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THE MODIFIED ASTROMETRIC PROCEDURE OF SATELLITE PLATE REDUCTION AS APPLIED AT THE KOOTWIJK OBSERVATORY OF THE DELFT GEODETIC INSTITUTE

A description with some results for SAOP

by

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CONTENTS

SUMMARY

The camera station of the Geodetic Institute of Delft University of Technology has participated in a number of internationally coordinated geodetic satellite observation programmes. In a previous publication [1] contributions to several international observation programmes were discussed. The present publication concentrates on the methods and formulas applied for the reduction of observations made for the European Short Arc Observation Programme (SAOP). These photographic satellite observations have been made at two observing-sites i.e. Delft-Ypenburg and Kootwijk, in the period April 1971–April 1974.

Some results of observations and computations are given.

THE MODIFIED ASTROMETRIC PROCEDURE OF SATELLITE PLATE REDUCTION, AS APPLIED AT THE KOOTWIJK OBSERVATORY OF THE DELFT GEODETIC INSTITUTE

1 Introduction

This publication was already announced in [1]. It is a follow-up of [1]; special attention will now be given to the system of modified reduction formulas by means of which the initial information of photographs taken for the current European Short Arc Observation Programme (SAOP) are reduced. The photographs referred to are those taken with the equatorially mounted TA-120 concentric-mirror type camera (Bouwers-Maksutov).

The correction-formulas, used in connection with the reduction-formulas (section 5) are to a great extent those used by SCHÜRER [2]. The precision aimed at is better than $0''.1$ in directions and better than $0^s.0001$ in time. For this reason the precision in computation is better than $0''.01$.

The observations have been made from two different observing-sites:

- DELFT, YPENBURG, until 1st December 1973;
- KOOTWIJK (Fig. 1), from 1st December 1973 onwards.

For the correct location data, see [3].

In this publication the parts dealing with the reduction methods are similar to those given in [1]; nevertheless for convenience of the reader these parts are repeated here. Quite different compared to [1] are the sections 5, 7 and 8; here the formulas for conversion to fixed-Earth reference, as used for SAOP, and results are discussed. Just like [1], the present publication must be considered as an account of work; in this case covering the period April 1971–April 1974.

2 Some technical details

An astrometric (short-Turner) method is applied to reduce stellar oriented photographic satellite plates to fixed-earth station-to-satellite directions. The plates considered are those taken with the TA-120 camera in use by the Delft Geodetic Institute.

The relevant optical features of the TA-120 camera are:

optics:	Bouwers-Maksutov concentric mirror
focal length:	120 cm
effective aperture:	21 cm
field:	$5° \times 5°$ spherical.

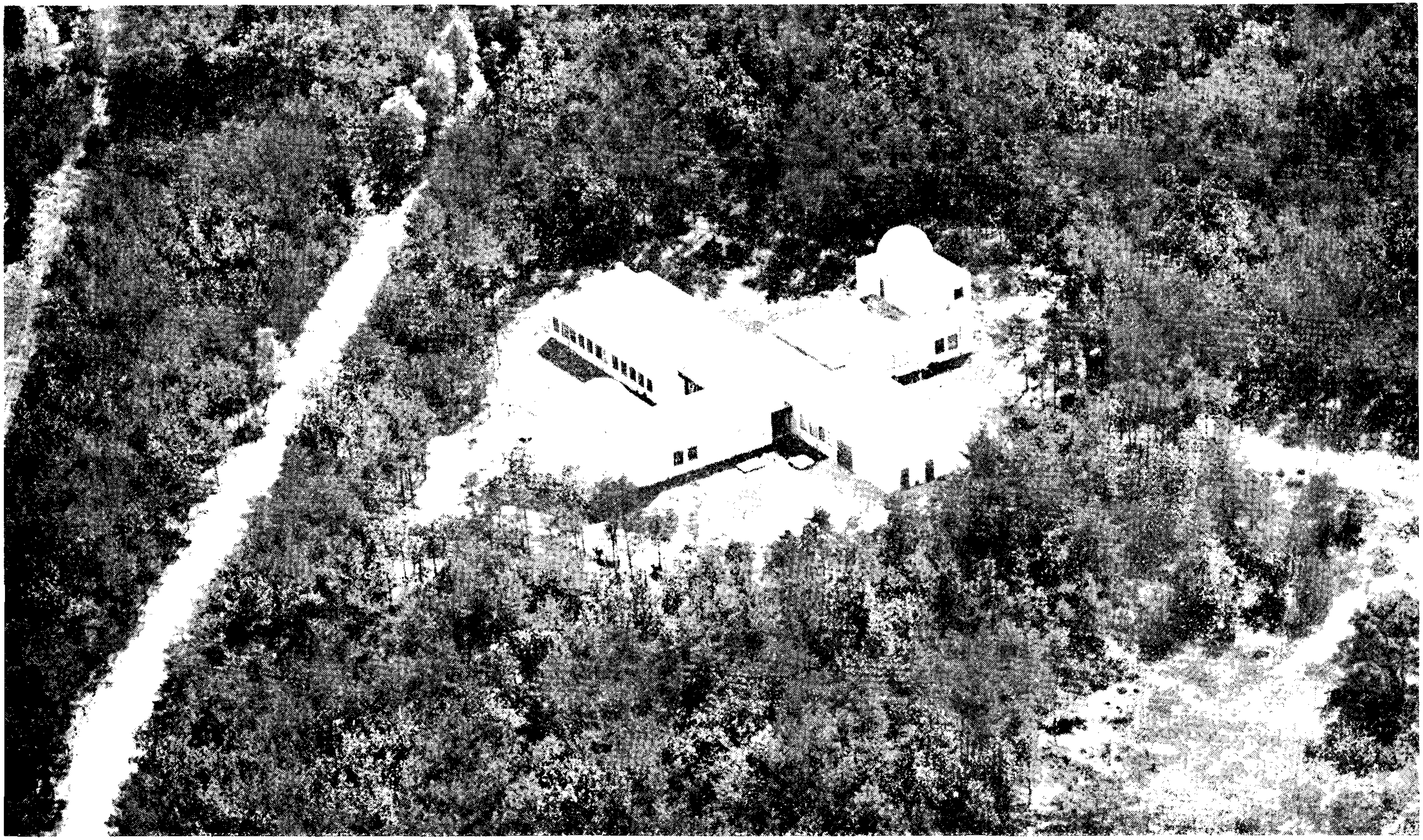

Fig. 1. The Kootwijk Observatory.

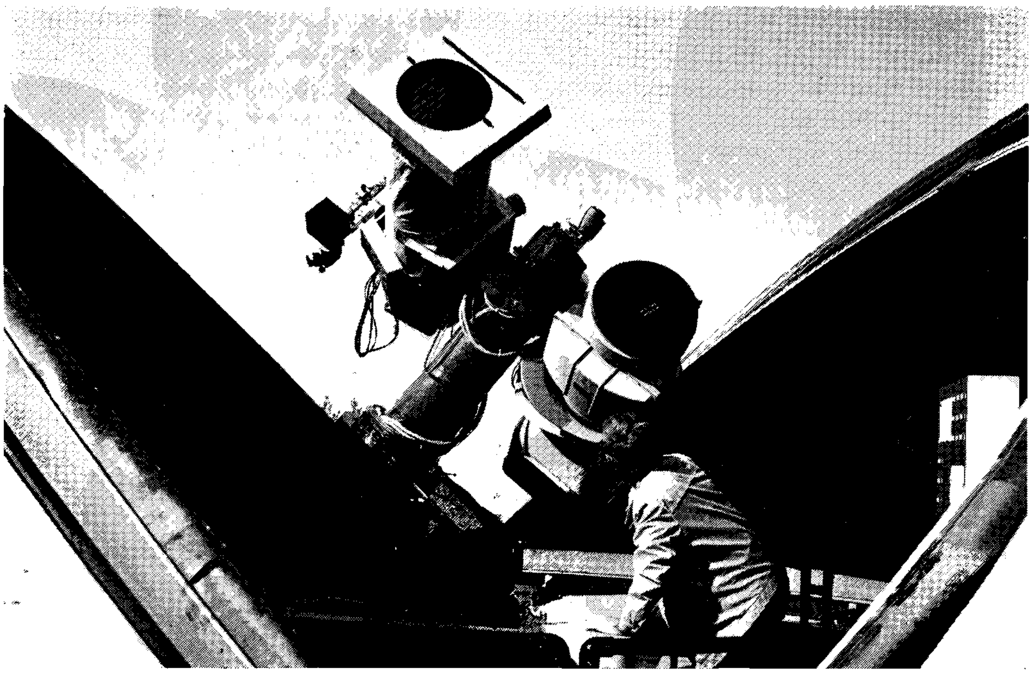

Fig. 2 Camera dome of Kootwijk Observatory

The camera operates on 5 inches wide roll-film (currently Kodak 2475 Estar Base), each frame before exposure being pressed to assume a spherical shape with about 240 cm radius of curvature.

