prints of a photo of the baseline measurement for the amount of f 27.25. None of these prints can be found in the archives of the Netherlands Geodetic Commission. Stamkart, however, sent one of them to the municipality of Haarlemmermeer to be kept in the townhall. On inquiry it appeared to be still present there. The municipality was even so kind as to have a reproduction made for me. It is pictured as fig. 11. According to a description belonging to the photo (1) is Mr. Stamkart and (2) is Mr. Van Hees. The three workmen are Dirk Utermark (3), Klaas Veerman (4), and Cornelis Witziers (5). The boat (n), already mentioned above, was used for the transport of the apparatus, the room (i) for putting away clothes and for keeping watch. The railings (h) to keep the public away are clearly visible just as the planks upon which the observers stood during the observation. The warning board "drive at a foot pace" (o) was so badly visible that it was touched up in this figure.

In the beginning Stamkart made several mistakes in the execution of his base line measurement. They must be ascribed to inexperience in this sort of measurements. The grass bank along the road was not so suited for the purpose as the springy underground influenced the accuracy of the measurements in an unfavourable way. Later on this inaccuracy could be forestalled by the use of a plank upon which the two observers at the two ends of the measuring bar could stand when the microscopes were read. The plank was supported at the two ends and it did not touch the ground in the middle [63].

It must also be imputed to inexperience that in the beginning during the lunch break the measuring bar remained in the open air, exposed to often strong heat and wind, without the precaution of marking the ends of the bar by temporary marks in the ground.

In his report over the year 1868, dated November 7th 1868 [64], Stamkart writes to the minister of home affairs that "notwithstanding all precautions taken one of the heavy bars of four metres length has been damaged. On August 6th, that bar fell by an imprudent hastiness of one of the workmen". As a consequence of the accident the glass plates upon which the end measures of the iron

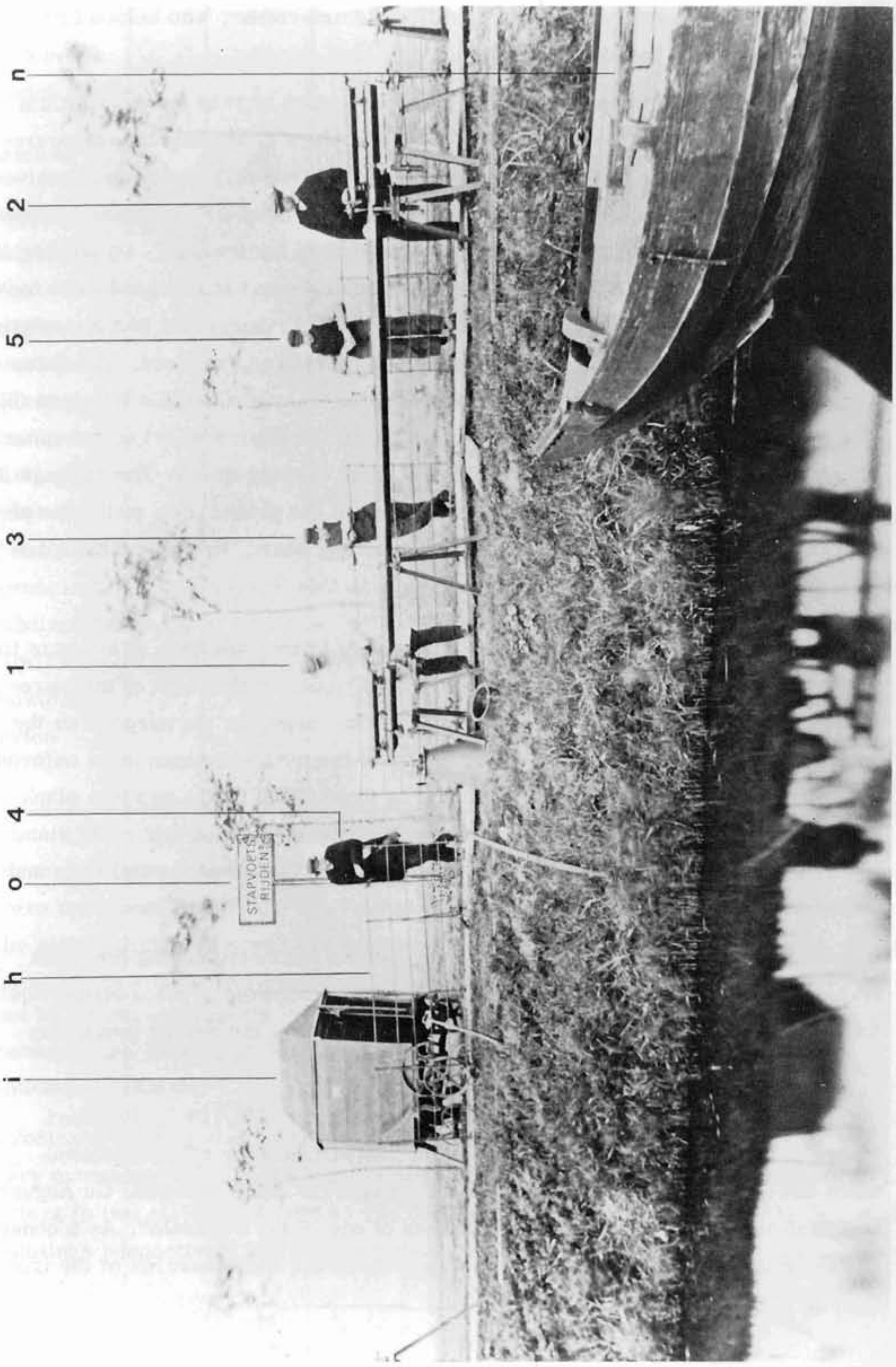


Fig. 11

and the zinc bar were etched, were smashed to pieces. Already the next day the instrument maker, Mr. Olland from Utrecht, was on the spot to survey the damage. In the same report to the minister Stamkart could write "that the damage is repaired, at any rate in a sufficient way".

It will be clear that by this accident which implicated the making of another measuring scheme with three bars and the influences already mentioned before the accuracy of the measurements in 1868 is less than in 1869. Table 5 gives a survey of this accuracy and the final result of the base line measurement. Moor mentions these results on page 51 of his thesis.

Table 5

Sections	"Normal" metres			Diff. 4-3=2-4 (mm)
	First measurement	Second measurement	Mean	
1	2	3	4	5
A	104.99699	105.00001	104.99850	-1.51
B	279.98422	279.98677	279.98550	-1.27
C	424.99721	424.99844	424.99782	-0.62
D	825.00812	825.00432	825.00622	+1.90
E	———	534.99920	534.99920	———
F	240.01058	239.99990	240.00524	+5.34
G	205.02382	205.01738	205.02060	+3.22
H	90.00838	90.00692	90.00765	+0.73
I	899.95960	899.96155	899.96058	-0.97
K	509.97531	509.97395	509.97463	+0.68
L	809.93734	809.93777	809.93756	-0.21
M	165.04762	165.04460	165.04611	+1.51
Bridge	100.07198	100.07198	100.07198	———
N	430.02607	430.02542	430.02574	+0.32
O	299.94415	299.94429	299.94422	-0.07
P	50.81618	50.81638	50.81628	-0.10
	5435.80757 534.99920	5970.78888	5970.79783	+8.95
	5970.80677	$m^2 \approx 54.63$	$m \approx \pm 7.4$ mm	

The sections in column 1 refer to the letters in fig. 10, the measures in columns 2 and 3 to the results of the first and second measurements respectively. They are expressed in the "normal" metres referring to the standard Repsold bar of the apparatus. Column 4 gives the mean of 2 and 3 and column 5 the differences (in mm) 4 - 3 = 2 - 4. Conformably to Moor the distance of about 100 m over the bridge is mentioned with its mean from the three measurements.

As can be seen the distance E in column 2 is missing; it should be 535. 02913 m but Stamkart rejected this result. According to Stamkart the mistake in the measurement was made on August 3rd and 4th 1868. "During the lunch break between noon and 2 o'clock p. m. a more than usual displacement of the bars had taken place. Moreover a tarpaulin that sheltered the bars against the wind was blown down and touched one of the bars" [65] . The difference with the second measurement in 1869 is very large indeed: 29. 93 mm. Later on, according to Moor, Stamkart rejected also the first measurements of the sections F and G because of their too great differences 2 - 3 = + 10. 68 mm and 2 - 3 = + 6. 44 mm respectively with the distances in 1869, but this cannot be found in [65] .

The final result of the base length is mentioned at the end of table 5: the mean length is 5970. 79783 (Repsold) metres. The two measurements deviate + 8. 95 and - 8. 95 mm from this mean. If it is assumed that the absolute amounts of the two missing differences for E and bridge in column 5 are about the mean 1. 32 mm of the 14 others, then the standard deviation in the mean length of the base line can be computed at about $\pm$ 7. 4 mm. It is of course only a rough estimate of the accuracy. It agrees, however, rather well with the passage in Stamkart's report to the minister of home affairs over the year 1870 [66] . He writes there that the result in 1868 is 5970. 7806 m, and in 1869 5970. 7866 m, "a difference of 6 mm in a distance of more than an hour's walk". The standard deviation in the mean distance is estimated in that report at about 5 mm.

In this report Stamkart also writes on the reduction of the "Repsold normal metre" to the Van Swinden standard glass metre. "The Repsold metre is, at a temperature of 65^{o}F, almost 0. 175 mm longer than the Van Swinden metre at the temperature of melting ice. Expressed in metres of Van Swinden the length of the base line is therefore (5970. 7835 + 1. 0449=) 5971. 828 metres. The computation of the ratio of the two metres from the great number of comparisons made is not quite ready yet. It may undergo a very small change of some thousandths of a mm".

The attentive reader will have noticed that the results of the measurements in the above report over 1870 are not quite the same as those in table 5. The latter refer to a revision of the computation of the same measurement in 1880 and 1881, about

12 years after its completion. With the same correction 0.175 mm per metre the base length according to these data is 5971.843 (Van Swinden) metres.

It is certain that the length of the base line was on Stamkart's mind till his death in 1882. Perhaps his profession of Inspector of Weights and Measures had to do with this lack of resolution which, again and again, made him compare the length of the Repsold metre with Van Swinden's glass metre and the platinum copy in Delft of the international standard metre in Paris.

In 1887 Oudemans re-computed the length of the base line [67]. He found 5971.740 international metres for it, a difference of 103 mm with the above result.

10. *THE EXTENSION OF THE MEASUREMENT OF THE BASE LINE TO THE TRIANGULATION POINTS SCHIPHOL AND LIJNDEN OF THE BASE EXTENSION NETWORK*

As was already said in the preceding section the distance Q of about 100 m in fig. 10 served the determination of the distance of about 700 m between the south-eastern terminal point of the base line and Stamkart's triangulation point 5, the flag-pole on the fort Schiphol of his base extension network. In his lecture on the base measurement in Amsterdam he already remarked that this had to be done "by small triangles" from which the required distance could be computed. The extension is represented in fig. 12.

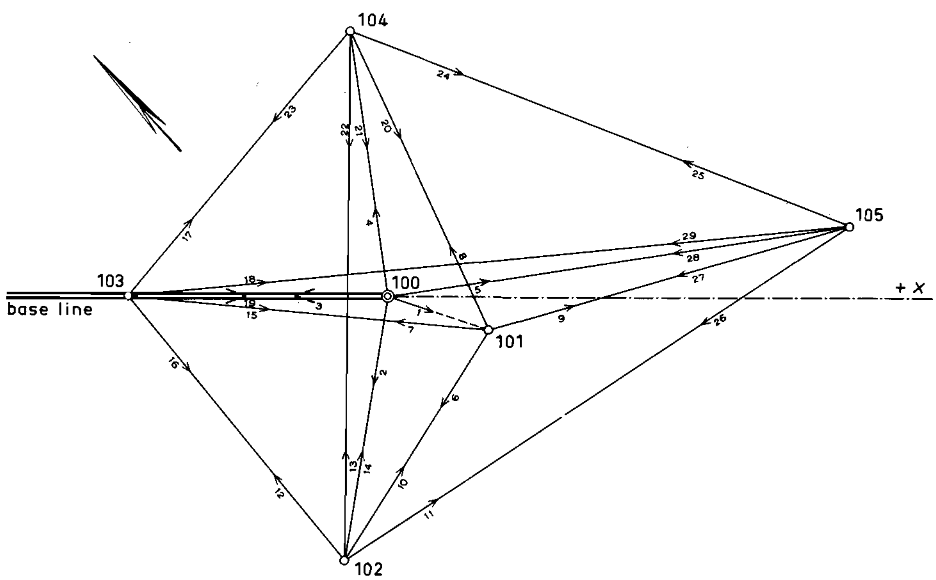


Fig. 12

Table 6

Stations i	Directions	Sighting points k	α_{ik}	$\alpha_{ik} + 0'_i$	ψ'_{ik}	f_{ik} (sec)	$\cos \psi'_{ik}$	$\sin \psi'_{ik}$
			$0'_i$	$\Delta 0_i$				
1	2	3	4	5	6	7=6−5	8	9
100	1	101	0°00'00".00	90°00'56".88	90°00'56".92	+0".04	−0.00028	+1.00000
	2	102	91 45 32.54	181 46 29.42	181 46 29.39	−0.03	−0.99952	−0.03097
	3	103	179 59 03.12	270 00 00.00	270 00 00.00	0.00	0	−1.00000
	4	104	268 40 47.52	358 41 44.40	358 41 44.41	+0.01	+0.99974	−0.02276
	5	105	359 16 55.61	89 17 52.49	89 17 45.83	−6.66	+0.01229	+0.99992
			90 00 56.88	+1.36				
101	6	102	0 00 00.00	195 37 45.82	195 37 42.78	−3.04	−0.96303	−0.26940
	7	103	74 22 20.24	270 00 06.06	270 00 11.74	+5.68	+0.00006	−1.00000
	8	104	149 11 44.25	344 49 30.07	344 49 34.79	+4.72	+0.96514	−0.26175
	9	105	253 32 52.94	89 10 38.76	89 10 23.77	−14.99	+0.01443	+0.99990
			195 37 45.82	+ 1.07				
102	10	101	0 00 00.00	15 37 43.62	15 37 42.78	−0.84	+0.96303	+0.26940
	11	105	43 55 55.74	59 33 39.36	59 33 39.35	−0.01	+0.50662	+0.86217
	12	103	301 32 54.96	317 10 38.58	317 10 38.57	−0.01	+0.73346	−0.67973
	13	104	344 36 18.94	0 14 02.56	0 14 04.34	+1.78	+0.99999	+0.00409
	14	100	346 08 46.84	1 46 30.46	1 46 29.39	−1.07	+0.99952	+0.03097
			15 37 43.62	+ 3.13				
103	15	101	0 00 00.00	90 00 06.06	90 00 11.74	+5.68	−0.00006	+1.00000
	16	102	47 10 32.52	137 10 38.58	137 10 38.57	−0.01	−0.73346	+0.67973
	17	104	313 02 22.22	43 02 28.28	43 02 28.29	+0.01	+0.73086	+0.68252
	18	105	359 32 50.82	89 32 56.88	89 32 56.76	−0.12	+0.00787	+0.99997
	19	100	359 59 53.94	90 00 00.00	90 00 00.00	0.00	0	+1.00000
			90 00 06.06	+ 4.23				
104	20	101	0 00 00.00	164 49 25.99	164 49 34.79	+8.80	−0.96514	+0.26175
	21	100	13 52 08.44	178 41 34.43	178 41 44.41	+9.98	−0.99974	+0.02276
	22	102	15 24 33.65	180 13 59.64	180 14 04.34	+4.70	−0.99999	−0.00409
	23	103	58 13 02.29	223 02 28.28	223 02 28.29	+0.01	−0.73086	−0.68252
	24	105	314 43 16.54	119 32 42.53	119 32 42.54	+0.01	−0.49311	+0.86997
			164 49 25.99	+ 6.64				
105	25	104	0 00 00.00	299 32 42.47	299 32 42.54	+0.07	+0.49311	−0.86997
	26	102	300 00 54.82	239 33 37.29	239 33 39.35	+2.06	−0.50662	−0.86217
	27	101	329 37 46.60	269 10 29.07	269 10 23.77	−5.30	−0.01443	−0.99990
	28	100	329 45 09.77	269 17 52.24	269 17 45.83	−6.41	−0.01229	−0.99992
	29	103	330 00 14.41	269 32 56.88	269 32 56.76	−0.12	−0.00787	−0.99997
			299 32 42.47	+ 6.06				

Table 6

l'_{ik} (metres)	For Δx_i Δy_i: $\dfrac{-\rho\cos\psi'_{ik}}{l'_{ik}} = a_{ik}$	For Δx_i Δy_i: $\dfrac{+\rho\sin\psi'_{ik}}{l'_{ik}} = b_{ik}$	For Δx_k Δy_k: $\dfrac{+\rho\cos\psi'_{ik}}{l'_{ik}} = -a_{ik}$	For Δx_k Δy_k: $\dfrac{-\rho\sin\psi'_{ik}}{l'_{ik}} = -b_{ik}$	$\Delta\psi_{ik}$ (sec)	$\psi_{ik} = \psi'_{ik} + \Delta\psi_{ik}$	v_{ik} (sec)
10	11	12	13	14	15	16=6+15	17
100.00				-20.626	-0.″19	90°00′56.″73	-1.″51
402.24			-5.125	+0.159	+3.83	181 46 33.22	+2.45
385.05					0.00	270 00 00.00	-1.36
402.61			+5.122	+0.117	-0.73	358 41 43.68	-2.08
686.14			+0.037	-3.006	+10.52	89 17 56.35	+2.50
							0
417.45		-1.331	-4.758	+1.331	+2.94	195 37 45.72	-1.17
485.05		-4.252			-0.04	270 00 11.70	+4.57
417.08		-1.294	+4.773	+1.294	-3.33	344 49 31.46	+0.33
586.15		+3.519	+0.051	-3.519	+12.33	89 10 36.10	-3.73
							0
417.45	-4.758	+1.331		-1.331	+2.94	15 37 45.72	-1.02
810.22	-1.290	+2.195	+1.290	-2.195	+3.84	59 33 43.19	+0.71
548.15	-2.760	-2.558			+3.42	317 10 41.99	+0.28
804.56	-2.564	+0.010	+2.564	-0.010	+1.74	0 14 06.08	+0.40
402.24	-5.125	+0.159			+3.83	1 46 33.22	-0.36
							0
485.05				-4.252	-0.04	90 00 11.70	+1.41
548.15			-2.760	-2.558	+3.42	137 10 41.99	-0.82
550.73			+2.737	-2.556	+5.44	43 02 33.73	+1.22
1071.17			+0.015	-1.926	+6.76	89 33 03.52	+2.42
385.05					0.00	90 00 00.00	-4.23
							0
417.08	+4.773	+1.294		-1.294	-3.33	164 49 31.46	-1.16
402.61	+5.122	+0.117			-0.73	178 41 43.68	+2.62
804.56	+2.564	-0.010	-2.564	+0.010	+1.74	180 14 06.08	-0.19
550.73	+2.737	-2.556			+5.44	223 02 33.73	-1.19
799.17	+1.272	+2.245	-1.272	-2.245	+6.56	119 32 49.10	-0.07
							0
799.17	-1.272	-2.245	+1.272	+2.245	+6.56	299 32 49.10	+0.56
810.22	+1.290	-2.195	-1.290	+2.195	+3.84	239 33 43.19	-0.16
586.15	+0.051	-3.519		+3.519	+12.33	269 10 36.10	+0.96
686.14	+0.037	-3.006			+10.52	269 17 56.35	-1.95
1071.17	+0.015	-1.926			+6.77	269 33 03.53	+0.58
							0

The point 100 - Stamkart calls it 0 in his computations [68] - is the southeastern terminal point of the base line; 103 (Stamkart calls it 3) is also a point of this line. The distance between the two points is the sum of the sections A and B of the base line measurement in fig. 10. According to table 5 this sum is 384.9840 (Repsold) metres. Conformably to section 9 it must be corrected with 0.175 mm per m in order to find the distance 385.0514 with the Van Swinden glass metre, the assumed standard. In an analogous way the distance 100-101 is the section Q in fig. 10. Its length, expressed in the same standard is $\{(99.9858 + 99.9863):2\}$ + 0.175 mm per m = 100.0035 m. According to Stamkart the distances are 385.0158 m and 99.9860 m respectively, just like the length of the base line too short in my opinion.

Table 6, column 4, gives a survey of the directions α_{ik} measured in the points i = 100 up to and including 105 and used for the determination of 105 (the triangulation point 5 on the fort Schiphol) with respect to the base line. Stamkart also used in his computations the direction in 101 to 100 but it was left out in the present computation as it is based on only one observation. "Stamkart's" computation - it was done twice, once by Mr. Bouten and once by Mr. Van Hees - is very confusing. Too many things on the 55 pages used have too little to do with the problem that had to be solved. Stamkart demonstrates here that geodesy is a new field for him in which he should have gone more deeply than he apparently did.

In a certain sense the unsatisfactory plan of the computation cannot be taken him amiss. Though he knew the adjustment according to the least squares, the technique at that time was hardly able to adjust the "great" number of contradictions in the observations in a satisfactory way. I therefore took the trouble to adjust the said observations according to the method of the least squares. In this adjustment the distances 100 - 103 and 100 - 101, measured with the Repsold apparatus had to remain unchanged.

In a local coordinate system xy in which 100 - 103 is the negative x-axis and 100 the origin, provisional coordinates of the points 101 up to and including 105 could be computed with the aid of the observations in column 4 of table 6. They are mentioned in the columns 2 and 5 of table 7.

For the ease of survey of the directions measured the scale in y-direction in fig. 12 is exaggerated for the points 101 and 105. Because of the small area in which the points are situated the curvature of the earth could be neglected in the computations.

Table 7

Points i	x'_i	Δx_i	x_i	y'_i	Δy_i	y_i
1	2	3	4	5	6	7
100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
101	+100.0035	0.0000	+100.0035	- 0.0276	+0.0001	- 0.0275
102	- 12.4580	-0.0076	- 12.4656	-402.0458	-0.0051	-402.0509
103	-385.0514	0.0000	-385.0514	0.0000	0.0000	0.0000
104	- 9.1646	-0.0009	- 9.1655	+402.5087	-0.0223	+402.4864
105	+686.0888	-0.0293	+686.0595	+ 8.4297	-0.0354	+ 8.3943

In column 5 of table 6 all directions measured are orientated provisionally in the same system (bearing 100 - 103 = 270^o). From the provisional bearings:

$$\psi'_{ik} = \text{arc tan} \left\{ (x'_k - x'_i) : (y'_k - y'_i) \right\}$$

(column 6) of the sides the amounts

$$f_{ik} = \psi'_{ik} - (\alpha_{ik} + 0'_i)$$

could be computed (column 7) and the amounts cos ψ'_{ik} (column 8), sin ψ'_{ik} (column 9) and $1'_{ik}$ (column 10), the provisional lengths between the several points. If Δx_i, Δy_i, Δx_k, Δy_k are the corrections to the amounts x'_i, y'_i, x'_k, y'_k in order to get the adjusted amounts x_i, y_i, x_k, y_k, $\Delta \psi_{ik}$ (column 15) the corrections given to ψ'_{ik} in order to find $\psi_{ik} = \psi'_{ik} + \Delta \psi_{ik}$ (column 16) and $\Delta 0_i$ (column 5) the corrections to the provisional orientations $0'_i$ in order to find $0_i = 0'_i + \Delta 0_i$, then the remaining differences:

$$v_{ik} = \psi_{ik} - (\alpha_{ik} + 0_i) \quad \text{(column 17)}$$

can be expressed by the correction equations

$$v_{ik} = f_{ik} + a_{ik} \Delta x_i + b_{ik} \Delta y_i - a_{ik} \Delta x_k - b_{ik} \Delta y_k - \Delta 0_i$$

with

$$a_{ik} = \frac{-\rho \cos \psi'_{ik}}{1'_{ik}} \qquad \text{(columns 11,13)} \quad ,$$

$$b_{ik} = \frac{+\rho \sin \psi'_{ik}}{1'_{ik}} \qquad \text{(columns 12,14)} \quad \text{and}$$

$$\Delta \psi_{ik} = a_{ik} \Delta x_i + b_{ik} \Delta y_i - a_{ik} \Delta x_k - b_{ik} \Delta y_k .$$

If Δx_i, Δy_i, Δx_k, Δy_k, and l'_{ik} are expressed in cm and $\rho = 206264.8$, then the $\Delta\psi_{ik}$'s are found in seconds of arc.

As the distance 100 - 103 must remain unchanged and therefore $\Delta x_{100} = \Delta y_{100} = \Delta x_{103} = \Delta y_{103} = 0$, the columns 11 and 12 for i = 100 and i = 103 are not filled in. The same holds for the columns 13 and 14 for the points k = 100 and k = 103. As also the distance 100 - 101 remains unchanged and $y'_{101} \approx 0$, the correction $\Delta x_{101} = 0$. Column 11 for i = 101 and column 13 for k = 101 got therefore also a dash.

As an example underneath some arbitrary correction equations are given. They can easily be read from the table:

$$v_1 = +0.04 - 20.626\,\Delta y_{101} - \Delta 0_{100}$$

$$v_8 = +4.72 - 1.294\,\Delta y_{101} + 4.773\,\Delta x_{104} + 1.294\,\Delta y_{104} - \Delta 0_{101}$$

$$v_{25} = +0.07 - 1.272\,\Delta x_{105} - 2.245\,\Delta y_{105} + 1.272\,\Delta x_{104} + 2.245\Delta y_{104} - \Delta 0_{105}.$$

The 29 correction equations have 13 unknowns: Δy_{101}, Δx_{102}, Δy_{102}, Δx_{104}, Δy_{104}, Δx_{105}, Δy_{105} and the six corrections $\Delta 0_{100}$ up to and including $\Delta 0_{105}$. They were computed with the IBM 360/65-computer of the Delft University of Technology for the condition $[vv]$ = minimum. It will be clear that the amounts $\psi_{ik} = \psi'_{ik} + \Delta\psi_{ik}$ derived from these unknowns (column 6) must agree with the amounts

$$\psi_{ik} = \text{arc tan} \left\{(x_k - x_i) : (y_k - y_i)\right\} .$$

These adjusted coordinates $x_i y_i$, $(x_k y_k)$ are mentioned in the columns 4 and 7 of table 7. The corrections Δx_i Δy_i (Δx_k Δy_k) are in the columns 3 and 6 respectively.

The v's in column 17 of table 6 are a measure for the accuracy of the measurement. As $[vv] = 101.86$ and $m^2 = [vv] : (29 - 13) = 6.37$, the standard deviation in a direction is $\pm\ 2''.52$, not too bad if one realizes that for the mean distance 564 m in the network this standard deviation represents 7 mm in a direction perpendicular to that direction.

As Stamkart computed in the same coordinate system his final results:

$$x_{105} = +685.9965\,, \quad y_{105} = +8.4170 \quad [69] \quad ,$$

can immediately be compared with mine:

$$x_{105} = +686.0595\,, \quad y_{105} = +8.3943.$$

From the above coordinates of 105 it follows that the distance between the triangulation point on the fort Schiphol and the base terminal point NW is

$$\left\{ (5971.843 + 686.060)^2 + 8.394^2 \right\}^{\frac{1}{2}} = 6657.908 \text{ m.}$$

A second side of the base extension network which lies close to the measured base line is the side between the triangulation points Schiphol (5), the flag-pole on the fort, and Lijnden (7), the chimney of the water pumping station. In his lecture in Amsterdam Stamkart already gave a solution for the determination of Lijnden with respect to the base line. I give this solution in fig. 13 [70] .

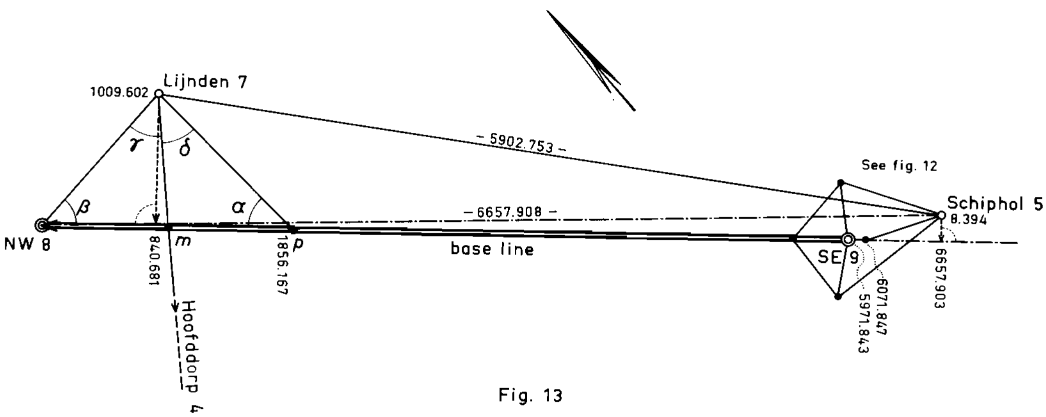


Fig. 13

p is a point in the base line. Its distance to the terminal point N.W. (8) must be known. m, first planned as the terminal point, is the intersection point of the base line with the connecting line Lijnden - Hoofddorp. p and m are the same letters as used in the report of the lecture. N.W.-p is the sum of the sections L up to and including P of the base line (see fig. 10). Conformably to table 5 this distance is 1855.842m. On page 106 in portfolio A2 Stamkart uses this distance indeed but he should have corrected it with 0.175 mm per metre (1856.167 m) already mentioned in section 9. As $\alpha = 44^\circ 50' 00''.53$, $\beta = 50^\circ 12' 58''.64$, $\gamma = 40^\circ 02' 14''.43$, and $\delta = 44^\circ 54' 45''.87$, the length of the perpendicular from Lijnden on the base line and the distance from N.W. to the foot of this perpendicular can then be computed in an easy way. These two distances are given in fig. 13. In the right part of the figure the position of Schiphol with respect to the base line is drawn once again (see also fig. 12). From the two parts the distance Schiphol - Lijnden = 5902.753 m follows in an easy way. This distance Schiphol - Lijnden, directly derived from the measurement of the base line, will be the measure in the now following comparison between the length of the side Amsterdam - Haarlem in Stamkart's main triangulation network in the system of the R(ijks) D(riehoeksmeting).

11. *THE EXTENSION OF THE SIDE SCHIPHOL - LIJNDEN TO THE SIDE AMSTERDAM - HAARLEM OF THE MAIN TRIANGULATION NETWORK*

The base extension network is represented in fig. 14. Its scale can be borrowed from the drawing. For its clearness, however, the position of the base line N.W. (8) - S.E. (9) with respect to the line Schiphol (5) - Liede (6) is represented in an exaggerated way; in reality the perpendicular distance between terminal point SE (9) and the connecting line 5-6 is only about 8.4 m, that between the terminal point NW (8) and 5-6 about 7.8 m.

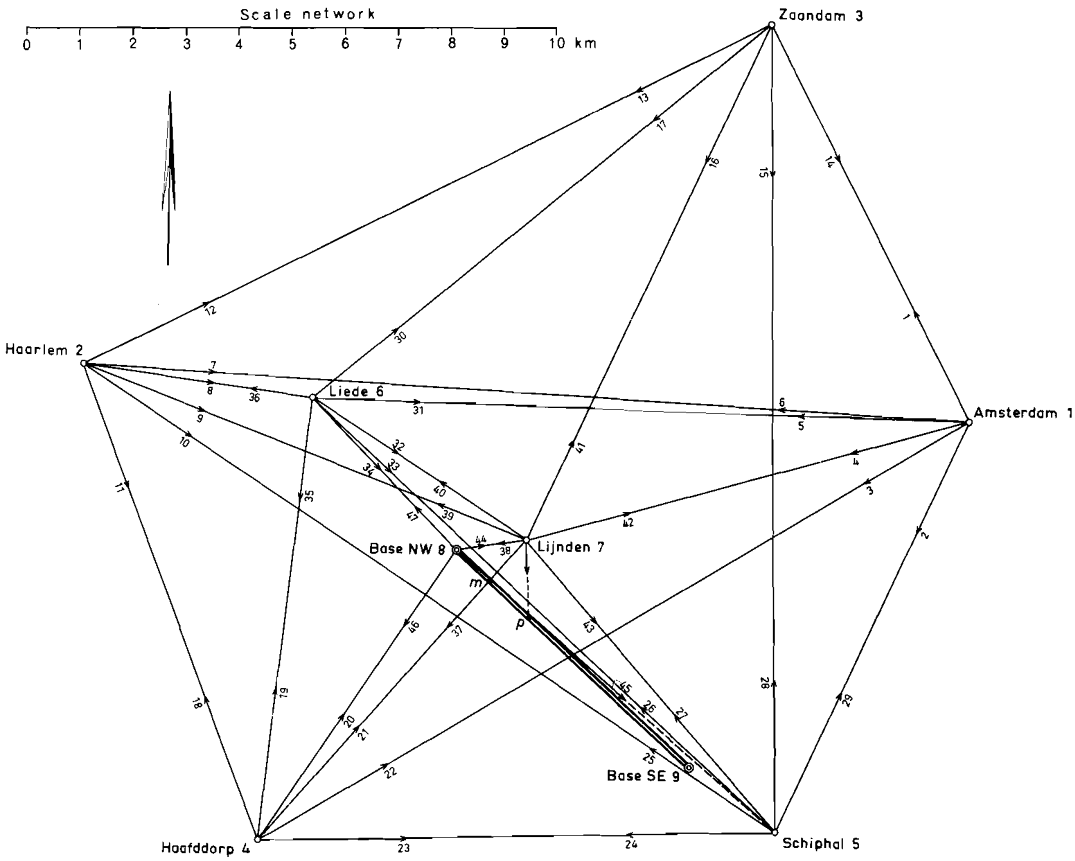


Fig. 14

5 and 6 are flagpoles on the forts Schiphol and Liede respectively, 3 and 4 are church towers at Zaandam and Hoofddorp, and 7 is the chimney of the water-pumping station Lijnden. 1 and 2, finally, are the so-called "centres" of the Western tower in Amsterdam and the St. Bavochurch at Haarlem. As already explained in section 8, these centres differ in general from the spires on the towers.

For the computation of the network 47 directions were measured in the various angular points. They are indicated with their numbers in fig. 14. The spherical directions α_{ik} are given in column 5 of table 9. The length 5902.753 m of the side Schiphol - Lijnden, computed in the preceding section, determines the distance Amsterdam - Haarlem.

"By the many redundant directions in the baseline extension network", Stamkart remarks on page 270 of the report of his lecture in Amsterdam, "there are plenty of opportunities to arrive at several computations of the distance Amsterdam - Haarlem and at the most probable mean distance".

In his report over the year 1871, dated May 15th 1872, to the minister of home affairs [71]he gave already a provisional amount: 16788.038 m. According to this report Krayenhoff found 16789.285 m for it "and therefore 1.247 or about $1\frac{1}{4}$ m more, a difference that cannot be called insignificant as it is about 1 : 13.000 of the length".

In the final computation Stamkart did not adjust this part of his triangulation network in a correct way according to the least squares, though he did some attempts, especially in the quadrangle bordered by the angular points Haarlem (2), Amsterdam (1), terminal point SE (9), and Hoofddorp (4) with the terminal point NW (8) about in its centre. Some spherical excesses in this part were computed, but it is not clear where they are used for. The computation of several other spherical excesses, however, is missing, also that of the largest triangle Schiphol (5) - Haarlem (2) - Zaandam (3) of the network. He should not have neglected this amount of $0''.45$ in his first order measurement.