The camera is mounted equatorially and driven at the sidereal rate.

With optically passive satellites, timing of satellite images is achieved by means of a focal plane chopper of special design. The essential point here is that during the exposure of the satellite trail two narrow strips of light weight material (about 2 cm wide), mutually separated by about 0.5 cm between the strips, periodically chop the light beam from the satellite just before it could reach the focal plane. This produces each time a satellite image in the center of a trail interruption. Time control of the chopper is obtained by recording photo-electronically the instants that the twin-strips assume four selected and equally spaced calibration positions, known geometrically with respect to the camera's fiducial marks. Finally numerical interpolation yields for each satellite image the time instant at which the chopper occupied the position in which it produced that image.

Each successful film frame is copied onto a glass plate, which is subsequently measured on a Mann 422F XY-comparator. A plate is measured in two positions, mutually rotated over about 180°. For each satellite image six reference stars are selected, evenly distributed with respect to the satellite image and as close to it as is practical. Throughout the measurements and the subsequent calculations each satellite image with its reference stars is treated individually. However, it proved inevitable to assign identical reference stars to adjacent satellite images. The sequence of measurement is as follows: satellite image-reference stars-reference stars in reversed order-satellite image. This cycle is repeated once, before the operator proceeds to the next satellite image. When all satellite images have been treated in this way the entire procedure is performed with the plate rotated through about 180°.

Reference star positions are taken from a magnetic tape version of the SAO star catalog by means of a computer search programme.

The computational part of the plate reduction is performed in three computer programmes briefly indicated by:

1. "Time reduction"
2. "Position reduction"
3. "Conversion to fixed-Earth reference".

In the following three sections these programmes are described consecutively, giving details of the formula's used.

3 Time reduction

This programme reduces the chopper-time records to time instants related to the satellite positions recorded on the photographic plate. Corrections are applied for receiver delay, propagation time of the 75 kHz HBG time signal from Neuchâtel to Delft and for the time difference between *UTC* and the HBG emission.

Suppose the chopper traverses the rectangular coordinate system defined by the fiducial marks in x-direction and specify the calibration positions by x^1, x^2, x^3, x^4 respectively.

Denote the instants recorded for these positions and for satellite image sub. i (approximate coordinates x_i, y_i) by respectively:

$$t_0+\Delta t_i^1, t_0+\Delta t_i^2, t_0+\Delta t_i^3, t_0+\Delta t_i^4$$

Then, disregarding receiver delay and emission and propagation corrections, a provisional time instant $\bar{t}_i$ for the recording of image sub. i is obtained from:

$$\bar{t}_i = t_0+\Delta t_i \quad (3.1)$$

in which:

$$\Delta t_i = ((x_i)^2, x_i, 1)\cdot \underline{a} \quad (3.2)$$

with:

$$\underline{a} = (M^*\cdot M)^{-1}\cdot M^*\cdot \underline{t} \quad (3.3)$$

if:

$$M = \begin{pmatrix} (x^1)^2 & x^1 & 1 \\ (x^2)^2 & x^2 & 1 \\ (x^3)^2 & x^3 & 1 \\ (x^4)^2 & x^4 & 1 \end{pmatrix} \quad \text{and} \quad \underline{t} = \begin{pmatrix} \Delta t_i^1 \\ \Delta t_i^2 \\ \Delta t_i^3 \\ \Delta t_i^4 \end{pmatrix} \quad (3.4)$$

Until 1st May 1972 a local time standard was, by means of a variable delay, brought in temporary synchronism with the received HBG time signals, just before a satellite observation.

* Indicating transposition.

Hence until that date, in order to relate the satellite image recording instants to *UTC*, $\bar{t}_i$ had to be corrected as follows:

$$t_i = \bar{t}_i + \Delta_d + \Delta_p + E \qquad (3.5)$$

in which:

Δ_d = receiver delay = 1.5 ms
Δ_p = propagation correction = 2.2 ms
E = "*UTC*-signal" as published by BIH in circular D.

Since 1st May 1972 a rubidium time and frequency standard (HP 5065 A) is used to keep *UTC* between periodic flying clock visits. This technique meets the needs of satellite photography to an extent that corrections from $\bar{t}_i$ to t_i could be omitted since that date.

4 Position reduction

This programme reduces plate measurements of satellite and star images to provisional topocentric geometric satellite directions referred to the astrometric system adopted for the SAO catalog (equinox 1950.0, system FK4). The directions are provisional in that no corrections will be applied for annual aberration, diurnal aberration, light travel time, parallactic refraction and satellite phase.

Denote plate measurement positions by I and II respectively.

Denote arithmetic means over all four satellite and star image measurements expressed in mm and after division by the focal length (1200 mm) as follows:

satellite image sub. i:

$$\bar{X}^{\mathrm{I}}_i, \bar{Y}^{\mathrm{I}}_i; \qquad \bar{X}^{\mathrm{II}}_i, \bar{Y}^{\mathrm{II}}_i$$

image star sub. k as related to satellite image sub. i:

$$\bar{x}^{\mathrm{I}}_{i,k}, \bar{y}^{\mathrm{I}}_{i,k}; \qquad \bar{x}^{\mathrm{II}}_{i,k}, \bar{y}^{\mathrm{II}}_{i,k}$$

If *MJD* is the Modified Julian Date of observation (integer number), then stellar positions updated for proper motion are:

$$\left.\begin{aligned} \alpha_k &= \alpha_{1950,k} + \tau\mu_k \\ \delta_k &= \delta_{1950,k} + \tau\mu'_k \end{aligned}\right\} \qquad (4.1)$$

where (omitting subscript k) α_{1950}, δ_{1950} and μ, μ' are taken from the SAO catalog, and:

$$\tau = \frac{MJD - 33282}{365.24} \qquad (4.2)$$

Adopt approximate right ascension A_i and approximate declination D_i for the direction associated with satellite image sub. i.

Then solve standard coordinates $\xi_{i,k}$, $\eta_{i,k}$ from:

$$\begin{pmatrix} \cos\eta_{i,k}\cos\xi_{i,k} \\ \cos\eta_{i,k}\sin\xi_{i,k} \\ \sin\eta_{i,k} \end{pmatrix} = T_i \cdot \begin{pmatrix} \cos\delta_k\cos\alpha_k \\ \cos\delta_k\sin\alpha_k \\ \sin\delta_k \end{pmatrix} \quad (4.3)$$

with:

$$T_i = \begin{pmatrix} \cos D_i\cos A_i & \cos D_i\sin A_i & \sin D_i \\ -\sin A_i & \cos A_i & 0 \\ -\sin D_i\cos A_i & -\sin D_i\sin A_i & \cos D_i \end{pmatrix} \quad (4.4)$$

Now, for both plate measurement positions, form:

$$\underline{l}_i = \begin{bmatrix} \vdots & \vdots \\ \xi_{i,k} - \bar{x}_{i,k} \\ \vdots & \vdots \\ \eta_{i,k} - \bar{y}_{i,k} \\ \vdots & \vdots \end{bmatrix} \quad (4.5)$$

and:

$$M_i = \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & & & \\ \bar{x}_{i,k} & \bar{y}_{i,k} & 1 & & 0 & \\ \vdots & \vdots & \vdots & & & \\ \hline & & & \vdots & \vdots & \vdots \\ & 0 & & \bar{x}_{i,k} & \bar{y}_{i,k} & 1 \\ & & & \vdots & \vdots & \vdots \end{array}\right] \quad (4.6)$$

Then, assuming a Gaussian (normal) probability distribution for the components of $\underline{l}_i$, with correlation freedom and constant variance, the most probable $\underline{a}_i$ of linear plate constants is obtained independently for both plate measurement positions:

$$\underline{a}_i = V\{\underline{a}_i\} \cdot M_i^* \cdot \underline{l}_i \quad (4.7)$$

with:

$$V\{\underline{a}_i\} = (M_i^* \cdot M_i)^{-1} \quad (4.8)$$

The standard coordinates for the station-to-satellite direction become:

$$\begin{pmatrix} \xi_i \\ \eta_i \end{pmatrix} = B_i \cdot \underline{a}_i + \begin{pmatrix} \bar{X}_i \\ \bar{Y}_i \end{pmatrix} \quad (4.9)$$

for both plate measurement positions independently, if:

$$B_i = \begin{pmatrix} \bar{X}_i & \bar{Y}_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{X}_i & \bar{Y}_i & 1 \end{pmatrix} \quad (4.10)$$