Table 8

i	X'_i	ΔX_i	X_i	Y'_i	ΔY_i	Y_i
1	2	3	4=2+3	5	6	7=5+6
1			-34298.674			+24525.678
2			-51065.405			+25399.028
3	-38139.495	+0.225	-38139.270	+31847.738	-0.045	+31847.693
4	-47652.856	-0.068	-47652.924	+16587.956	+0.162	+16588.118
5	-38991.981	-0.056	-38992.037	+17594.026	+0.115	+17594.141
6	-47455.073	-0.070	-47455.143	+25165.308	-0.035	+25165.273
7	-42659.106	-0.196	-42659.302	+22218.805	+0.069	+22218.874
8	-43958.772	-0.128	-43958.900	+22026.988	+0.105	+22027.093
9			-39508.864			+18045.320

Table 9

Stations i No.	Name	direction	Sighting points k	α_{ik}	ϵ_{ik} (sec)	$\alpha_{ik}+\epsilon_{ik}$ in plane of proj. $0'_i$	$\alpha_{ik}+\epsilon_{ik}+0'_i$ $\Delta 0_i$	ψ'_{ik}	$\Delta\psi_{ik}$ (sec)	ψ_{ik}	v_{ik} (sec)
1	2	3	4	5	6	7	8	9	10	11=9+10	12=11-8
1	Amsterdam	1	Zaandam	0°00′00.″00	+0.″20	0°00′00.″00	332°19′14.″13	332°19′13.″92	+4.″45	332°19′18.″37	+1.″17
		2	Schiphol	241 46 50.66	-0.45	241 46 50.21	214 06 04.14	214 06 04.58	+2.72	214 06 07.30	+0.10
		3	Hoofddorp	266 57 08.03	-0.76	266 57 07.27	239 16 21.20	239 16 21.95	+2.31	239 16 24.26	0.00
		4	Lijnden	282 15 13.74	-0.36	282 15 13.38	254 34 27.31	254 34 27.68	+2.81	254 34 30.49	+0.12
		5	Liede	300 27 41.73	-0.38	300 27 41.35	272 46 55.28	272 47 00.16	-0.60	272 46 59.56	+1.22
		6	Haarlem	300 39 40.37	-0.48	300 39 39.89	272 58 53.82	272 58 54.28	0.00	272 58 54.28	-2.60
						332 19 13.93	+ 3.06				0
2	Haarlem	7	Amsterdam	0 00 00.00	+0.48	0 00 00.48	92 58 54.75	92 58 54.28	0.00	92 58 54.28	-1.12
		8	Liede	0 43 20.01	+0.10	0 43 20.11	93 42 14.38	93 42 14.27	+2.23	93 42 16.50	+1.47
		9	Lijnden	17 44 24.95	+0.06	17 44 25.01	110 43 19.28	110 43 20.45	+0.12	110 43 20.57	+0.64
		10	Schiphol	29 53 57.69	-0.12	29 53 57.57	122 52 51.84	122 52 51.98	-0.95	122 52 51.03	-1.46
		11	Hoofddorp	65 50 48.30	-0.46	65 50 47.84	158 49 42.11	158 49 42.56	+0.12	158 49 42.68	-0.09
		12	Zaandam	330 30 13.45	+0.83	330 30 14.28	63 29 08.55	63 29 07.75	+2.01	63 29 09.76	+0.56
						92 58 54.27	+ 0.65				0
3	Zaandam	13	Haarlem	0 00 00.00	-0.83	359 59 59.17	243 29 08.57	243 29 07.75	+2.01	243 29 09.76	+0.57
		14	Amsterdam	268 50 09.43	-0.20	268 50 09.23	152 19 18.63	152 19 13.92	+4.45	152 19 18.37	-0.89
		15	Schiphol	299 56 14.12	-0.72	299 56 13.40	183 25 22.80	183 25 21.60	+4.19	183 25 25.79	+2.36
		16	Lijnden	321 39 40.96	-0.65	321 39 40.31	205 08 49.71	205 08 39.61	+8.33	205 08 47.94	-2.39
		17	Liede	350 51 41.48	-0.70	350 51 40.78	234 20 50.18	234 20 47.90	+3.26	234 20 51.16	+0.35
						243 29 09.40	+ 0.62				0
4	Hoofddorp	18	Haarlem	0 00 00.00	+0.46	0 00 00.46	338 49 42.11	338 49 42.56	+0.12	338 49 42.68	+0.06
		19	Liede	22 29 32.66	+0.52	22 29 33.18	1 19 14.83	1 19 15.34	+0.06	1 19 15.40	+0.06
		20	Base N.W.	55 21 18.33	+0.41	55 21 18.74	34 11 00.39	34 11 00.89	-0.55	34 11 00.34	-0.55
		21	Lijnden	62 44 23.63	+0.44	62 44 24.07	41 34 05.72	41 34 06.23	-0.93	41 34 05.30	-0.92
		22	Amsterdam	80 26 41.12	+0.76	80 26 41.88	59 16 23.53	59 16 21.95	+2.31	59 16 24.26	+0.22
		23	Schiphol	104 32 44.28	+0.24	104 32 44.52	83 22 26.17	83 22 26.68	+1.14	83 22 27.82	+1.14
						338 49 41.65	+ 0.51				0

Table 9 (continued)

1	2	3	4	5	6	7	8	9	10	11	12
5	Schiphol	24	Hoofddorp	0°00′00″.00	-0″.24	359°59′59″.76	263°22′26″.17	263°22′26″.68	+1″.14	263°22′27″.82	+0″.21
		25	Haarlem	39 30 21.05	+0.12	39 30 21.17	302 52 47.58	302 52 51.98	-0.95	302 52 51.03	+2.01
		26	Liede	48 26 29.83	+0.18	48 26 30.01	311 48 56.42	311 48 59.62	-2.20	311 48 57.42	-0.44
		27	Lijnden	58 12 45.81	+0.15	58 12 45.96	321 35 12.37	321 35 16.97	-4.85	321 35 12.12	-1.69
		28	Zaandam	100 02 57.28	+0.72	100 02 58.00	3 25 24.41	3 25 21.60	+4.19	3 25 25.79	-0.06
		29	Amsterdam	130 43 39.02	+0.45	130 43 39.47	34 06 05.88	34 06 04.58	+2.72	34 06 07.30	-0.02
						263 22 26.41	+ 1.44				0
6	Liede	30	Zaandam	0 00 00.00	+0.70	0 00 00.70	54 20 47.20	54 20 47.90	+3.26	54 20 51.16	+0.55
		31	Amsterdam	38 26 08.54	+0.38	38 26 08.92	92 46 55.42	92 47 00.16	-0.60	92 46 59.56	+0.74
		32	Lijnden	67 13 04.37	-0.03	67 13 04.34	121 33 50.84	121 33 55.27	-0.81	121 33 54.46	+0.21
		33	Schiphol	77 28 07.68	-0.18	77 28 07.50	131 48 54.00	131 48 59.62	-2.20	131 48 57.42	+0.01
		34	Base N.W.	77 33 50.54	-0.08	77 33 50.46	131 54 36.96	131 54 41.41	-2.88	131 54 38.53	-1.84
		35	Hoofddorp	126 58 24.40	-0.52	126 58 23.88	181 19 10.38	181 19 15.34	+0.06	181 19 15.40	+1.61
		36	Haarlem	219 21 27.96	-0.10	219 21 27.86	273 42 14.36	273 42 14.27	+2.23	273 42 16.50	-1.27
						54 20 46.50	+ 3.41				0
7	Lijnden	37	Hoofddorp	0 00 00.00	-0.44	359 59 59.56	221 34 06.23	221 34 06.23	-0.93	221 34 05.30	-0.25
		38	Base N.W.	40 02 14.42	-0.05	40 02 14.37	261 36 21.04	261 36 15.73	+4.11	261 36 19.84	-0.53
		39	Haarlem	69 09 14.64	-0.06	69 09 14.58	290 43 21.25	290 43 20.45	+0.12	290 43 20.57	-0.01
		40	Liede	79 59 48.30	+0.03	79 59 48.33	301 33 55.00	301 33 55.27	-0.81	301 33 54.46	+0.13
		41	Zaandam	163 34 42.09	+0.65	163 34 42.74	25 08 49.41	25 08 39.61	+8.33	25 08 47.94	-0.80
		42	Amsterdam	213 00 23.89	+0.36	213 00 24.25	74 34 30.92	74 34 27.68	+2.81	74 34 30.49	+0.24
		43	Schiphol	280 01 05.07	-0.15	280 01 04.92	141 35 11.59	141 35 16.97	-4.85	141 35 12.12	+1.21
			p (fig.13)	315 05 14.13							
						221 34 06.67	- 0.67				0
8	Base N.W.	44	Lijnden	81 12 27.36	+0.05	81 12 27.41	81 36 19.83	81 36 15.73	+4.11	81 36 19.84	+0.32
		45	Schiphol	131 21 05.52	-0.11	131 21 05.41	131 44 57.83	131 44 58.41	-1.71	131 44 56.70	-0 82
			Base S.E.	131 25 26.33	-0.11	131 25 26.22	131 49 18.64				
		46	Hoofddorp	213 47 08.88	-0.41	213 47 08.47	214 11 00.89	214 11 00.89	-0.55	214 11 00.34	-0.23
		47	Liede	311 30 45.61	+0.08	311 30 45.69	311 54 38.11	311 54 41.41	-2.88	311 54 38.53	+0.73
						0 23 52.42	- 0.31				0

The great number of unnumbered pages, even without any text, makes it impossible to follow the sequence of the computations and to form a judgement on these computations, apparently done in the plane of projection of the then used Bonne-projection. The length of the extended base line Schiphol - Lijnden in the computations is 5902.27 m, the distance Amsterdam - Haarlem 16789.28 m. Therefore the accuracy of Stamkart's triangulation was checked in the following way.

In an adjustment according to the least squares I started from the coordinates of Amsterdam (centre of the tower, 1) and Haarlem (centre of the tower, 2) in the R.D.-system. From the coordinates of Schiphol and Lijnden, to be found from this adjustment, follows the distance between these two points. It can be compared with the length 5902.753 m found in fig. 13. The standard deviation m in a direction of the triangulation network can also be compared with Stamkart's amount $m = \pm\ 1''.357$.

The coordinates of Amsterdam

$$X_1 = -\ 34298.674\ ,\ Y_1 = +\ 24525.678$$

are borrowed from [72] , those of Haarlem

$$X_2 = -\ 51065.405\ ,\ Y_2 = +\ 25399.028$$

from the coordinates of the spire in 1898 and the reduction of these coordinates to the centre of the tower treated as an example in section 8 (fig. 6). According to these coordinates the distance between Amsterdam and Haarlem is:

$$16789.461 + 1.287 + 0.005 = 16790.753 \text{ m.}$$

The first correction term is the correction from the chord 16789.461 in the plane of projection to the chord of the conformal sphere, the second term the correction to be given to the length of the chord of the sphere in order to find the arc on the sphere [73].

With the uncorrected amounts α_{ik} in column 5 of table 9 provisional coordinates $X'_i\ Y'_i$ of the angular points of the network were computed. They are mentioned in columns 2 and 5 of table 8. In contradistinction to the computations in table 6 the directions measured had to be reduced to directions of the chords in the plane of projection before they could be orientated in the same system. The corrections ϵ_{ik} from arc to chord in column 6 of table 9 were computed with the formula

$$\epsilon_{ik} = 0.0012658\ (X'_k\ Y'_i - X'_i\ Y'_k) \qquad [\ 74\ ]$$

(X' and Y' in km, ϵ in seconds of arc).

The rest of the table is analogous to table 6. The columns for f_{ik}, cos ψ'_{ik}, sin ψ'_{ik}, and l'_{ik} are left out.

The amounts $v_{ik} = \psi_{ik} - \left\{(\alpha_{ik} + \epsilon_{ik}) + (0'_i + \Delta 0_i)\right\}$ in column 12 are very small. As $[vv] = 50.5$ and the number of unknowns is 20 (12 unknown coordinate corrections ΔX_3, ΔY_3 up to and including ΔX_8, ΔY_8 and 8 corrections $\Delta 0_i$), the standard deviation in a direction is:

$$\left\{50.5 : (47 - 20)\right\}^{\frac{1}{2}} = \pm 1.''367.$$

It agrees excellently with the amount $m = \pm 1.''357$ found by Stamkart.

The computation of the adjusted coordinates $X_i Y_i$ is mentioned in columns 4 and 7 of table 8. From the coordinates i = 5 and i = 7 follows the distance 5902.287 + 0.469 + 0.000 = 5902.756 m between the points Schiphol and Lijnden. In fig. 13 it appeared to be 5902.753 m. The difference - 3 mm - is extraordinary small and can be neglected.

From i = 5 and i = 8 follows in an analogous way the distance Schiphol - base terminal point N.W. It is 6657.913 m, only 5 mm more than the amount 6657.908 derived from the base line and its extension to Schiphol (see fig. 13).

In table 8 are also mentioned the coordinates of the southeastern terminal point (9) of the base line. These coordinates were computed from those of Schiphol (5) and terminal point N.W. (8) and the measures in fig. 13. From the coordinates follows the length 5971.847 m of the base line. It is in perfect harmony with the measured distance 5971.843 m.

Though I agree that the perfect results mentioned above will also have been influenced by chance, the conclusion seems to be justified that the length of Stamkart's base line fits excellently in the R.D.-triangulation network, provided at least - I remarked it already - that the spires and "centres" of Amsterdam and Haarlem have not changed between 1870 and 1898.

12. *COMPARISON OF THE RESULTS OF STAMKART'S BASE EXTENSION NETWORK WITH THOSE OF THE R.D.*

Not only the coordinates of the points Amsterdam (1) and Haarlem (2) of Stamkart's network in fig. 14 are known in the R.D.-coordinate system, but also those of Zaandam (3), Hoofddorp (4), base line terminal point N.W. (8), and base line terminal point S.E. (9). These coordinates are mentioned in columns 6 and 7 of table 10, next to those (columns 4 and 5) computed in table 8. In column 3 the year of their determination is mentioned. The R.D.-point Lijnden

Table 10

Points		year det.	Stamkart		R(ijks)D(riehoeksmeting)		Diff. (cm)	
No.	Name		X	Y	X	Y	ΔX	ΔY
1	2	3	4	5	6	7	6-4 =8	7-5 =9
1	Amsterdam	1898	-34298.674	+24525.678	-34298.674	+24525.678	0	0
2	Haarlem	1898	-51065.405	+25399.028	-51065.405	+25399.028	0	0
3	Zaandam	1909	-38139.270	+31847.693	-38139.33	+31847.61	-6	-8
4	Hoofddorp	1916	-47652.924	+16588.118	-47652.95	+16588.10	-3	-2
8	Term. point NW	1928	-43958.900	+22027.093	-43958.68	+22027.13	+22	+4
9	" " SE	1928	-39508.864	+18045.320	-39508.86	+18045.32	0	0

(X = - 42659.38, Y = + 22218.70), a bar on the chimney of the waterpumping station and determined in 1909, can not be identical with Stamkart's Lijnden. The latter point is most probably the middle of the chimney as may be seen from the mean of four pointings in each series at the borders of the chimney [75] .

Column 3 shows that the R. D.-points are of different order. Amsterdam is a first order point, Haarlem a point of first order second rate, that is to say that it forms no part of the first order triangulation network but is determined from first order points only. Zaandam (3) and Hoofddorp (4) are second order points. Though they are situated at only 18km from each other, there are seven years between their respective determinations. The possibility is therefore not excluded that their position with repect to each other might be less good than in Stamkart's accurate network. It must be said, however, that it is not quite clear whether Stamkart determined the spires of these towers or the "centres". I assumed the spires. It is anyhow beyond dispute that the spires served him as sighting points.

The two terminal points (8) and (9) of the base line, finally, were determined by resection by the Cadastral landsurveying department at Haarlem [76] at the occasion of the intended transfer in 1928 of the points to the department of "Waterstaat" already discussed in section 9. From the measurements and the computations of these measurements - they are also kept in the archives of the Netherlands Geodetic Commission - the size and the shape of the standard ellipses of the two points were computed. They are represented in fig. 15. The ellips of (9) with a long semi axis a = 23. 0 mm and a short semi axis b = 10.4 mm,

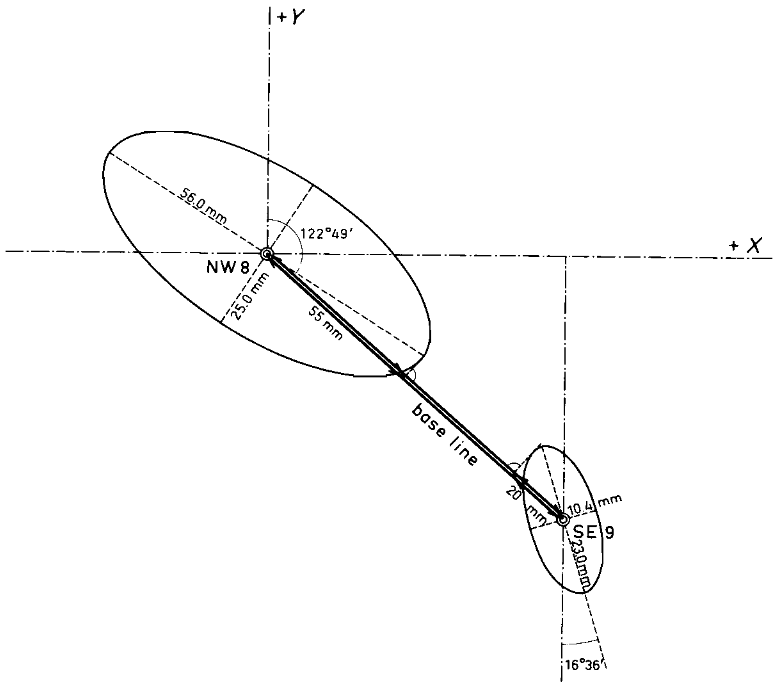


Fig. 15

appears to be much smaller that that of (8) (a = 56.0 mm, b = 25.0 mm). The inclination of the long axes with respect to the gridnorth in the two points is $163^\circ 24'$ and $122^\circ 49'$ respectively. From the drawing can be scaled-off that the standard deviation of (9) in the direction of the base line is $\pm$ 20 mm. That of (8) is $\pm$ 55 mm, so that the standard deviation in the computed length of the base line is $\pm$ 58.5 mm. Because of this great amount it seems unjustifiable to attach too great a value to the length 5971.71 m that can be computed from the cadastral coordinates of the two points.

The differences (in cm) between the two sets of coordinates in table 10 are mentioned in the columns 8 and 9. In fig. 16 they are marked as vectors with respect to the points with a single circle of the network. The R. D.-points have a double circle. The scales can be derived from the figure.

It is remarkable and of course a (lucky) chance that the coordinates of terminal point (9) of the base line, computed from Stamkart's measurements are exactly the same as those found from the cadastral determination in 1928. Not only for Amsterdam (1) and Haarlem (2), but also for terminal point S.E. (9) of the base line the single and the double circle therefore coincide (vector zero).

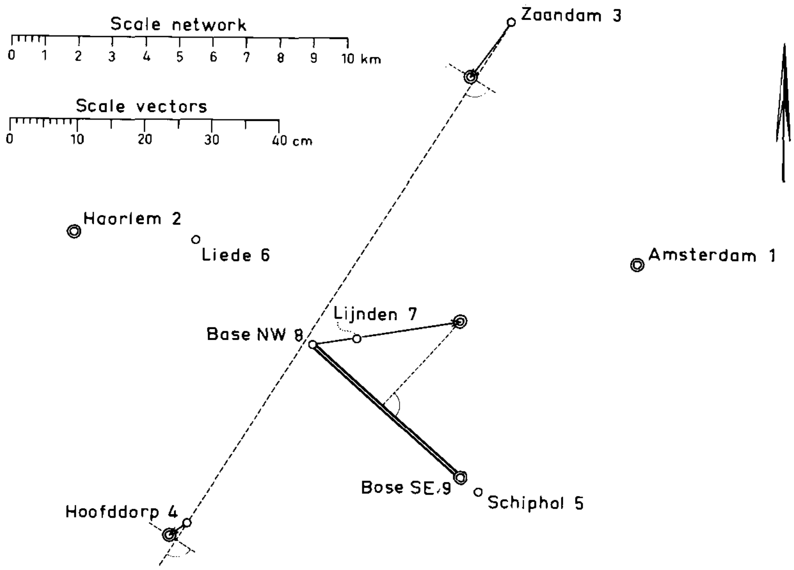


Fig. 16

The vector 3 cm in Hoofddorp (4) is very small. That in Zaandam (3) is bigger (10 cm). The distance (about 18 km) between Zaandam and Hoofddorp in Stamkart's system is (22.5 - 6.0) 5 mm = 8 cm longer than in the R.D.-system. Zaandam - Haarlem is 9 cm longer. As the vector in Hoofddorp is about perpendicular to the direction to Haarlem, the distance Hoofddorp - Haarlem (9.4 km) in the two systems is about alike.

The only dissonance is the great vector of 22 cm in the terminal point N.W. of the base line. The component of the vector in the direction of the base line is 14 cm, that is to say the length of the base line according to the cadastral measurement in 1928 is 14 cm shorter than that found from Stamkart's base line measurement.

As the cadastral coordinates of (8) with respect to the position of Amsterdam, Haarlem, Zaandam, and Hoofddorp are also bad, it seems plausible to reject these coordinates in the columns 6 and 7 of table 10. The other results are so good that they confirm the conclusion already drawn in section 11: the length of the base line fits excellently in the R.D.-triangulation network.

13. *ANALYSIS OF THE LOCAL TRIANGULATION NETWORK IN THE CITY OF LEIDEN*

As was already mentioned on page 43 , a local triangulation network in the city of Leiden served the determination of the mutual situation of the spires of three towers in that city and the Leiden observatory. The network is represented in fig. 17.

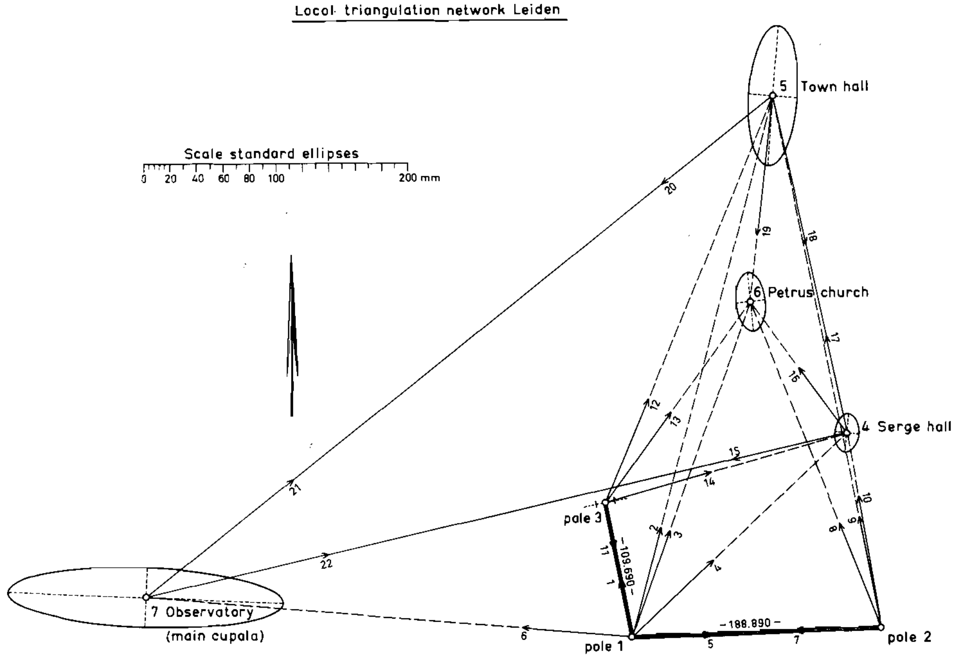


Fig. 17

The three towers mentioned above are those of the Saaihal (Serge hall), the present Lodewijk church (station 4), the Townhall (station 5), and the Catholic Petrus church (station 6). The main cupola of the Observatory in the sketch is station 7. The Serge hall was an angular point of Krayenhoff's triangulation network. That is the reason why Stamkart used it in his measurements. As Stamkart's measurements in Leiden for his main network were partly done on station 5 and partly on 6, the mutual situation of these towers was also necessary. The situation of the main cupola of the Observatory with respect to the town hall was necessary as the astronomical azimuth to Delft, measured in another observation room of the Observatory (see section 21), had to be reduced to the townhall, the angular point Leiden of the main network. The stations 1, 2, and 3, finally, are wooden poles in the terrain between which the distances 1-2 = 188. 890 m and 1-3 = 109. 690 m could be measured.

The 22 directions measured with the small instrument are indicated with their respective numbers on the sketch and in column 4 of table 11. Column 5 gives the mean of each of the two series. The amounts α_i are borrowed from the "couvert F" in file 11. They are already reduced to the spires of the towers.

The sketch demonstrates a very bad knowledge of simple landsurveying. It is e.g. incomprehensible that in none of the towers 4, 6, and 7 the terrestrial points 1, 2, and 3 were used as sighting points. Moreover the intersection of the directions 12, 2, 9, and 17, all of them situated within an angle of about 36^o, is much too sharp to guarantee a good determination of the townhall with respect to the Petrus church, situated about on the bisector of this angle. It affects the accuracy of the important distance 5-6 in an unfavourable way and by that the accuracy of the reductions of the measurements on the Petrus church in the main triangulation network to the centre townhall.

The determination of the Observatory is also bad: intersections too sharp and an extrapolation too great with respect to the base lines measured. They have an unfavourable influence on the accuracy of the distance townhall - observatory, of the utmost importance for the reduction to the centre townhall of the astronomical azimuth to Delft.

The choice of station 3, about 1.50 m south of and not on the line 4-7 makes it very difficult to determine the position of 3 with respect to this line as the very small angle in 4 between 3 and 7 was not measured.

In "cahier F" of the file Leiden Stamkart tried to adjust the network according to the method of the least squares. His adjustment, however, is wrong. His attempt refers to the part bordered by the points 1, 3, 5, 4, and 2. The adjusted triangle 4-5-7, its side 4-5 on the corresponding side of the "adjusted" pentagon, determined the position of the Observatory. The direction 6, measured to the Observatory in station 1, was left out in the adjustment.

The following results refer to a correct adjustment of the network. Just like in Stamkart's computation the coordinates of the seven stations in this adjustment were determined in a coordinate system, the origin of which is the pole 1 ($x_1 = y_1 = 0.000$). The side 1-2 is the positive x-axis. The measured distances 1-2 and 1-3 are assumed to be faultless. The first distance determines the coordinates of 2:

$$x_2 = +188.890, \quad y_2 = 0.000.$$

As the length 1-3 = 109.690 m must remain unchanged, there is a relation between the corrections Δx_3 and Δy_3 in station 3 of fig. 18.

Table 11

Stations		Sighting points	Direc-tions	Observations α_i	Corr. p_i	Adj. Observ. $\alpha_i + p_i$
No.	Name					
1	2	3	4	5	6	7=5+6
1	————	pole 3	1	260° 33′ 46″	- 1″	260° 33′ 45″
		Townhall	2	286 46 36	-27	286 46 09
		Petruschurch	3	291 52 58	+11	291 53 09
		Serge hall	4	319 10 20	+10	319 10 30
		pole 2	5	0 00 00	+ 5	0 00 05
		Observatory	6	186 42 06	+ 3	186 42 09
					+ 1	
2	————	pole 1	7	0 00 00	- 1	359 59 59
		Petruschurch	8	69 48 05	-27	69 47 38
		Townhall	9	80 10 12	+26	80 10 38
		Serge hall	10	81 49 08	+ 2	81 49 10
					0	
3	————	pole 1	11	0 00 00	+ 4	0 00 04
		Townhall	12	214 53 02	+ 4	214 53 06
		Petruschurch	13	229 47 25	+ 5	229 47 30
		Serge hall	14	268 12 18	-13	268 12 05
					0	
4	Serge hall	Observatory	15	0 00 00	- 3	359 59 57
		Petruschurch	16	66 03 50	+13	66 04 03
		Townhall	17	90 01 49	-10	90 01 39
					0	
5	Townhall	Serge hall	18	0 00 00	- 7	359 59 53
		Petruschurch	19	18 43 28	+ 8	18 43 36
		Observatory	20	64 41 17	0	64 41 17
					+ 1	
7	Observatory (main cupola)	Townhall	21	0 00 00	+ 4	0 00 04
		Serge hall	22	25 17 02	- 4	25 16 58
					0	

Table 12

Stations		Stamkart		Least squares		Rijksdriehoeksmeting		Least squares adapted at R. D.		Diff. (mm)	
No	Name	x	y	x	y	X	Y	X'	Y'	7-9	8-10
1	2	3	4	5	6	7	8	9	10	11	12
1	pole	0. 000	0. 000	0. 000	0. 000	—	—	-61449. 304	+323. 413	—	—
2	pole	+188. 890	0. 000	+188. 890	0. 000	—	—	-61260. 561	+329. 602	—	—
3	pole	- 17. 994	+108. 204	- 17. 989	+108. 205	—	—	-61470. 824	+430. 944	—	—
4	Serge hall	+168. 003	+145. 205	+168. 022	+145. 168	-61286. 167	+473. 976	-61286. 170	+473. 973	+ 3	+ 3
5	Town hall	+120. 028	+398. 213	+119. 958	+398. 135	-61342. 494	+725. 083	-61342. 485	+725. 167	- 9	-84
6	Petrus church	+ 98. 594	+245. 463	+ 98. 572	+245. 398	-61358. 82	+571. 94	-61358. 850	+571. 849	+ 30	+91
7	Observatory	—	—	-367. 400	+ 43. 167	-61817. 855	+354. 497	-61817. 831	+354. 507	-24	-10

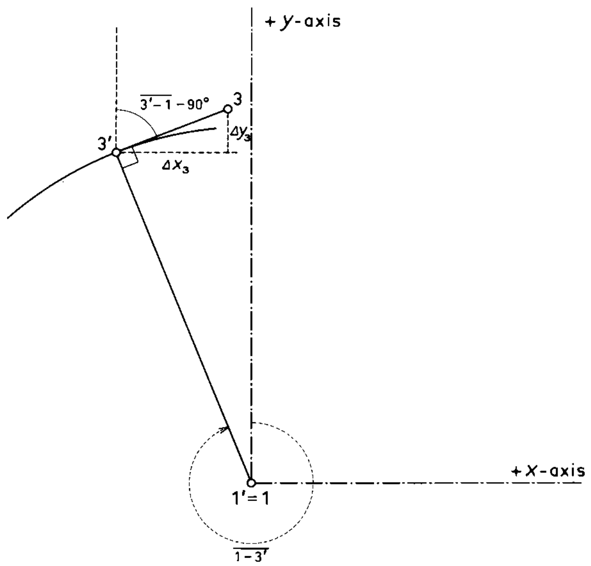


Fig. 18

It can be expressed by:

$$\Delta y_3 = - \Delta x_3 \tan \overline{3' - 1}$$
$$= - \Delta x_3 \tan \overline{1 - 3'} .$$

As (see table 11, column 5) the provisional gridbearing $\overline{1 - 3'}$ = $350^{\circ}33'46''$ ($\tan \overline{1 - 3'} \approx - 0.166$), $\Delta y_3 = + 0.166 \; \Delta x_3$.

The number of unknown coordinate corrections is therefore: (7-2)2 - 1 = 9, and, as the number of unknown orientation corrections $\Delta 0_i = 6$, there are 15 unknowns to be solved. As there are 22 directions measured, the number of redundant data is 7.

The result of the computation with the computer is partly given in table 11 and partly in table 12.

The corrections p_i to the observations α_i are shown in column 6 of table 11. As $[pp] = 3044$, $m^2 = 3044 : 7 = 435$, so that $m = \pm \; 21''$. This standard deviation in each of the 22 measured directions is very large, but the sides of the network are short.

On page 49 of his (wrong) computations Stamkart finds $m = \pm \; 54''$ in each of the angles measured or $54'' : \sqrt{2} = \pm \; 38''$ in a direction. The amount is much too high. In order to show the bad mutual situation of the stations in the local network, the standard ellipses of the points are given in fig. 17. Those of the poles 1 and 2 are of course zero. The short axis of 3 is also zero. Its semi long axis, perpendicular to 1-3 is 0.7 cm. Especially the ellipses of the Observatory and the Townhall are very bad. Their semi long axes are 10.4 cm and 5.2 cm respectively. They prove the correctness of the considerations already given in this section: the inexpert arrangement of the measurement causes much too large standard deviations in the very important distances Townhall - Petruschurch and Townhall - Observatory. That in the first distance appears to be 5.6 cm, that in the latter one 8.9 cm.

Table 12 (columns 3 and 4) gives a survey of the coordinates of the angular points of the network "adjusted" by Stamkart [77]. As already remarked, the Observatory was not included in Stamkart's adjustment. Its coordinates are therefore missing in the columns 3 and 4. The coordinates in columns 5 and 6 are those found from the adjustment according to the least squares.

Provisional computations showed that Stamkart's Observatory was situated about 15 m east and 8 m north of the centre of the meridian circle in the building of the Observatory. As the coordinates of the meridian circle, determined in the R. D.-system in 1897, are known (X = - 61832.511, Y = + 346.653), the place of Stamkart's station 7 could easily be identified. It is the main observation cupola of the Observatory. Unfortunately, however, it is no longer in the same state as in Stamkart's days. Mr. dr. Van Herk, formerly attached to Leiden Observatory, informed me that in 1922 the cupola was replaced by another one so that Stamkart's sighting point on the building does not exist any more. The observation pillar, however, remained unaltered. I assumed therefore - there is no alternative - that the centre of the observation pillar agrees with Stamkart's sighting point in 1872. In March 1974 Mr. Schipper, engineer of the Cadastral landsurveying department at Leiden was so kind as to determine its coordinates from a permanent mark in the balustrade of the building. With those of the Serge hall, Townhall, and Petrus-church, determined in 1897, 1897, and 1917 respectively in the R. D.-system, they are mentioned in the columns 7 and 8 of table 12. In the columns 9 and 10 the results are shown of a similarity transformation in which the coordinates in the columns 5 and 6 are adapted to those in columns 7 and 8. The remaining differences 7-9 and 8-10 are mentioned in mm in the columns 11 and 12.

From the computations it follows that the distances after the transformation are 25 mm per 100 m smaller than those computed from the coordinates in the columns 5 and 6. As, because of the stereographic map projection used, the distances in Leiden computed from the R. D.-coordinates are about 7 mm per 100 m smaller than in the terrain, a scale error of 18 mm per 100 m in Stamkart's network is found. I don't know how far the wooden rods used for the base measurement must be held responsible for this systematic error.

Thanks to the kind collaboration of Mr. ir. A. J. Kers of Topografische Dienst at Delft a map of a part of Leiden with the situation of the seven points in Stamkart's network is given in fig. 19. Pole 1 was situated at the corner of the present Douzastraat and Lange Raamsteeg (steeg = alley). Line 1-3 runs along the Douzastraat [78].

A survey of the distances in the network is given in table 13. Those in column 2 are the results of Stamkart's computations, those in column 3 are borrowed from the adjustment according to the least squares. The differences in column 4 are only caused by the different adjustments.