Unit weight variance in (seconds of arc)^2 is estimated from:

$$\hat{\sigma}_i^2 = \frac{\underline{v}_i^* \cdot \underline{v}_i}{2s_i - 6} \cdot (206265)^2 \quad (4.11)$$

where s_i is the number of reference stars used (usually six) and the correction vector $\underline{v}_i$ is obtained from:

$$\underline{v}_i = \underline{l}_i - M_i \cdot \underline{a}_i . \qquad (4.12)$$

Standard coordinates from both plate measurement positions are combined to mean values:

$$\xi_i = \frac{\xi_i^{\mathrm{I}} + \xi_i^{\mathrm{II}}}{2}; \qquad \eta_i = \frac{\eta_i^{\mathrm{I}} + \eta_i^{\mathrm{II}}}{2} \qquad (4.13)$$

These are transformed into right ascension and declination by solving α_i, δ_i from:

$$\begin{pmatrix} \cos\delta_i \cos\alpha_i \\ \cos\delta_i \sin\alpha_i \\ \sin\delta_i \end{pmatrix} = T_i^* \cdot \begin{pmatrix} \cos\eta_i \cos\xi_i \\ \cos\eta_i \sin\xi_i \\ \sin\eta_i \end{pmatrix} \qquad (4.14)$$

Now suppose the satellite trail makes an angle ψ with the positive declination axis and moreover suppose that along trail comparator measurements have a standard deviation g times that of across trail measurements, then the variance-covariance matrix of the mean standard coordinates will be:

$$V\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} = \tfrac{1}{4}(\hat{\sigma}_i^{\mathrm{I}})^2 B_i^{\mathrm{I}} \cdot V\{\underline{a}_i^{\mathrm{I}}\} \cdot (B_i^{\mathrm{I}})^* + \tfrac{1}{4}(\hat{\sigma}_i^{\mathrm{II}})^2 B_i^{\mathrm{II}} \cdot V\{\underline{a}_i^{\mathrm{II}}\} \cdot (B_i^{\mathrm{II}})^* + \tfrac{1}{2}\sigma^2 R \cdot \begin{pmatrix} g^2 & 0 \\ 0 & 1 \end{pmatrix} \cdot R^* \qquad (4.15)$$

if:

$$R = \begin{pmatrix} \sin\psi & -\cos\psi \\ \cos\psi & \sin\psi \end{pmatrix} \qquad (4.16)$$

and σ is the standard deviation of across trail comparator measurements, expressed in seconds of arc.

Defining

$$E = \begin{pmatrix} \sec D_i & 0 \\ 0 & 1 \end{pmatrix} \qquad (4.17)$$

the variance-covariance matrix of α_i, δ_i becomes:

$$V\begin{Bmatrix} \alpha_i \\ \delta_i \end{Bmatrix} = \begin{pmatrix} \sigma_{\alpha_i}^2 & \sigma_{\alpha_i\delta_i} \\ \sigma_{\delta_i\alpha_i} & \sigma_{\delta_i}^2 \end{pmatrix} = E \cdot V\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} \cdot E \qquad (4.18)$$

Because of the simplifying assumptions as regards the statistical properties of the components of $\underline{l}_i$, the off-diagonal elements of both

$$V\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} \quad \text{and} \quad V\begin{Bmatrix} \alpha_i \\ \delta_i \end{Bmatrix}$$

should be zero.

The essential output of this programme consists of

$$\alpha_i, \delta_i \quad \text{and} \quad \sigma_{\alpha_i}, \sigma_{\delta_i}$$

α_i, δ_i should be interpreted as indicated in the beginning of this section.

5 Conversion to fixed-Earth reference

This programme transforms the station-to-satellite directions as derived in the previous section to a fixed-Earth reference frame and also applies corrections for annual aberration, diurnal aberration, light travel time, parallactic refraction and satellite phase.

Time instants t_i as obtained from programme "time reduction" are converted into *MJD*, taking the observation date into account. This yields $(MJD/\text{station})_i$.

The correction for light travel time is applied to form $(MJD/\text{satellite})_i$:

$$(MJD/\text{satellite})_i = (MJD/\text{station})_i - \frac{r_i}{2.5902072 \times 10^{10}} \quad \text{(5.1)}$$

where r_i stands for the estimated station-to-satellite range at t_i in km.

$(MJD/\text{satellite})_i$ will be abbreviated to *MJD*.

MJD, which was calculated in terms of *UTC* is reduced to *MJD*1 in terms of *UT*1, by application of differences *UT*1–*UTC* obtained from a linear interpolation in the smoothed values as listed in circular D issued by the BIH.

Next define:

$$T = MJD1 - 33282.0 \quad \text{(5.2)}$$

Precession is taken into account by matrix:

$$P = \begin{pmatrix} -\sin\varkappa\sin\omega + \cos\varkappa\cos\omega\cos\nu & -\cos\varkappa\sin\omega - \sin\varkappa\cos\omega\cos\nu & -\cos\omega\sin\nu \\ +\sin\varkappa\cos\omega + \cos\varkappa\sin\omega\cos\nu & +\cos\varkappa\cos\omega - \sin\varkappa\sin\omega\cos\nu & -\sin\omega\sin\nu \\ \cos\varkappa\sin\nu & -\sin\varkappa\sin\nu & +\cos\nu \end{pmatrix} \quad \text{(5.3)}$$

in which:

$$\begin{aligned} \varkappa &= 2.3436\cdot10^{-8} + 3.05953200167\cdot10^{-7}\cdot T + 0.109\cdot10^{-14}\cdot T^2 \\ \omega &= 2.3436\cdot10^{-8} + 3.05953200607\cdot10^{-7}\cdot T + 0.396\cdot10^{-14}\cdot T^2 \\ \nu &= 2.0379\cdot10^{-8} + 2.66039999761\cdot10^{-7}\cdot T - 0.156\cdot10^{-14}\cdot T^2 \end{aligned}$$

Nutation is accounted for by:

$$N = \begin{pmatrix} 1 & -\Delta\mu & -\Delta\nu \\ \Delta\mu & 1 & -\Delta\varepsilon \\ \Delta\nu & \Delta\varepsilon & 1 \end{pmatrix} \quad \text{(5.4)}$$

with:

$$\Delta\mu = \cos\varepsilon\cdot\Delta\psi$$
$$\Delta\nu = \sin\,\varepsilon\cdot\Delta\psi$$

with:

$$\varepsilon = 23^\circ.44578 - 3^\circ.5627\cdot 10^{-7}\cdot T$$

and:

$$\begin{aligned}\varrho''.\Delta\psi = {} & -17''.2445\sin\Omega - 1.2730\sin(2F-2D+2\Omega) + 0.2088\sin 2\Omega - 0.2037\sin(2F+2\Omega)\\ & + 0.1259\sin L' + 0.0675\sin L - 0.0496\sin(L'+2F-2D+2\Omega) - 0.0342\sin(2F+\Omega)\\ & - 0.0261\sin(L+2F+2\Omega) + 0.0214\sin(-L'+2F-2D+2\Omega) - 0.0149\sin(L-2D)\\ & + 0.0124\sin(2F-2D+\Omega) + 0.0114\sin(-L+2F+2\Omega) + 0.0060\sin 2D\\ & + 0.0058\sin(L+\Omega) - 0.0057\sin(-L+\Omega) - 0.0052\sin(-L+2F+2D+2\Omega)\\ & + 0.0045\sin(2L-2D) + 0.0045\sin(-2L+2F+\Omega) - 0.0044\sin(L+2F+2\Omega)\end{aligned}$$

$$\begin{aligned}\varrho''.\Delta\varepsilon = {} & 9''.2106\cos\Omega + 0.5520\cos(2F-2D+2\Omega) - 0.0904\cos 2\Omega + 0.0884\cos(2F+2\Omega)\\ & + 0.0216\cos(L'+2F-2D+2\Omega) + 0.0183\cos(2F+\Omega) + 0.0113\cos(L+2F+2\Omega)\\ & - 0.0093\cos(-L'+2F-2D+2\Omega) - 0.0066\cos(2F-2D+\Omega) -\\ & - 0.0050\cos(-L+2F+2\Omega)\end{aligned}$$

in which:

$$\begin{aligned} L &= 215^\circ.52915 + 13^\circ.0649924465\cdot T\\ L' &= 358^\circ.00067 + 0^\circ.9856002669\cdot T\\ F &= 52^\circ.26346 + 13^\circ.2293504490\cdot T\\ D &= 144^\circ.29461 + 12^\circ.1907491914\cdot T\\ \Omega &= 12^\circ.11282 - 0^\circ.0529538652\cdot T + 0^\circ.156\cdot 10^{-11}\cdot T^2\end{aligned}$$

The α, δ-output of the programme "position reduction" (section 4) defined in unit-vector form gives:

$$\underline{z} = \begin{pmatrix}\cos\delta\cos\alpha\\ \cos\delta\sin\alpha\\ \sin\delta\end{pmatrix} \qquad (5.5)$$

The resultant rotation due to precession and nutation is applied to vector $\underline{z}$ to give direction $\underline{x}$:

$$\underline{x} = N\cdot P\cdot\underline{z} \qquad (5.6)$$

Now, solve $\bar{\alpha}$, $\bar{\delta}$ from:

$$\begin{pmatrix}\cos\bar{\delta}\cos\bar{\alpha}\\ \cos\bar{\delta}\sin\bar{\alpha}\\ \sin\bar{\delta}\end{pmatrix} = \underline{x} \qquad (5.7)$$

where $\bar{\alpha}$ and $\bar{\delta}$ are the direction components of the directions to the satellite in the true sidereal system, starting from the 1950.0 SAO mean celestial system. The following corrections are applied to the values ($\bar{\alpha}$, $\bar{\delta}$).

Annual aberration is corrected for by adding corrections $\Delta_1\alpha$, $\Delta_1\delta$:

$$\left.\begin{aligned}\Delta_1\alpha &= \frac{-20''.50(\sin\bar{\alpha}\sin\lambda_{\odot}+\cos\varepsilon\cos\bar{\alpha}\cos\lambda_{\odot})}{\cos\bar{\delta}}\\ \Delta_1\delta &= -20''.50(\cos\bar{\alpha}\sin\bar{\delta}\sin\lambda_{\odot}-\cos\lambda_{\odot}\sin\bar{\alpha}\sin\bar{\delta}\cos\varepsilon+\cos\lambda_{\odot}\cos\bar{\delta}\sin\varepsilon)\end{aligned}\right\} \qquad (5.8)$$

where $\lambda_{\odot}$ is the true longitude of the sun.

If:

$$\begin{aligned} g &= 358^{\circ}.001479+0^{\circ}.9856003\cdot T\\ e &= 0.01673011-1.1478\cdot 10^{-9}\cdot T-9.4\cdot 10^{-17}\cdot T^2\\ L_{\odot} &= \text{(mean longitude)} = 280^{\circ}.081216+0^{\circ}.9856473437\cdot T+2.267\cdot 10^{-13}\cdot T^2\end{aligned}$$

then:

$$\lambda_{\odot} = L_{\odot}+\tfrac{1}{4}(8e-e^3)\sin g+\tfrac{5}{4}e^2\sin 2g+\tfrac{13}{12}e^3\sin 3g$$

Corrections for diurnal aberration are [4]:

$$\left.\begin{aligned}\Delta_2\alpha &= 0''.32\cos\varphi\cos h\sec\bar{\delta}\\ \Delta_2\delta &= 0''.32\cos\varphi\sin h\sin\bar{\delta}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5.9)$$

where φ is the latitude of the station,

$$\left.\begin{aligned} h &= \theta+\lambda-\bar{\alpha}\\ \lambda &= \text{the east-longitude and}\\ \theta &= 100^{\circ}.0755410+360.9856473459\cdot T+0^{\circ}.2902\cdot 10^{-12}\cdot T^2+\Delta\mu\ldots\end{aligned}\right\} \quad \ldots \quad (5.10)$$

The correction for parallactic refraction is calculated as follows (see [2] and [6])

$$\begin{aligned} \cos z &= \sin\varphi\sin\bar{\delta}+\cos\varphi\cos\bar{\delta}\cos h\\ \sin z &= \sqrt{1-\cos^2 z}\\ \sin q &= \frac{\sin h\cos\varphi}{\sin z}\\ \cos q &= \frac{\sin\varphi-\sin\bar{\delta}\cos z}{\cos\bar{\delta}\sin z}\\ \Delta R &= -483''.8\frac{\sin z}{\cos^2 z}\cdot\frac{1}{r}\left\{1-0.00126\frac{2+\sin^2 z}{\cos^2 z}\right\}\frac{B}{B_0} \quad \ldots\ldots\ldots \quad (5.11)\end{aligned}$$

with:

B = the barometric pressure in mm Hg

B_0 = the barometric pressure at mean sealevel (here 760 mm Hg).

$$\left.\begin{aligned}\Delta_3\alpha &= -\Delta R''\cdot\sec\bar{\delta}\sin q\\ \Delta_3\delta &= -\Delta R''\cdot\cos q\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5.12)$$

Incidentally, z stands for the zenith-angle of the station-to-satellite direction.

Satellite phase is corrected for as follows:

$$\left.\begin{aligned} \Delta_4\alpha &= \frac{206''.265\,\varrho\cos\frac{1}{2}u\sin v}{r\cos\delta} \\ \Delta_4\delta &= \frac{206''.265\,\varrho\cos\frac{1}{2}u\cos v}{r} \end{aligned}\right\} \qquad (5.13)$$

Here ϱ is the radius of the satellite in metres, and u and v are to be solved from:

$$\begin{aligned} \sin u\cos v &= \sin\bar{\delta}(\cos\lambda_\odot\cos\bar{\alpha}+\cos\varepsilon\sin\lambda_\odot\sin\bar{\alpha})-\cos\bar{\delta}\sin\varepsilon\sin\lambda_\odot \\ \sin u\sin v &= \sin\bar{\alpha}\cos\lambda_\odot-\cos\bar{\alpha}\cos\varepsilon\sin\lambda_\odot \\ \cos u &= \cos\bar{\delta}(\cos\lambda_\odot\cos\bar{\alpha}+\cos\varepsilon\sin\lambda_\odot\sin\bar{\alpha})+\sin\bar{\delta}\sin\varepsilon\sin\lambda_\odot \end{aligned}$$

With:

$$\left.\begin{aligned} [\alpha] &= \bar{\alpha}+\Delta_1\alpha+\Delta_2\alpha+\Delta_3\alpha+\Delta_4\alpha \\ [\delta] &= \bar{\delta}+\Delta_1\delta+\Delta_2\delta+\Delta_3\delta+\Delta_4\delta \end{aligned}\right\} \qquad (5.14)$$

the corrected direction components $\underline{x}'$ of the directions to the satellite in the true sidereal system are determined:

$$\underline{x}' = \begin{pmatrix} \cos[\delta]\cos[\alpha] \\ \cos[\delta]\sin[\alpha] \\ \sin[\delta] \end{pmatrix} \qquad (5.15)$$

Now, a final rotation due to earth rotation and polar motion can be applied to unit-vector $\underline{x}'$ to give fixed-Earth direction $\underline{y}$:

$$\underline{y} = S\cdot R\cdot\underline{x}' \qquad (5.16)$$

Earth rotation is expressed by:

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (5.17)$$

Polar motion components x, y are taken from the smoothed values listed in circular *D* issued by BIH by means of a linear interpolation.

The polar motion matrix used in 5.16 is:

$$S = \begin{pmatrix} 1 & 0 & +x \\ 0 & 1 & -y \\ -x & +y & 1 \end{pmatrix} \qquad (5.18)$$

Now, solve $\bar{\bar{\alpha}}$, $\bar{\bar{\delta}}$ from:

$$\begin{pmatrix} \cos\bar{\bar{\delta}}\cos\bar{\bar{\alpha}} \\ \cos\bar{\bar{\delta}}\sin\bar{\bar{\alpha}} \\ \sin\bar{\bar{\delta}} \end{pmatrix} = \underline{y} \qquad (5.19)$$

where $\bar{\bar{\alpha}}$ and $\bar{\bar{\delta}}$ are the direction components of the directions to the satellite in the fixed-Earth (Greenwich) system.

The procedure contained in most of the formulas of this section follows [2] and [5].

The variance-covariance matrix of $\underline{y}$ is estimated as:

$$V\{\underline{y}\} = G \cdot V\begin{Bmatrix} \alpha \\ \delta \end{Bmatrix} \cdot G^{*} \qquad (5.20)$$

in which:

$$G = \begin{pmatrix} -\cos[\delta]\sin[\alpha] & -\sin[\delta]\cos[\alpha] \\ +\cos[\delta]\cos[\alpha] & -\sin[\delta]\sin[\alpha] \\ 0 & \cos[\delta] \end{pmatrix} \qquad (5.21)$$

and where

$$V\begin{Bmatrix} \alpha \\ \delta \end{Bmatrix}$$

is taken from the output of programme "position reduction" (see section 4).