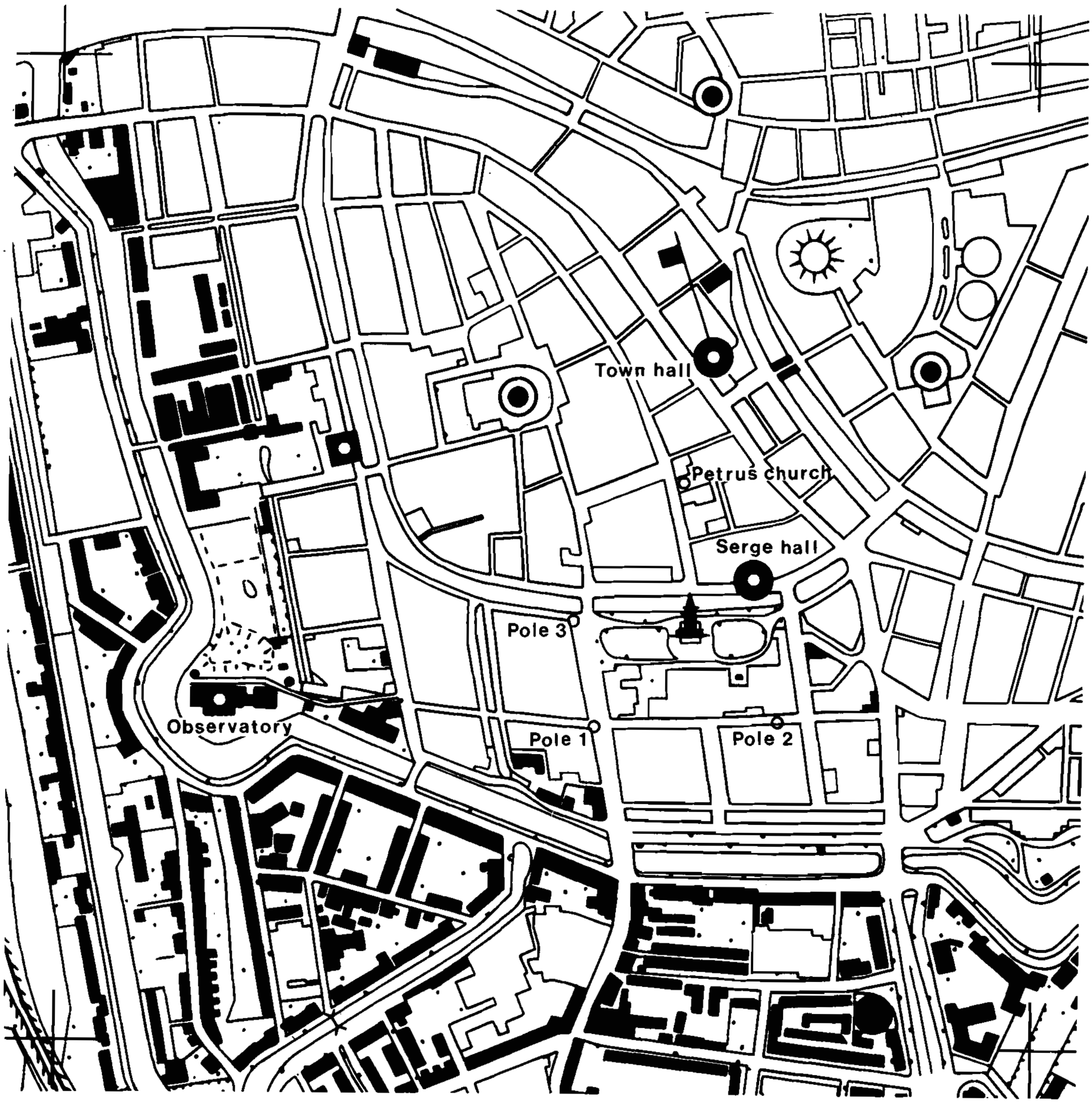

Town hall
Petrus church
Serge hall
Pole 3
Observatory
Pole 1
Pole 2

Fig. 19

Table 13

Distances	Stamkart	Least squares	Diff. (mm)
1	2	3	3-2=4
1-2	188.890	188.890	0
1-3	109.690	109.690	0
1-4	222.057	222.048	- 9
1-5	415.909	415.814	-95
1-6	264.524	264.455	-69
1-7	———	369.927	——
2-3	233.472	233.468	- 4
2-4	146.700	146.660	-40
2-5	404.123	404.058	-65
2-6	261.544	261.491	-53
3-4	189.642	189.648	+ 6
3-5	321.178	321.074	-104
3-6	180.091	180.023	-68
4-5	257.516	257.493	-23
4-6	121.940	121.940	0
4-7	———	545.051	——
5-6	154.246	154.227	-19
5-7	602.932	602.926	- 6
6-7	———	507.964	——

Apart of course from the standard deviations in the stations, caused by the inaccurate measurements and the bad intersection of several directions, Stamkart's wrong adjustment gives deviations of - 19 mm and - 6 mm in the important distances Townhall - Petruschurch (5-6) and Townhall - Observatory (5-7) respectively.

14. *THE MAIN TRIANGULATION NETWORK AND THE CONDITIONS THE MEASURED DIRECTIONS HAVE TO COMPLY WITH*

The main triangulation network (see fig. 2) has 47 angular points. The stations Leer and Onstwedde, also shown on the map, are not included in this number. The measurement of these stations was on the programme for the season 1882, but Stamkart's death in January of that year prevented this. Measurements were made in all stations, except in Pilsum in the utmost northeastern part of the network. The three missing directions at that station to Uithuizermeden, Holwierde, and Emden would also be measured in 1882. The numbers of the stations in fig. 2 are not the same as Stamkart's file numbers in table 14. Lommel e.g. in the utmost south of the network has the station number 1; its file number is 21. In table 14 the stations (column 2) are given in the sequence of their numbers (column 1). The file

Table 14

Stations No.	Stations Name	Sighting points	Pointings	Adjusted directions station measurements spire - spire	Directions	p_i	Adjusted directions triang. network
1	2	3	4	5	6	7	8 = 5 + 7
1	Lommel (21) (catholic church)	Hoogstraten	12	0°00'00".00	1	+0".022	0°00'00".02
		Hilvarenbeek	20	39 54 38.37	2	-0.541	39 54 37.83
		Helmond	17	103 44 51.07	3	+0.455	103 44 51.52
		Borkel	47	—	—	—	—
		Nederweert	13	141 13 17.95	4	+0.064	141 13 18.01
			109			0	
2	Nederweert (18) (catholic church)	Lommel	19	0 00 00.00	5	-0.064	359 59 59.94
		Helmond	22	85 10 07.94	6	+0.064	85 10 08.00
		Peer	10	—	—	—	—
			51			0	
3	Helmond (17) (St. Lambertus church)	's-Hertogenbosch	39	0 00 00.00	7	+0.900	0 00 00.90
		Nederweert	22	209 11 13.05	8	-0.064	209 11 12.99
		Lommel	18	266 32 40.32	9	-0.149	266 32 40.17
		Hilvarenbeek	26	317 15 13.67	10	-0.688	317 15 12.98
			105			0	
4	Hilvarenbeek (19) (catholic church)	Helmond	31	0 00 00.00	11	+0.243	0 00 00.24
		Lommel	23	65 27 16.18	12	+0.147	65 27 16.33
		Hoogstraten	24	159 22 12.14	13	+1.615	159 22 13.76
		Breda	21	203 54 06.38	14	-2.158	203 54 04.22
		's-Hertogenbosch	25	296 43 55.38	15	+0.153	296 43 55.53
			124			0	
5	Hoogstraten (20) (catholic church)	Willemstad	23	0 00 00.00	16	+0.876	0 00 00.88
		Breda	15	37 07 35.47	17	+0.002	37 07 35.47
		Hilvarenbeek	15	104 38 55.95	18	-1.360	104 38 54.59
		Lommel	15	150 49 21.05	19	+0.482	150 49 21.53
		Rijckevorssel	75	—	—	—	—
			143			0	
6	Breda (16) (great church)	Willemstad	14	0 00 00.00	20	+0.131	0 00 00.13
		Dordrecht	39	46 10 09.89	21	-1.437	46 10 08.45
		Gorinchem	21	90 42 23.90	22	+0.535	90 42 24.44
		's-Hertogenbosch	20	136 53 11.18	23	-0.128	136 53 11.05
		Hilvarenbeek	22	178 18 29.53	24	+1.408	178 18 30.94
		Hoogstraten	16	246 15 23.22	25	-0.509	246.15 22.71
			132			0	
7	's-Hertogenbosch (15) (St. John church)	Helmond	21	0 00 00.00	26	-0.580	359 59 59.42
		Hilvarenbeek	28	73 59 08.95	27	-0.075	73 59 08.88
		Breda	16	119 43 59.56	28	-0.101	119 43 59.46
		Gorinchem	19	170 30 29.38	29	+0.757	170 30 30.14
			84			0	

Table 14 (continued

1	2	3	4	5	6	7	8 = 5 + 7
8	Willemstad (23)	Rotterdam	12	0°00′00.″00	30	-0.077	359°59′59.″92
	(reformed church)	Dordrecht	27	41 06 43.08	31	+0.145	41 06 43.22
		Breda	21	108 47 38.44	32	-0.448	108 47 37.99
		Hoogstraten	14	137 55 26.86	33	+0.379	137 55 27.24
		Numansdorp	34	—	—	—	—
			108			0	
9	Dordrecht (14)	Rotterdam	26	0 00 00.00	34	+0.407	0 00 00.41
	(great church;	Gouda	34	54 15 14.43	35	-1.003	54 15 13.43
	centre)	Gorinchem	23	130 33 57.21	36	+0.706	130 33 57.92
		Breda	11	207 34 19.21	37	+0.341	207 34 19.55
		Willemstad	11	273 43 18.16	38	-0.451	273 43 17.71
			105			0	
10	Gorinchem (13)	Breda	35	0 00 00.00	39	-0.151	359 59 59.85
	(reformed church)	Dordrecht	25	58 27 24.28	40	-0.647	58 27 23.63
		Gouda	19	111 09 16.12	41	+1.541	111 09 17.66
		Utrecht	16	172 10 25.14	42	-0.365	172 10 24.78
		's-Hertogenbosch	18	276 57 15.41	43	-0.379	276 57 15.03
			113			0	
11	Rotterdam (3)	Delft	27	0 00 00.00	44	+0.511	0 00 00.51
	(St. Laurens	Leiden Obs.	11	40 09 45.77	—	—	—
	church)	Leiden townhall	14	41 11 21.64	45	+0.448	41 11 22.09
		Gouda	12	97 36 31.51	46	-1.289	97 36 30.22
		Dordrecht	10	174 58 56.87	47	-0.544	174 58 56.33
		Willemstad	16	227 35 30.33	48	+0.874	227 35 31.20
			90			0	
12	Gouda (4)	Nieuwkoop	24	0 00 00.00	49	+0.617	0 00 00.62
	(St. John church)	Utrecht	9	55 23 27.78	50	-1.250	55 23 26.53
		Bergambacht	19	—	—	—	—
		Gorinchem	13	121 10 01.51	51	-1.269	121 10 00.24
		Dordrecht	10	172 09 22.89	52	+0.013	172 09 22.90
		Rotterdam	16	220 31 42.57	53	+1.979	220 31 44.55
		Delft	15	253 34 18.68	54	-0.580	253 34 18.10
		Leiden	15	300 36 27.80	55	+0.490	300 36 28.29
			121			0	
13	Delft (2)	Leiden townhall	30	0 00 00.00	56	-0.459	359 59 59.54
	(new church)	Nieuwkoop tower	10	—	—	—	—
		Nieuwkoop abbey	18	32 54 39.56	57	+1.473	32 54 41.03
		Gouda	21	61 27 40.43	58	-0.943	61 27 39.49
		Rotterdam	19	110 48 36.91	59	-0.071	110 48 36.84
		Leiden Obs.	29	359 15 59.56	—	—	—
			127			0	
14	Leiden (11)	Rotterdam	23	0 00 00.00	60	-1.831	359 59 58.17
	(townhall)	Delft	25	27 59 59.34	61	+0.535	27 59 59.88
		Leiden Obs.	13	50 28 43.58	—	—	—
		Haarlem	24	201 02 12.79	62	+1.296	201 02 14.09
			85			0	
	Measured on Pe-	Rotterdam	19	0 00 00.00	63	+0.842	0 00 00.84
	truschurch and	Nieuwkoop	15	271 32 26.01	64	-0.965	271 32 25.04
	reduced to townhall	Gouda	24	316 29 51.57	65	+0.122	316 29 51.69
			58			0	

Table 14 (continued)

1	2	3	4	5	6	7	8 = 5 + 7
15	Nieuwkoop (5)	Haarlem	23	0°00′00.″00	66	-0.552	359°59′59.″45
	(abbey)	Amsterdam	19	36 32 07.92	67	+0.424	36 32 08.34
		Utrecht	31	126 27 44.36	68	+0.656	126 27 45.02
	/	Gouda	28	217 42 01.29	69	-1.137	217 42 00.15
		Leiden	33	293 21 01.36	70	+0.609	293 21 01.97
			134			0	
16	Utrecht (12)	Amersfoort	24	0 00 00.00	71	+0.330	0 00 00.33
	(tower of the	Gorinchem	15	131 03 28.23	72	+0.745	131 03 28.98
	cathedral)	Gouda	15	184 15 49.29	73	+0.703	184 15 49.99
		Nieuwkoop	20	217 38 10.43	74	-0.486	217 38 09.94
		Amsterdam	14	264 29 50.85	75	-1.691	264 29 49.16
		Naarden	19	298 35 59.57	76	+0.399	298 35 59.97
			107			0	
17	Haarlem (1)	Leiden	19	0 00 00.00	77	-1.222	359 59 58.78
	(St. Bavochurch)	Alkmaar	18	173 19 39.80	78	-0.585	173 19 39.22
		Amsterdam	29	250 22 08.37	79	+1.598	250 22 09.97
		Nieuwkoop	13	317 09 03.07	80	+0.210	317 09 03.28
			79			0	
18	Amsterdam (7)	Utrecht	27	0 00 00.00	81	+1.311	0 00 01.31
	(Western tower)	Nieuwkoop	32	43 12 47.31	82	-0.290	43 12 47.02
		Haarlem	28	119 53 47.35	83	-1.465	119 53 45.88
		Alkmaar	27	189 46 30.24	84	+0.016	189 46 30.26
		Edam	16	243 10 32.93	85	-0.044	243 10 32.89
		Naarden	24	321 58 49.35	86	+0.472	321 58 49.82
			154			0	
19	Naarden (22)	Harderwijk	19	0 00 00.00	87	+0.471	0 00 00.47
	(reformed church)	Huizen	42	—	—	—	—
		Amersfoort	18	56 26 16.02	88	-0.633	56 26 15.39
		Utrecht	20	108 02 44.49	89	+0.047	108 02 44.54
		Amsterdam	18	215 55 23.21	90	+0.186	215 55 23.40
		Edam	13	263 02 45.30	91	-0.072	263 02 45.23
			130			0	
20	Edam (10)	Naarden	35	0 00 00.00	92	+0.082	0 00 00.08
	(chimes tower)	Amsterdam	26	54 04 22.52	93	-0.221	54 04 22.30
		Alkmaar	32	141 32 34.14	94	+0.172	141 32 34.31
		Hoorn	18	201 36 26.76	95	+0.049	201 36 26.81
		Enkhuizen	19	235 58 50.53	96	-0.083	235 58 50.45
			130			0	
21	Alkmaar (16)	Schagen	23	0 00 00.00	97	-0.428	359 59 59.57
	(weighhouse	Hoorn	22	76 56 12.37	98	-0.644	76 56 11.73
	tower)	Edam	10	113 02 50.22	99	+0.150	113 02 50.37
		Amsterdam	18	152 10 36.28	100	+0.600	152 10 36.88
		Haarlem	17	185 15 22.63	101	+0.322	185 15 22.95
			90			0	

Table 14 (continued)

1	2	3	4	5	6	7	8 = 5 + 7
22	Hoorn (9) (great church)	Edam	36	0° 00′ 00.″ 00	102	-0.390	359° 59′ 59.″ 61
		Alkmaar	10	83 49 29.07	103	+0.154	83 49 29.22
		Schagen	16	128 56 57.01	104	+0.419	128 56 57.43
		Medemblik	20	187 41 24.12	105	+0.031	187 41 24.15
		Enkhuizen	16	242 15 02.13	106	-0.214	242 15 01.92
			98			0	
23	Schagen (8) (tower)	Alkmaar	20	0 00 00.00	107	+0.772	0 00 00.77
		Medemblik	16	263 47 51.55	108	+0.406	263 47 51.96
		Hoorn	19	302 03 41.40	109	-1.179	302 03 40.22
			55			0	
24	Medemblik (31) (reformed church)	Enkhuizen	42	0 00 00.00	110	-0.540	359 59 59.46
		Hoorn	17	70 11 15.20	111	+0.272	70 11 15.47
		Schagen	12	153 11 01.12	112	+0.152	153 11 01.27
		Hindeloopen	14	285 13 59.71	113	+0.117	285 13 59.83
		Staveren	15	293 49 10.99	—	—	—
			100			0	
25	Enkhuizen (30) (southern church)	Edam	18	0 00 00.00	114	+0.271	0 00 00.27
		Hoorn	46	27 52 40.26	115	-0.768	27 52 39.49
		Medemblik	24	83 07 45.58	116	+0.670	83 07 46.25
		Staveren	23	154 49 32.83	—	—	—
		Hindeloopen	15	156 51 41.08	117	+0.162	156 51 41.24
		Lemmer	14	202 39 43.34	118	+0.297	202 39 43.64
		Urk	30	244 33 29.69	119	-0.632	244 33 29.06
			170			0	
26	Hindeloopen (32) (reformed church)	Harlingen	25	0 00 00.00	120	-0.341	359 59 59.66
		Sneek	43	58 22 56.62	121	+0.612	58 22 57.23
		Lemmer	26	115 19 01.63	122	-0.018	115 19 01.61
		Enkhuizen	20	193 01 21.92	123	-1.020	193 01 20.90
		Staveren	42	198 44 48.87	—	—	—
		Medemblik	25	224 31 26.51	124	+0.766	224 31 27.28
			181			0	
27	Lemmer (33) (reformed church)	Hindeloopen	21	0 00 00.00	125	+0.426	0 00 00.43
		Sneek	24	52 49 08.44	126	-0.271	52 49 08.17
		Oldeholtpa	34	138 35 30.65	127	+0.266	138 35 30.92
		Blokzijl	29	190 10 29.41	128	-1.330	190 10 28.08
		Urk	46	263 49 45.75	129	+1.177	263 49 46.93
		Enkhuizen	20	303 30 20.76	130	-0.268	303 30 20.49
			174			0	
28	Urk (29) (reformed church)	Enkhuizen	32	0 00 00.00	131	-0.074	359 59 59.93
		Staveren	10	44 27 15.35	—	—	—
		Lemmer	13	98 25 43.48	132	-1.400	98 25 42.08
		Blokzijl	13	150 46 44.46	133	+1.430	150 46 45.89
		Kampen	14	194 41 35.44	134	+0.262	194 41 35.70
		Harderwijk	11	254 32 52.48	135	-0.218	254 32 52.26
			93			0	

Table 14 (continued)

1	2	3	4	5	6	7	8 = 5 + 7
29	Blokzijl (28)	Meppel	29	0°00′00.″00	136	-0.620	359°59′59.″38
	(reformed church)	Kampen	27	87 13 02.12	137	+0.468	87 13 02.59
		Urk	12	151 58 34.82	138	-1.662	151 58 33.16
		Lemmer	12	205 58 09.77	139	+1.861	205 58 11.63
		Oldeholtpa	21	276 06 50.25	140	-0.047	276 06 50.20
			101			0	
30	Oldeholtpa (35)	Meppel	21	0 00 00.00	141	+0.175	0 00 00.17
	(reformed church)	Blokzijl	15	41 11 29.24	142	+0.271	41 11 29.51
		Wolvega	26	—	—	—	—
		Lemmer	18	99 27 55.22	143	-0.446	99 27 54.77
			80			0	
31	Meppel (34)	Blokzijl	30	0 00 00.00	144	+0.810	0 00 00.81
	(reformed church)	Oldeholtpa	32	54 55 23.49	145	-0.410	54 55 23.08
		Beilen	18	128 36 52.29	—	—	—
		Kampen	27	308 13 57.44	146	-0.400	308 13 57.04
			107			0	
32	Kampen (27)	Veluwe	16	0 00 00.00	147	-1.487	359 59 58.51
	(new tower)	Harderwijk	18	34 55 33.01	148	+2.127	34 55 35.14
		Urk	35	111 32 39.04	149	-0.827	111 32 38.21
		Blokzijl	19	182 52 18.92	150	+0.033	182 52 18.95
		Genemuiden	83	—	—	—	—
		Meppel	15	223 53 12.58	151	+0.154	223 53 12.73
			186			0	
33	Harderwijk (26)	Urk	14	0 00 00.00	152	-0.097	359 59 59.90
	(great church)	Kampen	11	43 31 43.44	153	-1.286	43 31 42.15
		Veluwe	22	130 06 13.08	154	+1.422	130 06 14.50
		Ermelo	59	—	—	—	—
		Amersfoort	11	218 34 03.11	155	-0.553	218 34 02.56
		Naarden	14	261 51 56.51	156	+0.514	261 51 57.02
			131			0	
34	Veluwe (25)	Amersfoort	30	0 00 00.00	157	+0.074	0 00 00.07
	(stone)	Harderwijk	26	52 56 27.15	158	-1.142	52 56 26.01
		Kampen	26	111 26 17.60	159	+1.067	111 26 18.67
			82			0	
35	Amersfoort (24)	Utrecht	20	0 00 00.00	160	-0.626	359 59 59.37
	(Our Lady tower)	Naarden	20	66 59 30.22	161	+0.644	66 59 30.86
		Harderwijk	12	147 15 22.47	162	+0.469	147 15 22.94
		Veluwe	17	185 51 10.84	163	-0.486	185 51 10.35
			69			0	
36	Sneek (36)	Lemmer	21	0 00 00.00	164	+0.271	0 00 00.27
	(St. Martini	Hindeloopen	27	70 14 49.77	165	-0.612	70 14 49.16
	church)	Harlingen	16	143 31 27.84	166	+0.434	143 31 28.27
		Scharnegoutum	46	—	—	—	—
		Leeuwarden	23	214 19 59.76	167	+0.275	214 20 00.03
		Drachten	21	263 50 42.86	168	-0.369	263 50 42.49
			154			0	

Table 14 (continued)

1	2	3	4	5	6	7	8 = 5 + 7
37	Harlingen (41) (Western church)	Leeuwarden	22	0°00'00."00	169	+0.093	0°00'00."09
		Sneek	33	51 05 44.58	170	-0.434	51 05 44.15
		Hindeloopen	19	99 26 08.23	171	+0.341	99 26 08.57
		Franeker	22	———	—	———	———
			96			0	
38	Leeuwarden (37) (Oldehove)	Kollum	25	0 00 00.00	172	+0.114	0 00 00.11
		Drachten	22	47 01 19.49	173	+0.254	47 01 19.74
		Goutum	56	———	—	———	———
		Sneek	26	134 37 38.04	174	-0.275	134 37 37.76
		Harlingen	31	192 43 23.19	175	-0.093	192 43 23.10
			160			0	
39	Drachten (38) (reformed church)	Sneek	27	0 00 00.00	176	+0.369	0 00 00.37
		Leeuwarden	30	42 53 01.39	177	-0.254	42 53 01.14
		Kollum	19	114 49 50.53	178	-1.003	114 49 49.53
		Buitenpost	62	———	—	———	———
		Groningen	23	173 19 26.81	179	+0.889	173 19 27.70
			161			0	
40	Kollum (39) (reformed church)	Hornhuizen	26	0 00 00.00	180	-0.708	359 59 59.29
		Groningen	19	54 02 09.46	181	-0.181	54 02 09.28
		Buitenpost	60	———	—	———	———
		Drachten	16	140 04 22.46	182	+1.003	140 04 23.46
		Leeuwarden	25	201 06 16.66	183	-0.114	201 06 16.55
			146			0	
41	Groningen (42/3) (Martini tower)	Drachten	36	0 00 00.00	184	-0.889	359 59 59.11
		Kollum	45	35 28 08.00	185	+0.181	35 28 08.18
		Hornhuizen	45	75 13 53.95	186	+0.808	75 13 54.76
		Bedum	136	———	—	———	———
		Uithuizermeden	40	135 50 23.34	187	+0.481	135 50 23.82
		Holwierde	42	164 15 40.65	188	-0.654	164 15 40.00
		Midwolda	33	206 59 26.80	189	+0.073	206 59 26.87
		Onstwedde	20	234 56 52.14	—	———	———
			397			0	
42	Hornhuizen (44) (reformed church)	Uithuizermeden	22	0 00 00.00	190	+0.100	0 00 00.10
		Groningen	34	59 02 09.98	191	-0.808	59 02 09.17
		Leens	37	———	—	———	———
		Kollum	24	145 14 12.99	192	+0.708	145 14 13.70
			117			0	
43	Uithuizermeden (45) (reformed church)	Pilsum	24	0 00 00.00	193	+0.176	0 00 00.18
		Holwierde	12	47 24 31.09	194	+0.285	47 24 31.38
		Groningen	24	133 52 29.49	195	-0.361	133 52 29.13
		Hornhuizen	23	194 13 52.28	196	-0.100	194 13 52.18
			83			0	

Table 14 (continued)

1	2	3	4	5	6	7	8 = 5 + 7
44	Holwierde (46) (reformed church)	Uithuizermeden	46	0°00'00."00	197	-0.346	359°59'59."65
		Pilsum	42	104 33 07.19	198	-0.130	104 33 07.06
		Emden townhall	34	149 21 45.07	199	-0.141	149 21 44.93
		"Gasthaus church	28	—	—	—	—
		Midwolda	26	214 57 35.53	200	+0.148	214 57 35.68
		Groningen	14	294 53 12.41	201	+0.470	294 53 12.88
			190			0	
45	Midwolda (47) (reformed church)	Holwierde	26	0 00 00.00	202	-0.222	359 59 59.78
		Emden townhall	26	61 17 39.87	203	+0.187	61 17 40.06
		"Gasthaus church	26	—	—	—	—
		Leer	19	109 25 27.37	—	—	—
		Onstwedde	13	199 51 01.88	—	—	—
		Groningen	22	302 39 22.53	204	+0.035	302 39 22.57
			132			0	
46	Emden (48) (townhall)	Leer	30	0 00 00.00	—	—	—
		Midwolda	23	81 00 27.46	205	-0.106	81 00 27.35
		Holwierde	28	134 06 57.33	206	+0.052	134 06 57.38
		Pilsum	47	190 11 17.70	207	+0.054	190 11 17.75
		Mitlum	57	—	—	—	—
		Hage	22	237 37 00.07	—	—	—
		Aurich	32	284 19 33.03	—	—	—
			239			0	
47	Pilsum (Pfarr kirche)	No observations					

numbers between brackets in column 2 refer to table 4. In columns 3 and 4 of table 14 the sighting points are mentioned and the number of pointings at these sighting points. At Lommel (No. 1) e.g. the number of pointings at Hoogstraten, Hilvarenbeek, Helmond, Borkel, and Nederweert is 12, 20, 17, 47, and 13 respectively. The total number (109) of the pointings agrees of course with that in column 6 of table 4. Borkel, not belonging to the network, is a common reference point in the series measured at Lommel, just like Leens in the series at the station Hornhuizen (No. 42), already discussed in section 7.

The adjusted directions at every station in table 14 are mentioned in column 5. As already remarked at the end of section 7, they are reduced to directions from the spire of the station to the spires of the surrounding stations. All directions measured - except those to the "reference" points in the series and some other directions - are numbered in sequence in column 6 of the table and on the map of the network in fig. 2. Their total number is 207.

As already discussed in detail in section 13 the measurements at the station Leiden (No. 14) (file 11) were partially done on the townhall and partially on the Petrus church. At the first station the directions were measured to Rotterdam, Delft, Leiden Observatory, and Haarlem, at the latter station those to Rotterdam, Nieuwkoop, and Gouda. Because of the great eccentricity (about 154 m) the common direction Rotterdam in the two adjusted series in column 5 is numbered twice, 60 and 63 respectively. On the map the numbers 63, 64, and 65 are indicated with double arrows.

The station Dordrecht (No. 9) was visited twice; once in 1872 and once in 1877. On July 9th 1877, after his measurements from July 3 - July 6, Stamkart and his assistant Bouten, according to Stamkart's diary, looked for "a considerable difference between the measurements in 1872 and those in 1877. The difference in the angle between Rotterdam and Gouda was about -11", that between Gorinchem and Breda about + 14'' ". A mistake in the measurements could not be found. According to the diary, however, he discovered that "between 1872 and 1877 the tower sagged 20.5 mm to the North and 2.5 mm to the East". It pleads for Stamkart's sense for accuracy that he found this sagging of the tower already in 1877 (the main direction "to the North" is correct [79]), whereas Heuvelink [80] in his publication Topografische kaart en Rijksdriehoeksmeting (Delft 1920, pages 8-16) assumes that the station Dordrecht had not changed since Krayenhoff's measurements in 1810.

As the measurements at the stations Rotterdam and Gouda took place in 1871, those at Gorinchem and Willemstad in 1872 and those at Breda in 1873 - at all these stations Dordrecht was a sighting point - it will be clear that Stamkart's measurements at Dordrecht in 1877 cannot be used in the adjustment of the triangulation network. They are therefore not included in the results of the adjustment of the directions measured in column 5 of table 14.

As can be seen in fig. 2 by far the greatest part of the directions measured form "normal" triangles, that is to say that in a quadrangle the directions of only one of the two diagonals were measured. The only exception is the construction in the pentagon bordered by the points Nieuwkoop (No. 15) - Gouda (No. 12) - Rotterdam (No. 11) - Delft (No. 13) - Leiden (No. 14). Possibly Stamkart chose this construction in order to obtain as well as possible a solid mutual position of Leiden, Delft, and Rotterdam. For, according to the programme, in Leiden the astronomical azimuth would be measured either to Delft or to Rotterdam. The directions to Leiden Observatory in the series at the stations Delft and Rotterdam seem to confirm this supposition.

The directions in Delft (No. 13) to Nieuwkoop (tower) and Nieuwkoop (abbey) must be attributed to a mistake in the measurements. In the adjustment of the series the mistake could be corrected.

The directions measured in Medemblik (No. 24), Enkhuizen (No. 25), and Hindeloopen (No. 26) to Staveren are not used in the computation of the network. They relate to the reformed church at Staveren, determined by the R.D. in 1914. The tower is situated about 20 m north of Krayenhoff's station.

In fig. 2 forty six stations ($p = 46$) of the network are connected by 101 full drawn lines ($l = 101$). Only Pilsum (No. 47) where no measurements took place is an exception. The number of polygon conditions is therefore $l - p + 1 = 101 - 46 + 1 = 56$. 55 of them are triangle conditions, relating to the triangles 1 up and including 19 and 21 up to and including 56 in the figure 2. The triangles are also mentioned in table 15. Triangle 1 (column 1) e.g. refers to the stations Helmond (No. 3), Nederweert (No. 2), and Lommel (No. 1) (columns 3 and 2). After the adjustment of the triangulation network the sum of its spherical angles must be 180° plus the spherical excess $1''.74$ of the triangle.

If p_i ($i = 1, 2, \ldots 206, 207$) is the correction in seconds to a direction measured then $(57^\circ 21' 27''.27 + p_9 - p_8) + (85^\circ 10' 07''.94 + p_6 - p_5) + (37^\circ 28' 26''.88 + p_4 - p_3) =$ $= 180^\circ 00' 01''.74$ or $-p_3 + p_4 - p_5 + p_6 - p_8 + p_9 = -0''.35$, etc.