6 Quality assessment by means of curve-fitting

The combined "observations" t_i from (3.5) and α_i, δ_i from (4.14) are being checked on their internal precision by means of a curve fitting procedure.

Define:

$$\underline{v}_i = \begin{pmatrix} \cos\delta_i\cos\alpha_i \\ \cos\delta_i\sin\alpha_i \\ \sin\delta_i \end{pmatrix} \qquad (6.1)$$

The direction cosines from (6.1) are referenced to a right-handed rectangular Cartesian frame which is defined by the first and the last directions observed on one plate, as follows:

$$\left.\begin{aligned} \underline{x} &\equiv \underline{v}_1 \\ \underline{z} &= \frac{\underline{v}_1 \times \underline{v}_n}{|\underline{v}_1 \times \underline{v}_n|} \\ \underline{y} &= \underline{z} \times \underline{x} \end{aligned}\right\} \qquad (6.2)$$

$$\underline{v}_i' = \begin{pmatrix} \underline{x}^{*} \\ \underline{y}^{*} \\ \underline{z}^{*} \end{pmatrix} \cdot \underline{v}_i \qquad (6.3)$$

Determine spherical coordinates along track and across track ξ_i and η_i from:

$$\begin{pmatrix} \cos\eta_i \cos\xi_i \\ \cos\eta_i \sin\xi_i \\ \sin\eta_i \end{pmatrix} = \underline{v}'_i \qquad (6.4)$$

$$0 \leqslant \xi_i < 360^\circ$$
$$-90^\circ \leqslant \eta_i \leqslant +90^\circ$$

Now, to ξ_i, η_i together with the t_i a curve fitting procedure is applied, as follows:

$$\tau_i = t_i - t_1 \qquad (6.5)$$

Define:

$$T_k = \begin{bmatrix} 1 & \tau_1 & \tau_1^2 & \tau_1^3 \ldots \tau_1^k \\ 1 & \tau_2 & \tau_2^2 & \tau_2^3 \ldots \tau_2^k \\ 1 & \tau_3 & \tau_3^2 & \tau_3^3 \ldots \tau_3^k \\ \vdots & & & \\ 1 & \tau_i & \tau_i^2 & \tau_i^3 \ldots \tau_i^k \\ \vdots & & & \\ 1 & \tau_n & \tau_n^2 & \tau_n^3 \ldots \tau_n^k \end{bmatrix} \qquad (6.6)$$

Then, assuming correlation freedom and unit weight within both observation vectors:

$$\underline{\xi} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{pmatrix}; \qquad \underline{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{pmatrix} \qquad (6.7)$$

least squares solutions for the coefficient-vectors $\underline{a}$ and $\underline{b}$ are obtained from:

$$\left.\begin{aligned} \underline{a} &= Q_k \cdot T_k^* \cdot \underline{\xi} \\ \underline{b} &= Q_k \cdot T_k^* \cdot \underline{\eta} \end{aligned}\right\} \qquad (6.8)$$

in which:

$$Q_k = (T_k^* \cdot T_k)^{-1} \qquad (6.9)$$

The correction vectors are:

$$\left.\begin{aligned} \underline{\varepsilon}_\xi &= T_k \cdot \underline{a} - \underline{\xi} \\ \underline{\varepsilon}_\eta &= T_k \cdot \underline{b} - \underline{\eta} \end{aligned}\right\} \qquad (6.10)$$

Here it has been tacitly assumed that the t_i are non-stochastic quantities.

Finally

$$\left.\begin{aligned} \hat{\sigma}_\xi^2 &= \frac{\underline{\varepsilon}_\xi^* \cdot \underline{\varepsilon}_\xi}{n-k-1} \\ \hat{\sigma}_\eta^2 &= \frac{\underline{\varepsilon}_\eta^* \cdot \underline{\varepsilon}_\eta}{n-k-1} \end{aligned}\right\} \qquad (6.11)$$

where n stands for the number of reduced satellite images and k for the degree of the polynomial applied.

The estimates $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ are used for judging the "quality" of the photographic observations on each individual plate.

Noticing the small camerafield $k = 2$ was adopted invariably.

7 Reporting for SAOP

In Fig. 3 a number of computation steps in the programme "conversion to fixed-Earth reference" are explicitly mentioned.

At a meeting in Bern, Switzerland, in October 1972, it was agreed that stations participating in the SAOP-campaigns would report the so-called geometric directions 1950.0, corrected for: annual aberration, diurnal aberration, parallactic refraction and satellite phase. Hence (see Fig. 3):

(A)+(3)+(4)+(5)+(6), and seperately:
(3), (4), (5), (6).

The corresponding time instants are in *UTC*, and corrected for light travel time.

8 Results

All plates contributed to SAOP have been listed in tables I through V (station YPENBURG) and table VI (station KOOTWIJK). Simultaneous observations for geometric network computations are indicated with "gm"; observations for geocentric network computations (using the short-arc method [7]) are indicated with "gc" (column 9).

Also given are $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ (colums 6 and 7) for each individual plate, together with the number n of individual satellite images from which $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ have been calculated.

$\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ are taken as indicators of gross-errors either in the observations or in their reduction. Experience has suggested that observations with either $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta > 1''$ should be suspected. Adopting this rather arbitrary criterion it is meaningful to consider the percentage of observations (plates) with $\hat{\sigma}_\xi$ and/or $\hat{\sigma}_\eta < 1''$. The following conclusions are then to be drawn (see figs. 4, 5, 6 and 7):

1. The fraction of observations (in the SAOP-programme all made on PAGEOS) with $\hat{\sigma}_\eta < 1''$ does not exceed that with $\hat{\sigma}_\xi < 1''$. So the difference between the mentioned fractions is here less evident than in [1]. This may be due to the improved timing equipment.
2. The fraction of observations with both $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta < 1''$ is $\approx 80\%$. This is, for observations on PAGEOS, a confirmation of the results published in [1].

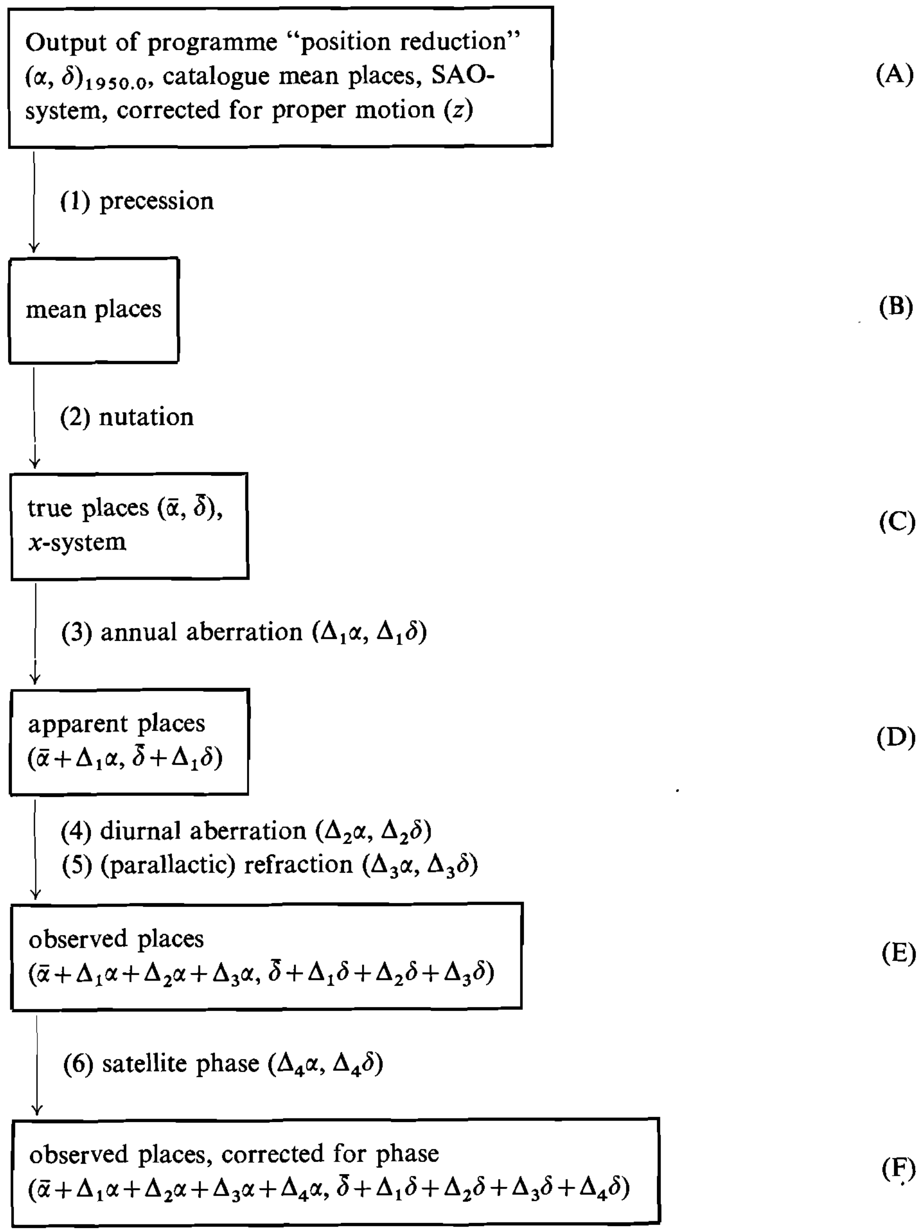

Fig. 3. Reduction/correction scheme.