The closing errors in the various triangles can be found from the bottom rows column 10 (or 11) minus column 7 in table 15. Though that for triangle 1 $180^\circ 00' 01''.74 -$ $- 180^\circ 00' 02''.09 = -0''.35$ is very small, there are several large closing errors. That in triangle 4, e.g., is $-7''.05$; that in triangle 21 is $-7''.53$. Those in the triangles 36 and 41 are even $+8''.86$ and $+8''.53$ respectively. A possible error in Stamkart's computation of the reductions to centre at the stations could not be found. The large closing error in triangle 21 might be imputed to the angle in Leiden between Nieuwkoop and Haarlem. The angle had to be derived from the directions (64-63) - (62-60), that is to say from the measurements at two stations, situated at about 154 m from each other. The 56th polygon condition is determined by the angles of the Zuiderzee pentagon Edam (No. 20) - Naarden (No. 19) - Harderwijk (No. 33) - Urk (No. 28) - Enkhuizen (No. 25). As the spherical excess of the pentagon is $6''.63$ (see the end of table 15), the condition runs:

$$p_{87} - p_{91} + p_{92} - p_{96} + p_{114} - p_{119} + p_{131} - p_{135} + p_{152} - p_{156} = +1''.14 .$$

Table 15

No. triangle	Stations No.	Stations Name	File	Directions right	Directions left	Directions right-left	Corrections p: p_r	Corrections p: p_l	Adj. spherical angles: Stamkart	Adj. spherical angles: R. D.	Diff. v (sec)	Opposite chords (m): Stamkart	Opposite chords (m): R. D.	Diff. v (cm)
1	2	3	4	5	6	7	8	9	10=7+8-9	11	12=11-10	13	14	15=14-13
1	3	Helmond	17	9	8	57°21′27.″27	-0.″149	-0.″064	57°21′27.″18	57°21′23.″87	-3.″31	30932.45	30932.26	-19
	2	Nederweert	18	6	5	85 10 07.94	+0.064	-0.064	85 10 08.07	85 10 10.50	+2.43	36604.10	36604.30	+20
	1	Lommel	21	4	3	37 28 26.88	+0.064	+0.455	37 28 26.49	37 28 27.37	+0.88	22349.34	22349.56	+22
						180 00 02.09	-0.021	+0.327	180 00 01.74	180 00 01.74	0			
2	3	Helmond	17	10	9	50 42 33.35	-0.688	-0.149	50 42 32.81	50 42 33.26	+0.45	31143.80	31143.99	+19
	1	Lommel	21	3	2	63 50 12.70	+0.455	-0.541	63 50 13.70	63 50 12.79	-0.91	36117.70	36117.78	+ 8
	4	Hilvarenbeek	19	12	11	65 27 16.18	+0.147	+0.243	65 27 16.08	65 27 16.54	+0.46	36604.10	36604.30	+20
						180 00 02.23	-0.086	-0.447	180 00 02.59	180 00 02.59	0			
3	1	Lommel	21	2	1	39 54 38.37	-0.541	+0.022	39 54 37.80	39 54 38.47	+0.67	27696.45	27696.85	+40
	5	Hoogstraten	20	19	18	46 10 25.10	+0.482	-1.360	46 10 26.94	46 10 26.00	-0.94	31143.80	31143.99	+19
	4	Hilvarenbeek	19	13	12	93 54 55.96	+1.615	+0.147	93 54 57.43	93 54 57.71	+0.28	43067.80	43068.25	+45
						179 59 59.43	+1.556	-1.191	180 00 02.18	180 00 02.18	0			
4	4	Hilvarenbeek	19	14	13	44 31 54.24	-2.158	+1.615	44 31 50.47	44 31 51.11	+0.64	20956.39	20956.82	+43
	5	Hoogstraten	20	18	17	67 31 20.48	-1.360	+0.002	67 31 19.12	67 31 19.92	+0.80	27612.32	27612.84	+52
	6	Breda	16	25	24	67 56 53.69	-0.509	+1.408	67 56 51.77	67 56 50.33	-1.44	27696.45	27696.85	+40
						180 00 08.41	-4.027	+3.025	180 00 01.36	180 00 01.36	0			
5	4	Hilvarenbeek	19	15	14	92 49 49.00	+0.153	-2.158	92 49 51.31	92 49 48.96	-2.35	38503.19	38503.43	+24
	6	Breda	16	24	23	41 25 18.35	+1.408	-0.128	41 25 19.89	41 25 19.46	-0.43	25504.86	25504.94	+ 8
	7	's-Hertogenbosch	15	28	27	45 44 50.61	-0.101	-0.075	45 44 50.58	45 44 53.36	+2.78	27612.32	27612.84	+52
						179 59 57.96	+1.460	-2.361	180 00 01.78	180 00 01.78	0			
6	3	Helmond	17	7	10	42 44 46.33	+0.900	-0.688	42 44 47.92	42 44 47.87	-0.05	25504.86	25504.94	+ 8
	4	Hilvarenbeek	19	11	15	63 16 04.62	+0.243	+0.153	63 16 04.71	63 16 05.69	+0.98	33559.74	33559.94	+20
	7	's-Hertogenbosch	15	27	26	73 59 08.95	-0.075	-0.580	73 59 09.45	73 59 08.52	-0.93	36117.70	36117.78	+ 8
						179 59 59.90	+1.068	-1.115	180 00 02.08	180 00 02.08	0			
7	6	Breda	16	20	25	113 44 36.78	+0.131	-0.509	113 44 37.42	113 44 38.22	+0.80	39405.69	39406.04	+35
	5	Hoogstraten	20	17	16	37 07 35.47	+0.002	+0.876	37 07 34.60	37 07 32.69	-1.91	25983.53	25983.50	- 3
	8	Willemstad	23	33	32	29 07 48.42	+0.379	-0.448	29 07 49.24	29 07 50.35	+1.11	20956.39	20956.82	+43
						180 00 00.67	+0.512	-0.081	180 00 01.26	180 00 01.26	0			
8	6	Breda	16	21	20	46 10 09.89	-1.437	+0.131	46 10 08.32	—	—	20494.26	—	—
	8	Willemstad	23	32	31	67 40 55.36	-0.448	+0.145	67 40 54.77	—	—	26281.44	—	—
	9	Dordrecht	14	38	37	66 08 58.95	-0.451	+0.341	66 08 58.16	—	—	25983.53	25983.50	- 3
						180 00 04.20	-2.336	+0.617	180 00 01.25	180 00 01.25				
9	6	Breda	16	22	21	44 32 14.01	+0.535	-1.437	44 32 15.98	—	—	21629.07	—	—
	9	Dordrecht	14	37	36	77 00 22.00	+0.341	+0.706	77 00 21.64	—	—	30048.30	30048.45	+15
	10	Gorinchem	13	40	39	58 27 24.28	-0.647	-0.151	58 27 23.78	—	—	26281.44	—	—
						180 00 00.29	+0.229	-0.882	180 00 01.40	180 00 01.40				

Table 15 (continued)

1	2	3	4	5	6	7	8	9	10=7+8−9	11	12= 11−10	13	14	15= 14−13
10	7	's-Hertogenbosch	15	29	28	50°46'29".82	+0".757	−0".101	50°46'30".68	50°46'30".37	−0.31	30048.30	30048.45	+15
	6	Breda	16	23	22	46 10 47.28	−0.128	+0.535	46 10 46.62	46 10 47.14	+0.52	27986.42	27986.66	+24
	10	Gorinchem	13	39	43	83 02 44.59	−0.151	−0.379	83 02 44.82	83 02 44.60	−0.22	38503.19	38503.43	+24
						180 00 01.69	+0.478	+0.055	180 00 02.11	180 00 02.11	0			
11	9	Dordrecht	14	34	38	86 16 41.84	+0.407	−0.451	86 16 42.70	—	—	25740.24	25740.10	−14
	8	Willemstad	23	31	30	41 06 43.08	+0.145	−0.077	41 06 43.30	—	—	16960.81	—	—
	11	Rotterdam	3	48	47	52 36 33.46	+0.874	−0.544	52 36 34.88	—	—	20494.26	—	—
						179 59 58.38	+1.426	−1.072	180 00 00.88	180 00 00.88				
12	9	Dordrecht	14	35	34	54 15 14.43	−1.003	+0.407	54 15 13.02	—	—	18415.95	18416.14	+19
	11	Rotterdam	3	47	46	77 22 25.36	−0.544	−1.289	77 22 26.10	—	—	22141.91	—	—
	12	Gouda	4	53	52	48 22 19.68	+1.979	+0.013	48 22 21.65	—	—	16960.81	—	—
						179 59 59.47	+0.432	−0.869	180 00 00.77	180 00 00.77				
13	10	Gorinchem	13	41	40	52 41 51.84	+1.541	−0.647	52 41 54.03	—	—	22141.91	—	—
	9	Dordrecht	14	36	35	76 18 42.78	+0.706	−1.003	76 18 44.49	—	—	27044.99	27044.85	−14
	12	Gouda	4	52	51	50 59 21.38	+0.013	−1.269	50 59 22.66	—	—	21629.07	—	—
						179 59 56.00	+2.260	−2.919	180 00 01.18	180 00 01.18				
14	12	Gouda	4	54	53	33 02 36.11	−0.580	+1.979	33 02 33.55	—	—	13235.28	—	—
	11	Rotterdam	3	46	44	97 36 31.51	−1.289	+0.511	97 36 29.71	—	—	24059.53	—	—
	13	Delft	2	59	58	49 20 56.48	−0.071	−0.943	49 20 57.35	—	—	18415.95	18416.14	+19
						180 00 04.10	−1.940	+1.547	180 00 00.61	180 00 00.61				
15	12	Gouda	4	55	54	47 02 09.12	+0.490	−0.580	47 02 10.19	—	—	18565.45	—	—
	13	Delft	2	58	56	61 27 40.43	−0.943	−0.459	61 27 39.95	—	—	22287.48	22287.42	− 6
	14	Leiden	11	61−60	65−63	71 30 07.77	+2.366	−0.720	71 30 10.86	—	—	24059.53	—	—
						179 59 57.32	+1.913	−1.759	180 00 00.99	180 00 00.99				
16	12	Gouda	4	49	55	59 23 32.20	+0.617	+0.490	59 23 32.33	—	—	19799.97	—	—
	14	Leiden	11	65	64	44 57 25.56	+0.122	−0.965	44 57 26.65	—	—	16255.02	—	—
	15	Nieuwkoop	5	70	69	75 39 00.07	+0.609	−1.137	75 39 01.82	—	—	22287.48	22287.42	− 6
						179 59 57.83	+1.348	−1.612	180 00 00.79	180 00 00.79				
17	12	Gouda	4	50	49	55 23 27.78	−1.250	+0.617	55 23 25.91	—	—	24321.36	—	—
	15	Nieuwkoop	5	69	68	91 14 16.93	−1.137	+0.656	91 14 15.14	—	—	29543.68	29543.76	+ 8
	16	Utrecht	12	74	73	33 22 21.14	−0.486	+0.703	33 22 19.95	—	—	16255.02	—	—
						180 00 05.85	−2.873	−1.976	180 00 01.00	180 00 01.00				
18	10	Gorinchem	13	42	41	61 01 09.02	−0.365	+1.541	61 01 07.11	61 01 07.97	+0.86	29543.68	29543.76	+ 8
	12	Gouda	4	51	50	65 46 33.73	−1.269	−1.250	65 46 33.71	65 46 34.38	+0.67	30799.06	30799.12	+ 6
	16	Utrecht	12	73	72	53 12 21.06	+0.703	+0.745	53 12 21.02	53 12 19.49	−1.53	27044.99	27044.85	−14
						180 00 03.81	−0.931	+1.036	180 00 01.84	180 00 01.84	0			
19	14	Leiden	11	61	60	27 59 59.34	+0.535	−1.831	28 00 01.71	—	—	13235.28	—	—
	11	Rotterdam	3	45	44	41 11 21.64	+0.448	+0.511	41 11 21.58	—	—	18565.45	—	—
	13	Delft	2	59	56	110 48 36.91	−0.071	−0.459	110 48 37.30	—	—	26352.30	26352.41	+11
						179 59 57.89	+0.912	−1.779	180 00 00.58	180 00 00.58				

Table 15 (continued)

1	2	3	4	5	6	7	8	9	10=7+8−9	11	12= 11−10	13	14	15= 14−13
20	12	Gouda	4	55	53	80°04′45.″23	+0.″490	+1.″979	80°04′43.″74	80°04′44.″47	+0.73	26352.30	26352.41	+11
	11	Rotterdam	3	46	45	56 25 09.87	−1.289	+0.448	56 25 08.13	56 25 06.15	−1.98	22287.48	22287.42	− 6
	14	Leiden	11	63	65	43 30 08.43	+0.842	+0.122	43 30 09.15	43 30 10.40	+1.25	18415.95	18416.14	+19
						180 00 03.53	+0.043	+2.549	180 00 01.02	180 00 01.02	0			
21	15	Nieuwkoop	5	66	70	66 38 58.64	−0.552	+0.609	66 38 57.48	—	—	26730.21	26730.54	+33
	14	Leiden	11	64−63	62−60	70 30 13.22	−1.807	+3.127	70 30 08.29	—	—	27445.24	—	—
	17	Haarlem	1	77	80	42 50 56.93	−1.222	+0.210	42 50 55.50	—	—	19799.97	—	—
						180 00 08.79	−3.581	+3.946	180 00 01.26	180 00 01.26				
22	15	Nieuwkoop	5	67	66	36 32 07.92	+0.424	−0.552	36 32 08.90	—	—	16790.29	16790.38	+ 9
	17	Haarlem	1	80	79	66 46 54.70	+0.210	+1.598	66 46 53.31	—	—	25919.34	—	—
	18	Amsterdam	7	83	82	76 41 00.04	−1.465	−0.290	76 40 58.86	—	—	27445.24	—	—
						180 00 02.66	−0.834	+0.756	180 00 01.07	180 00 01.07				
23	16	Utrecht	12	75	74	46 51 40.42	−1.691	−0.486	46 51 39.22	—	—	25919.34	—	—
	15	Nieuwkoop	5	68	67	89 55 36.44	+0.656	+0.424	89 55 36.67	—	—	35520.79	35520.76	− 3
	18	Amsterdam	7	82	81	43 12 47.31	−0.290	+1.311	43 12 45.71	—	—	24321.36	—	—
						180 00 04.17	−1.325	+1.249	180 00 01.60	180 00 01.60				
24	16	Utrecht	12	76	75	34 06 08.72	+0.399	−1.691	34 06 10.81	34 06 07.53	−3.28	20926.25	20925.77	−48
	18	Amsterdam	7	81	86	38 01 10.65	+1.311	+0.472	38 01 11.49	38 01 13.86	+2.37	22988.42	22988.77	+35
	19	Naarden	22	90	89	107 52 38.72	+0.186	+0.047	107 52 38.86	107 52 39.77	+0.91	35520.79	35520.76	− 3
						179 59 58.09	+1.896	−1.172	180 00 01.16	180 00 01.16	0			
25	19	Naarden	22	91	90	47 07 22.09	−0.072	+0.186	47 07 21.83	—	—	18937.65	—	—
	18	Amsterdam	7	86	85	78 48 16.42	+0.472	−0.044	78 48 16.94	—	—	25350.70	—	—
	20	Edam	10	93	92	54 04 22.52	−0.221	+0.082	54 04 22.22	—	—	20926.25	20925.77	−48
						180 00 01.03	+0.179	+0.224	180 00 00.98	180 00 00.98				
26	20	Edam	10	94	93	87 28 11.62	+0.172	−0.221	87 28 12.01	—	—	29979.30	29978.96	−34
	18	Amsterdam	7	85	84	53 24 02.69	−0.044	+0.016	53 24 02.63	—	—	24091.59	—	—
	21	Alkmaar	6	100	99	39 07 46.06	+0.600	+0.150	39 07 46.51	—	—	18937.65	—	—
						180 00 00.37	+0.728	−0.055	180 00 01.15	180 00 01.15				
27	18	Amsterdam	7	84	83	69 52 42.89	+0.016	−1.465	69 52 44.37	69 52 45.40	+1.03	28885.18	28885.00	−18
	17	Haarlem	1	79	78	77 02 28.57	+1.598	−0.585	77 02 30.75	77 02 27.99	−2.76	29979.30	29978.96	−34
	21	Alkmaar	6	101	100	33 04 46.35	+0.322	+0.600	33 04 46.07	33 04 47.81	+1.74	16790.29	16790.38	+ 9
						179 59 57.81	+1.936	−1.450	180 00 01.20	180 00 01.20	0			
28	20	Edam	10	95	94	60 03 52.62	+0.049	+0.172	60 03 52.50	—	—	20999.32	—	—
	21	Alkmaar	6	99	98	36 06 37.85	+0.150	−0.644	36 06 38.64	—	—	14281.17	—	—
	22	Hoorn	9	103	102	83 49 29.07	+0.154	−0.390	83 49 29.61	—	—	24091.59	—	—
						179 59 59.54	+0.353	−0.862	180 00 00.75	180 00 00.75				
29	22	Hoorn	9	104	103	45 07 27.94	+0.419	+0.154	45 07 28.21	—	—	17559.02	—	—
	21	Alkmaar	6	98	97	76 56 12.37	−0.644	−0.428	76 56 12.15	—	—	24137.20	—	—
	23	Schagen	8	107	109	57 56 18.60	+0.772	−1.179	57 56 20.55	—	—	20999.32	—	—
						179 59 58.91	+0.547	−1.453	180 00 00.91	180 00 00.91				

Table 15 (continued)

1	2	3	4	5	6	7	8	9	10=7+8-9	11	12= 11-10	13	14	15= 14-13
30	22	Hoorn	9	105	104	58°44′27.″11	+0.″031	+0.″419	58°44′26.″72	—	—	20788.28	—	—
	23	Schagen	8	109	108	38 15 49.85	-1.179	+0.406	38 15 48.27	—	—	15060.00	—	—
	24	Medemblik	31	112	111	82 59 45.92	+0.152	+0.272	82 59 45.80	—	—	24137.20	—	—
						180 00 02.88	-0.996	+1.097	180 00 00.79	180 00 00.79				
31	22	Hoorn	9	106	105	54 33 38.01	-0.214	+0.031	54 33 37.77	—	—	14932.84	14932.86	+ 2
	24	Medemblik	31	111	110	70 11 15.20	+0.272	-0.540	70 11 16.01	—	—	17243.73	—	—
	25	Enkhuizen	30	116	115	55 15 05.32	+0.670	-0.768	55 15 06.76	—	—	15060.00	—	—
						179 59 58.53	+0.728	-1.277	180 00 00.54	180 00 00.54				
32	20	Edam	10	96	95	34 22 23.77	-0.083	+0.049	34 22 23.64	—	—	17243.73	—	—
	22	Hoorn	9	102	106	117 44 57.87	-0.390	-0.214	117 44 57.69	—	—	27029.92	—	—
	25	Enkhuizen	30	115	114	27 52 40.26	-0.768	+0.271	27 52 39.22	—	—	14281.17	—	—
						180 00 01.90	-1.241	+0.106	180 00 00.55	180 00 00.55				
33	25	Enkhuizen	30	117	116	73 43 55.50	+0.162	+0.670	73 43 54.99	73 43 56.76	+1.77	—	—	—
	24	Medemblik	31	110	113	74 46 00.29	-0.540	+0.117	74 45 59.63	74 45 56.87	-2.76	—	—	—
	26	Hindeloopen	32	124	123	31 30 04.59	+0.766	-1.020	31 30 06.38	31 30 07.37	+0.99	14932.84	14932.86	+ 2
						180 00 00.38	+0.388	-0.233	180 00 01.00	180 00 01.00	0			
34	25	Enkhuizen	30	118	117	45 48 02.26	+0.297	+0.162	45 48 02.40	45 47 58.61	-3.79	—	—	—
	26	Hindeloopen	32	123	122	77 42 20.29	-1.020	-0.018	77 42 19.29	77 42 22.82	+3.53	32310.78	32310.55	-23
	27	Lemmer	33	125	130	56 29 39.24	+0.426	-0.268	56 29 39.93	56 29 40.19	+0.26	—	—	—
						180 00 01.79	-0.297	-0.124	180 00 01.62	180 00 01.62	0			
35	25	Enkhuizen	30	119	118	41 53 46.35	-0.632	+0.297	41 53 45.42	41 53 44.48	-0.94	21812.03	21811.78	-25
	27	Lemmer	33	130	129	39 40 35.01	-0.268	+1.177	39 40 33.57	39 40 33.24	-0.33	20853.85	20853.68	-17
	28	Urk	29	132	131	98 25 43.48	-1.400	-0.074	98 25 42.15	98 25 43.42	+1.27	32310.78	32310.55	-23
						180 00 04.84	-2.300	+1.400	180 00 01.14	180 00 01.14	0			
36	28	Urk	29	133	132	52 21 00.98	+1.430	-1.400	52 21 03.81	52 21 06.86	+3.05	21348.59	21348.61	+ 2
	27	Lemmer	33	129	128	73 39 16.34	+1.177	-1.330	73 39 18.85	73 39 16.01	-2.84	25873.51	25873.13	-38
	29	Blokzijl	28	139	138	53 59 34.95	+1.861	-1.662	53 59 38.47	53 59 38.26	-0.21	21812.03	21811.78	-25
						179 59 52.27	+4.468	-4.392	180 00 01.13	180 00 01.13	0			
37	29	Blokzijl	28	140	139	70 08 40.48	-0.047	+1.861	70 08 38.57	70 08 42.22	+3.65	23607.02	23607.13	+11
	27	Lemmer	33	128	127	51 34 58.76	-1.330	+0.266	51 34 57.16	51 34 52.62	-4.54	19665.30	19664.92	-38
	30	Oldeholtpa	35	143	142	58 16 25.98	-0.446	+0.271	58 16 25.26	58 16 26.16	+0.90	21348.59	21348.61	+ 2
						180 00 05.22	-1.823	+2.398	180 00 01.00	180 00 01.00	0			
38	29	Blokzijl	28	136	140	83 53 09.75	-0.620	-0.047	83 53 09.18	83 53 09.12	-0.06	23892.91	23892.61	-30
	30	Oldeholtpa	35	142	141	41 11 29.24	+0.271	+0.175	41 11 29.34	41 11 31.49	+2.15	15825.34	15825.33	- 1
	31	Meppel	34	145	144	54 55 23.49	-0.410	+0.810	54 55 22.27	54 55 20.17	-2.10	19665.30	19664.92	-38
						180 00 02.48	-0.759	+0.938	180 00 00.78	180 00 00.78	0			
39	29	Blokzijl	28	137	136	87 13 02.12	+0.468	-0.620	87 13 03.21	87 12 59.75	-3.46	24086.18	24085.95	-23
	31	Meppel	34	144	146	51 46 02.56	+0.810	-0.400	51 46 03.77	51 46 05.68	+1.91	18942.21	18942.18	- 3
	32	Kampen	27	151	150	41 00 53.66	+0.154	+0.033	41 00 53.78	41 00 55.33	+1.55	15825.34	15825.33	- 1
						179 59 58.34	+1.432	-0.987	180 00 00.76	180 00 00.76	0			

Table 15 (continued)

1	2	3	4	5	6	7	8	9	10=7+8-9	11	12=11-10	13	14	15=14-13
40	28	Urk	29	134	133	43° 54′ 50″.98	+0″.262	+1″.430	43° 54′ 49″.81	43° 54′ 51″.75	+1″.94	18942.21	18942.18	- 3
	29	Blokzijl	28	138	137	64 45 32.70	-1.662	+0.468	64 45 30.57	64 45 30.63	+0.06	24703.27	24703.00	-27
	32	Kampen	27	150	149	71 19 39.88	+0.033	-0.827	71 19 40.74	71 19 38.74	-2.00	25873.51	25873.13	-38
						180 00 03.56	-1.367	+1.071	180 00 01.12	180 00 01.12	0			
41	28	Urk	29	135	134	59 51 17.04	-0.218	+0.262	59 51 16.56	59 51 13.36	-3.20	31017.62	31016.94	-68
	32	Kampen	27	149	148	76 37 06.03	-0.827	+2.127	76 37 03.08	76 37 06.19	+3.11	34894.80	34894.55	-25
	33	Harderwijk	26	153	152	43 31 43.44	-1.286	-0.097	43 31 42.26	43 31 42.34	+0.08	24703.27	24703.00	-27
						180 00 06.51	-2.331	+2.292	180 00 01.89	180 00 01.89	0			
42	33	Harderwijk	26	154	153	86 34 29.64	+1.422	-1.286	86 34 32.35	86 34 32.10	-0.25	36314.20	36313.62	-58
	32	Kampen	27	148	147	34 55 33.01	+2.127	-1.487	34 55 36.62	34 55 38.44	+1.82	20828.10	20827.95	-15
	34	Veluwe	25	159	158	58 29 50.45	+1.067	-1.142	58 29 52.66	58 29 51.09	-1.57	31017.62	31016.94	-68
						179 59 53.10	+4.616	-3.915	180 00 01.63	180 00 01.63	0			
43	33	Harderwijk	26	155	154	88 27 50.03	-0.553	+1.422	88 27 48.05	88 27 44.96	-3.09	33375.42	33375.03	-39
	34	Veluwe	25	158	157	52 56 27.15	-1.142	+0.074	52 56 25.93	52 56 27.73	+1.80	26643.49	26643.39	-10
	35	Amersfoort	24	163	162	38 35 48.37	-0.486	+0.469	38 35 47.42	38 35 48.71	+1.29	20828.10	20827.95	-15
						180 00 05.55	-2.181	+1.965	180 00 01.40	180 00 01.40	0			
44	33	Harderwijk	26	156	155	43 17 53.40	+0.514	-0.553	43 17 54.47	43 17 57.18	+2.71	21927.80	21928.08	+28
	35	Amersfoort	24	162	161	80 15 52.25	+0.469	+0.644	80 15 52.07	80 15 50.59	-1.48	31513.68	31513.70	+ 2
	19	Naarden	22	88	87	56 26 16.02	-0.633	+0.471	56 26 14.92	56 26 13.69	-1.23	26643.49	26643.39	-10
						180 00 01.67	+0.350	+0.562	180 00 01.46	180 00 01.46	0			
45	19	Naarden	22	89	88	51 36 28.47	+0.047	-0.633	51 36 29.15	51 36 27.93	-1.22	19575.06	19575.23	+17
	35	Amersfoort	24	161	160	66 59 30.22	+0.644	-0.626	66 59 31.49	66 59 32.66	+1.17	22988.42	22988.77	+35
	16	Utrecht	12	71	76	61 24 00.43	+0.330	+0.399	61 24 00.36	61 24 00.41	+0.05	21927.80	21928.08	+28
						179 59 59.12	+1.021	-0.860	180 00 01.00	180 00 01.00	0			
46	27	Lemmer	33	126	125	52 49 08.44	-0.271	+0.426	52 49 07.74	52 49 09.45	+1.71	—	—	—
	26	Hindeloopen	32	122	121	56 56 05.01	-0.018	+0.612	56 56 04.38	56 56 03.76	-0.62	—	—	—
	36	Sneek	36	165	164	70 14 49.77	-0.612	+0.271	70 14 48.89	70 14 47.80	-1.09	—	—	—
						180 00 03.22	-0.901	+1.309	180 00 01.01	180 00 01.01	0			
47	36	Sneek	36	166	165	73 16 38.07	+0.434	-0.612	73 16 39.12	73 16 41.18	+2.06	—	—	—
	26	Hindeloopen	32	121	120	58 22 56.62	+0.612	-0.341	58 22 57.57	58 22 55.73	-1.84	—	—	—
	37	Harlingen	41	171	170	48 20 23.65	+0.341	-0.434	48 20 24.42	48 20 24.20	-0.22	—	—	—
						179 59 58.34	+1.387	-1.387	180 00 01.11	180 00 01.11	0			
48	36	Sneek	36	167	166	70 48 31.92	+0.275	+0.434	70 48 31.76	70 48 33.08	+1.32	—	—	—
	37	Harlingen	41	170	169	51 05 44.58	-0.434	+0.093	51 05 44.05	51 05 42.06	-1.99	—	—	—
	38	Leeuwarden	37	175	174	58 05 45.15	-0.093	-0.275	58 05 45.33	58 05 46.01	+0.68	—	—	—
						180 00 01.65	-0.252	+0.252	180 00 01.15	180 00 01.15	0			
49	36	Sneek	36	168	167	49 30 43.10	-0.369	+0.275	49 30 42.46	49 30 38.74	-3.72	—	—	—
	38	Leeuwarden	37	174	173	87 36 18.55	-0.275	+0.254	87 36 18.02	87 36 18.51	+0.49	—	—	—
	39	Drachten	38	177	176	42 53 01.39	-0.254	+0.369	42 53 00.77	42 53 03.99	+3.22	—	—	—
						180 00 03.04	-0.898	+0.898	180 00 01.24	180 00 01.24	0			

Table 15 (continued)

1	2	3	4	5	6	7	8	9	10=7+8-9	11	12= 11-10	13	14	15= 14-13
50	39	Drachten	38	178	177	71° 56′ 49″.14	″ -1.003	″ -0.254	71° 56′ 48″.39	71° 56′ 50″.02	″ +1.63	—	—	—
	38	Leeuwarden	37	173	172	47 01 19.49	+0.254	+0.114	47 01 19.63	47 01 18.04	-1.59	—	—	—
	40	Kollum	39	183	182	61 01 54.20	-0.114	+1.003	61 01 53.08	61 01 53.05	-0.03	—	—	—
						180 00 02.83	-0.863	+0.863	180 00 01.11	180 00 01.11	0			
51	39	Drachten	38	179	178	58 29 36.28	+0.889	-1.003	58 29 38.17	58 29 36.47	-1.70	—	—	—
	40	Kollum	39	182	181	86 02 13.00	+1.003	-0.181	86 02 14.18	86 02 14.84	+0.66	—	—	—
	41	Groningen	42/3	185	184	35 28 08.00	+0.181	-0.889	35 28 09.07	35 28 10.12	+1.05	—	—	—
						179 59 57.28	+2.073	-2.073	180 00 01.43	180 00 01.43	0			
52	41	Groningen	42/3	186	185	39 45 45.95	+0.808	+0.181	39 45 46.58	39 45 47.97	+1.39	—	—	—
	40	Kollum	39	181	180	54 02 09.46	-0.181	-0.708	54 02 09.99	54 02 10.61	+0.62	—	—	—
	42	Hornhuizen	44	192	191	86 12 03.01	+0.708	-0.808	86 12 04.53	86 12 02.51	-2.02	—	—	—
						179 59 58.42	+1.335	-1.335	180 00 01.09	180 00 01.09	0			
53	41	Groningen	42/3	187	186	60 36 29.39	+0.481	+0.808	60 36 29.06	60 36 30.01	+0.95	—	—	—
	42	Hornhuizen	44	191	190	59 02 09.98	-0.808	+0.100	59 02 09.07	59 02 09.91	+0.84	—	—	—
	43	Uithuizermeden	45	196	195	60 21 22.79	-0.100	-0.361	60 21 23.05	60 21 21.27	-1.78	—	—	—
						180 00 02.16	-0.429	+0.547	180 00 01.19	180 00 01.19	0			
54	41	Groningen	42/3	188	187	28 25 17.31	-0.654	+0.481	28 25 16.18	28 25 17.80	+1.62	—	—	—
	43	Uithuizermeden	45	195	194	86 27 58.40	-0.361	+0.285	86 27 57.75	86 27 58.85	+1.10	—	—	—
	44	Holwierde	46	197	201	65 06 47.59	-0.346	+0.470	65 06 46.77	65 06 44.05	-2.72	—	—	—
						180 00 03.30	-1.361	+1.236	180 00 00.70	180 00 00.70	0			
55	41	Groningen	42/3	189	188	42 43 46.15	+0.073	-0.654	42 43 46.88	42 43 46.58	-0.30	—	—	—
	44	Holwierde	46	201	200	79 55 36.88	+0.470	+0.148	79 55 37.20	79 55 38.06	+0.86	—	—	—
	45	Midwolda	47	202	204	57 20 37.47	-0.222	+0.035	57 20 37.21	57 20 36.65	-0.56	—	—	—
						180 00 00.50	+0.321	-0.471	180 00 01.29	180 00 01.29	0			
56	45	Midwolda	47	203	202	61 17 39.87	+0.187	-0.222	61 17 40.28	—	—	—	—	—
	44	Holwierde	46	200	199	65 35 50.46	+0.148	-0.141	65 35 50.75	—	—	—	—	—
	46	Emden	48	206	205	53 06 29.87	+0.052	-0.106	53 06 30.03	—	—	—	—	—
						180 00 00.20	+0.387	-0.469	180 00 01.06	180 00 01.06				
57	46	Emden	48	207	206	56 04 20.37	+0.054	+0.052	56 04 20.37	—	—	—	—	—
	44	Holwierde	46	199	198	44 48 37.88	-0.141	-0.130	44 48 37.87	—	—	—	—	—
	47	Pilsum	—	—	—	—	—	—	79 07 02.52	—	—	—	—	—
									180 00 00.76	180 00 00.76				
58	44	Holwierde	46	198	197	104 33 07.19	-0.130	-0.346	104 33 07.41	104 33 08.05	+0.64	—	—	—
	43	Uithuizermeden	45	194	193	47 24 31.09	+0.285	+0.176	47 24 31.20	47 24 31.28	+0.08	—	—	—
	47	Pilsum	—	—	—	—	—	—	28 02 21.95	28 02 21.23	-0.72	—	—	—
									180 00 00.56	180 00 00.56	0			
Zuiderzee pentagon	20	Edam	10	92	96	124 01 09.47	+0.082	-0.083	124 01 09.64	—	—	25350.70	—	—
	19	Naarden	22	87	91	96 57 14.70	+0.471	-0.072	96 57 15.24	—	—	31513.68	31513.70	+ 2
	33	Harderwijk	26	152	156	98 08 03.49	-0.097	+0.514	98 08 02.88	98 08 02.64	-0.24	34894.80	34894.55	-25
	28	Urk	29	131	135	105 27 07.52	-0.074	-0.218	105 27 07.66	105 27 05.14	-2.52	20853.85	20853.68	-17
	25	Enkhuizen	30	114	119	115 26 30.31	+0.271	-0.632	115 26 31.21	—	—	27029.92	—	—
						540 00 05.49	+0.653	-0.491	540 00 06.63	540 00 06.63				

The total number of sides L in the network is 105, viz. the l = 101 sides mentioned above, the side Delft (No. 13) - Nieuwkoop (No. 15), and the three rays in Pilsum (No. 47) to Uithuizermeden (No. 43), Holwierde (No. 44), and Emden (No. 46) respectively. As the number of points P = 47, the number of side (sine) conditions is L - 2P + 3 = 105 - 94 + 3 = 14. Nine of them can be found from the base angles of the triangles around the central points Hilvarenbeek (No. 4), Breda (No. 6), Dordrecht (No. 9), Gouda (No. 12), Nieuwkoop (No. 15), Amsterdam (No. 18), Hoorn (No. 22), Blokzijl (No. 29), and Holwierde (No. 44).

The tenth condition relates to the uncommon form of the network bordered by the stations N(ieuwkoop), G(ouda), R(otterdam), D(elft), and L(eiden). If NG is the spherical distance Nieuwkoop - Gouda, etc., then:

$$\frac{\sin NG.\ \sin ND.\ \sin NL}{\sin ND.\ \sin NL.\ \sin NG} = 1.$$

If sin NG : sin ND, sin ND : sin NL, and sin NL : sin NG in this relation are replaced by the proportions of the sines of the opposite spherical angles in the triangles NGD, NDL, and NLG respectively, then follows in a logarithmic form:

$$\log\sin\left\{28^{\circ}33'00''.87 + (p_{58} - p_{57})\right\} + \log\sin\left\{116^{\circ}27'33''.33 + (-p_{60} + p_{61} + p_{63} - p_{64})\right\} + \log\sin\left\{59^{\circ}23'32''.20 + (p_{49} - p_{55})\right\} = \log\sin\left\{106^{\circ}25'41''.32 + (p_{49} - p_{54})\right\} + \log\sin\left\{32^{\circ}54'39''.56 + (p_{57} - p_{56})\right\} + \log\sin\left\{44^{\circ}57'25''.56 + (p_{65} - p_{64})\right\} \text{ with e.g.}$$

$$\log\sin\left\{28^{\circ}33'00''.87 + (p_{58} - p_{57})\right\} = \log\sin 28^{\circ}33'00''. + \frac{M}{\rho}(p_{58} - p_{57})\cot 28^{\circ}33'00''.87\ .$$

As M, the modulus of Brigg's system of logarithms, is 0.43429448, $\rho'' = 206264.806$, $M : \rho'' = 0.0000021055$ and $\cot 28^{\circ}33'00''.87 \approx +1.83792$, $10^6 \log\sin\left\{28^{\circ}33'00''.87 + (p_{58} - p_{57})\right\} = 10^6(9.679363562 - 10) + 3.870\,(p_{58} - p_{57})$.

Worked out the condition runs:

$$+1.867\,p_{49} - 0.621\,p_{54} - 1.246\,p_{55} + 3.253\,p_{56} - 7.123\,p_{57} + 3.870\,p_{58} + 1.048\,p_{60} - 1.048\,p_{61} - 1.048\,p_{63} + 3.157\,p_{64} - 2.109\,p_{65} = -21.397.$$

It will be clear that the multiplication by 10^6 saves the writing of a great number of ciphers zero.

In the same area and with the pole Gouda (No. 12) the eleventh sine condition follows in an analogous way from the relation:

$$\frac{\sin GR.\ \sin GD.\ \sin GL}{\sin GD.\ \sin GL.\ \sin GR.} = 1.$$

The 12th sine condition can be found from a successive application of the sine rule in the closed chain of triangles 45 - 44 - 43 - 42 - 41 - 40 - 36 - 35 - 34 - 33 - 31 - 32 - 28 - 26 - 25 - 24 around the former Zuiderzee. The analogous condition 52 in Krayenhoff's triangulation network was already discussed in detail in [81] .

The remaining 13th and 14th sine conditions, finally, are analogous to the conditions 53 and 54 in [81] : in the plane of projection of the R. D. the sum of the projections of the sides (chords) of the Zuiderzee pentagon on two arbitrary perpendicular axes must be zero. The points Urk, Enkhuizen, Edam, and Naarden of the pentagon are the same as those in Krayenhoff's triangulation. The points Harderwijk in the two networks differ about 19 metres as Krayenhoff's station was on the roof of the reformed church, whereas Stamkart's triangulation point coincides with the spire of the church tower.

The total number 56 + 14 = 70 of the conditions in the network can be verified in an easy way. Starting from the coordinates of two arbitrary points in the network the coordinates of the 45 other points must be determined from the 207 directions in the 47 series at 46 stations (in Leiden there are two series). As there are 45 x 2 unknown coordinates and 47 unknown orientations the number of unknowns is 137 and the number of conditions 207 - 137 = 70. This number is of course the same as found before.

15. *ADJUSTMENT OF THE SPHERICAL DIRECTIONS IN THE TRIANGULATION NETWORK ACCORDING TO THE METHOD OF THE LEAST SQUARES*

With the I. B. M. 360/65-computer of the Delft University of Technology the 207 corrections p_i to the spherical directions measured were determined in such a way that $[pp]$= minimum. The computation is analogous to that given in section 19 of [1]. The result is given in column 7 of table 14. For every station $[p]$= 0 (check). The adjusted directions are given in column 8.

For the computation of the corrections $p_{r(ight)} - p_{l(eft)}$ to the angles of the triangles the p's are once again mentioned in the columns 8 (p_r) and 9 (p_l) of table 15. Those to the directions 4 (right) and 3 (left) at the station Lommel of the triangle 1 e.g. are $+ 0''.064$ and $+ 0''.455$ respectively. As $p_r - p_l = - 0''.39$, the adjusted spherical angle at Lommel of this triangle is:

$$37^o 28' 26''.88 - 0''.39 = 37^o 28' 26''.49 .$$

As $[pp] = 108.8$ and $m^2 = [pp] : (207 - 137) = 1.554$, the internal accuracy of the triangulation can therefore be characterized by the standard deviation $m = \pm 1''.247$ in each of the 207 directions in column 8 of table 14. It is considerably higher than the amount $m \approx 0''.44$ found as a rough estimate at the end of section 7. As, however, the latter amount refers partly to measurements in only one eccentric point, several inaccuracies or even errors in the determination and the computation of the reductions to centre and in the reduction from centre to spire are not included in these standard deviations. The large closing errors e.g. in the triangles 4, 21, 36, and 41 (see page 83) , the great eccentricities in e.g. Urk (see end section 8) and Leiden (see section 13) and the inexpert determination of the mutual situation of centre and spire (see section 8) of a station do expect that Stamkart probably has made several errors in this respect.

In the histogram of fig. 20 the 207 p's are arranged in a surveyable manner. The class interval is $0''.4$. The number of p's between $-0''.2$ and $+0''.2$ is 61, that between $+0''.2$ and $+0''.6$ = 38, etc. At the same scale and for the same class interval the figure also gives the number of p's to be expected according to Gauss' law of probabilities ($m = \pm 1''.247$) if the distribution of errors would have been a normal one (27 between $-0''.2$ and $+0''.2$, 25 between $+0''.2$ and $+0''.4$, etc.)

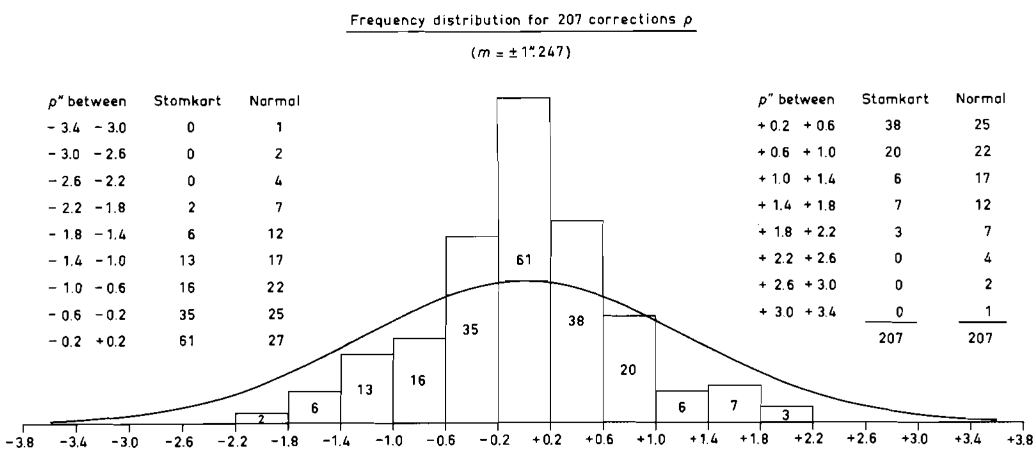


Fig. 20

As can be seen the numbers in the corresponding classes match badly. Just like in Krayenhoff's triangulation [82] the number of small p's in Stamkart's adjusted network is too high, that of the larger p's too small. I cannot find an explanation for this phenomenon, the more so as Stamkart did not compute his triangulation and the closing errors in his triangles don't agree with my computation because of his wrong adjustment of the directions measured at the various stations. An intentional influencing of the observations must therefore be ruled out.

16. *PROVISIONAL ADAPTATION OF THE ADJUSTED NETWORK TO THE POINTS BREDA AND 's-HERTOGENBOSCH OF THE R.D.-TRIANGULATION*

The computation of the coordinates of Stamkart's stations might be started from the R. D.-coordinates of the angular points Haarlem and Amsterdam of the network. As was already shown in section 11, the distance (about 16.8 km) between these points is in excellent agreement with that derived from Stamkart's base measurement.

In this section, however, will be started from the much longer side Breda (No. 6) - 's-Hertogenbosch (No. 7). The length of its chord on the conformal sphere in the R.D.-system is 38503,43 m and it forms part of the triangles 5 and 10. It was selected because of the almost ideal accordance of the adjusted angles of triangle 10 with those found from the R. D. - coordinates of its angular points (see table 15, triangle 10, columns 10 and 11). They prove - apart of course from very small alterations in the spires which cannot be found at such large a distance - that the stations have not changed between Stamkart's determination in 1872 - 1873 and the R. D. measurements in 1894 - 1895.

The coordinates $X'_6 Y'_6$ and $X'_7 Y'_7$ of the stations Breda and 's-Hertogenbosch respectively in the plane of projection of the R. D.-triangulation are:

$$X'_6 = -42438.967 = X_6, \qquad Y'_6 = -62806.846 = Y_6$$
$$X'_7 = -5494.576 = X_7, \qquad Y'_7 = -51970.337 = Y_7$$

From these coordinates and the adjusted spherical angles α and β in the left and the right base point L and R respectively the coordinates of the apex T = Hilvarenbeek (No. 4) in Stamkart's system XY can be determined in the way already described in detail in [83].

The computation (see fig. 21) begins with the (very) provisional computation by intersection of T from L and R and the uncorrected base angles α and β. The result is:

$$X_T' = -17430.-, \quad Y_T' = -74508.-.$$

Thereafter the angles

$$\beta' = \beta + (\epsilon''_{RT} - \epsilon''_{RL}) \text{ and}$$
$$\alpha' = \alpha + (\epsilon''_{LR} - \epsilon''_{LT})$$

can be computed. In these expressions ϵ_{RT}, ϵ_{RL}, $\epsilon_{LR} = -\epsilon_{RL}$ and ϵ_{LT} are the small angles (in seconds) between arc and chord in the stereographic map projection (from arc to chord in a clockwise direction is positive).

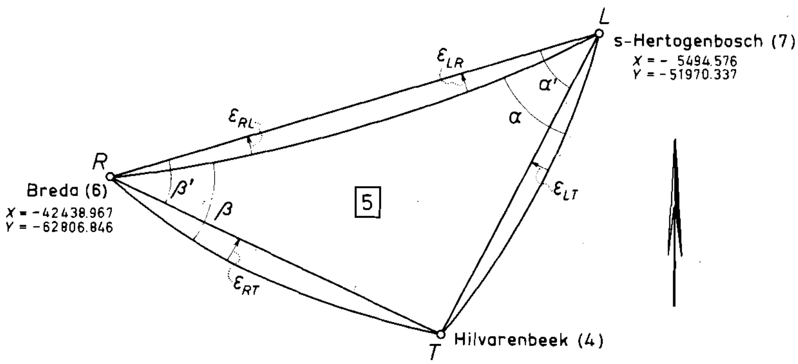


Fig. 21

ϵ_{RT} is: $\epsilon''_{RT} = 0.0012658\,(X_T Y_R - X_R Y_T) = -2''.617$ (X and Y in km),
and analogously:

$$\epsilon''_{RL} = 0.0012658\,(X_L Y_R - X_R Y_L) = -2''.355,$$

and

$$\epsilon''_{LT} = 0.0012658\,(X_T Y_L - X_L Y_T) = +0''.628.$$

Because of the very small amounts ϵ_{RT}, etc. in the above formulae the coordinates X_T' and Y_T' may be used instead of the still unknown values $X_T Y_T$.

From the coordinates $X_6 Y_6$, $X_7 Y_7$ and the angles α' and β' in the plane of projection finally follow by intersection the coordinates:

$$X_4 = -17430.197, \qquad Y_4 = -74508.335$$

of the station Hilvarenbeek.

In an analogous way the coordinates:

$$X_3 = +18678.165, \qquad Y_3 = -75246.978$$

of the apex Helmond of triangle 6 can be computed from the base angles and the coordinates of the base points 's-Hertogenbosch (L) and Hilvarenbeek (R). As the spires of Hilvarenbeek and Helmond in Stamkart's time are assumed to be identical with the R. D.-points determined in 1894 and 1892 respectively, the coordinates in the R. D.-system:

$$X'_4 = -17429.858, \qquad Y'_4 = -74508.430$$

$$X'_3 = +18678.366, \qquad Y'_3 = -75246.754$$

already give some impression of the accuracy of Stamkart's measurement.

Proceeding in the same way all the angular points of Stamkart's network were computed in the XY-system. The computation was done with the computer in the sequence of the stations 4, 3, 1, 2, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 35, 33, 34, 32, 28, 29, 31, 30, 27, 25, 26, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 24, 22, 23, 21, 20.

For an effective check on the adjustment of the network the coordinates of the stations Nos. 6 (5-4), 7 (10-6), 12 (14-11), 15 (13-12), 10 (16-12), 16 (18-15), 28 (29-27), 47 (43-44), 22 (20-25), and 18 (20-21) were computed again, but in another way. The numbers between brackets are the stations serving in these computations as left and right basepoints respectively. Apart from very small rounding off errors the results are of course the same.

The coordinates in the system XY of the 47 stations are mentioned in the columns 3 and 4 of table 16. The sequence of the stations is the same as the sequence in table 14. In columns 5 and 6 are given the (same) R. D.-coordinates of the points Breda (No. 6) and 's-Hertogenbosch (No. 7) from which the computation in columns 3 and 4 was started. The other coordinates in columns 5 and 6 - in total 38 pairs - are those of the points which are identical or are assumed to be identical with the spires in Stamkart's network.

Only for a few points the time elapsed between Stamkart's determination and the R. D.-measurements is rather long. For Hindeloopen (No. 26), used as a station in the triangulation instead of Krayenhoff's Staveren, it is 39 years (from 1875 till 1914) and for Midwolda 41 years (from 1880 till 1921). For Pilsum it is very short: only four years (from 1881 till 1885). For the other stations the period is about 20 years from about 1875 till the R. D. first order measurements in 1895.

Notwithstanding this rather short period it is dangerous to rely too much on the unchangeableness of the spires. Especially in the western part of the country some towers tend to sag as they are built on weak peat-moor. The sagging about to the north of Dordrecht (No. 9) was already given as an example on page 82 . It will be clear that the coordinates of Dordrecht are therefore not inserted in the columns 5 and 6 of table 16. The same holds for the stations Nieuwkoop (No. 15) and Edam (No. 20). As was already shown in [84] the towers sag to the southwest and the northeast respectively. In 1972 the sagging of the station Edam even appeared to be so serious that the tower was in danger of falling down.

Table 16

Stations i		System Stamkart		System R. D.		System XY adapted to System X'Y'		Differences (mm)	
No.	Name	X_i	Y_i	X'_i	Y'_i	X''_i	Y''_i	v_i	w_i
1	2	3	4	5	6	7	8	9	10
1	Lommel	- 5075.316	-103095.478	- 5074.792	-103095.491	- 5075.120	-103095.096	+328	-395
2	Nederweert	+25173.342	- 96630.822	+25173.457	- 96630.713	+25173.317	- 96630.328	+140	-385
3	Helmond	+18678.165	- 75246.978	+18678.366	- 75246.754	+18678.072	- 75246.650	+294	-104
4	Hilvarenbeek	-17430.197	- 74508.335	-17429.858	- 74508.430	-17430.068	- 74508.194	+210	-236
5	Hoogstraten	-43544.034	- 83733.222	-43543.860	- 83733.527	-43543.695	- 83733.156	-165	-371
6	Breda	-42438.967	- 62806.846	-42438.967	- 62806.846	-42438.741	- 62806.905	-226	+ 59
7	's-Hertogenbosch	- 5494.576	- 51970.337	- 5494.576	- 51970.337	- 5494.635	- 51970.277	+ 59	- 60
8	Willemstad	-65637.046	- 51105.147	-65636.929	- 51105.357	-65636.734	- 51105.396	-195	+ 39
9	Dordrecht	-50150.391	- 37683.737	—	—	-50150.244	- 37683.992	—	—
10	Gorinchem	-28577.303	- 36148.957	-28577.386	- 36148.952	-28577.298	- 36149.112	- 88	+160
11	Rotterdam	-62066.073	- 25615.135	-62065.996	- 25615.636	-62065.912	- 25615.526	- 84	-110
12	Gouda	-46450.863	- 15854.571	-46450.827	- 15854.872	-46450.849	- 15854.944	+ 22	+ 72
13	Delft	-70504.967	- 15420.136	—	—	-70504.805	- 15420.633	—	—
14	Leiden	-61342.549	+ 725.634	-61342.494	+ 725.083	-61342.526	+ 725.082	+ 32	+ 1
15	Nieuwkoop	-41573.183	- 349.883	—	—	-41573.278	- 350.328	—	—
16	Utrecht	-18222.452	- 7145.319	-18222.582	- 7145.440	-18222.659	- 7145.603	+ 77	+163
17	Haarlem	-51065.638	+ 25399.450	-51065.625	+ 25399.097	-51065.804	+ 25398.796	+179	+301
18	Amsterdam	-34299.271	+ 24525.839	-34299.277	+ 24525.501	-34299.537	+ 24525.275	+260	+226
19	Naarden	-15345.549	+ 15660.447	-15346.008	+ 15660.579	-15345.889	+ 15660.035	-119	+544
20	Edam	-23098.673	+ 39794.380	—	—	-23099.086	+ 39793.778	—	—
21	Alkmaar	-43132.775	+ 53172.117	-43132.499	+ 53171.311	-43133.131	+ 53171.330	+632	- 19
22	Hoorn	-22152.809	+ 54043.203	—	—	-22153.300	+ 54042.516	—	—
23	Schagen	-39877.137	+ 70425.767	—	—	-39877.600	+ 70424.888	—	—
24	Medemblik	-19153.525	+ 68800.599	-19154.008	+ 68799.851	-19154.110	+ 68799.835	+102	+ 16
25	Enkhuizen	- 6394.900	+ 61042.949	- 6395.432	+ 61042.257	- 6395.525	+ 61042.298	+ 93	- 41

Table 16 (continued)

1	2	3	4	5	6	7	8	9	10
26	Hindeloopen	+ 756.949	+ 87672.016	+ 756.49	+ 87670.84	—	—	—	—
27	Lemmer	+21817.817	+ 76788.258	+ 21816.730	+ 76787.673	+21816.936	+76787.652	-206	+ 21
28	Urk	+13945.036	+ 56447.890	+ 13944.285	+ 56447.586	+13944.307	+56447.371	- 22	+215
29	Blokzijl	+38752.962	+ 63791.393	+ 38751.731	+ 63790.810	+38752.041	+63790.953	-310	-143
30	Oldeholtpa	+44714.902	+ 82530.314	+ 44713.926	+ 82529.121	+44713.849	+82529.787	+ 77	-666
31	Meppel	+54257.903	+ 60626.932	+ 54256.502	+ 60626.036	+54256.901	+60626.591	-399	-555
32	Kampen	+35870.714	+ 45070.869	+ 35869.416	+ 45070.446	+35869.905	+45070.532	-489	- 86
33	Harderwijk	+15601.659	+ 21594.994	+ 15601.011	+ 21595.167	+15601.096	+21594.701	- 85	+466
34	Veluwe	+32150.205	+ 8950.397	+ 32149.330	+ 8950.746	+32149.602	+ 8950.267	-272	+479
35	Amersfoort	+ 0.202	- 0.172	0.000	0.000	- 0.155	- 0.409	+155	+409
36	Sneek	+18205.878	+ 97586.731	+ 18204.882	+ 97585.410	—	—	—	—
37	Harlingen	+ 1659.739	+113382.871	+ 1659.309	+113381.106	—	—	—	—
38	Leeuwarden	+26895.551	+116671.025	+ 26894.377	+116668.837	—	—	—	—
39	Drachten	+47799.761	+106076.090	+ 47796.979	+106074.532	—	—	—	—
40	Kollum	+50806.438	+125442.373	+ 50803.703	+125439.213	—	—	—	—
41	Groningen	+78888.543	+119068.067	+ 78883.131	+119065.167	—	—	—	—
42	Hornhuizen	+64685.209	+137613.001	+ 64681.415	+137608.698	—	—	—	—
43	Uithuizermeden	+87953.223	+140257.454	+ 87947.553	+140252.598	—	—	—	—
44	Holwierde	+98756.555	+134825.039	+ 98750.120	+134820.326	—	—	—	—
45	Mildwolda	+108459.063	+116837.758	+108451.41	+116834.15	—	—	—	—
46	Emden	+121117.393	+136365.268	—	—	—	—	—	—
47	Pilsum	+111243.044	+149064.040	+111236.221	+149058.285	—	—	—	—

After the observations at the station Delft in 1871 the upper part of the tower was destroyed by lightning and fire on September 29th, 1872. The present spire dates from July 11th, 1876 [85]. It was determined in the R. D.-system in 1897 and it is not identical with the spire of Stamkart's measurements. As, however, the astronomical azimuth in Leiden to Delft was measured in 1870, the destruction of the spire in 1872 is of no influence on the results of the determination of this azimuth.

Krayenhoff's station Hoorn burned down in 1838. About ten metres east of it another tower was built with a church in the form of a Greek temple. It burned down in September 1871, some time before Stamkart determined his station Hoorn (No. 22) on the ruins of the tower. A photo of the ruins is in Stamkart's file No. 9. Thereafter the present tower was built, about at the same place as that of the "Grecian temple" [86]. The R.D. point Hoorn, determined in 1911, is therefore not identical with Stamkart's station No. 22.

Stamkart's station Schagen (No. 23), measured in September 1871, is identical with Krayenhoff's station. The tower dated from 1460. By the imprudence of a plumber it burned down in 1895 [87] . At about the same place the present tower was built. The spire is not identical with that of Stamkart's station.

It is beyond doubt that Stamkart's station Veluwe (No. 34) is identical with the R. D.-point of the same name. It is the only point of the network marked with a stone. Krayenhoff determined the stone already in his triangulation as the station Observatoire in 1805. In order to show something of Stamkart's sense for perfection underneath a rather detailed description is given of his dealings with this stone for his own triangulation.

In his letter to the Administrator of the Crownland, dated April 30th, 1875 [88] Stamkart writes that he found it back in the summer of 1874, "no longer in an upright position but in an inclined position and lying in a pit filled with sand. The inscription on the stone can no longer be read. It is clear that the inquisitiveness and vandalism of the shepherds who traversed this part of the heath with their sheep for half a century gradually caused this. I wished, with the approval of His Excellency the minister of home affairs, to re-erect the stone and to fix it in a better way by means of a subterranean brickwork in order to preserve for the future the place of Krayenhoff's measurements on the Veluwe and that of my own observations".

On June 3rd, 1875, the administration of the Crownland in its letter No. 832 [89] informed Stamkart "that there are no objections to replacing the stone in the way as described in your letter".

As already mentioned in [90] the stone was replaced on June 8th, 1875 [91]. "Ten (square?) metres of land around the stone were taken over in subtenancy from the Hoog Soerense Mark for five guilders". The rent would end in 1898.

From the list of expenses over the year 1875 it appears that Stamkart paid f 48. 68 to H. Wegerig at Apeldoorn for "making some brickwork around a stone on the Veluwe" and f 23. 50 to G.J. Ackerstaff at Deventer "for an ashlar with inscriptions". A railing around the stone, finally, was made by the contractor H. van Baaren at Apeldoorn [92] . It cost f 350.--. According to the minister of home affairs these expenses had to be paid from the allocation for the year 1877.

Twelve years later - during the reconnaissance of the first order R. D.-triangulation in 1889 - the stone was found again in an excellent condition and was used as a first order point.

An insight into the mutual situation of the various stations in the two systems X_iY_i and $X'_iY'_i$ of table 16 can be found in table 17. For 44 sides of the network (columns 1-4) the amounts in column 5 represent the relative differences (in cm per km) between the lengths computed from the columns 5 and 6 (R. D.) of table 16 and those computed from the columns 3 and 4 (Stamkart). That for Breda - 's-Hertogenbosch is of course zero. That for Haarlem - Amsterdam appears to be -0. 1 cm per km, that for Amsterdam - Utrecht - 0. 7 cm per km, etc. For Oldeholtpa - Drachten it is -2. 5 cm per km. It is true that this side forms no part of Stamkart's triangulation network but it would have been a side in the projected double chain of triangles north of the line Hindeloopen - Lemmer - Oldeholtpa. As already remarked before this double chain could not be realized because of the bad reconnaissance of the triangulation at the station Oldeholtpa.

The amounts in column 6 of table 17 represent the differences between the gridbearings $\overline{P'Q'}$ computed from the coordinates $X'Y'$ (R. D.) in table 16 and the gridbearings $\overline{PQ}$ computed from the coordinates XY (Stamkart). As

$$\tan\,(\overline{P'Q'} - \overline{PQ}) = \frac{(\overline{P'Q'} - \overline{PQ})''}{206264.''8} = \frac{\tan \overline{P'Q'} - \tan \overline{PQ}}{1 + \tan \overline{P'Q'} \tan \overline{PQ}} = \frac{\cot \overline{PQ} - \cot \overline{P'Q'}}{1 + \cot \overline{PQ} \cot \overline{P'Q'}}$$

the amounts $(\overline{P'Q'} - \overline{PQ})''$ can easily be found. For P(Q) = Haarlem and Q(P) = Amsterdam the rotation is $-0.''17$. For the side Amsterdam - Utrecht it is $+ 0.''07$. For Breda - 's-Hertogenbosch from which the computation of the coordinates XY was started the rotation is of course zero.

For the double chain of triangles in the southern part of the network and the narrow strip between the North Sea and the former Zuidersea with the central points

Table 17

Stations				Dist. RD minus Stkrt. in cm/km	Rotation RD minus Stkrt. in sec.	Stations				Dist. RD minus Stkrt. in cm/km	Rotation RD minus Stkrt. in sec.
No.	Name	No.	Name			No.	Name	No.	Name		
1	2	3	4	5	6	1	2	3	4	5	6
1	Lommel	3	Helmond	-0.1	-2.25	30	Oldeholtpa	27	Lemmer	-0.2	+5.39
1	Lommel	5	Hoogstraten	+0.4	-2.00	27	Lemmer	25	Enkhuizen	-1.3	-2.32
4	Hilvarenbeek	3	Helmond	-0.4	+1.81	25	Enkhuizen	24	Medemblik	-0.5	-0.31
5	Hoogstraten	8	Willemstad	+0.3	+0.03	25	Enkhuizen	21	Alkmaar	-2.0	-1.54
6	Breda	7	's-Hertogenbosch	0.0	0.00	21	Alkmaar	18	Amsterdam	-1.8	-0.91
7	's-Hertogenbosch	10	Gorinchem	+0.3	-0.32	27	Lemmer	26	Hindeloopen	-3.5	-2.06
8	Willemstad	11	Rotterdam	-1.1	+0.01	26	Hindeloopen	37	Harlingen	-2.3	+0.40
11	Rotterdam	12	Gouda	+0.4	-2.14	37	Harlingen	36	Sneek	-3.1	+0.63
12	Gouda	16	Utrecht	-0.4	-1.54	36	Sneek	27	Lemmer	-3.5	-0.35
16	Utrecht	10	Gorinchem	-0.4	-0.01	36	Sneek	30	Oldeholtpa	-0.2	-0.82
11	Rotterdam	14	Leiden	-0.2	-0.16	30	Oldeholtpa	39	Drachten	-2.5	-15.14
14	Leiden	17	Haarlem	+0.6	-0.89	39	Drachten	36	Sneek	-5.8	-1.77
17	Haarlem	18	Amsterdam	-0.1	-0.17	26	Hindeloopen	36	Sneek	-2.7	-1.43
18	Amsterdam	16	Utrecht	-0.7	+0.07	36	Sneek	38	Leeuwarden	-4.1	+1.94
16	Utrecht	19	Naarden	+0.9	-3.21	38	Leeuwarden	39	Drachten	-7.3	+1.45
16	Utrecht	35	Amersfoort	+0.2	-3.15	39	Drachten	40	Kollum	-8.0	+3.08
35	Amersfoort	33	Harderwijk	-1.0	-2.80	40	Kollum	41	Groningen	-9.3	+2.43
33	Harderwijk	32	Kampen	-2.8	-0.68	41	Groningen	42	Hornhuizen	-9.0	+3.81
32	Kampen	31	Meppel	-1.6	+2.52	42	Hornhuizen	43	Uithuizermeden	-8.2	+2.98
29	Blokzijl	28	Urk	-2.1	+1.05	43	Uithuizermeden	41	Groningen	-8.2	+4.76
28	Urk	19	Naarden	-1.1	+0.07	41	Groningen	45	Midwolda	-7.4	+6.08
31	Meppel	30	Oldeholtpa	-1.9	+2.34	43	Uithuizermeden	47	Pilsum	-5.6	+3.59

Amsterdam and Hoorn the amounts in the columns 5 and 6 of table 17 are very small. In column 5 almost all of them are negative which might point to a small scale error in the side Breda - 's-Hertogenbosch. The single chain of triangles south and east of the Zuiderzee is less good. The large negative rotation $-3''.15$ of the side Utrecht - Amersfoort changes into $+2''.34$ for Meppel - Oldeholtpa and even into $+5''.39$ for Oldeholtpa - Lemmer.

North of the line Medemblik - Enkhuizen - Lemmer - Oldeholtpa a comparison between the coordinates in the two systems XY and X'Y' has hardly any sense. In the single chain of triangles, formed by the numbers 34, 46, 47, 48, 49, 50, 51, 52, and 53 the scale error increases from -1.3 cm per km in the side Enkhuizen - Lemmer (triangle 34) to even - 9.3 cm per km and -9.0 cm per km in the sides Kollum - Groningen and Hornhuizen - Groningen of triangle 52.

In the sides Sneek - Drachten and Leeuwarden - Drachten of triangle 49 this error is -5.8 and -7.3 cm per km respectively. The amounts affect the situation of Drachten in Stamkart's network in a considerable way. As Stamkart's distances are much too large, "his" Drachten (No. 39) will be situated much more to the east than the R.D.-Drachten. It will be clear that in Oldeholtpa (No. 30), at a distance of about 23.7 km south of Drachten, the amount for the rotation of the side Oldeholtpa - Drachten in column 6 of table 17 will therefore be negative and very large. The amount appears to be $-15''.14$.

Still more to the north the sign of the rotation becomes positive. For the sides Uithuizermeden - Groningen (triangles 53 and 54) and Groningen - Midwolda (triangle 55) the rotation is $+4''.76$ and $+6''.08$ respectively.

17. *FINAL ADAPTATION OF THE ADJUSTED NETWORK TO 28 IDENTICAL POINTS OF THE R.D.-TRIANGULATION*

By the great number of identical points in the two networks the choice of the "baseline" Breda - 's-Hertogenbosch for the computation of the coordinates of Stamkart's network is of course rather arbitrary. In order to adapt the adjusted network as well as possible to the R.D.-triangulation not only the identical stations Breda and 's-Hertogenbosch should therefore be used for a similarity transformation but also the other identical stations mentioned in table 16. Because of the bad agreement of Stamkart's adjusted measurements with the R.D.-results in the single chain of triangles 34, 46, 47, 48, 49, 50, 51, 52, and 53, already described in the previous section, the stations Hindeloopen (No. 26), Sneek (No. 36), Harlingen (No. 37), Leeuwarden (No. 38), Drachten (No. 39), Kollum (No. 40), Groningen (No. 41), Hornhuizen (No. 42), Uithuizermeden (No. 43), Holwierde (No. 44), Midwolda (No. 45), and Pilsum (No. 47) must be left out from this transformation, notwithstanding the excellent results of the adjustment of the observations at several stations in this part of the triangulation. According to table 4 Hornhuizen (No. 42) and Uithuizermeden (No. 43) even belong to the most accurate stations of the triangulation. The weak construction north of the line Medemblik - Enkhuizen - Lemmer - Oldeholtpa, however, would spoil the good results in the southern part of Stamkart's network.

The results of the similarity transformation are shown as the coordinates $X_i'' Y_i''$ in the columns 7 and 8 of table 16. The remaining differences $v_i = X'_i - X''_i$ and $w_i = Y'_i - Y''_i$ (in mm) in the points used for the transformation are given in the columns 9 and 10 respectively.

In fig 22 these differences are represented as vectors. Their lengths can be read from the drawing. The smallest (in Leiden, No. 14) is 32 mm, the largest (in Meppel, No. 31) is 684 mm. The vector in Amsterdam is 344 mm, that in Haarlem 350 mm. From the projections of these vectors on the side Amsterdam - Haarlem it appears that, after the transformation of Stamkart's adjusted network, the distance Amsterdam - Haarlem is 8.5 cm smaller than in the R. D. - system. The relative difference +0.5 cm per km is worse than the amount -0.1 cm per km found in table 17 and agrees somewhat less with the result of the baseline measurements found at the end of section 11. The difference +0.6 cm per km between the two amounts agrees of course with the relative difference between the distances Breda - 's-Hertogenbosch computed from the coordinates in the $X'Y'$ and $X''Y''$ - system respectively.

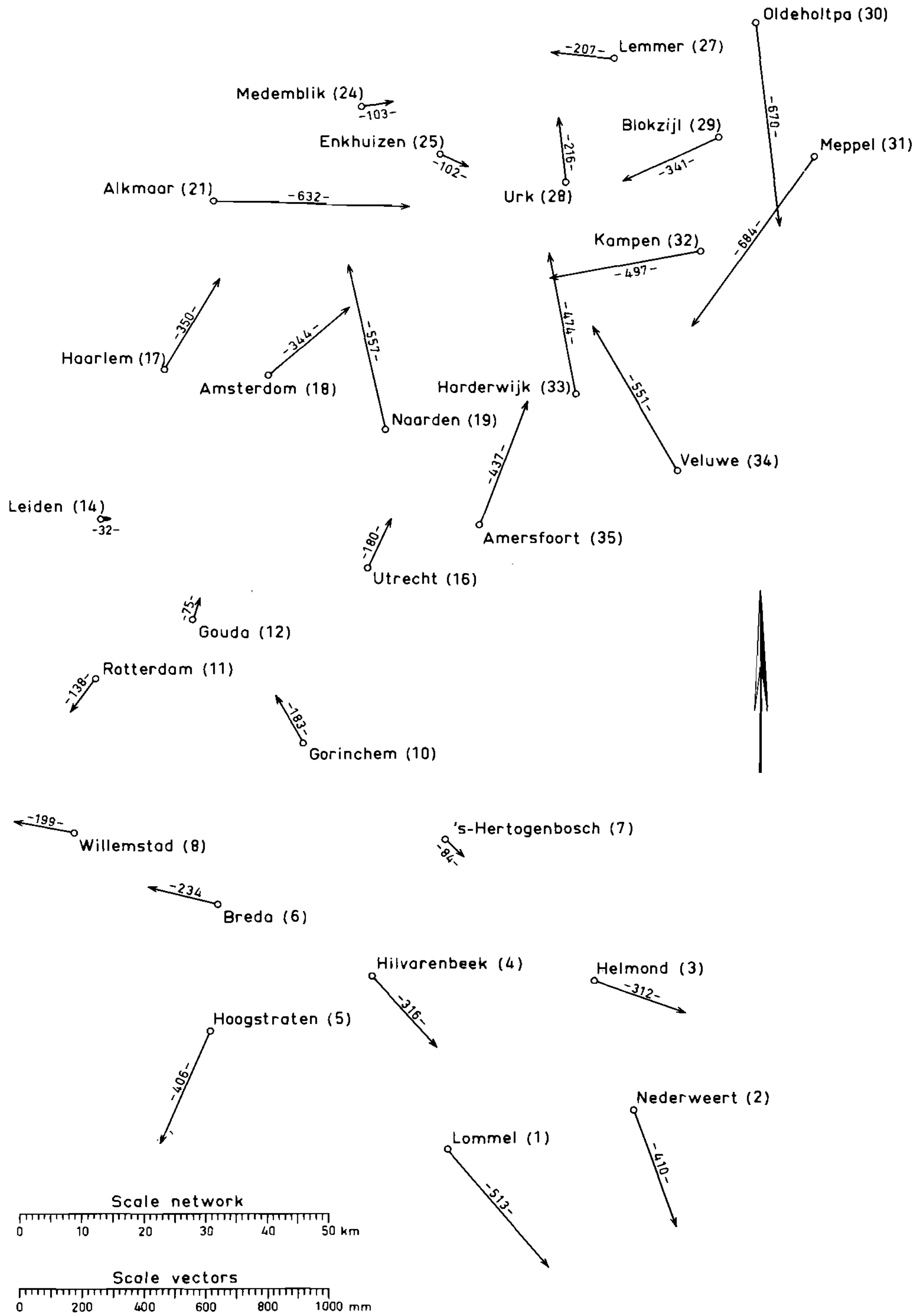
The difference between the gridbearings of the side Haarlem - Amsterdam in the two systems is $+ 0''.87$. The difference $+1''.04$ with the corresponding amount $-0''.17$ in column 6 of table 17 agrees of course with the rotation computed from the cotangents of the gridbearings of the side Breda - 's-Hertogenbosch in the $X'Y'$ and $X''Y''$-system respectively.

If the transformation would also have been used for the northern part of the triangulation the rotations for the sides Uithuizermeden - Groningen and Groningen - Midwolda would have led to the unacceptable amounts:

$+ 4''.76 + 1''.04 = + 5''.80$ and
$+ 6''.08 + 1''.04 = + 7''.12$ respectively.

The very great rotation $-15''.14$ of the side Oldeholtpa - Drachten (see table 17) would then be reduced to the also much too high amount:

$- 15''.14 + 1''.04 = - 14''.10.$



Oldeholtpa (30)
-207- Lemmer (27)
Medemblik (24)
-103-
Blokzijl (29)
-670-
Enkhuizen (25)
-216-
Meppel (31)
-102-
-341-
Alkmaar (21)
-632-
Urk (28)
Kampen (32)
-684-
-497-
-350-
-344-
-557-
-474-
Haarlem (17)
-551-
Amsterdam (18)
Harderwijk (33)
Naarden (19)
-437-
Veluwe (34)
Leiden (14)
-32-
-180-
Amersfoort (35)
Utrecht (16)
-75-
Gouda (12)
-138-
Rotterdam (11)
-183-
Gorinchem (10)
-199-
Willemstad (8)
's-Hertogenbosch (7)
-84-
-234
Breda (6)
Hilvarenbeek (4)
Helmond (3)
-312-
-316-
Hoogstraten (5)
-406-
Nederweert (2)
Lommel (1)
-410-
-513-
Scale network
0 10 20 30 40 50 km
Scale vectors
0 200 400 600 800 1000 mm

Fig. 22

18. *COMPARISON BETWEEN THE ANGLES AND SIDES (CHORDS) OF THE ADJUSTED NETWORK WITH THE RESULTS OF THE R.D.*

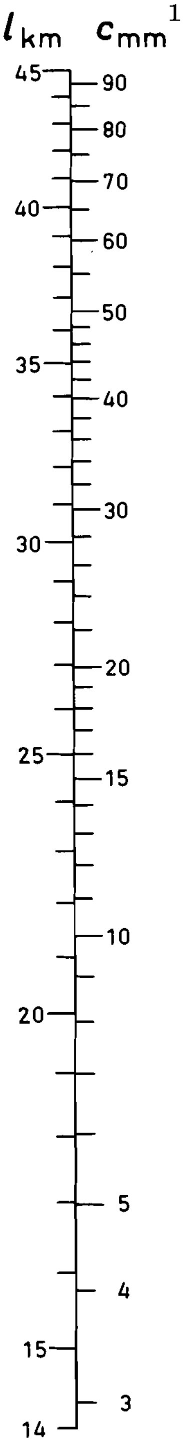


Fig. 23

A good survey of the external accuracy of Stamkart's triangulation can be obtained by comparing the adjusted angles of the triangles with the angles of the triangles in the R. D.-system which are identical or are assumed to be identical with those of Stamkart. The same can be done by comparing Stamkart's side lengths of the triangles south of the line Medemblik - Enkhuizen - Lemmer - Oldeholtpa with those of the R. D.