satellite : pageos) passive

station : Ypenburg
programme : SAOP

no. of observations : 195

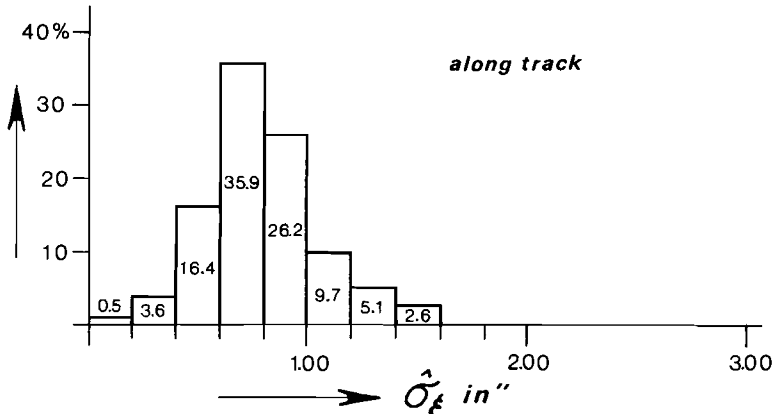

fig. 4

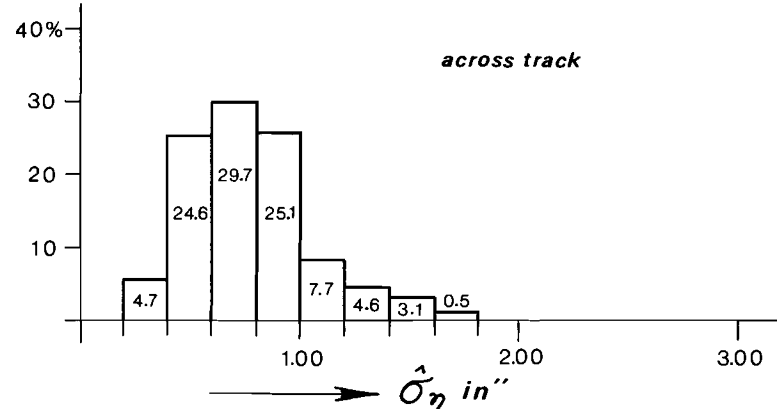

fig. 5

satellite : pageos) passive

station : Kootwijk
programme : SAOP

no. of observations : 34

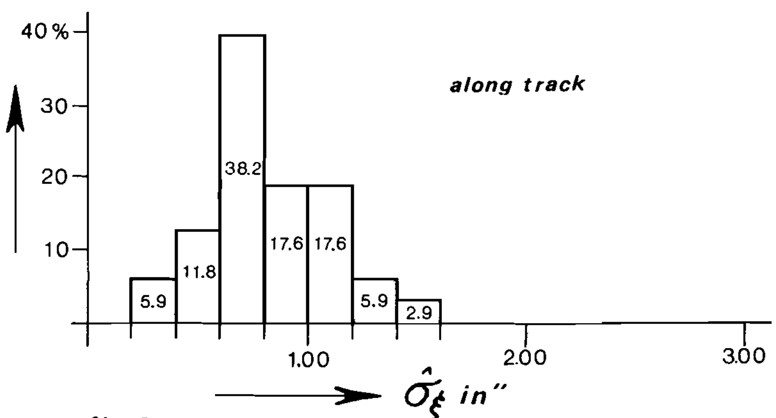

fig. 6

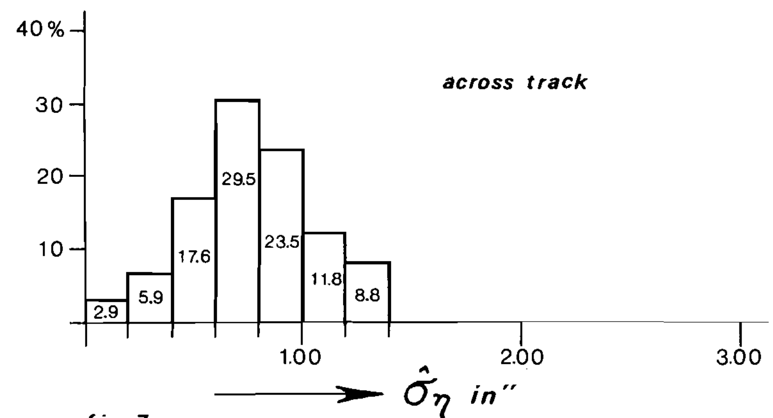

fig. 7

References

[1] L. Aardoom, D. L. F. van Loon and T. J. Poelstra – The astrometric procedure of satellite plate reduction as applied at the Delft Geodetic Institute. A description with some results for WEST, NGSP and ISAGEX. Netherlands Geodetic Commission, Publications on Geodesy, New Series, Vol. 5, No. 4. Delft, 1975.

[2] M. Schürer – Internal publications and private communication:
a. Notizen zu einer Vorlesung über Satellitengeodäsie, gehalten im Sommersemester 1971 an der ETH Zürich;
b. Plate-reduction at Zimmerwald, October 1972;
c. Die parallaktische Refraktion, October 1972.

[3] NASA directory of observing station locations, third ed., 1973.

[4] Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac. London, 1961.

[5] G. Veis – Precise aspects of terrestrial and celestial reference frames. In: The use of artificial satellites for geodesy. (ed. G. Veis). North-Holland Publishing Co., Amsterdam, 1963, pp. 201–216.

[6] G. Veis – Geodetic uses of artificial satellites. Smithsonian Contributions to Astrophysics, Vol. 3, No. 9, 1960.

[7] E. Tengström – Über das Short-Arc-satelliten-beobachtungsvorhaben im Bereich des "Europäischen Datums". Mitteilungen aus dem Institut für theoretische Geodäsie der Universität Bonn, No. 16, Bonn, 1973.

Table I. Station: Ypenburg Covering period: April 1971
Campaign no.: 1 Satellite: Pageos

revolution no.	frame no.	observation date	time from	time to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
14033	01	710415	00h32m59s	00h34m45s	0.7	0.9	9	gc, gm
	02		00 37 03	00 38 34	0.6	0.5	10	gc
	03		00 41 15	00 42 35	0.4	0.3	10	gc, gm
	04		00 45 18	00 46 17	0.5	0.5	8	gc
	05		00 48 45	00 50 01	0.3	0.7	10	gc, gm
	06		00 52 34	00 53 56	0.7	0.5	11	gc
	07		00 56 34	00 57 52	0.4	0.5	10	gc
	08		01 00 25	01 01 43	0.8	0.4	10	gc
	09		01 04 19	01 05 43	0.4	0.4	11	gc
14081	01	710421	00 34 50	00 36 31	0.6	0.5	9	gc
	02		00 42 14	00 43 53	0.8	0.9	8	gc
	03		00 50 08	00 51 31	0.8	0.6	10	gc
	04		00 58 02	00 59 05	0.6	0.6	10	gc, gm
	05		01 01 54	01 03 05	0.6	0.4	11	gc
	06		01 13 23	01 14 52	0.7	0.8	7	gc
14089	01	710422	00 37 11	00 38 53	0.5	0.5	10	gc
	02		00 41 10	00 42 45	0.7	0.8	10	gc
	03		00 45 26	00 46 38	0.5	0.7	9	gc
	04		00 53 04	00 54 24	0.3	0.5	13	gc
	05		00 57 09	00 58 03	0.2	0.3	6	gc, gm
	06		01 00 55	01 02 03	1.1	0.9	11	gc
	07		01 04 43	01 05 55	1.1	0.9	12	gc
	08		01 08 38	01 09 57	1.4	1.3	10	gc, gm
	09		01 12 29	01 13 58	1.1	1.3	8	gc, gm