The survey is given in table 15, for the angles in the columns 10, 11, and 12, for the sides in the columns 13, 14, and 15. Columns 11 and 13 give the R. D.-values of the angles and opposite sides (chords) respectively, columns 12 and 15 the differences in seconds and centimeters respectively, between the R. D.-results of the angles and the sides with those of Stamkart's triangulation; for the sides after its adaptation to the 28 identical points mentioned in the previous section. The differences c between the arcs on the sphere and the chords l are very small for the triangles in Stamkart's network. They can be computed with the formula $c_{mm} = (l_{km})^3 : 977.72$ (see also [1], page 80, formula 10).
For l = 17 km, c is about 5 mm. As, however, c is directly proportional to the third power of l, c increases rapidly when l increases. For l = 34 km c is therefore about 4 cm. The relation between l and c can be read from fig. 23.

In several triangles, e.g. the numbers 2, 3, 4, 6, 7, 10, 18, 20, 35, 45, 46, 47, 48, 50, 51, 52, 53, 55, and 58 the agreement between the angles is very good. In the numbers 2, 3, 6, 10, 55, and 58 the differences in column 12 are even smaller than one second. Other triangles, e.g. the numbers 33, 36, 37, 39, 41, and 49 give rather big differences. By far the biggest are found in triangle 37 with the angular points Blokzijl, Lemmer and Oldeholtpa. The difference for the angle Blokzijl is $+ 3''.65$, that for the angle Lemmer $- 4''.54$, and that for the angle Oldeholtpa $+ 0''.90$. It is striking that also in Krayenhoff's triangulation this same triangle is the poorest of the whole network. His differences appear to be + 6".02, - 11".71, and + 5".69 respectively | 93] . As Krayenhoff's measurements of these stations date from 1807 and 1810, those of Stamkart from 1875, 1876, and 1877 and those of the R. D. from 1899, 1900, and 1901, the amounts of the differences are hardly comparable. The signs, however, agree. It might therefore be possible - I remarked it already in the investigation of Krayenhoff's network | 94] - that similar changes as described for Edam, Dordrecht, and Nieuwkoop affected the results of the computations.

Fig. 24, however, does not affirm this possibility. In this figure the sides of triangle 37 and the adjacent triangle 36 connect the R. D.-stations Lemmer, Oldeholtpa, Blokzijl, and Urk. In each of the angular points the vectors are drawn to the points as determined by Krayenhoff and Stamkart respectively [95] . Their ends are indicated with a single circle and a K (Krayenhoff) and with a double circle and an S (Stamkart) respectively. A connection between the three pairs of coordinates of the four stations cannot be shown.

The absolute differences in the side lengths (chords on the sphere) are, as already remarked, only given for the triangles 1 up to and including 45 of table 15. That

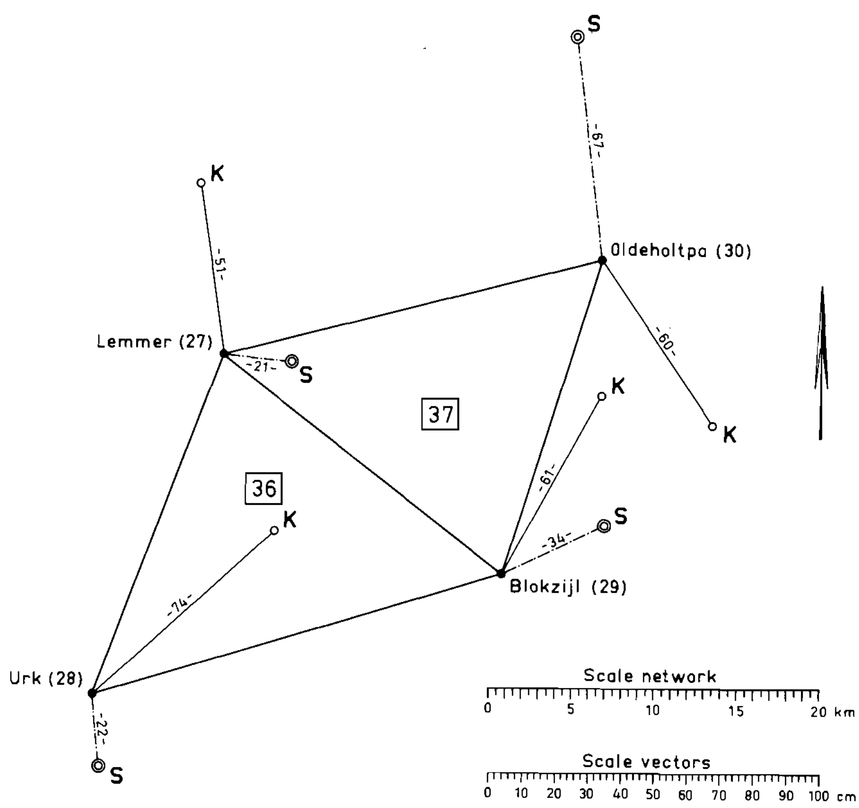


Fig. 24

for the side Amsterdam - Utrecht e.g. is - 3 cm, that for Enkhuizen - Medemblik is + 2 cm, and that for Harderwijk - Kampen - 68 cm. The latter amount is by far the greatest in the part of the network adapted to the points of the R.D. The differences are in general much smaller than the analogous amounts in column 9 of table 29 of Krayenhoff's network. The lengths of the vectors in fig. 22 show of course the same tendency.

The relative differences for the sides Amsterdam - Utrecht, Enkhuizen - Medemblik and Harderwijk - Kampen are -0.1 cm per km, + 0.1 cm per km, and -2.2 cm per km respectively. The amounts can also be found from table 17 by adding the constant amount +0.6 cm per km for the side Breda - 's-Hertogenbosch to the amounts in column 5 of that table. Moreover the differences +0.6 - 9.3 = - 8.7 cm per km for the distance Kollum - Groningen and + 0.6 - 9.0 = - 8.4 cm per km for the distance Groningen - Hornhuizen show once more that the northern part of the network does not fit at all in the adaptation to the R.D.-points of the southern part.

From table 14 it appears that in the series measured at the stations Rotterdam (No. 11) and Delft (No. 13) was pointed at Leiden Observatory. The directions were not used in the adjustment of the triangulation. How the observations to the Observatory in column 5 of that table were checked is given below for the station Delft.

As the gridbearing of the chord Delft - Leiden (townhall) in the plane of projection is

$$\text{arc tan}\ \frac{X''_{14} - X''_{13}}{Y''_{14} - Y''_{13}} = 29^{\circ}34'25''.67$$

and the small angle between arc and chord for this direction is $-1''.26$, the gridbearing of the arc Delft - Leiden (townhall) is $29^{\circ}34'24''.41$. As the adjusted direction to Leiden (townhall) in the series is $359^{\circ}59'59''.54$ (see column 8), the orientation of the series is $29^{\circ}34'24''.87$. From the analogous amounts $(91^{\circ}02'03''.85 + 0''.51) - 61^{\circ}27'39''.49 = 29^{\circ}34'24''.87$ for the direction Delft - Gouda follows of course the same orientation of the series (check). From this orientation and the spherical direction $359^{\circ}15'59''.56$ measured to Leiden Observatory in the series (see column 5 of table 14) the gridbearing of the arc Delft - Leiden Observatory is:

$$29^{\circ}34'24''.87 + 359^{\circ}15'59''.56 = 28^{\circ}50'24''.43 .$$

It differs but $0''.15$ from the gridbearing

$$28^{\circ}50'25''.52 - 1''.24 = 28^{\circ}50'24''.28$$

that can be computed from the R.D.-coordinates of the Observatory (see table 12, columns 7 and 8) and "Stamkart's" coordinates of Delft in the columns 7 and 8 of table 16.

Analogous computations in the series at Rotterdam to Leiden Observatory give gridbearings $0^\circ 32' 44''.82$ and $0^\circ 32' 48''.08$ respectively. The difference is $3''.26$. If, instead of "Stamkart's" coordinates of Rotterdam the R.D.-coordinates are used, the gridbearing $0^\circ 32' 48''.08$ changes into $0^\circ 32' 48''.74$. The difference between the two determinations is then $3''.92$.

19. *CONCLUSIONS ON THE GEODETIC PART OF THE TRIANGULATION*

As already stated in the Introduction of this publication, the death-blow to Stamkart's triangulation was the letter, dated February 28th 1885 to the minister of home affairs and to the members of the States General. The letter was drawn up by Schols and it was published in Tijdschrift voor Kadaster en Landmeetkunde [14]. Because of its importance I repeat below the most incriminating sentence in the letter in which Schols remarks that, "since before 1866 Stamkart was never engaged with accurate angle measurements, the result of his angle measurements were far below the precision adopted for the European triangulation and even less good than Krayenhoff's measurement, rejected previously as insufficient".

Was Schol's judgement correct? First of all he should have realized that it is of very little importance whether "before 1866 Stamkart was never engaged with accurate angle measurements". What is really of importance is that Stamkart, for more than thirty years (since 1833),was an inspector of weights and measures. In this position he dealt with accurate measurements though, it is true, not with angular measurements. This profession must have made him a good observer with an excellent sense for accuracy. Because of this quality it can be assumed that he was the right man for the triangulation, provided of course that this sense for accuracy should not be cancelled by a lack of knowledge of geodesy.

Could, as he wrote, Schols really judge the accuracy of Stamkart's measurements and conclude that the results were "even less good than those of Krayenhoff"?. It seems that this question must be answered in the negative.

From Krayenhoff's triangulation it follows [96] that for the "accurate" instrument the standard deviation in the measurement of a retained (not re-jected) angle can be estimated at about $2''.4$ and for the less accurate instrument at about $4''.9$. These amounts, derived from a rather small number of observations, implicate standard deviations in a direction of $1''.7$ and $3''.5$ respectively. From Stamkart's measurements, however, it appears (see table 4) that the standard deviation in each of the 5881 directions measured at the various stations is about $m = \pm 2''.19$. The amount - Schols did not know it and could not know it - agrees with the mean of the smallest

$m = \pm 1''.48$ (at Hornhuizen , file 44) and the largest $m = \pm 2''.98$ (at Groningen, files 42 and 43). It is not worse and at any rate much more reliable than the amount $1''.7 + 3''.5) : 2 = 2''.6$ derived from Krayenhoff's measurements.

It is also important that, for a considerable number of Stamkart's stations, the influence of the centering errors is included in the standard deviation of a direction measured. The large (largest) standard deviation $m = \pm 2''.98$ at Groningen e.g. would have been considerably better if Stamkart could have measured his directions at only one eccentric station instead of at eccentric stations at the fourth and second gallery of the tower. The determination of the mutual situation of the spire of the Martini tower and the stations at two different galleries was a difficult problem for him. However, he found a solution and apparently with good success as is shown in section 8 (see page 42) for the station Amsterdam. The situation of the spire in Stamkart's measurements with respect to Krayenhoff's "centre" of the tower at the height of the first gallery (42.8 m below the spire) differs but a few centimetres from the results of the R.D.-measurements in 1898. The difference, however, between the projection of the spire on the first gallery and the "centre" of the tower at that place is about 63 cm [97] but Krayenhoff took no account of this difference, neither in Amsterdam nor elsewhere in his triangulation. Also for this reason it seems impossible that Stamkart's measurements should be worse than those of Krayenhoff.

After the adaptation of the southern part of Stamkart's network at 28 identical points of the R.D.-triangulation the remaining vectors are much smaller than in Krayenhoff's network as can be seen from a comparison between fig. 22 in this publication and fig. 20 (page 139) of [1]. This also pleads for a higher accuracy of Stamkart's measurements. It is true that for the adaptation of his network the part north of the R.D.-points Medemblik - Enkhuizen - Lemmer - Oldeholtpa had to be rejected but the main reason for this rejection was the poor construction of the single chain of triangles 34, 46, ..., 52, and 53, already discussed in section 16. It was due to the bad reconnaissance in Oldeholtpa with the result that Krayenhoff's station Onstwedde could not be used in the triangulation (see the dotted lines in fig. 2).

The accordance between the R.D.-length Amsterdam - Haarlem and Stamkart's distance between these two stations computed from his baseline extension network is excellent if this part of the triangulation is adjusted according to the method of the least squares. As the network (see fig. 14) has 27 redundant data, Stamkart could not execute this adjustment as it requires the solution of 27 normal equations. The very small v's in the directions measured at the 8 stations of the baseline extension network (see column 12 in table 9) have a standard deviation $m = \pm 1''.367$. The accuracy can hardly be surpassed as can also be seen from the very small vectors for the R. D.-points Hoofddorp and Zaandam and the vector zero in the base terminal point southeast of fig. 16.

The objections to Stamkart's triangulation are not limited to the bad reconnaissance, to the single chain of triangles in the northern part of the triangulation and to the primitive reduction to centre of the directions measured. They also relate to the insufficient investigation of the theodolite used for the first order measurements, to the poor distribution of the series measured on the limb of the theodolite and to the lack of insight into simple problems of land surveying which nowadays can easily be solved by landsurveyors of secondary level. A striking example of this lack of insight is that Stamkart, for the elimination of the remaining adjusting errors of his instrument, never used the theodolite with face right and face left. Moreover consistent changing face after each half of a series could not be carried out since in several cases some towers could not be seen in the second half of the - often too long - series. The practical objections to this theoretical blunder, however, will have been of little influence on the accuracy of the observations because of the small deviation of the telescope from the horizontal position in flat country.

The adjusted observations computed from these irregular and incomplete series are correlated. Apparently Stamkart did not know that but he can hardly be blamed for it. The determination of uncorrelated "series" of several directions by means of the measurement of the angles between the rays in those series in all combinations was treated theoretically by Hansen in 1871 [98], but only about 1880 applied in practice by General Schreiber for the first order triangulation in Prussia. The R. D.-first order measurements were also carried out according to "Schreiber's" method.

The worst example of poor knowledge of simple landsurveying is the local triangulation network in the city of Leiden, described in section 13. The mutual situation of the townhall and the Petrus church (see fig. 17) is very bad. It affects in a serious way the accuracy of the reduction of the series measured on the Petrus church to the "centre" townhall. In the same way the badly determined distance townhall - observatory considerably affects the azimuth townhall - Delft, computed from the astronomical azimuth to Delft measured at the Observatory.

Up till now Schols' slashing criticism on Stamkart's triangulation was never contested. As no one after Schols studied his work seriously and took the trouble to compute it, this bad judgement still stands. Stamkart's triangulation should be forgotten as soon as possible or, as Heuvelink remarks in a letter: "it should be covered with the well-known cloak of charity; Stamkart's work is null and void" [99] .

In my opinion this is an undiscerning judgement and it is tragic that nobody was willing to see or was able to see the merits of Stamkart's lifework, carried out with an obstinate energy and at the sacrifice of all his personal interests till his death at the age of almost 77. His handicap, however, was that he was no geodesist. He sinned therefore against the most elementary geodetic principles, and for the lack of this knowledge he should be reproached. His results, except his excellent baseline extension network, are therefore below the level demanded at the end of the 19th century (not below the level of Krayenhoff's measurements as Schols suggested in his letter already mentioned before). For this reason they were rightly rejected.

The Netherlands Commission for Triangulation and Levelling (the present Netherlands Geodetic Commission) replaced Stamkart's network by an excellent triangulation. Still in our days it satisfies the demands in the domain of science and technique.

The considerations in this section form the final judgement on Stamkart's geodetic part of the triangulation. Notwithstanding the title of this publication "Stamkart's triangulation in the Netherlands", Kaiser's astronomical part will be treated in the next sections.

ASTRONOMICAL PART OF THE TRIANGULATION

20. *GENERAL SURVEY OF THE ASTRONOMICAL MEASUREMENTS ; DETERMINATION OF LONGITUDES AND LATITUDES*

The operations carried out at Leiden Observatory are mentioned in "Annalen der Sternwarte in Leiden, zweiter Band (1870)" [100] . They consisted of:

a. the determination of the declinations of 202 stars, used for the determination of latitudes,
b. the determination of the difference in longitude with Göttingen, carried out in 1867,
c. the determination of the difference in longitude with Brussels, carried out in 1868,
d. the determination of the difference in longitude with Bonn, carried out in 1870,
e. the determination of the latitude of Leiden,
f. the determination of the azimuth Leiden - Delft.

The planned determination of the difference in longitude between Dangast [101] and Leiden was not carried out. The meridian circle, the main instrument of the Observatory, was used for the measurements, mentioned in a, c, and e. It was made by Pistor and Martins, delivered to Leiden Observatory in 1861, and mounted in the meridian hall. This was a room of 10.39 x 6.75 x 4.81 m, situated eccentrically from the main cupola of the building. Kaiser describes the instrument and gives a drawing of it in [102] . It had a telescope with a focus length of eight feet (about 2.50 m) and two vertical limbs (A and B) with a diameter of three feet (about 0.90 m). They were calibrated in parts of 5' (about 0.65 mm). By means of prisms and lenses the parts where the limbs had to be read with microscopes were illuminated with a paraffin lamp.

For the determination of the difference in longitude between Leiden and Bonn (d) Kaiser used a transit instrument, also made by Pistor and Martins, and delivered to Leiden Observatory in 1869. It had a broken telescope with a focus length of 32 inches (about 0.85 m) and it was mounted eccentrically from the main cupola and the meridian hall in a small building in the garden of the Observatory.

The Universal-instrument, made by Repsold, used for the determination of the difference in longitude between Göttingen and Leiden (b) and of the azimuth Leiden - Delft (f), finally, will be treated in section 21.

The determination of the declinations of the 202 "triangulation stars" mentioned under a is a pure astronomical subject. The interested reader will find all details in Annalen II, mentioned before. The description of the other activities with the exception of the determination of the difference in longitude between Leiden and Göttingen (b) can also be found in Annalen II. Concerning this latter difference Kaiser remarks on page IV that "at the request of the central office the observations made in Leiden and the corrections to these observations were sent to this office in the autumn of 1868. Though the Bureau promised me that I should get them back for a further processing next winter, I heard nothing more about the reduction of the observations at Göttingen and about the determination of the results".

These results, however, are published in [103]. According to that publication the difference in longitude is $21^m50^s.06$. The reduction from the "small meridian room" at Leiden Observatory where the observations were done to the meridian circle of the observatory is included in this amount. The result was derived from the observations of 30 stars in the meridian of Leiden and the same 30 stars in the meridian of Göttingen. Their moments of transit at Leiden were transmitted by a special telegraphic connection to Göttingen Observatory and compared there with the local time. In the same way the moments of transit at Göttingen were registered at Leiden Observatory. A comparison between the personal errors of the observers completed the measurements.

As the now known longitude of Leiden (meridian circle) is $-0^h17^m56^s.15$ and that of Göttingen $-0^h39^m46^s.22$, the difference $+21^m50^s.07$ agrees excellently with the above result, notwithstanding the bad weather conditions during the measurements in 1867.

In an analogous way the differences Leiden - Brussels (c) (18 stars) and Leiden - Bonn (d) (24 stars) were determined. The first difference in longitude appeared to be $-0^m27^s.49$, the latter $+10^m26^s.96$. The second amount is also reduced to the meridian circle of Leiden Observatory.

According to the now known longitudes of Brussels, Bonn, and Leiden the differences are $-0^m28^s.15$ and $+10^m27^s.03$ respectively. The deviating result between Brussels and Leiden must be imputed to a wrong position of the Leiden instrument in the meridian. Kaiser gives a detailed speculation on the instrumental error on the pages [154] and [155] of his Annalen II.

The determination of the latitude of Leiden (meridian circle) is discussed in detail on the pages [110] up to and including [120] of Annalen II. It was carried out between 1863 and 1868 by the determination of the meridian altitudes of

a. 21 circum polar stars in upper and lower culmination (in total 996 transits).
   The observations were done in the years 1864-1868. They had the drawback that, per transit, only one observation could be made.

b. polaris (pole star, α Ursae minoris) in upper and lower transit. The 252 meridian altitudes were computed from circum meridian altitudes (several observations per transit). The observations were done in the years 1864-1868.

c. δ Ursae Minoris in upper and lower culmination. The 34 meridian altitudes were computed from circum meridian altitudes of the star (several observations per transit). The observations were done in 1863.

The meridian (circum meridian) altitudes were read on the limbs A and B of the meridian circle and for each of these limbs and for each of the determinations a, b, and c Kaiser computed the latitude φ of the Observatory. The result is given in table 18. From the results shown in this table and their standard deviations follows for:

limb A : $\varphi = 52^\circ 09' 19''.99$ and for
limb B : $\varphi = 52^\circ 09' 19''.93$

with a mean:

$$\varphi = 52^\circ 09' 19''.96.$$

Table 18

Method	Limb A	Limb B
a	$52^\circ 09' 20''.03$	$52^\circ 09' 19''.95$
b (year 1864)	———	$52^\circ 09' 19''.98$
b (year 1865/6)	$52^\circ 09' 19''.89$	$52^\circ 09' 19''.91$
b (year 1867)	$52^\circ 09' 19''.87$	$52^\circ 09' 19''.81$
b (year 1868)	$52^\circ 09' 20''.07$	$52^\circ 09' 20''.03$
c	$52^\circ 09' 19''.90$	$52^\circ 09' 19''.84$

It is an excellent result deviating only $0''.2$ from the amount $\varphi = 52^\circ 09' 19''.8$ given in the Astronomical Ephemeris for 1974. Dr. G. van Herk of Leiden Observatory informed me that the slight difference in latitude is due to additional observations and to the use of refined coefficients for the refraction.

A supplementary consideration on Leiden's longitude and latitude will be given in section 23.

21. *DETERMINATION OF THE AZIMUTH LEIDEN (SMALL MERIDIAN ROOM OF THE OBSERVATORY) – DELFT (NEW CHURCH)*

"When, at the end of the year 1861, I sent a nice photo of the new observatory, made by my son dr. P. J. Kaiser, to his excellency General Baeyer, he immediately saw that this observatory, because of its free location, would give an excellent opportunity for the measurement of an azimuth for the Middle European triangulation. He asked me to take this determination of azimuth upon me and I was quite willing to satisfy this request" [104].

From the above quotation it appears that, long before his official letter [8] to the Dutch government (see section 1), Kaiser, in a personal contact with General Baeyer, promised to do the astronomical part of the triangulation. It would, however, take another six years before the measurement of the azimuth could be realized (autumn 1870). It was done by dr. E. Becker. The universal instrument used for it was made by Repsold at Hamburg. It is the same instrument with which, in 1867, the difference in longitude between Leiden and Göttingen was determined (see section 20). Kaiser describes it on page XLIX of his Annalen I. Just like Stamkart's Pistor and Martins' theodolite (see section 5) it is at present in the Rijksmuseum voor de geschiedenis der natuurwetenschappen (National museum for the history of natural sciences) at Leiden. Thanks to the kind offices of Dr. Van Herk and the collaboration of the museum a photo of the instrument could be reproduced as fig. 25. The instrument - it dates from 1853 - is in a very bad state as it was kept in the part of the museum that was destroyed by bombing during world war II. The object glass of the broken telescope is in o, its diameter is 48 mm. The eye piece is in e. The focus length of the telescope is 564 mm, its magnification 60. The diameter of the horizontal limb h is 315 mm, that of the vertical limb v is 245 mm. Each of the limbs, divided into parts of 4', could be read with micrometer microscopes. Those on the vertical limb are still present. They are marked on the photo with an a. One unit on the drum d of the microscopes represents one second of arc. Readings were therefore done in tenths or twentieths of a second.

The microscopes on the horizontal limb are missing. They were placed perpendicular to the line connecting the magnifier m with the centre of the limb. The magnifier was used for a rough reading of the orientation of the limb when in a series the telescope was pointed at the first object in that series with the clamping screw c and the slow motion screw s. The instrument was mounted on a pillar in the "small meridian room C" on the first floor of the observatory [105]. The room had slits in the meridian and three big windows at the south, west and north side [106]

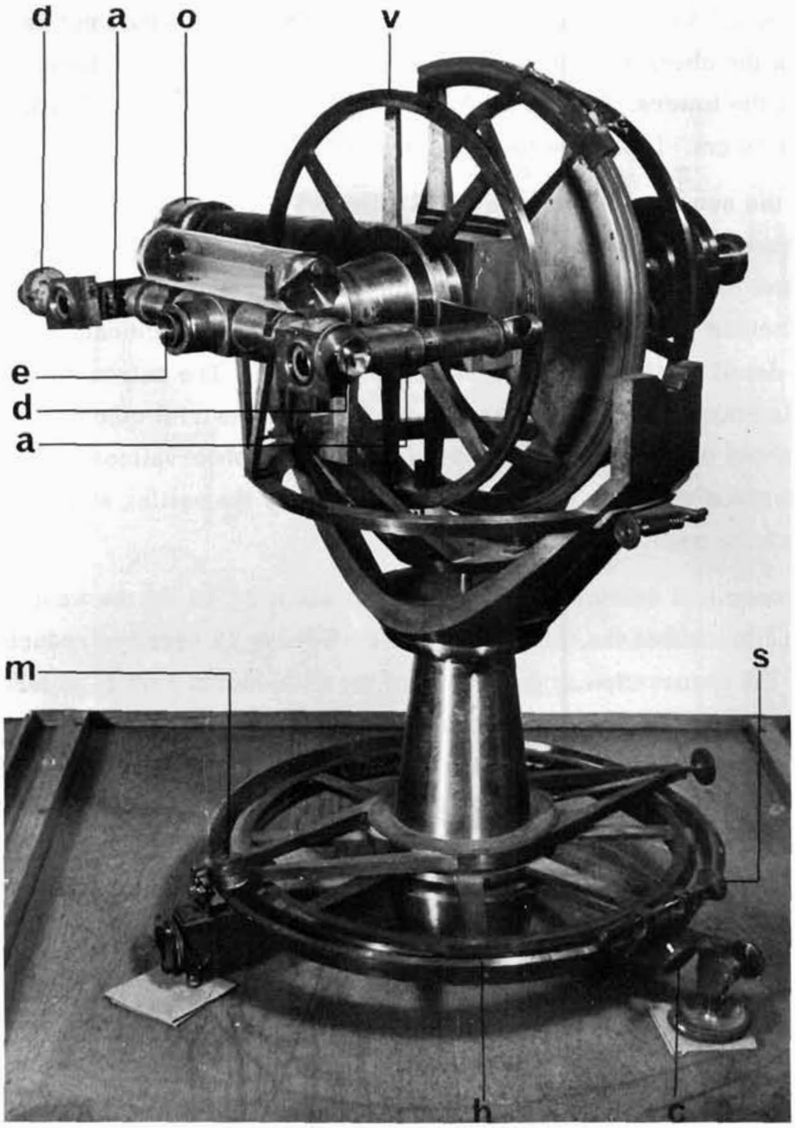


Fig. 25

In 1861 the tower of the St. Laurens Church at Rotterdam (station No. 11), about in the meridian of Leiden, could be seen from the observatory. Because of its rather long distance to Leiden (26. 0 km) it would have been very suitable for an azimuth determination. An objection, however, was that the tower had no sighting point and that, according to Kaiser, it was very often "wrapped in mist". Moreover, in 1870 it could no longer be seen because the trees had grown too high.
At the southwest side of the observatory, however, were the towers of the Hague (Jacobschurch, 14. 9 km) and Delft (New Church, 18. 0 km, station No. 13) which

could be used for the determination of an azimuth. As they could not be seen from the pillar in the observation room C, the walls of the room were pierced in the direction of the towers. In each of the two holes an iron tube was fixed. It had a diameter of 10 cm. The tubes could be closed with iron covers.

Already in the summer of 1868 the naval officer T. C. Gobée, practising astronomy at Leiden observatory, tried to determine the azimuth to The Hague but he failed. In the autumn of 1870, finally, the azimuth to Delft was measured by Dr. E. Becker of Leiden observatory. Kaiser describes the measurements and the computation in detail in Annalen II, pages [207] – [222] . The astronomical object was the pole star (Polaris, α Ursae minoris), the terrestrial object the gilt sphere (diameter about 0. 70 m) on the top of the tower. The observations were done in the late afternoon when the sphere was still lit up by the setting sun and Polaris could already be seen.

As the astronomical azimuth Leiden - Delft is about $28^{\circ}07'$ to the west of the south and the sun illuminates the western half of the sphere, a negative reduction must be given to the observation to the centre of the illuminated part in order to find the direction to the centre of the sphere. On page |209| of his description Kaiser defines the centre of the illuminated part (see fig. 26). O is the centre of the sphere with

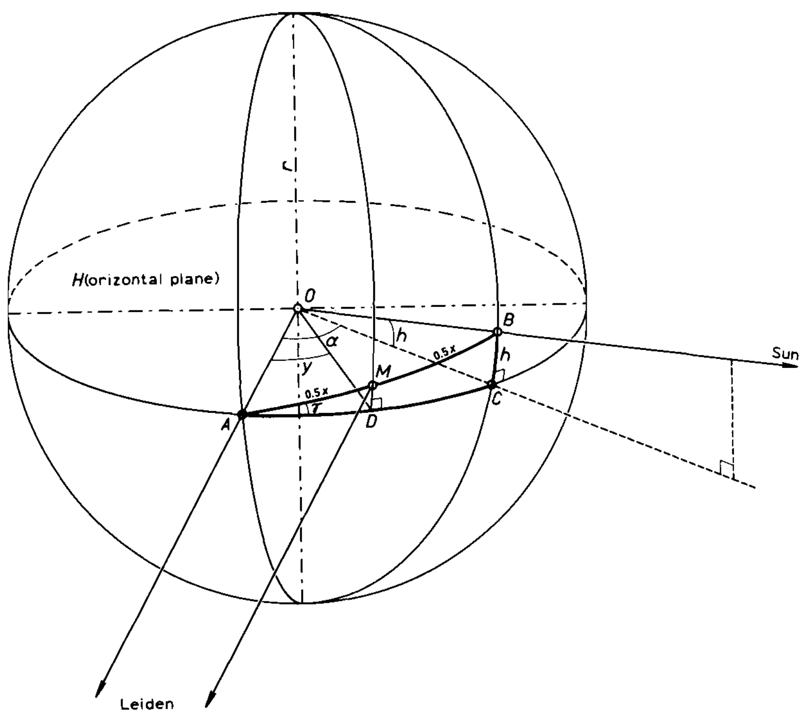


Fig. 26

a radius r. H is the horizontal plane through O. As the distance l between Leiden and Delft is about 18. 0 km and the area is flat, the direction O - Leiden intersects the sphere about in the horizontal plane (in A). The line O - Sun intersects the sphere in B. The altitude of the sun is supposed to be h, the difference of the azimuths, O - Leiden minus O - Sun, is the angle AOC = α. AMB is the great circle connecting A and B. Its length, in arc measure, is x. According to Kaiser M, its centre (AM = MB = 0.5 x), is the centre of the illuminated part of the sphere. If angle AOD = arc AD = y, the reduction R in azimuthal sense from M to the centre of the sphere can easily be computed. It is

$$R = -206265'' \frac{(r \cos MD) \sin y}{l}$$

$$= -206265'' (r : l) \cos MD \sin y \quad \ldots \quad (a)$$

r cos MD in this formula is the radius of the circle through M parallel to the horizontal plane through O. r and l in (a) are of course expressed in the same unit of length.

In the right-angled spherical triangles ABC and AMD holds respectively:

$$\cos x = \cos h \cos \alpha \text{ and } \cos \tfrac{1}{2} x = \cos y \cos MD \quad \ldots \quad (b)$$

and in triangle ABC:

$$\sin \alpha \cot h = \cot \nu \text{ or}$$
$$\tan \nu = \tan h : \sin \alpha \quad \ldots \quad (c)$$

In triangle AMD holds also:

$$\tan y = \cos \nu \tan \tfrac{1}{2} x \quad \ldots \quad (d)$$

y and MD in (a) can therefore be expressed in the known elements α and h at the moment of an observation. As

$$2 \sin^2 \tfrac{1}{2} x = 1 - \cos x \text{ and}$$
$$2 \cos^2 \tfrac{1}{2} x = 1 + \cos x$$
$$\tan^2 \tfrac{1}{2} x = \frac{1 - \cos x}{1 + \cos x} = \frac{1 - \cos h \cos \alpha}{1 + \cos h \cos \alpha} \quad \ldots \quad (e)$$

According to (c) holds:

$$1 + \tan^2 \nu = 1 + \frac{\tan^2 h}{\sin^2 \alpha} = (\sin^2 \alpha + \tan^2 h) : \sin^2 \alpha$$

so that:

$$\cos^2 \nu = \sin^2 \alpha : (\sin^2 \alpha + \tan^2 h) \quad \ldots \quad (f)$$

If the value $\cos^2 \nu$ in (f) and the amount $\tan^2 \frac{1}{2} x$ in (e) are substituted in the square of (d) the result is:

$$\tan^2 y = \frac{\sin^2 \alpha}{\sin^2 \alpha + \tan^2 h} \cdot \frac{1 - \cos h \cos \alpha}{1 + \cos h \cos \alpha} \quad \ldots \quad (g)$$

If the amount $\cos MD = \frac{1}{2} \cos x : \cos y$ from (b) and the value:

$$\cos \tfrac{1}{2} x = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 + \cos h \cos \alpha}{2}}$$

are substituted in (a) then (a) runs:

$$R = - 206265'' \, (r : l) \tan y \sqrt{\frac{1 + \cos h \cos \alpha}{2}}$$

In connection with (g) R therefore is:

$$R = - 206265'' \left(\frac{r}{l}\right) \sqrt{\frac{1 + \cos h \cos \alpha}{2} \, \frac{\sin^2 \alpha}{\sin^2 \alpha + \tan^2 h} \, \frac{1 - \cos h \cos \alpha}{1 + \cos h \cos \alpha}}$$

or

$$R = - 206265'' \left(\frac{r}{l}\right) \sqrt{\frac{\sin^2 \alpha \, (1 - \cos h \cos \alpha)}{2 \, (\sin^2 \alpha + \tan^2 h)}} \quad . \; . \; . \; . \; . \; . \; . \; . \qquad \text{(h)}$$

Strictly speaking R must be computed from the amounts α and h at the moment of the pointing at the terrestrial object. An approximation, however, of this moment, deduced from the sidereal time at the moment of the observation of Polaris in the same series will do.

The formula (h) for R does not agree with Kaiser's formula which, apart from the different notations used, can be found on page [210] of Annalen II. It runs:

$$R = - 206265'' \left(\frac{r}{l}\right) \frac{\cos h \sin \alpha}{\sqrt{2 + 2 \cos h \cos \alpha}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad \text{(k)}$$

As the above deduction is correct, Kaiser must have made a mistake. According to Kaiser $206265''$ (r : l) is $4''.40$. He found the amount by measuring in Leiden the diameter of the sphere ($8''.80$). The amount could be checked as the (half of the) sphere is still kept in the Delft townhall. Though it is heavily damaged and deformed by its fall from the 90 m high tower during the fire in 1872, I could determine its circumference at about 2.15 m. The diameter is therefore about 0.685 m. As the distance Leiden - Delft is about 18009.8 m the radius r of the sphere in arc measure is $3''.92$.