Table II. Station: Ypenburg Covering period: April–May 1972
Campaign no.: 3 Satellite: Pageos

revolution no.	frame no.	observation date	time from	time to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
16947	02	720415	01h29m22s	01h30m05s	1.2	1.0	7	gm
17097	01	720502	20 44 40	20 46 08	0.8	0.9	11	gc
	02		20 49 00	20 49 45	0.8	0.8	8	gc
	03		20 52 35	20 53 26	1.0	0.8	7	gc
	04		20 56 21	20 57 22	0.6	0.9	9	gc, gm
	05		21 01 07	21 01 15	—	—	3	gc
	06		21 03 58	21 05 10	1.8	1.0	9	gc
	07		21 07 45	21 09 05	0.6	0.7	13	gc
17098	01	720502	23 46 59	23 47 56	0.9	0.7	6	gc
	02		23 50 36	23 51 51	0.9	0.8	10	gc
	03		23 54 28	23 55 35	0.9	0.8	10	gc
	04	720503	00 02 11	00 03 08	0.8	0.8	8	gc, gm
	05		00 06 15	00 07 23	1.5	1.7	8	gc

Table III. Station: Ypenburg Covering period: Sept.–Oct. 1972
Campaign no.: 4 Satellite: Pageos

		observation						
			time					
revolution no.	frame no.	date	from	to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
18207	01	720919	00h27m28s	00h27m48s	0.8	0.4	5	gm
18215	01	720920	00 28 08	00 29 16	0.7	0.8	13	gc
	02		00 32 38	00 33 26	0.7	1.3	7	gc, gm
	03		00 36 43	00 37 34	0.5	0.7	10	gc
	04		00 40 58	00 41 46	0.6	1.3	8	gc, gm
	05		00 45 30	00 46 27	0.7	0.2	8	gc, gm
18223	01	720921	00 32 35	00 33 32	0.8	1.1	11	gc, gm
	02		00 36 55	00 37 44	1.0	0.7	10	gc, gm
	03		00 41 15	00 41 48	0.6	0.6	7	gc, gm
	04		00 45 26	00 46 21	0.7	0.5	11	gc, gm
	05		00 50 00	00 50 55	0.7	0.7	9	gc
18238	03	720922	21 57 27	21 58 22	0.9	0.9	9	gm
18263	01	720926	00 55 41	00 56 37	0.6	0.4	9	gc, gm
	02		00 59 59	01 00 55	0.5	0.8	11	gc
	03		01 04 18	01 05 08	0.8	0.4	10	gc, gm
	04		01 08 32	01 09 20	0.8	0.8	7	gc
	05		01 12 51	01 14 06	0.7	0.8	12	gc, gm
18279	01	720928	01 05 19	01 06 23	0.4	0.7	10	gc
	02		01 08 27	01 09 23	0.6	1.1	9	gc, gm
	03		01 11 33	01 12 28	0.5	0.8	9	gc, gm
	04		01 14 37	01 15 31	0.5	0.8	11	gc
	05		01 17 42	01 18 42	1.0	0.7	10	gc, gm
	06		01 20 39	01 21 25	0.7	1.4	8	gc
	07		01 23 17	01 23 51	1.0	0.6	6	gc
18302	01	720930	22 18 34	22 19 30	0.8	0.5	9	gm
18334	01	721004	22 35 48	22 36 27	0.4	0.4	8	gc
	02		22 38 36	22 39 37	0.8	0.4	12	gc
	03		22 41 54	22 42 49	0.9	0.5	11	gc, gm
	04		22 45 10	22 46 04	0.8	0.7	11	gc, gm
	05		22 48 26	22 49 21	0.9	0.6	9	gc
	06		22 51 42	22 52 44	0.6	0.9	10	gc
18342	02	721005	22 44 00	22 44 55	0.5	0.6	10	gc, gm
	03		22 47 13	22 47 57	0.9	0.4	9	gc
	04		22 50 24	22 51 19	0.7	0.8	10	gc
	05		22 53 38	22 54 40	0.6	0.6	10	gc
	06		22 57 04	22 58 05	0.5	1.3	10	gc, gm
18343	01	721006	01 41 25	01 41 47	1.1	0.2	4	gc
	02		01 43 50	01 44 53	1.2	1.2	10	gc
	03		01 47 07	01 47 56	1.1	1.0	8	gc
	04		01 50 19	01 51 08	0.7	0.7	8	gc
	05		01 53 40	01 54 16	0.3	1.1	6	gc

Table III. (continued)

revolution no.	frame no.	observation date	time from	time to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
18358	01	721007	22h50m22s	22h51m02s	0.4	0.4	8	gc, gm
	02		22 53 37	22 54 16	0.6	0.3	8	gc, gm
	03		22 56 53	22 57 43	0.6	0.4	10	gc, gm
	04		23 00 15	23 01 10	0.7	0.7	9	gc, gm
	05		23 03 34	23 04 29	0.7	0.3	9	gc
	06		23 07 14	23 08 16	0.5	0.2	10	gc
18382	02	721010	23 05 54	23 06 45	0.7	0.2	10	gm
18390	01	721011	23 10 25	23 11 16	0.2	0.9	10	gm
	03		23 16 35	23 17 32	0.8	0.5	9	gm
	04		23 20 10	23 21 14	0.4	0.7	10	gm
18398	03	721012	23 16 48	23 17 39	0.6	0.9	10	gm
18406	01	721013	23 18 02	23 18 54	0.9	1.0	10	gc
	02		23 21 15	23 22 07	0.8	0.7	10	gc
	03		23 24 24	23 25 07	1.4	0.5	7	gc, gm
	04		23 27 45	23 28 42	0.7	0.8	9	gc
	05		23 31 11	23 31 57	0.8	1.3	8	gc
	06		23 34 10	23 35 21	0.8	1.4	9	gc
18414	02	721014	23 22 26	23 23 17	0.7	0.4	10	gm
18437	01	721017	20 31 08	20 32 12	0.8	0.7	10	gc, gm
	02		20 34 46	20 35 48	0.7	0.6	10	gc, gm
	03		20 37 47	20 38 50	0.7	0.6	10	gc
	04		20 41 06	20 42 09	0.9	0.5	10	gc, gm
	05		20 44 00	20 44 48	0.6	0.5	8	gc
	06		20 46 52	20 47 55	1.0	1.4	8	gc
18445	01	721018	20 48 25	20 49 28	0.6	0.6	10	gm
18446	01	721018	23 35 15	23 36 13	0.7	1.4	8	gc, gm
	02		23 38 42	23 39 30	0.6	0.7	7	gc, gm
	03		23 40 14	23 41 08	0.7	0.9	8	gc
	04		23 42 17	23 43 06	0.4	0.7	7	gc, gm
	05		23 44 19	23 45 14	0.8	0.8	8	gc, gm
	06		23 47 22	23 48 09	0.7	0.7	7	gc, gm
	07		23 50 57	23 52 09	1.1	0.6	10	gc, gm

Table IV. Station: Ypenburg Covering period: March–April 1973
Campaign no.: 5 Satellite: Pageos

revolution no.	frame no.	observation date	time from	time to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
19627	01	730315	00h00m01s	00h01m08s	0.9	0.9	11	gm
19682	03	730322	21 36 43	21 37 03	0.4	0.7	8	gm
19690	01	730323	21 19 00	21 20 02	0.8	1.1	10	gm
	02		21 22 31	21 23 30	0.9	0.4	10	gm
	03		21 26 48	21 27 11	0.6	0.5	5	gm
	04		21 30 45	21 31 07	1.1	1.3	5	gm
	05		21 34 35	21 35 23	0.6	0.9	10	gm
	06		21 38 19	21 39 06	0.3	0.7	10	gm
19706	01	730325	21 36 34	21 37 22	0.6	0.4	10	gm
19738	02	730329	21 58 46	21 59 41	0.6	0.6	9	gm
19802	02	730406	22 02 29	22 02 55	1.1	0.9	6	gm
	03		22 09 48	22 10 20	0.4	0.9	7	gm
19834	03	730410	22 26 38	22 27 26	0.6	0.8	10	gm