The following example illustrates the computation of R.
On July 22nd, the first day of the determination of the azimuth, the sun at Delft ($\varphi \approx 52^\circ 01'$) is 2° above the horizon ($h = + 2^\circ$) when its azimuth is about $58^\circ 21'$ west of the north. As the astronomical azimuth Delft - Leiden is about $28^\circ 01'$ east of the north, the angle α in (h) and (k) is $86^\circ 22'$. With $206265''$ (r : l) = $3''.92$ the reduction according to (h) appears to be then $- 2''.68$. According to (k) it is also $-2''.68$. Because of the very small amounts R it makes hardly any difference whether, for the computation of α and h in (h), the azimuth and the altitude of the sun are computed in Leiden instead of in Delft and for the azimuth Delft - Leiden the azimuth Leiden - Delft $\pm$ 180° is used.

An alternative determination of the reduction R with the aid of analytic geometry will be given underneath.

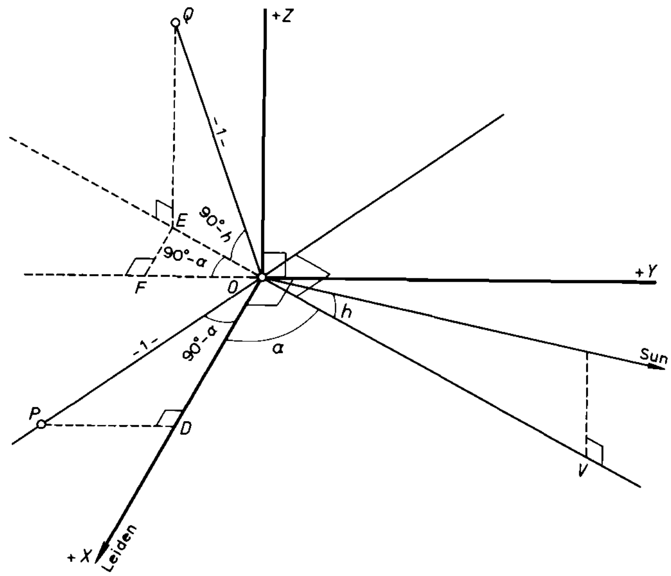


Fig. 27

In fig. 27 O is the centre of the sphere of the tower at Delft. XOY is the horizontal plane through O. The X-axis of the right-angled coordinate system is pointed at the pillar in the small meridian room of Leiden observatory. The Z-axis is perpendicular to the XOY-plane. OV is the intersection of the vertical plane through O and the sun with the XOY-plane. The altitude of the sun is h. The difference of the azimuths O-Leiden and O-Sun is α . P and Q are points at a distance 1 from O in the flat plane in O perpendicular to the line O-Sun. From the figure follow the coordinates of O, P and Q:

$X_O = 0, \quad Y_O = 0, \quad Z_O = 0;$

$X_P = OD = \sin\alpha \ , \quad Y_P = DP = -\cos\alpha \ , \quad Z_P = 0 \ ;$

$X_Q = FE = -\sin h \cos\alpha \ , \quad Y_Q = OF = -\sin h \sin\alpha$, and $Z_Q = EQ = \cos h.$

They satisfy the general equation of a flat plane:

$AX + BY + CZ + D = 0.$

The equation of the plane in O perpendicular to the line O - Sun runs therefore:

$\cos\alpha \ . \ X + \sin\alpha \ . \ Y + \tan h \ . \ Z = 0,$ or

$X + \tan\alpha \ . \ Y + (\tan h : \cos\alpha) \ . \ Z = 0$ . . . . (1)

If the amount X^2 from (l) is substituted in the equation:

$$X^2 + Y^2 + Z^2 = r^2 \quad \text{(m)}$$

of the sphere, then the equation is found of the projection in the YOZ-plane of the circle that forms the border between the illuminated and the dark half of the sphere. The equation runs as follows:

$$Y^2 + (\cos^2\alpha + \tan^2 h)\, Z^2 + 2 \sin\alpha \tan h \,.\, YZ - r^2\cos^2\alpha = 0 \quad \text{(n)}$$

It represents an ellips. For the example already given above ($\alpha = 86^\circ 22'$, $h = +2^\circ$ and $r = 3''.92$) the left half of the ellips is represented in fig. 28. Its semi long axis a and semi short axis b are $3''.92$ and $0''.25$ respectively.

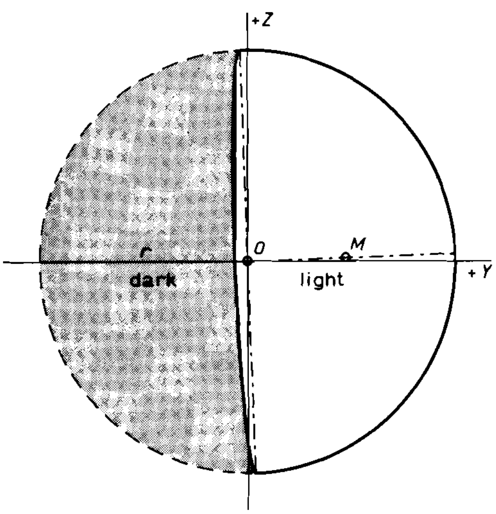


Fig. 28

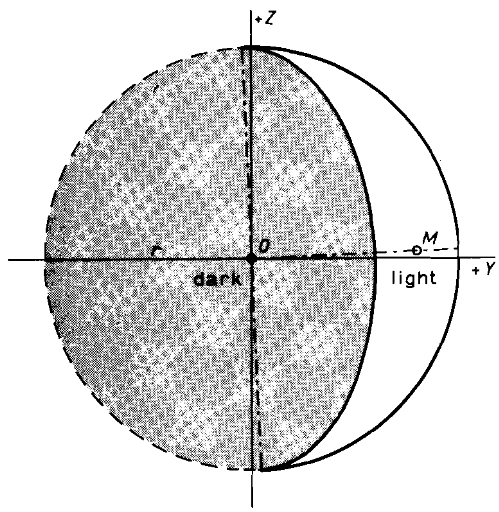


Fig. 29

For the sake of clearness the right half of the ellips at the back of the sphere is omitted. The Y-axis of the coordinate system must turn $\beta = 92^\circ 00'$ to the left in order to coincide with the long axis of the ellips. As the west (right) side of the sphere is fully visible, the width of the illuminated part is $r + b = 3''.92 + 0''.25 = 4''.17$. M, in the centre of the illuminated part, is determined by the distance

$$OM = (r + b) : 2 - b = (r - b) : 2 = 3''.67 : 2$$

and the angle $\beta = 92^\circ 00'$.

The reduction R is therefore:

$$R = - Y_M = - \left\{(r - b) : 2\right\} \sin 92^\circ 00' = - 1''.83 \quad \text{(o)}$$

The amount differs considerably from $R = - 2''.68$ found with the formulae (h) and (k). Moreover Kaiser's amount $r = 4''.40$ is about ten percent too large. For $R = - 2''.68$ he would therefore have found $- 3''.01$.

Another determination of the reduction R, relating to a fictitious observation on the last day (October 2nd; $\delta \approx -3^{\circ}40'$) of the measurements, is shown in fig. 29. For $h = +2^{\circ}$, α appears to be about $126^{\circ}32'$ sothat $\beta \approx 92^{\circ}29'$, $a = 3''.92$ and $b = 2''.33$. According to (o) the reduction R is therefore

$$R = -\left\{(r + b) : 2\right\} \sin 92^{\circ}29' = -3''.12.$$

According to (h) and (k) it is $-3''.50$ and $-3''.89$ respectively.

As already remarked the celestial object in the determination of the azimuth Leiden - Delft was the polestar. The method used was that by hour angle. The observations were carried out between July 22nd 1870 and September 4th 1870 (reading Delft on the horizontal limb about 90°, 120°, 150°, 180°, 210° and 240°) and between September 15th and October 2nd 1870 (reading Delft on the horizontal limb about 225°, 195°, 165°, 135°, 105° and 255°). The observations on July 22nd (two separate series) and the computation of the azimuth in these series are given in table 19.

Column 1 in the table gives the position of the vertical limb: to the R(ight) or to the L(eft) of the observer and column 2 the object pointed at. The pointings were done with respect to two vertical wires (1 and 2, see column 3) at a short distance ($19''.2$) from each other in the centre of the field of view of the telescope. Column 4 gives the sidereal time at the moment Polaris intersects each of these wires. The readings on the horizontal limb are given in column 5 and the deviations of the bubble of the level in column 6. The plus (minus) sign means that the end of the horizontal axis of the instrument at the side of the vertical limb is above (below) the horizon. As the sensitivity of the level was $2''.01$ per Paris' line the correction to the reading $62^{\circ}31'22''.10$ in column 5 is:

$$-1.95 \times 2''.01 \tan h = -4''.81 \text{ (column 7).}$$

h in this computation is the altitude of the pole star during the pointing. The reduced readings are given in column 8.

Column 9 gives the azimuth A of Polaris. It can be computed from:

$$\cot A = \frac{\sin \varphi \cos t - \cos \varphi \tan \delta}{\sin t}$$

in which φ is the latitude of the "small meridian room" in Leiden observatory, t the hour angle of the pole star and δ its declination (δ 1870. 0 = $+ 88^{\circ}36'58''.42$), φ was known from the amount $\varphi = 52^{\circ}09'19''.96$ given in section 20 and the small reduction from the place of the meridian circle to the "small meridian room".

Table

Limb	Object	Wire	Sidereal time	Reading hor. limb	Level	Corr.	Corr. readings	Azimuth Polaris
1	2	3	4	5	6	7	8=5+7	9
R	Delft	1		$270^{\circ}00'43''.20$			$270^{\circ}00'43''.20$	
"	"	2		270 01 01.80			270 01 01.80	
"	Polaris	1	$14^h18^m02^s.70$	62 31 22.10	+1.95	$-4''.81$	62 31 17.29	$180^{\circ}37'39''.45$
"	"	2	14 25 41.70	62 36 05.10	+2.00	-4.94	62 36 00.16	180 41 51.97
L	"	2	14 33 50.20	242 39 44.40	+0.72	+1.78	242 39 46.18	180 46 17.81
"	"	1	14 40 34.70	242 43 52.90	+0.40	+0.99	242 43 53.89	180 49 55.57
"	Delft	2		90 00 38.00			90 00 38.00	
"	"	1		90 00 57.75			90 00 57.75	
"	"	2		90 00 38.65			90 00 38.65	
"	"	1		90 00 58.00			90 00 58.00	
"	Polaris	2	15 03 53.80	242 55 35.75	+0.92	+2.28	242 55 38.03	181 02 09.28
"	"	1	15 08 57.80	242 58 41.25	+0.79	+1.96	242 58 43.21	181 04 44.23
R	"	1	15 16 31.30	63 02 16.60	+2.20	-5.45	63 02 11.15	181 08 32.12
"	"	2	15 21 22.80	63 05 09.75	+2.22	-5.51	63 05 04.24	181 10 56.45
"	Delft	1		270 00 43.80			270 00 43.80	
"	"	2		270 01 01.75			270 01 01.75	

For the computation of t = L(ocal) S(idereal) T(ime) minus R(ight) A(scension) L. S. T. was of course necessary. It was found from the determinations of longitude described also in the previous section. According to Kaiser the R. A. 1870. 0 of Polaris was $1^h11^m17^s.51$.

Column 10 in table 19 gives the orientation of the series. The amount for wire 1 in the upperhalf is $(118^{\circ}06'22''.16 + 118^{\circ}06'01''.68) : 2 = 118^{\circ}06'11''.92$ and analogously for wire 2: $118^{\circ}06'11''.72$. Behind the directions to Delft they are mentioned in brackets . The azimuths to Delft (centre of the illuminated part of the sphere) in columns 11 and 12 need no explanation. The computation of the reduction $-3''.29$ $(-3''.17)$ to the centre of the sphere (column 13) was already discussed in detail. The reduced azimuths in the two series in column 14 have a mean $28^{\circ}06'58''.73$ (column 15).

19

Orientation	Azimuth Delft	Mean az. Delft (luminous point)	Red. to centre sphere	Mean az. Delft (centre sphere)	Mean of the series
10=9-8	11=8+10	12	13	14=12+13	15
(118°06'11".92)	28°06'55".12	28°07'02".40	-3".29	28°06'59".11	28°06'58".73
(118 06 11.72)	28 07 13.52	28 07 01.62	-3.29	28 06 58.33	
118 06 22.16					
118 05 51.81					
298 06 31.63					
298 06 01.68					
(298 06 11.72)	28 06 49.72				
(118 06 11.92)	28 07 09.67				
(298 06 11.73)	28 06 50.38	28 07 01.93	-3.17	28 06 58.76	
(298 06 11.00)	28 07 09.00	28 07 01.90	-3.17	28 06 58.73	
298 06 31.25					
298 06 01.02					
118 06 20.97					
118 05 52.21					
(118 06 11.00)	28 06 54.80				
(118 06 11.73)	28 07 13.48				

The result is given once more in table 20 (first amount, reading Delft on the limb about 90°) with the results of the other series. The mean azimuth in the left part of the table is 28°06'57".87 ± 0".329, the mean azimuth in the right part 28°06'56".59 ± 0".522 [107] . The weighted mean of the two amounts is 28°06'57".50, the standard deviation in this weighted mean ± 0".278.

Table 20

Position limb	Reduced azimuth to Delft	v	Position limb	Reduced azimuth to Delft	v
90°	28°06'58".73	-0".86	225°	28°06'57".94	-1".35
120	58.04	-0.17	195	56.97	-0.38
150	57.50	+0.37	165	55.92	+0.67
180	56.45	+1.42	135	55.81	+0.78
210	58.31	-0.44	105	58.04	-1.45
240	58.22	-0.35	255	54.85	+1.74
	28°06'57".87	-0".03		28°06'56".59	+0".01
m^2 = 3.24:5= 0.648; m = ± 0".805			m^2 = 8.15:5= 1.630; m = ± 1".277		
M^2 = 0.648:6=0.108; M= ± 0".329			M^2 = 1.630:6= 0.272; M=± 0".522		

22. *REDUCTION AND CONVERSION OF THE ASTRONOMICAL AZIMUTH FROM THE SMALL MERIDIAN ROOM OF LEIDEN OBSERVATORY TO DELFT INTO GRID-BEARINGS FROM THE MAIN CUPOLA AND THE TOWNHALL TO DELFT. COMPARISON OF THE GRIDBEARINGS WITH THE R.D.-RESULTS*

The azimuth $28^{\circ}06'57''.50$, measured on the pillar in the "small meridian room", had to be reduced first to the main cupola of the observatory, Stamkart's station 7 in the local triangulation network of Leiden in fig. 17, and thereafter from the main cupola to the townhall, Stamkart's station No. 14 of the main network.

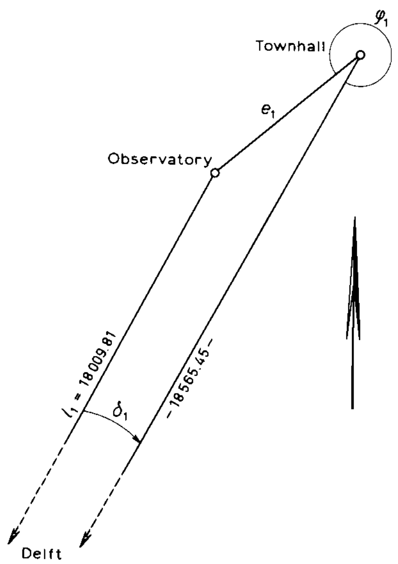


Fig. 30

The latter reduction δ_1, (see fig. 30) can be found from:

$$\sin\ \delta_1 = -e_1 \sin\ \varphi_1 : l_1$$

with $\varphi_1 = 337^{\circ}31'15''.76$ (see table 14, station No. 14) and $e_1 = 602.926\ m \pm 0.089$ (see for the distance table 13 and for the standard deviation page 71).

l_1 the distance Observatory (main cupola) - Delft is 18009.81. It was computed from e_1, φ_1 and the distance townhall - Delft = 18565.45 of the main triangulation network (see triangle 15 in table 15):

$$\delta_1 = \text{arc sin } (-e_1 \sin\ \varphi_1 : l_1) = +44'00''.26.$$

The standard deviation in δ_1 because of the standard deviation m_e in e_1 is:

$$\pm \delta_1'' m_e : e_1 = \pm (2640''.26 \text{ x } 0.089): 602.926 = = \pm 0''.39.$$

The less important standard deviations in l_1 and φ_1 are left out of consideration. The reduction of the azimuth to Delft from Becker's station in the "small meridian room" to the main cupola of the observatory was determined by Kaiser. He describes the measurements on page [220] of Annalen II. They are shown in fig. 31. The figure represents a cross section through the balustrade around the flat roof above the first floor of the observatory near the main cupola and the pillar where the azimuth to Delft was measured.

A and B are the terminal points of a 14. 058 m long baseline. Kaiser marked the points with small copper pins in the wooden balustrade. The balustrade is still intact. As, however, it is now covered with zinc and Kaiser did not describe the exact place of A and B, a local investigation was necessary in order to verify Kaiser's measurements. It could be done thanks to the kind help of Dr. Van Herk and of Mr. K. P. H. Jasperse, engineer of the Cadastral land survey department at Leiden.

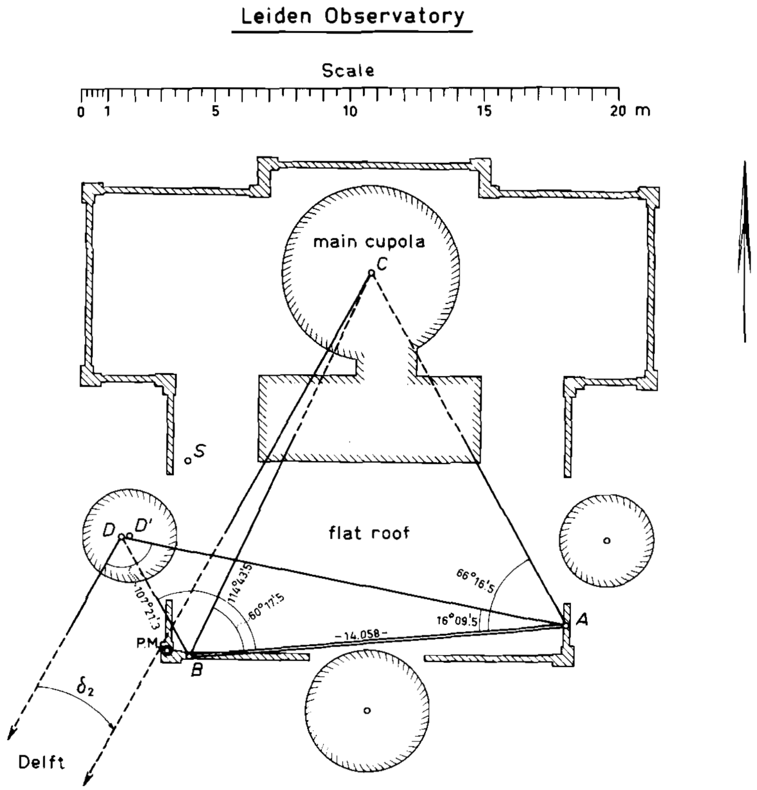


Fig. 31

In A and B Kaiser measured the four angles indicated in figure 31. As, however, Becker's station D in the small meridian room was situated on the first floor of the building and the base line AB was on the roof, the angles with D as a sighting point had to be measured by pointing at the string of a plumb perpendicular above Becker's Repsold instrument.

In D, finally, Kaiser measured the angle $107^{\circ}21'.3$ between A and Delft with a sextant (!).

The unchecked measurement makes it possible, indeed, to determine the reduction δ_2 from D to C for the direction Delft, if at least the distance C - Delft is known. According to Stamkart's measurements, it is 18009.81 m (see fig. 30), but in 1870 Kaiser could not possibly know this distance for the simple reason that Stamkart's measurements at Delft were only done in 1871.

In the consideration on page [221] of Annalen II Kaiser uses 17905.8 m for the distance, though he remarks "that this distance remained uncertain up to 100 metres".

On account of this error Van der Plaats in his paper | 108 | asks in a footnote on page 295: "Would anyone abroad really have believed that still in the year of Our Lord 1870 the distance between two of the oldest cities in Holland was uncertain up to one hundred metres ?"

It is the first error in the triangle Delft - D - C that affects the accuracy of the reduction δ_2. From the length of the baseline (14. 058 m) and the four angles in the base points A and B follows the distance CD. Kaiser found 13. 657 m for it but he must have made a mistake in his computation. It should be 13. 678 m. His computation of the angle CDA = $58^\circ 16'$, necessary for the determination of angle Delft - D - C, is also wrong. It should be $58^\circ 31' 47''$. Apart from a possible error in the angle Delft - D - A, measured with the sextant, the result of his computation of:

$$\delta''_2 = \frac{13.657 \sin (107^\circ 21'.3 + 58^\circ 16'.0)}{17905.8}\, 206265'' = +\, 39''.07$$

is therefore uncertain; an error in one of the base angles already used for the computation of CD would of course affect again the result. Some new measurements were therefore necessary. They were done by Mr. Jasperse and they were facilitated by the presence of a permanent mark of the R. D.-first order triangulation about in the southwestern corner of the balustrade. In Fig. 31 it is marked PM. Its coordinates are:

$$X_{PM} = -\,61825.498, \qquad Y_{PM} = +\,340.221$$

It is the same mark from which the R. D.-coordinates:

$$X_C = -\,61817.855, \qquad Y_C = +\,354.497$$

of the pillar C in the main cupola of the observatory were determined by Mr. Schipper (see fig. 17 and table 12, columns 7 and 8).

From the coordinates of PM those of S, a point on the flat roof, were determined. They run: $X_S = -\,81624.698$, $Y_S = +\,347.407$. Southwest of S is a small observation cupola, situated perpendicular above the former "small meridian room". Its pillar is in line with that on which Becker's instrument was mounted for the determination of his azimuth to Delft. In the former meridian room its cross-section is a rectangle, in the cupola on the flat roof, however, it is an equilateral triangle, the angular points of which are bevelled-off. The centre, marked with a hole in the pillar, is the point D' in fig. 31. Its coordinates, determined from S are:

$$X_{D'} = -\,61826.891, \qquad Y_{D'} = +\,344.559.$$

D is the centre of the pillar that is assumed to be Becker's station in 1870. Its coordinates:

$X_D = -61827.194, \quad Y_D = +344.518$

were already determined as pillar I by the R.D.-first order triangulation in 1897. From the R.D.-coordinates of C and D follows a distance of 13.667 m between the two points. It agrees excellently with the amount 13.678 m found from Kaiser's measurements. Kaiser made therefore no mistake in the measurement of the distance AB and in the angles BAC, BAD, ABC and ABD respectively. From these angles and the coordinates of C and D follow the coordinates of A and B. They run:

$$X_A = -61810.659, \quad Y_A = +341.105,$$
$$X_B = -61824.673, \quad Y_B = +339.996.$$

They appear to be situated on the balustrade. For an accurate representation of the balustrade in fig. 31 its southwestern and southeastern angular points were determined in coordinates. From its gridbearing $90^\circ 43'$ and the convergence of the meridians $+42'49''$ at that spot (astronomical north minus grid north) it appears that, at least for this part of the observatory, it was built very accurately in a west-east direction.

From Becker's astronomical azimuth $208^\circ 06' 57''.50$ in D to Delft and the angle $0^\circ 42' 48''.82$ in D between grid north and astronomical north follows that, according to Becker's measurements, the gridbearing D - Delft is $208^\circ 49' 46''.32$. As the gridbearing to A is $101^\circ 39' 46''$ and that to C $43^\circ 06' 10''$ | 109 | , the angle Delft - D - A is $107^\circ 10' 00''$ and the angle Delft - D - C $165^\circ 43' 46''$. The first amount deviates $11'.3$ from the angle $107^\circ 21'.3$ measured by Kaiser with a sextant, the second amount $6'.5$ from Kaiser's angle $107^\circ 21'.3 + 58^\circ 16'.0 = 165^\circ 37'.3$. Perhaps the error of $11'.3$ is due to the fact that it is impossible to judge in the telescope of the sextant the accurate coincidence of the left object at a distance of 17 m and the right one at a distance of 18 km. A better determination of the reduction δ_2 is therefore:

$$\delta''_2 = \frac{13.667 \sin 165^\circ 43' 46''}{18009.81} \, 206265'' = +38''.59 .$$

Notwithstanding the very large error in the distance to Delft the reduction deviates only $0''.48$ from the amount $\delta_2 = +39''.07$ found by Kaiser.

According to Becker's measurements the gridbearing from Stamkart's station Leiden townhall to Delft therefore is:

$$208^\circ 49' 46''.32 + \delta_1 + \delta_2 = 208^\circ 49' 46''.32 + 0^\circ 44' 00''.26 + 38''.59 =$$
$$= 209^\circ 34' 25''.17.$$

According to the R.D.-coordinates in columns 7 and 8 of table 16 it is:

$$\text{arc tan }(+0.56747434) = 209^\circ 34' 25''.67 + 1''.26 = 209^\circ 34' 26''.93.$$

The amount $+ 1''.26$ in this computation is the small angle between chord and arc in the stereographic map projection. The two gridbearings differ $1''.76$. This difference might partly be explained by Becker's (Kaiser's) too large negative reductions from the centre of the illuminated part of the sphere on the tower at Delft to the centre of the sphere. The azimuths in table 20 tend therefore to be too small. If the standard deviation $M = \pm 0''.278$ in Becker's mean azimuth in table 20, the standard deviation $m = \pm 0''.39$ in the determination of δ_2 and the influence of the inaccuracy of Stamkart's measurements on the determination of the gridbearing Leiden (townhall) - Delft are also taken into account, then the difference of $1''.76$ is fully acceptable.

A more direct check on the accuracy of Becker's measurements in D can be obtained by comparing "his" gridbearing of the arc C - Delft $= 208^\circ 06' 57''.50 +$ $+ 0^\circ 42' 48''.82 + 38''.59 = 208^\circ 50' 24''.91$ with the gridbearing: $\overline{\text{C - Delft}}$ = arc tan $0.55067375 + 1''.24 = 208^\circ 50' 25''.52 + 1''.24 = 208^\circ 50' 26''.76$, computed from the R.D.-coordinates of C and "Stamkart's" coordinates of Delft in columns 7 and 8 of table 16. Becker's result is now $1''.85$ less than that of my computation using R.D.-coordinates, almost the same amount ($1''.76$) as found in the previous computation.

Still another check, finally, on Becker's azimuth to Delft can be obtained with the aid of some angle measurements with the Repsold-instrument on the pillar C (see fig. 31) in the main cupola of the observatory. They were executed between September 29th and October 6th 1860 by Dr. Kam and they relate to 22 towers that could be seen from there. Kaiser describes the measurements in Annalen II, pages [216] - [219] and mentions the angles (in five series each) between the top of the New Church at Delft and each of the towers Voorschoten, The Hague, Wassenaar, Katwijk, the castle Endegeest, Noordwijk, Zoeterwoude and Pijnacker, all of them situated at several kilometres from the observatory. If Kam's sighting points Voorschoten, The Hague, Wassenaar, Noordwijk and Zoeterwoude are assumed to be identical with the stations determined in the R.D.-system between 1897 and 1915 (C is identical indeed), then the azimuth C-Delft can be computed five times from the coordinates of the six given points and the five measured angles. The result of the computation of the adjusted azimuth depends on the accuracy of the measured angles and on the significance that must be attached to the given coordinates. Since 1938 when the "Handleiding voor de technische werkzaamheden bij de kadastrale hermetingen" (H.T.W. 1938, page 88) was published, the ideas in this field have changed. The H. T. W. 1956 (page 15) gives therefore other views on this subject. In their turn they are superseded by Baarda's "S-transformations and criterion matrices" (publication of the Nederlands Geodetic Commission, Delft 1973).

The above problem was solved using the methods indicated in each of the publications of 1938, 1956 and 1973. The 1973-solution was done by Mr. Van Mierlo and Mr. Gravesteyn of the Computing Centre of the Delft Geodetic Institute. The computations according to the directives in the H. T. W. 's 1938 and 1956 were done by myself. The solution according to the H. T. W. 1938 is compiled in table 21. In my opinion it gives the most satisfying results.

Column 2 of the table gives the names of the right sighting points with the years of their determination (Delft is the left sighting point) and column 3 Kam's observations in the five series. The means of the angles are given in column 4. Column 5 gives the gridbearings

$$\overline{CI} = \text{arc tan}\left\{(X'_I - X'_C) : (Y'_I - Y'_C)\right\} - \epsilon''_{CI}$$

of the arcs CI in the plane of projection of the R. D.-system. ϵ''_{CI} (see also page 94) is determined by:

$$\epsilon''_{CI} = 0.0012658\ (X'_I Y'_C - X'_C Y'_I)\ .$$

The five derived gridbearings of the arc C-Delft are shown in column 6 = 5 - 4. The standard deviation M_i in each of them depends on:

a. the standard deviation m_i in the mean of each of the angles mentioned in column 4. As for Voorschoten [pp] = 10.87, m_1^2 = 10.87 : (4 x 5) = 0.54.

b. the standard deviation $n_i\sqrt{2}$ in the computed gridbearing C-I. As the H. T. W. 1938 assumed that the standard ellipses of C and the five points I are situated within circles with a radius of 3 cm,

$$n''_i = \frac{0.03\ (m)}{l_i\ (m)}\ 206265'' \quad \text{and} \quad 2\,n_i^2 = \frac{76.581}{l_i^2\,(km^2)}$$

For Voorschoten (l_1 = 4.197 km) $2\,n_1^2$ = 4.35. The amounts l_i, m_i^2 and $2n_i^2$ are given in the columns 7, 8 and 9 respectively of table 21.

$M_i^2 = m_i^2 + 2\,n_i^2$ (column 10) gives the square of the standard deviations in each of the five gridbearings to Delft. The weights of these gridbearings are $g_i = \sigma^2 : M_i^2$. σ^2 in this formula is the estimated square of the standard deviation in the "observation" with the weight g = 1. For $\sigma^2 = 1.7$ - in that case g = 1 for the mean distance l $\approx$ 7.7 km - the amounts $g_i = 1.7 : M_i^2$ are mentioned in column 11. For Voorschoten g_1 = 0.35.

Table 21

No.	Right sighting point	Angle between Delft and right sighting point	Mean	Computed R.D. gridbearings	Gridbearings Delft	l_{km}	m^2	$2n^2$	M^2	$g=\frac{1.7}{M^2}$	v''	gv	gvv
1	2	3	4	5	6=5−4	7	8	9	8+9=10	11	12	13	14
1	Voorschoten (1915)	$9^{\circ}33'32''.6$ 32.9 32.8 32.3 36.3	$9^{\circ}33'33''.38$	$218^{\circ}24'03''.25$	$208^{\circ}50'29''.87$	4.197	0.54	4.35	4.89	0.35	+2.70	+0.95	2.56
2	The Hague (1897)	$26^{\circ}32'53''.6$ 54.6 53.2 56.8 54.2	$26^{\circ}32'54''.48$	$235^{\circ}23'21''.86$	$208^{\circ}50'27''.38$	14.896	0.39	0.35	0.74	2.30	+0.21	+0.48	0.10
3	Wassenaar (1915)	$52^{\circ}24'44''.9$ 48.1 45.2 45.9 46.5	$52^{\circ}24'46''.12$	$261^{\circ}15'14''.16$	$208^{\circ}50'28''.04$	6.476	0.32	1.83	2.15	0.79	+0.87	+0.69	0.60
4	Noordwijk (1910)	$136^{\circ}09'35''.3$ 34.3 34.8 36.5 33.7	$136^{\circ}09'34''.92$	$345^{\circ}00'00''.65$	$208^{\circ}50'25''.73$	9.147	0.23	0.92	1.15	1.48	−1.44	−2.13	3.07
5	Zoeterwoude (1915)	$318^{\circ}20'04''.9$ 04.5 02.4 03.2 01.5	$318^{\circ}20'03''.30$	$167^{\circ}10'30''.48$	$208^{\circ}50'27''.18$	3.847	0.40	5.17	5.57	0.31	+0.01	0.00	0.00
					$208^{\circ}50'27''.17$	7.713				5.23		−0.01	6.33

The weighted mean gridbearing O_{mean} of the arc C-Delft is determined by:

$$O_{mean} = [gO] : [g] = 208^{o}50'27''.17 .$$

It differs hardly from the amount $208^{o}50'26''.76$ computed from the R.D.-coordinates of C and "Stamkart's" coordinates of Delft. The corrections v_i to the observations in column 3 of table 21 are mentioned in column 12, the $g_i v_i$'s ($[gv] = 0$) and the $g_i v_i v_i$'s in columns 13 and 14 respectively. As $[gvv] = 6.33$, $\hat{\sigma}^2 = 6.33 : (5-1) = 1.58$. The amount agrees excellently with $\sigma^2 = 1.7$ from which the computation was started. From $m^2_{O(mean)} = \sigma^2 : [g] = 0.33$ it follows that the standard deviation in the gridbearing to Delft is $m_{O(mean)} = \pm 0''.57$.

The result of the computations according to the directives of H.T.W. 1956 is $O_{mean} = 208^{o}50'\ 27''.19$. It agrees with the above mentioned result. For this special case $O_{mean} = 208^{o}50'\ 30''.77$ according to Baarda's publication and considerably larger than the five amounts in column 6, seems less convincing.

After the measurements in 1860, according to Dr. Kam, the spire of The Hague was repaired. Its position might therefore have altered a little. If, however, The Hague is left out from the computation in table 21, then the gridbearing to Delft becomes $208^{o}50\ 27.00 \pm 0''.76$; it differs only $0''.17$ from the previous result.

I cannot tell whether Kaiser had much confidence in the accuracy of Becker's azimuth to Delft but I am sure that, after Kaiser's death in 1872, his successor Van de Sande Bakhuyzen missed this confidence. Already soon after his arrival at Leiden in 1873 he found out that the universal instrument with which the azimuth was measured, had an instrumental error that already must have existed in 1860. "The azimuth to Delft can therefore be wrong for several seconds; as a matter of course it must be measured again".

It appears from Annalen IV page II that this instrumental error was a variable eccentricity of the centre of the horizontal limb with respect to the vertical axis of the instrument. The Repsold factory rectified the error for the greater part but not fully. "By reading on the two opposite micrometer microscopes on the limb, however", according to Van de Sande Bakhuyzen, "and by using the mean of these readings in the further computations, the error could be fully eliminated". This explanation is not very clear. As for his measurements Becker always read the two opposite microscopes, the error will not have affected the accuracy of his result. However, it might have affected Kam's measurements in table 21, as the angles in column 3 are borrowed from readings with only one of the two micrometer microscopes.

Apart from Becker's reductions to the centre of the sphere and from Kaiser's inaccurate reduction of the azimuth to the main cupola of the observatory, Becker's result is in my opinion excellent. Moreover, a new measurement of the azimuth to Delft would have been senseless as, because of the fire in the tower in 1872, it would not have fitted in Stamkart's triangulation. The stations Delft and Leiden of this triangulation were namely measured before the fire.