Table V. Station: Ypenburg Covering period: Sept.–Oct. 1973
Campaign no.: 6 Satellite: Pageos

revolution no.	frame no.	observation date	time from	time to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
21080	01	730913	20h30m38s	20h31m48s	0.6	0.7	9	gc
	02		20 34 45	20 35 46	0.6	0.7	10	gc, gm
	03		20 39 00	20 39 55	0.9	0.8	9	gc
	04		20 43 13	20 44 14	0.5	0.4	10	gc, gm
	05		20 47 11	20 48 28	0.7	0.7	10	gc, gm
21081	01	730913	23 22 07	23 22 40	0.8	0.8	5	gc
	04		23 36 59	23 37 59	0.8	0.9	10	gc, gm
	05		23 41 07	23 42 09	0.6	0.9	10	gc, gm
	06		23 45 24	23 46 26	0.8	1.0	10	gc, gm
21088	01	730914	20 25 09	20 25 32	1.4	1.0	4	gc
	02		20 28 04	20 29 23	1.2	0.7	10	gc
	03		20 32 24	20 33 25	0.6	0.6	10	gc, gm
	04		20 36 32	20 37 32	0.8	0.8	10	gc, gm
	05		20 40 47	20 41 47	0.5	0.7	10	gc
	06		20 44 36	20 45 35	1.2	0.8	10	gc, gm
	07		20 49 03	20 50 12	1.2	0.4	9	gc
	08		20 57 11	20 58 03	1.2	0.6	7	gc
21089	01	730914	23 30 18	23 31 20	0.9	1.0	10	gc
	02		23 34 33	23 35 35	0.8	0.6	10	gc
	03		23 38 44	23 39 46	0.6	0.7	10	gc
	04		23 42 53	23 43 56	0.7	1.4	10	gc
	05		23 47 01	23 48 03	0.8	0.4	10	gc
	06		23 51 28	23 52 46	0.5	0.5	10	gc
	07		23 55 09	23 56 25	1.4	0.9	10	gc
21224	01	731001	20 34 24	20 35 24	1.3	0.7	10	gc, gm
	02		20 39 57	20 40 47	0.4	0.6	10	gc, gm
	03		20 45 57	20 46 46	0.7	0.7	10	gc, gm
	04		20 52 29	20 53 24	0.8	0.6	10	gc, gm
	05		20 59 43	21 00 32	0.7	0.4	10	gc
	06		21 07 34	21 08 37	0.9	1.5	9	gc, gm
21240	01	731003	20 35 26	20 36 29	0.7	0.5	10	gc, gm
	02		20 41 03	20 41 53	0.9	1.0	10	gc, gm
	03		20 47 29	20 48 20	0.8	0.6	10	gc, gm
	04		20 54 38	20 55 28	0.6	0.2	10	gc, gm
	05		21 08 48	21 09 50	1.0	1.1	9	gc, gm

Table V. (continued)

revolution no.	frame no.	observation date	time from	time to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
21248	01	731005	20h31m18s	20h32m12s	0.9	0.9	7	gc
	02		20 35 25	20 36 32	0.7	0.7	9	gc, gm
	03		20 38 25	20 39 24	0.5	0.5	10	gc
	04		20 42 40	20 43 26	0.1	0.5	8	gc, gm
	05		20 46 25	20 47 17	0.4	0.5	9	gc, gm
	06		20 50 26	20 51 15	0.9	0.5	9	gc, gm
	07		20 54 26	20 55 11	0.4	0.3	8	gc, gm
	08		20 58 41	20 59 35	0.6	0.4	8	gc, gm
	09		21 02 42	21 03 40	0.3	0.4	8	gc, gm
21304	01	731011	20 37 34	20 38 38	1.0	0.5	8	gc
	02		20 41 33	20 42 33	0.9	1.2	8	gc
	03		20 46 51	20 47 31	0.9	0.6	7	gc, gm
	04		20 49 45	20 50 39	0.6	0.6	9	gc, gm
	05		20 53 55	20 54 53	1.0	0.7	10	gc
	06		20 58 05	20 59 07	0.6	0.5	10	gc
	07		21 02 19	21 03 17	0.5	0.9	10	gc
21312	01	731012	20 41 56	20 43 11	1.3	1.0	9	gc, gm
	02		20 46 10	20 47 12	0.7	1.0	10	gc, gm
	03		20 50 16	20 51 17	1.1	0.9	10	gc, gm
	04		20 54 21	20 55 23	1.2	0.9	10	gc, gm
	05		20 58 29	20 59 31	0.6	0.6	10	gc
	06		21 02 37	21 03 39	0.9	0.9	10	gc, gm
	07		21 06 57	21 07 58	0.6	0.6	8	gc, gm
21416	01	731025	20 50 59	20 52 17	0.7	0.6	10	gc
	02		20 54 48	20 56 07	0.8	0.8	10	gc
	03		20 59 27	21 00 36	0.5	0.8	9	gc
	04		21 03 37	21 04 55	0.7	0.5	10	gc
	05		21 07 52	21 09 10	0.5	0.6	10	gc, gm
	06		21 11 45	21 13 03	0.5	0.5	10	gc, gm
21423	01	731026	17 41 42	17 43 03	1.3	0.8	10	gc
	02		17 45 41	17 46 59	1.0	0.9	10	gc
	03		17 49 48	17 51 06	0.7	0.8	10	gc
	04		17 53 50	17 55 09	0.4	0.7	10	gc
	05		17 57 57	17 58 58	0.9	0.4	10	gc
	06		18 02 10	18 03 11	0.7	0.8	10	gc
	07		18 06 05	18 07 24	1.0	0.6	10	gc
	08		18 10 27	18 11 46	0.8	0.7	10	gc
21424	01	731026	20 51 16	20 52 18	0.7	0.5	8	gc
	02		20 54 53	20 56 12	0.6	0.6	10	gc

Table VI. Station: Kootwijk Covering period: March 1974
Campaign no.: 7 Satellite: Pageos

		observation						
			time					
revolution no.	frame no.	date	from	to	$\hat{\sigma}_\zeta$ in″	$\hat{\sigma}_\eta$ in″	n	use
22567	01	740318	20h00m25s	20h00m58s	0.7	0.5	9	gc, gm
	02		20 04 13	20 04 49	0.2	0.5	8	gc, gm
	03		20 11 38	20 12 40	1.0	0.7	10	gc, gm
	04		20 15 14	20 16 35	0.7	1.3	10	gc, gm
22575	04	740319	20 09 31	20 10 33	0.5	0.5	10	gm
22591	01	740321	19 59 47	20 00 21	1.1	1.2	8	gc, gm
	02		20 07 28	20 08 29	0.6	0.7	10	gc, gm
	03		20 11 06	20 11 51	0.5	0.7	8	gc
22623	01	740325	19 46 54	19 47 39	1.3	0.8	8	gc
	02		19 50 59	19 51 37	0.7	0.3	8	gc, gm
	03		19 55 01	19 55 37	0.7	0.4	8	gc, gm
	04		19 58 55	19 59 31	0.4	0.4	8	gc, gm
	05		20 03 00	20 03 45	0.4	0.8	8	gc, gm
	06		20 07 18	20 08 11	0.6	0.7	9	gc, gm
	07		20 10 59	20 12 00	0.8	1.0	10	gc
22631	01	740326	19 47 35	19 48 21	0.8	0.4	8	gc
	02		19 51 32	19 52 17	1.0	0.9	8	gc, gm
	03		19 55 41	19 56 17	0.6	0.7	8	gc
	04		20 00 06	20 00 48	1.0	0.8	9	gc, gm
	05		20 03 49	20 04 37	0.7	0.7	8	gc, gm
	06		20 07 18	20 08 18	0.9	0.7	10	gc, gm
	07		20 11 03	20 11 49	0.7	0.8	7	gc
22639	01	740327	19 46 56	19 47 56	1.1	1.1	10	gc
	02		19 54 59	19 55 33	0.8	0.3	8	gc, gm
	03		19 59 33	20 00 10	0.7	0.6	8	gc
	04		20 04 01	20 04 43	0.6	1.3	9	gc, gm
	05		20 07 24	20 08 16	1.2	0.8	9	gc, gm
22647	01	740328	19 50 58	19 51 38	0.8	0.6	7	gc, gm
	02		19 55 19	19 55 53	1.1	0.9	8	gc
	03		19 58 57	19 59 36	0.6	1.1	9	gc
	04		20 02 46	20 03 31	0.9	0.8	8	gc, gm
	05		20 06 11	20 07 10	1.4	1.1	10	gc, gm
22655	03	740329	19 59 34	20 00 16	0.6	0.7	9	gm
	04		20 03 36	20 04 28	0.3	0.1	9	gm