I think that, after 1872, a new determination of the astronomical azimuth, now to Haarlem (27.3 km from the main cupola), Nieuwkoop (20.3 km), Gouda (22.3 km) or Rotterdam (26.0 km) (see fig. 2) could only have been successful if the observations would have been done on the pillar in the main cupola, the highest station of the observatory. Though Stamkart's work for the triangulation proceeded till 1882, an attempt for such a determination was never made.

23. *RELATIONS BETWEEN THE COORDINATES X, Y IN THE STEREOGRAPHIC MAP PROJECTION IN THE NETHERLANDS AND THE GEOGRAPHIC COORDINATES φ, λ ON BESSEL'S ELLIPSOID. PLUMBLINE DEFLECTIONS*

The relations between the R.D.-coordinates X, Y in the stereographic map projection and the geographic coordinates φ (latitude) and λ (longitude with respect to Greenwich on Bessel's ellipsoid) are given in [110]. They run as follows:

$$\begin{aligned}\varphi &= 52^{\circ}09'22''.178 + 3236.033\,Y - 32.592\,X^2 - 0.247\,Y^2 - 0.850\,X^2Y \\ &\quad - 0.065\,Y^3 + 0.005\,X^4 - 0.017\,X^2Y^2, \\ \lambda &= -5^{\circ}23'15''.500 - 5261.305\,X - 105.979\,XY - 2.458\,XY^2 + 0.819\,X^3 \\ &\quad - 0.056\,XY^3 + 0.056\,X^3Y \quad \ldots\ldots\ldots\ldots \qquad (1)\end{aligned}$$

The unit for X and Y in the formulae is 100 km. The constant terms are the latitude φ and the longitude λ respectively of Amersfoort, the centre of the map projection (X = Y = 0). The corrections to the constant terms are expressed in seconds of arc. If the above formulae are used for the conversion of the R.D.-coordinates:

$$X = -61832.511, \qquad Y = +346.653$$

of the centre of the (former) meridian circle of Leiden observatory, then the geographic coordinates on Bessel's ellipsoid are:

$$\varphi_{\text{ellips}} = 52^{\circ}09'20''.935, \qquad \lambda_{\text{ellips}} = -4^{\circ}29'02''.270.$$

The latitude differs $\varphi_{\text{astro}} - \varphi_{\text{ellips}} = -1''.13$ from the astronomical latitude $\varphi_{\text{astro}} = 52^{\circ}09'19''.80$ in the Astronomical Ephemeris. The astronomical longitude $\lambda_{\text{astro}} = -0^{h}17^{m}56^{s}.15 = -4^{\circ}29'02''.25$ agrees excellently with the above amount $-4^{\circ}29'02''.27$.

The difference $\varphi_{\text{astro}} - \varphi_{\text{ellips}} = -1''.13$ in Leiden compels to a consideration of the constant terms in (1). The latitude of Amersfoort is derived from the astronomical latitudes φ_i determined in each of the stations 1 up to and including 14,

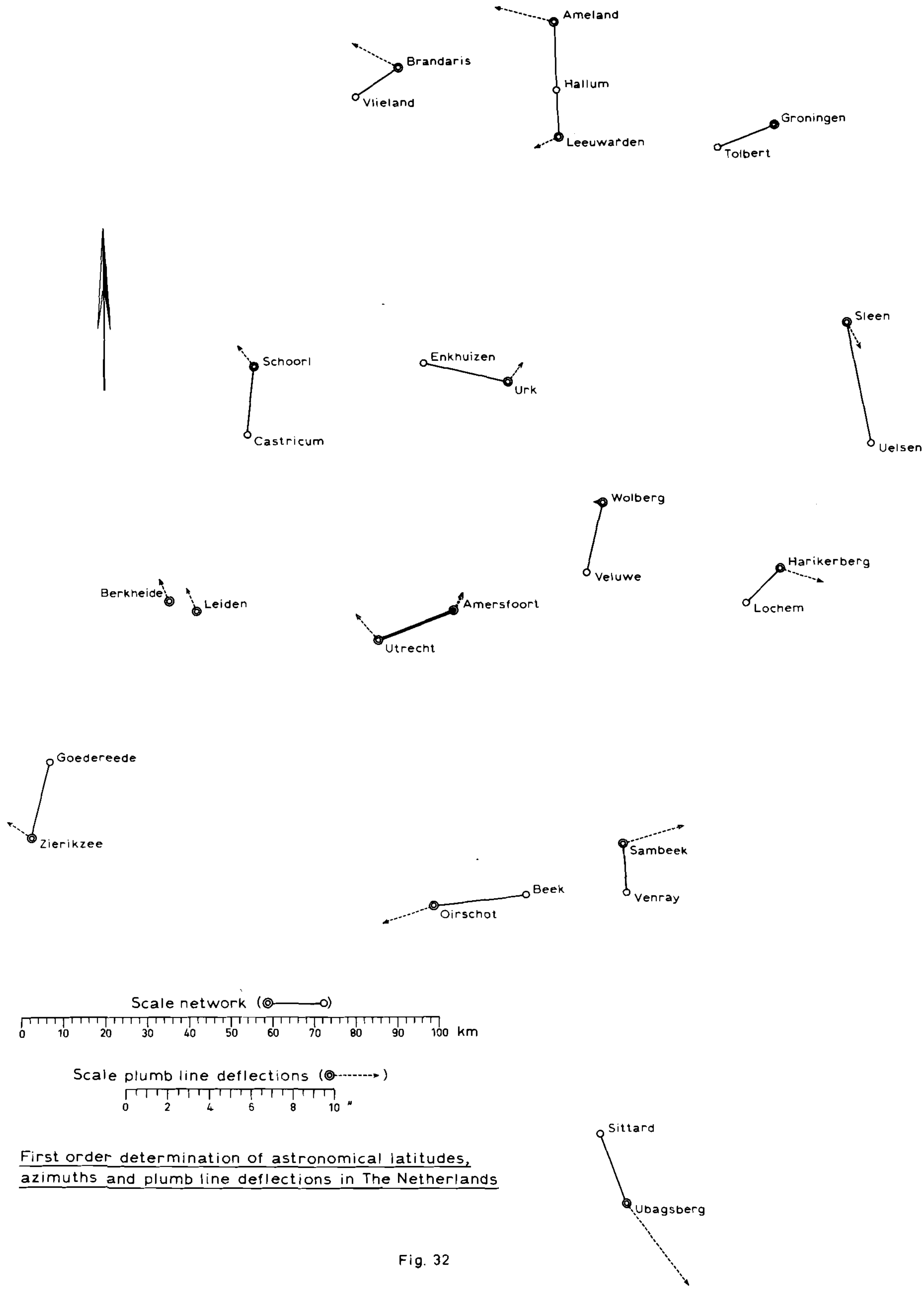

Ameland
Brandaris
Hallum
Vlieland
Groningen
Leeuwarden
Tolbert
Sleen
Schoorl
Enkhuizen
Urk
Castricum
Uelsen
Wolberg
Veluwe
Harikerberg
Berkheide
Leiden
Amersfoort
Lochem
Utrecht
Goedereede
Zierikzee
Sambeek
Beek
Venray
Oirschot
Scale network
0 10 20 30 40 50 60 70 80 90 100 km
Scale plumb line deflections
0 2 4 6 8 10 "
Sittard
Ubagsberg
First order determination of astronomical latitudes, azimuths and plumb line deflections in The Netherlands

Fig. 32

indicated with double circles in fig. 32. All of them are points of the R.D. first order (first order second rate) triangulation. The measurements at the station Ubagsberg (No. 14) were carried out in 1893 by J. Weeder, J.H. Wilterdink and the Leiden astronomer H.G. van de Sande Bakhuyzen [111], those at the stations 1 up to and including 13 between 1896 and 1899 by A. Pannekoek (later on professor of astronomy in Amsterdam) and R. Posthumus Meijjes [112]. The instruments used were the Repsold theodolite already used for Becker's determination of the azimuth Leiden - Delft (see fig. 25 in section 21) and the almost identical Repsold theodolite belonging to Utrecht observatory. The results of the measurements, reduced to the centre of the stations, are given on the pages 226 of [111] and 280 of [112]. They are also mentioned in column 3 of table 22. The astronomical latitude of Brandaris (the lighthouse on the isle of Terschelling) e.g. is: $\varphi_9$astro = $53^\circ 21' 40''.005 \pm 0''.059$. That of Utrecht is: $\varphi_2$astro = $52^\circ 05' 28''.583 \pm 0''.050$.

In the same fourteen stations astronomical azimuths $A_i$astro were measured to the stations marked with a single circle on the map in fig. 32. A résumé of the azimuths, reduced to the centres of the stations, is given on the pages 226^b of [111] and 281 of [112]. They are copied in column 7 of table 22. That from Brandaris to Vlieland is:

$$A_9\text{astro} = 235^\circ 26' 09''.450 \pm 0''.43.$$

The azimuth from Utrecht to Amersfoort is:

$$A_2\text{astro} = 68^\circ 22' 45''.588 \pm 0''.29.$$

From the amounts $\varphi_i$astro, $A_i$astro (i = 1,2,.......11,12,14) and the data of the triangulation network were determined 13 "astronomical" latitudes of Amersfoort and 13 "astronomical" azimuths Amersfoort - Utrecht. They are mentioned in the columns 4 and 8 respectively of table 22 and they are copied from the table on page 26 of [113]. φ_{13}astro and A_{13}astro, relating to the station Groningen, could not be used for the reduction to Amersfoort as a part of the data for this reduction was not available yet when the computation was done.

The mean "astronomical" latitude of Amersfoort is $52^\circ 09' 22''.178$, the mean "astronomical" azimuth Amersfoort - Utrecht is $248^\circ 35' 19''.891$. The two means - they are mentioned on the bottom row of table 22 in the columns 4 and 8 - are assumed to be the ellipsoidical latitude of Amersfoort and the ellipsoidical azimuth of the side Amersfoort - Utrecht of the triangulation network because $\varphi_{\text{amersfoort}}$ (φ_{17} in table 22) is the constant factor for φ in formula (1) of this section. The ellipsoidical latitudes of the other stations can be computed with formula (1). They can also be borrowed from the pages 205 - 208 of [114]. They are mentioned in column 5 of table 22; the ellipsoidical azimuths are given in column 9. The differences $\varphi_{\text{astro}} - \varphi_{\text{ellips}}$ and $A_{\text{astro}} - A_{\text{ellips}}$ are in the columns 6 and 10 respectively.

Table 22

Stations i No.	Name	φ_i astro	φ Amersfoort	φ_i ellips	φ_i astro minus φ_i ellips $= \xi$	A_i astro	A Amersfoort – Utrecht	A_i ellips	A_i astro minus A_i ellips	η	λ_i astro minus λ_i ellips
1	2	3	4	5	3–5=6	7	8	9	7–9=10	11	12
1	Oirschot	51°30′16″.849±0″.037	52°09′22″.947	16″.042	+0″.81	83°22′58″.287±0″.31	248°35′22″.992	55″.174	+3″.113	+2″.48	−3″.98
2	Utrecht	52 05 28.583±0.050	20.856	29.868	−1.28	68 22 45.588±0.29	21.188	44.206	+1.382	+1.08	−1.75
3	Sambeek	51 38 11.173±0.078	21.276	12.024	−0.85	176 28 02.515±0.29	16.253	06.206	−3.691	−2.92	+4.71
4	Wolberg	52 23 27.577±0.123	22.107	27.602	−0.02	193 28 52.167±0.29	20.302	51.607	+0.560	+0.43	−0.71
5	Harikerberg	52 14 12.788±0.071	22.851	12.065	+0.72	225 53 32.296±0.39	17.126	34.963	−2.667	−2.07	+3.37
6	Sleen	52 46 38.653±0.044	23.466	37.334	+1.32	170 49 27.868±0.45	18.871	28.780	−0.912	−0.69	+1.15
7	Schoorl	52 41 54.834±0.093	21.082	55.873	−1.04	185 33 01.225±0.34	20.804	00.208	+1.017	+0.77	−1.28
8	Zierikzee	51 39 03.558±0.085	21.339	04.346	−0.79	12 40 07.387±0.31	21.105	05.987	+1.400	+1.11	−1.79
9	Brandaris	53 21 40.005±0.059	20.937	41.225	−1.22	235 26 09.450±0.43	22.442	06.470	+2.980	+2.22	−3.71
10	Ameland	53 27 30.249±0.026	21.431	30.999	−0.75	179 05 52.922±0.20	22.424	49.066	+3.856	+2.86	−4.80
11	Leeuwarden	53 12 15.283±0.075	22.707	14.750	+0.53	358 31 57.632±0.35	20.291	56.056	+1.576	+1.18	−1.97
12	Urk	52 39 42.879±0.082	21.178	43.830	−0.95	283 18 02.563±0.35	18.769	03.489	−0.926	−0.71	+1.16
13	Groningen	53 13 13.746±0.041	———	13.782	−0.04	249 20 06.902±0.50	———	07.129	−0.227	−0.17	+0.28
14	Ubagsberg	50 50 53.20±0.045	26.134	49.180	+4.02	340 37 42.88±0.20	16.012	46.500	−3.62	−2.95	+4.67
15	Leiden	52 09 19.80	———	20.93	−1.13	———	———	———	———	+0.48	−0.78
16	Berkheide	(52 10 37.45)	———	38.640	(−1.19)	———	———	———	———	(+0.48)	(−0.78)
17	Amersfoort		52°09′22″.178	22″.178	0.00		248°35′19″.891			0.00	0.00

The disharmony between the sometimes large differences in the columns 6 and 10 and the small standard deviations in the columns 3 and 7, especially for Ubagsberg in the hilly southern part of the province of Limburg, points to plumbline deflections. They can be computed from the differences $\varphi_{astro} - \varphi_{ellips}$ (column 6), $A_{astro} - A_{ellips}$ (column 10), and φ_{ellips} (column 5) in table 22 with the formulae:

$$\xi = \varphi_{astro} - \varphi_{ellips} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\eta = (A_{astro} - A_{ellips}) \cot \varphi_{ellips} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\eta = -(\lambda_{astro} - \lambda_{ellips}) \cos \varphi_{ellips} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

ξ and η in these formulae are the components of the relative plumbline deflections with respect to Amersfoort on Bessel's ellipsoid in the directions south-north and west-east respectively. From the two expressions (3) and (4) for η follows the well-known equation of Laplace [115] :

$$(A_{astro} - A_{ellips}) + (\lambda_{astro} - \lambda_{ellips}) \sin \varphi_{ellips} = 0 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the above formulae northern latitudes φ and western longitudes λ are counted positive. For Ubagsberg, e.g. $\xi = +4''.02$ (column 6 of table 22), $\eta = -2''.95$ (column 11) and $\lambda_{astro} - \lambda_{ellips} = +4''.67$ (column 12).

The constant term $-5^\circ 23' 15''.500$ in the expression for λ in (1) is the "astronomical" longitude of Amersfoort. It is the rounded-off sum of the differences between the astronomical longitudes of the meridian circle at Leiden Observatory and Greenwich ($-0^h 17^m 56^s.15 = -4^\circ 29' 02''.250$) and the ellipsoidical longitudes of Amersfoort and Leiden ($-0^\circ 54' 13''.228$). The first amount was determined in 1880 and 1881 by H. G. van de Sande Bakhuyzen, the then director of Leiden Observatory [116] . The second amount was computed from the results of the first order R. D. -measurements on Bessel's ellipsoid.

The above computation of Amersfoort's longitude is indeed correct if the differences $\lambda_{astro} - \lambda_{ellips}$ in Amersfoort and Leiden respectively are the same. At Leiden Observatory this could have been checked by measuring an accurate astronomical azimuth as the accurate astronomical latitude of the station was already known ($\varphi_{astro} - \varphi_{ellips} = \xi = -1''.13$). Both the latitude and the azimuth would have given the opportunity to join Leiden to the stations 1 - 12 and 14, already mentioned in table 22 and to determine the components of the plumbline deflection in that station. For the latter determination, however, the measurement of the azimuth is not necessary as the component η can also be computed from the difference between the astronomical longitudes of Ubagsberg and Leiden, given in page 101 of [111] . It amounts to:

$$-5^m 52^s.314 \pm 0^s.015 = -1^\circ 28' 04''.71 \pm 0''.22 \ .$$

The ellipsoidical difference in longitude between the stations according to [114] (pages 206 and 208) is:

$$-0^{\circ}33'56''.926 - 0^{\circ}54'13''.229 = -1^{\circ}28'10''.16 .$$

The difference between the two amounts is:

$$(\lambda_{astro} - \lambda_{ellips})\ \text{Ubagsberg} - (\lambda_{astro} - \lambda_{ellips})\ \text{Leiden} = +5''.45 .$$

As for Ubagsberg $(\lambda_{astro} - \lambda_{ellips}) = +4''.67$ (see above), the corresponding amount for Leiden is $-0''.78$ sothat, according to (4), $\eta = +0''.48$ (table 22, column 11). ξ and η correspond with the amounts $\xi = -\theta \cos \epsilon$ and $\eta = -\theta \sin \epsilon$, given on page 9 of [117] (Delft 1927). $\theta = 1''.231$ in these expressions is the plumbline deflection, $\epsilon = 337^{\circ}13'$ is the azimuth of the vertical.

If the plumbline deflection in Leiden would have been taken into account, then Becker's astronomical azimuth in the preceding section would possibly have been somewhat larger than the ellipsoidical azimuth. For, according to (3):

$$A_{astro} - A_{ellips} = \eta \tan \varphi = +0''.48 \tan \varphi = +0''.62.$$

It is, however, about $1''.8$ smaller. The conclusion can therefore be maintained, that the mean reason of the too small astronomical azimuth is the wrong reduction of the measurements to the centre of the sphere on the tower of the church at Delft. In each of the stations 1 - 15 the plumbline deflections are shown in fig. 32. A similar map was already given in Appendix 3 of Vening Meinesz' [118] publication Observations de pendule dans les Pays Bas 1913 - 1921 (Delft 1923).

An important contribution to the knowledge of plumbline deflections in the Netherlands was made in 1974 by the Netherlands Geodetic Commission. As a supplement of the stations 1 - 14 in table 22 the astronomical latitudes and longitudes of 14 other first order R. D. stations were determined simultaneously applying the method of Gauss. The measurements were carried out by Mr. C. de Vries and the instrument was a Zeiss - Ni2 astrolabe.

The constant zenith distance for carrying out the observations is about 30°. In eccentric points of each of the fourteen R. D. -stations the instrument was set up on a tripod. The transit of each of the selected stars in the programme was observed on each of the five wires of the instrument. The moment of transit was registered with a signal key on an Omega-chronograph. The astronomical coordinates φ and λ of the eccentric stations were reduced to centre by centering measurements .

Of the stations Berkheide and Amersfoort provisional results were computed. Provisional results, not only because the astronomical measurements will be repeated in 1975, but also because Mr. De Vries' personal error could not be determined yet owing to the bad weather conditions in the autumn of 1974. Berkheide was chosen because it is situated at only 6.9 km westnorthwest of Leiden Observatory (former meridian circle). The choice of Amersfoort is obvious. Mr. De Vries' measurements give an excellent check on the latitude $\varphi = 52^{\circ}09'22''.178$ derived in table 22 and on the longitude $\lambda = -5^{\circ}23'15''.500$ used in the R.D.-computations according to formula (1), page 132. Mr. De Vries' provisional results run:

Berkheide : $\varphi = 52^{\circ}10'37''.45 \pm 0''.27$, $\lambda = -4^{\circ}23'18''.69 \pm 0''.22$,
Amersfoort : $\varphi = 52^{\circ}09'21''.31 \pm 0''.19$, $\lambda = -5^{\circ}23'10''.35 \pm 0''.16$.

The latitude of the stations is about the same as that of Leiden ($\varphi = 52^{\circ}09'19''.80$).

Because of the measuring method used the personal error of the observer is of little or no influence on the determinations of the astronomical latitudes but it does affect the results of the astronomical longitudes. Berkheide's latitude can easily be verified if, because of the small distance to Leiden observatory, it is assumed that the south-north component φ_{astro}Leiden $- \varphi_{ellips}$Leiden $= \xi =$ $= -1''.13$ (see table 22, column 6) is equal to the corresponding amount at Berkheide. As for Berkheide $\varphi_{ellips} = 52^{\circ}10'38''.64$ (see [114], page 205), φ_{astro} for that station is $52^{\circ}10'38''.64 - 1''.13 = 52^{\circ}10'37''.51$, which corresponds excellently with the latitude $\varphi = 52^{\circ}10'37''.45$ found by De Vries.

If also (see table 22, column 12):

$$\lambda_{astro}\text{Leiden} - \lambda_{ellips}\text{Leiden} = -0''.78 = \lambda_{astro}\text{Berkheide} - \lambda_{ellips}\text{Berkheide},$$

then:

$$\lambda_{astro}\text{Berkheide} = \lambda_{astro}\text{Leiden} - (\lambda_{ellips}\text{Leiden} - \lambda_{ellips}\text{Berkheide}) - 0''.78 =$$
$$= -4^{\circ}29'02''.25 + 5'40''.69 - 0''.78 = -4^{\circ}23'22''.34 .$$

The difference $-3''.65$ $(-0^{s}.24)$ with the longitude $-4^{\circ}23'18''.69$ found by De Vries might be due to his personal error. I assumed that in the difference between the longitudes of Amersfoort and Berkheide: $(-5^{\circ}23'10''.35 \pm 0''.16) - (-4^{\circ}23'18''.69 \pm$ $\pm 0''.22) = -0^{\circ}59'51''.66 \pm 0''.27$, this personal error is eliminated. As Berkheide and Amersfoort are the first and the second station measured (on June 4th and June 14th 1974 respectively), this supposition is quite acceptable. Amersfoort's astronomical longitude in that case is:

$$\lambda_{astro}\text{Amersfoort} = -4^{\circ}23'22''.34 - 0^{\circ}59'51''.66 = -5^{\circ}23'14''.00.$$

It can be used for a provisional determination of the plumbline deflection at Amersfoort. For λ_{ellips} Amersfoort in that computation the amount $-5^{\circ}23'14''.70$ should be used instead of $-5^{\circ}23'15''.50$ because for the ellipsoidical longitude of Leiden in the R.D.-computation a value of $-4^{\circ}29'02''.25$ was used instead of $-4^{\circ}29'02''.25 +$ $+0''.78$. As, according to (4), page 136:

$$\eta = -(\lambda_{astro} - \lambda_{ellips}) \cos \varphi_{ellips},$$
$$\eta = -0''.70 \cos 52^{\circ}09'22'' = -0''.43.$$

The south-north component ξ is:

$$\xi = \varphi_{astro} - \varphi_{ellips} = 52^{\circ}09'21''.31 - 52^{\circ}09'22''.18 = -0''.87\ .$$

The (provisional) plumbline deflection is drawn with a dashed bold line on the map in fig. 32. The bold line marks the difference with the plumbline deflections at the other astronomical stations, drawn with respect to a plumbline deflection zero at Amersfoort. If Mr. De Vries' measurements and the above suppositions are confirmed by the measurements in 1975 then:

$$\lambda_{astro}\text{Amersfoort} - \lambda_{astro}\text{Leiden} = -0^{\circ}54'11''.75$$

deviates $1''.48$ from the corresponding difference $-0^{\circ}54'13''.23$ between the ellipsoidical longitudes found from the R.D.-network. As the plumbline deflections at Leiden and Utrecht have about the same size and the same direction, the considerable difference $1''.75 + 0''.70 = 2''.45$ between the astronomical and ellipsoidical differences in longitude between Amersfoort and Utrecht must be imputed to the opposite directions of the η-components of the plumbline deflections at Utrecht and Amersfoort ($\eta = +1''.08$ and $\eta = -0''.43$ respectively), though the stations are situated at only 19.6 km from each other.

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[2] N. VAN DER SCHRAAF: De toetreding van Nederland tot de Middeneuropese Graadmeting. This paper will be published in the first half of 1975.

[3] J.J. BAEYER (1794-1885), alumnus of the German geodesist F.W. Bessel (1784-1846) and later his collaborator in the East Prussian triangulation.

[4] F. KAISER en L. COHEN STUART: De Eischen der medewerking aan de ontworpen graadmeting in Midden Europa voor het Koningrijk der Nederlanden, Amsterdam, 1864.

[5] F. KAISER (1808-1872), Dutch astronomer and director of Leiden Observatory.

[6] LEWIS COHEN STUART, born July 11th 1827 at The Hague, died at Delft July 24th 1878. In 1864 professor of geodesy and director of the Delft Polytechnical School. In 1874 charged with the first first-order levelling in the Netherlands.

[7] Archives Netherlands Geodetic Commission, file Kaiser, letter No. 65.

[8] Archives Netherlands Geodetic Commission, file Kaiser, letter No. 7.

[9] The letter is dated February 8th 1866. It is in the archives of the Netherlands Geodetic Commission, file Kaiser, letter 110a.

[10] Archives Netherlands Geodetic Commission, Collection Stamkart. Corr. file I.

[11] Archives Netherlands Geodetic Commission, Collection Schols, Correspondence. Triangulation I, letter No. 201.

[12] CHARLES MATHIEU SCHOLS, born March 28th 1849 at Maastricht, died March 17th 1897 at Delft. Professor of Geodesy at the Delft Polytechnical School from 1878 till his death.

[13] JEAN ABRAHAM CHRÉTIEN OUDEMANS, born December 16th 1827 at Amsterdam, died December 14th 1906 at Amsterdam. Dutch astronomer; director of the triangulation of Java in the former Dutch East Indies. Director of Utrecht Observatory.

[14] Year 1885, pages 92-98.

[15] Library of the Subdepartment of Geodesy of the Delft University of Technology (Delft 1953, pages 37-59).

[16] The R.D.-first order triangulation network was measured by the Netherlands Commission for Triangulation and Levelling.

[17] Nieuw Nederlands biografisch Woordenboek, volume I, columns 1486-1487.

[18] A. HUET: F.J. Stamkart (periodical "Eigen Haard", year 1882, pages 113-115, with portrait).

[19] Bouwstoffen voor de geschiedenis van de Levensverzekeringen en Lijfrenten in Nederland, bijeengebracht door de directie van de Algemeene Maatschappij van Levensverzekering en Lijfrente, gevestigd te Amsterdam, Damrak 74 (year 1897, pages 200-211, with portrait).

[20] By its letter No. 8851, dated September 26th 1974, the Greek Embassy at The Hague informed me that near the western coast of Chio(s) island an islet exists with the name Aspro Ke Kokkino. It is indicated as Aspronisos on the British Admiralty Chart No. 1645, 1967 edition. Its latitude is + 38°19'05", its longitude 25°56'36" east of Greenwich.

[21] Information from the Municipal Landsurvey department in Amsterdam.

[22] Archives Netherlands Geodetic Commission, Correspondence file O, letter 3-b.

[23] Achtergracht 808 = Frederiksplein 2 (plein = square). Information received from the Municipal Landsurvey department in Amsterdam.

[24] Royal Decree dated July 26th 1867, No. 65.

[25] Address on some envelopes in file Groningen 1879. The Municipal Landsurvey department informed me that the number is still the same as in 1879. The cadastral number of the lot is E 8328.

[26] Diary III, October 8th 1878.

[27] Correspondence Van Hees, file He letter 30.Archives Netherlands Geodetic Commission.

[28] The archivist of the town of Delft informed me that "the terrible disease" was the cholera. In June 1866 it was at its worst and it made a great many of victims.

[29] Archives Netherlands Geodetic Commission, letter dated May 29th 1873 in file O 11-k.

[30] Decree dated June 12th 1873 No. 33 Registration and Domains in File O 11-n. Archives Netherlands Geodetic Commission.

[31] Diary II, July 14th 1874.

[32] Letter minister of war , dated June 21st 1876, department Staff and Military affairs in file O 14-c.Archives Netherlands Geodetic Commission.

[33] Report over the year 1874, dated July 12th 1875 in file O 13-a. Archives Netherlands Geodetic Commission.

[34] Diary II, July 1st 1874.

[35] [15] pages 39-40.

[36] Diary II, Oldeholtpa, July 17th - July 22nd 1877.

[37] [1], page 17.

[38] [1], fig. 2.

[39] Description of the instrument by mr. P. van der Star, custodian of the museum.

[40] File O, letter 4-a, dated April 10th 1866.

[41] [15], page 42.

[42] Diary II, June 18th and 19th 1873.

[43] Diary II, September 15th 1875.

[44] Bill dated May 15th 1872 in file O, 10-a.

[45] Register of observations 1870-1871, page 51.

[46] e.g. the direction Hornhuizen at the station Uithuizermeden in Register of observations 1878-1880 page 272.

[47] Register of observations 1870-1871, pages 86, 87.

[48] Register of observations 1878-1880, pages 264-281.

[49] From 9.45 - 12.00 a.m. and from 2.00 - 4.15 p.m.; see Register of observations 1875, 1876 I, pages 261-266.

[50] They are not the same as those on page 3 of "Couvert A", as Stamkart reduced each of his 55 observations to the centre of the tower. Thereafter he computed

the mean of the reduced directions. It would have been much easier if he had given the reduction to centre after the computation of the mean directions in the eccentric point. It would have saved writing work and lessened the possibility of making calculation errors. If, however, the reductions in question are taken into account, Stamkart's amounts agree with mine.

[51] They are given in [1], table 15, column 11, pages 88-108.

[52] File 1, cahier C, page 8.

[53] File 1, cahier C, page 4.

[54] File 1, cahier C, page 5.

[55] Stamkart made a mistake of 45° in the computation on page 3 of his cahier D, so that all δ_2's found on that page are wrong.

[56] A. P., at present called N. A. P. (Normaal Amsterdams Peil) agrees with the mean high tide in summer of the Y in Amsterdam when this water was still in open communication with the former Zuiderzee. See e. g. P. I. van der Weele: De geschiedenis van het N. A. P. (Publication of the Netherlands Geodetic Commission). The German Normal Null corresponds with (N.) A. P.

[57] Section of Physics, pages 267-294.

[58] Diary, part I, page 59.

[59] Correspondence file O, letter 6-l.

[60] Correspondence file O, letter 14-d, dated October 7th 1876.

[61] Correspondence file O, letter 15-k, dated January 26th 1877.

[62] Correspondence file O, letters 6-p, 6-q and 7-d.

[63] Diary part I, July 20th 1868, page 73.

[64] Correspondence file O, letter 6-i, page 6.

[65] Portfolio Base measurement 1869, file Computation of the large difference in section E in 1868 and 1869.

[66] Correspondence file O, letter 9-1, pages 7 and 8.

[67] Report of the Rijkscommissie voor Graadmeting en Waterpassing over the year 1889, page 26.

[68] In portfolio 1870, Computation A1, pages 93-103; see also the pages 49-92.

[69] [68], page 101.

[70] Portfolio A2, page 106.

[71] Correspondence file O, letter 10-a, last page.

[72] [1], page 40.

[73] See formulae 8, 9 and 10 on page 80 of [1].

[74] [1], page 133.

[75] Register of Observations 1870-1871, pages 49, 59.

[76] They are known there as Haarlemmermeer 10 and 11.

[77] File 11, cahier F, page 50.

[78] See also: N. D. HAASBROEK: Gemma Frisius, Tycho Brahe and Snellius and their triangulations (Publication of the Netherlands Geodetic Commission), fig. 55, page 112.

[79] [1] , page 135.

[80] HENDRIK JAN HEUVELINK , born September 3rd 1861, died July 26th 1949. In 1884 he joined the Rijkscommissie voor Graadmeting en Waterpassing and worked on the first order triangulation under the direction of Schols. After Schols' death in 1897 he was appointed professor of geodesy at the Polytechnical School and leader of the triangulation.

[81] [1] , pages 77-84.

[82] [1] , fig. 18, page 131.

[83] [1] , section 20, page 132.

[84] [1] , page 135.

[85] Information from the archivist of the town of Delft.

[86] Information from the archivist of the town of Hoorn.

[87] Mr. dr. J. BREGMAN : Schagen door de eeuwen heen (Wormerveer 1965), page 165. Pictures of the tower next to the pages 196 and 197.

[88] Correspondence, file O, letter 13-h.

[89] Correspondence, file O, letter 13-i.

[90] [1] , page 17.

[91] Diary volume II.

[92] Correspondence, file O, letter 16-d.

[93] [1] , page 147, triangle 116.

[94] [1] , page 138.

[95] For Krayenhoff's differences with the R. D.-points see [1] , table 26, page 137. Stamkart's differences with the R. D.-points are borrowed from table 16.

[96] [1] , pages 38 and 39.

[97] [1] , page 48, fig. 9.

[98] Jordan's Handbuch der Vermessungskunde I, Stuttgart 1920, page 324.

[99] Letter No. 3889, dated June 10th 1929, Archives Netherlands Geodetic Commission.

[100] Annalen der Sternwarte in Leiden, zweiter Band (1870), page IV.

[101] Dangast-observatory, about 6 km north of Varel in northwestern Germany. Varel is the station No. 103 of Krayenhoff's triangulation (see [1], fig. 2). See General Bericht über die Mittel Europäische Gradmessung für das Jahr 1866, page 22, and Appendix IA and IB of that publication.

[102] Annalen der Sternwarte in Leiden, erster Band, pages V and VI.

[103] Astronomisch-geodätische Arbeiten in den Jahren 1869-1872, page 194.

[104] Annalen II, page [207] .

[105] See the plan of the building (picture II, fig. 4) in Annalen I.

[106] Annalen I page XLIX and Annalen II page [207] .

[107] Kaiser in his computations uses "probable errors" r instead of standard deviations M, not only on page [210] , but everywhere in his publication. The relation between the two can be expressed by r = 0. 6745 M. (See e. g. Jordan's Handbuch der Vermessungskunde, erster Band Ausgleichungsrechnung, Stuttgart 1920, page 595).

[108] J. D. VAN DER PLAATS: Overzicht van de graadmetingen in Nederland (Tijdschrift voor Kadaster en Landmeetkunde, 1889, pages 217-243, 256-306, and 1891, pages 65-101, 109-133).

[109] Because of the small distances DC and DA and the small deviation between the amounts CD = 13.678 m (Kaiser) and CD = 13.667 m (R. D.), the R. D.-angle CDA = 58°33'36" differs slightly from that derived from Kaiser's observations.

[110] Handleiding voor de Technische Werkzaamheden van het Kadaster (H. T. W. 1956), page 6.

[111] Déterminations de la différence de longitude Leyde-Ubagsberg, de l'azimut de la direction Ubagsberg-Sittard et de la latitude d'Ubagsberg (Delft 1905).

[112] Détermination de latitude et d'azimut dans les Pays Bas (Delft 1904).

[113] Hk. J. HEUVELINK: De stereografische kaartprojectie in hare toepassing bij de Rijksdriehoeksmeting (Delft 1918).

[114] Rijksdriehoeksmeting 1885-1928. Staten van waarnemingen en uitkomsten (Lists of observations and results).

[115] PIERRE SIMON, MARQUIS DE LAPLACE (1748-1827), famous French mathematician and astronomer.

[116] Détermination de la différence de longitude Leyde-Greenwich, exécutée en 1880 et 1881 par H. G. et E. F. van de Sande Bakhuyzen, published in Annalen der Sternwarte Leiden, Band VII (1897), page 245.

[117] Travaux géodésiques exécutées aux Pays Bas 1924, 1925 et 1926. Note présentée à la troisième assemblée générale de la section de géodésie de l'Union géodésique et géophysique internationale. Prague, September 1927 (Delft 1927).

[118] FELIX ANDRIES VENING MEINESZ (July 30th 1887-August 10th 1966), Dutch geophysicist, especially known for his investigations of gravity at sea.